The Power of Asking Questions: Resolving Financial Market Rumors through Public Inquiries*

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Abstract

Should regulators require firms to publicly confirm or deny rumors? In a sequential trading model, such "rumor regulation" may enhance pricing efficiency through two mechanisms: (i) an increase in the number of informed traders brought about by the public inquiry (ii) a shortened information advantage period. However, data on rumor-disclosure events in the Korea Exchange shows that such regulations may actually reduce fairness: informed traders earn higher profits than in the absence of the regulation due to an increase in noise trading and false alerts.

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Asymmetric information and how information influences prices are central questions in finance. Models based on strategic traders (e.g., Kyle (1985)) and slow diffusion of information (e.g., Hong and Stein (1999)) explain how information is reflected by prices. Although theoretical research appears in the literature, empirical examination remains a challenge because observing asymmetric information is difficult. In the Korea Exchange (KRX), a mechanism exists whereby a market regulator can alert market participants to possible asymmetric information among traders. The KRX market requires listed firms to disclose information relevant to firms’ material business information. Firms are also obligated to validate inquiries the KRX market requests regarding rumors and news. This inquiry is conducted through public disclosure so that once the disclosure occurs, everyone gets notified about the rumor. After the inquiry, firms must respond through disclosure. The response disclosure must contain information confirming the rumor if true, or denying it if false. Additional information may be attached to the disclosure to resolve uncertainty regarding the rumor.¹

This unique feature provides a natural experiment for studying market behavior when asymmetric information exists. Figure 1 shows price movements and trading volume of a stock around inquiry/response disclosures regarding rumors. In the top panel, the response confirms the rumor, and in the bottom panel, the response denies the rumor. In both examples, between the two disclosures, the stock price quickly moves in the direction of the value that is stabilized after inquiry, which suggests informed trading activity. More trading volume after inquiry disclosure are common to both cases, and the level of trading volume remains high the day after the response disclosure which suggests increased noise trading activity. That is, people’s attention has been attracted by the rumor inquiry. These observations motivates one to write a model with entry of informed traders as well as entry of noise traders.

I generalize the Kyle (1985) model to consider the aforementioned effects of inquiry/response disclosure regulation. To my knowledge, this paper is first to derive a Kyle-style theoretical model when the number of informed traders may increase during a sequence of trading periods. From observation of stock prices shown in Figure 1, I argue that two affect the price process: 1) the number of informed traders increases after inquiry, and the competition among them pushes price to the fundamental value faster, and 2) the information advantage period is likely to end in a short period due to a pending response disclosure, and this makes

¹See Appendix for more background on the regulation of the KRX market.
them trade more aggressively. For the first argument, one can think of a person, related closely to a high executive in a firm, making a phone call to determine whether a rumor is true. However, she can do this only if she knows such rumors are in the market. When the rumor inquiry is disclosed, everyone acknowledges the rumor, which informs the previously potentially informed traders. This relates to the increase in information asymmetry after disclosure in Kim and Verrecchia (1994). I show that expected profits of informed traders decrease and price convergence is faster than the original Kyle (1985) model, holding all else constant. Intuitively, the initial informed trader foresees that she might lose monopolistic informational advantage and earns less profit even when competition (inquiry disclosure event) does not occur.

Using the general model introduced in the paper, I show that the number of informed traders after inquiry on average is greater than one, so that there may exist competition among informed traders after the inquiry disclosure. As Figure 1 shows, the price convergence is not shown prior to the inquiry suggesting one or no informed trader prior at that period. This supports the increase in the number of informed traders after the inquired disclosure.

I also calculate expected profits earned by informed traders per inquiry. I show that the expected profits earned by informed traders depend crucially on the level of noise trading after the market regulator posts the inquiry. My empirical estimates of the profits earned by informed traders suggest that the disclosure regulation increases the profit of informed investor in the market. Even in a world in which every rumor is true, informed traders may earn higher expected profits if the inquiry induces a sufficiently large positive change in the amount of noise trading. While investor attention does alleviate informational frictions as in Fang and Peress (2009), it also increases profit opportunities available to the informed traders, thus offsetting the effect of increased competition among informed traders.

When estimating profits, time-varying disparities in noise trading are considered after inquiry. This paper also contributes by introducing an enhanced method to back out the variance of noise trading. Although Cho (2007) uses total trading volume as noise trading in Kyle-style models, this is imprecise since total trading consists of both noise and informed trading. Adding a normality assumption on buy and sell noise trading, I estimate the variance of noise trading using Generalized Method of Moments by Hansen (1982) using the moment conditions from the theoretical model. I show that the net variance of noise trading on average increases four times after inquiry.
The estimate of noise trading as well as informed trading allows an explanation that the inquiry/response disclosure attracts all traders. Uninformed speculators can trade buy or sell and risk averse traders can clear their position before the response disclosure. With the herding behavior of retail investors as in Barber, Odean, and Zhu (2009), noise trading activity varies more after the inquiry disclosure. The trading volume remains high one day after the response disclosure as in 1 where information asymmetry is resolved, supporting higher noise trading activity around rumor disclosure events.

This paper relates to the asymmetric information literatures in financial markets. In the Grossman and Stiglitz (1980) model, informed trader demand depends on the price of a risky asset and an information signal received, whereas uniformed trader demand depends on the price of a risky asset. Thus, the uninformed trader cannot distinguish variation in price due to supply of a security and that due to a value signal. Kyle (1985) explores the dynamic model of information trading using the auction framework, illustrating that the strategic behavior of informed traders are consistent with prices set efficiently during semi-strong efficiency. Glosten and Milgrom (1985) demonstrate similar results; the presence of noise traders facilitates trading, and market makers cannot distinguish informed and noise traders.

This paper is closely related to Kyle (1985), Holden and Subrahmanyam (1992), and Caldentey and Stacchetti (2012). Holden and Subrahmanyam (1992) revisit Kyle (1985) with multiple informed investors. They show price converges more quickly, and asymmetry between informed and uninformed traders reduces more quickly, as the number of informed traders increases. Caldentey and Stacchetti (2012) shows the effects of insider trading when the timing of inside information becomes public is random, also using the Kyle (1985) model. I model the after inquiry periods similar to Holden and Subrahmanyam (1992) and before inquiry periods similar to Kyle (1985), adopting the random deadline of Caldentey and Stacchetti (2012).

This paper also contributes by aggregating the diffusion of information of Hong and Stein (1999) in a Kyle (1985) setting. The theoretical model can be extended where the number of informed traders can increase multiple times during the sequence. Consequent increase in the number of informed traders will force the price to converge faster to the fundamental value of the asset due to fierce competition. In related literature, van Bommel (2003) modifies Kyle (1985) model and shows that informed traders spread rumors to maximize their expected profits. Hong and Stein (1999) uses gradual information diffusion among bounded rational agents to explain under-reaction and overreaction in stock prices.

Various papers have studied rumors in financial markets. Difonzo and Bordia (1997) conduct an experiment in which subjects were presented with a valid news item and published and unpublished rumors. Even when subjects believed that the rumors they read were incredible, they traded on rumors as an informative source. Spiegel, Tavor, and Templeman (2010) use Israeli stock market rumors to show that during the five preceding days and the event day, abnormal stock returns are significant and positive. Kiymaz (2002) also show that positive, significant abnormal returns are observable before the event date, but non-significant, negative abnormal returns are observed after the event date. Chou, Tian, and Yin (2012) argue that stock prices of rumored firms before rumor publication can be used to distinguish the truthfulness of a rumor.

Increase in noise trading after the inquiry disclosure is related to investors’ attention. Huberman and Regev (2001), Fang and Peress (2009), and Da, Engelberg, and Gao (2011) show that investor attention is related to movements in security prices. Hirshleifer and Teoh (2003) show that informationally equivalent forms of disclosure can lead to different consequences due to limited attention. Barber and Odean (2008) show that retail investors are net buyers of stocks that catches their attention and the retail investors herding behavior is shown in Barber, Odean, and Zhu (2009).

There has been analyses on disclosure regulations. Huddart, Hughes, and Levine (2001) analyzes the effects of public disclosure when insiders must disclose trades immediately using a Kyle (1985) model. They demonstrate that due to some revelation of insider information, price becomes more efficient and insider profit lessens. Morris and Shin (2002) show that when the agents have independent sources of information, the public disclosure may reduce welfare.

The remainder of the paper is organized as follows. Section 1 develops a generalized
Kyle-style model with structural change where the number of informed traders increases. In Section 2, I describe the data and estimate parameters to fit the model. Then in Section 3, I show I assess the inquiry/response disclosure and provide policy implications. I suggest further discussion in Section 4 and conclude the paper in Section 5.

1 Theoretical Model of Inquiry/Response Disclosure

I now introduce a model that describes informed traders’ behavior that is similar to KRX market inquiry/response disclosure regulations when rumors are present. I describe the second stage of the model, which is after an inquiry is made. I follow notations from Kyle (1985), with uncertain but finite end periods when multiple informed investors participate in the market. I then introduce the first stage of the model, which incorporates results from the second stage.

1.1 Preliminaries

A security is traded in a time interval $t = 0$ to $t = 1$ by $N$ sequential auctions. $M_n$ number of insiders at period $n = 1, \ldots, N$ exist who know the ex-post liquidation value of risky asset $v \sim N(p_0, \Sigma_0)$. An insider trade quantity $\Delta x_n$ at price $p_n$ at each $n = 1, \ldots, N$. Noise traders also exist who trade quantity $\Delta u_n \sim N(0, \sigma_n^2 \Delta t_n)$ at each $n = 1, \ldots, N$, independent of $v$. I assume noise trading during which buy and sell orders can be distinguished. Noise traders buy and sell according to the bivariate normal distribution as:

$$
\begin{bmatrix}
  \text{buy}_n \\
  \text{sell}_n
\end{bmatrix}
\sim N
\begin{bmatrix}
  \mu_{Bn} \\
  \mu_{Sn}
\end{bmatrix}
, 
\begin{bmatrix}
  \sigma_{Bn}^2 & 0 \\
  0 & \sigma_{Sn}^2
\end{bmatrix}
$$

$$
= N
\begin{bmatrix}
  \mu \\
  \mu
\end{bmatrix}
, 
\begin{bmatrix}
  \frac{1}{2} \sigma_n^2 & 0 \\
  0 & \frac{1}{2} \sigma_n^2
\end{bmatrix}
. \tag{1}
$$

Setting $\mu_{Sn} = \mu_{Bn} = \mu$ and $\sigma_{Bn}^2 = \sigma_{Sn}^2 = \frac{1}{2} \sigma_n^2$ allows us to keep the usual assumption since the net noise trading volume becomes $\text{buy} - \text{sell} \sim N(\mu_{Bn} - \mu_{Sn}, \sigma_{Bn}^2 + \sigma_{Sn}^2) = N(0, \sigma_n^2)$. Although this assumption is not used in this section, it is helpful when analyzing the variance of noise trading $\sigma_n^2$ from the data, which appears in section 4. In this section, I assume
\( \sigma_n^2 = \sigma^2 \) which is constant for all \( n \).

First, liquidation value \( v \) and the noise trader’s aggregate quantity \( \Delta u \) is realized. At the beginning of each period \( n \), \( \Delta u_n \) is unobserved by the insider, and the insider chooses \( \Delta x_n \) by only observing \( v \). Then, market makers determine the price \( p_n \) that clears the market after observing \( y_n = \Delta X_n + \Delta u_n \), where \( \Delta X_n \) is the number of orders submitted by all informed traders. The market efficiency condition is assumed to be

\[
p_n = \mathbb{E}[v \mid y_1, y_2, \ldots, y_n] \quad \text{for} \quad n = 1, 2, \ldots, N. \tag{2}
\]

The difference from the original Kyle (1985) model is that I introduce two types of disclosure processes between discrete time intervals when the market receives the rumor; one is voluntary disclosure and the other is inquiry/response disclosure. After the market clears, the firm may voluntarily disclose information that resolves information asymmetry with probability \( 1 - \theta_n \). Conditional on the firm not making a disclosure, market regulators can inquire using public disclosure about a rumor in the market with probability \( 1 - \eta_n \). Once the inquiry is made, the firm discloses relevant information, which eliminates information asymmetry among traders with probability \( 1 - \theta_m^I \) at some period \( m = n+1, n+2, \ldots, N \). All disclosures are made at the end of each period. \( 1 - \theta_n \) and \( 1 - \theta_m^I \) are probabilities that the information is disclosed at the end of period \( n \), conditional at the start of period \( n \), with the superscript \( I \) implying an after-inquiry state. At the beginning of period \( n \), \( \theta_n(1-\eta_n) \) is the probability that an inquiry disclosure occurs at the end of period \( n \). Figure 2 describes this sequence.

Since the market is efficient, market prices immediately adjust to the true liquidation value \( v \) immediately after voluntary/response disclosures. Insiders are uncertain about future expected profits because they have no additional informational advantage in the next period if a voluntary/response disclosure is made. If a response disclosure is made in period \( m \), profits from informational advantage can be reaped only up to \( m \). Since this period is unknown to informed investors, we will not achieve the same results as with the original model with \( m \) periods. This feature affects future expected profits and other parameters observed along with the effects of inquiry disclosures. I solve the model backward, starting with the model’s second stage.

\footnote{When \( \sigma_n^2 \) varies over time, informed traders submit higher volumes of trade since they can disguise themselves more as noise traders. Analysis when noise variance changes appears in Section 4.}
1.2 Second Stage Equilibrium

I now report results, which provide the difference equation system that describes equilibrium with a stochastic end period. Assume $t = 0$ is the first period after an inquiry.

**Proposition 1.** A unique linear equilibrium exists in our model, in which there are constants $\alpha^I_n$, $\beta_n$, $\delta^I_{n-1}$, $\lambda_n$, and $\Sigma_n$ characterized by the following:

\[
\Delta X_n = M_n \beta_n (v - p_{n-1}) \Delta t_n \hspace{1cm} (3)
\]

\[
\Delta p_n = \lambda_n (\Delta X_n + \Delta u_n) \hspace{1cm} (4)
\]

\[
\Sigma_n = Var(v | \Delta X_1 + \Delta u_1, \ldots, \Delta X_n + \Delta u_n) \hspace{1cm} (5)
\]

\[
E(\pi^I_n | p_1, \ldots, p_{n-1}, v) = \alpha^I_{n-1} (v - p_{n-1})^2 + \delta^I_{n-1} \hspace{1cm} (6)
\]

for all auctions $n = 1, \ldots, N$ and for all informed traders $i = 1, \ldots, M_n$.

The constants $\alpha^I_n$, $\beta_n$, $\delta^I_{n-1}$, $\lambda_n$, and $\Sigma_n$ are the unique solutions to the difference equation system

\[
\alpha^I_{n-1} = \frac{1 - \theta^I_n \alpha^I_n \lambda_n}{\lambda_n [M_n (1 - 2 \theta^I_n \alpha^I_n \lambda_n) + 1]^2} \hspace{1cm} (7)
\]

\[
\beta_n \Delta t_n = \frac{1 - 2 \theta^I_n \alpha^I_n \lambda_n}{\lambda_n [M_n (1 - 2 \theta^I_n \alpha^I_n \lambda_n) + 1]} \hspace{1cm} (8)
\]

\[
\delta^I_{n-1} = \theta^I_n (\delta^I_n + \alpha^I_n \lambda_n \sigma_n^2 \Delta t_n) \hspace{1cm} (9)
\]

\[
\lambda_n = \frac{M_n \beta_n \Sigma_n}{\sigma_n^2} \hspace{1cm} (10)
\]

\[
\Sigma_n = (1 - M_n \beta_n \lambda_n \Delta t_n) \Sigma_{n-1} \hspace{1cm} (11)
\]

for auctions $n = 1, \ldots, N - 1$, subject to the boundary conditions

\[
\alpha^I_N = 0 \hspace{1cm} \beta_N \Delta t_N = \frac{1}{\lambda_N (M_N + 1)} \hspace{1cm} \delta^I_N = 0 \hspace{1cm} \lambda_N = \frac{1}{\sigma_n} \left[ \frac{M_N \Sigma_N}{(M_N + 1) \Delta t_n} \right]^{1/2}
\]
at the Nth and final auction, the condition

$$\Sigma_1 = (1 - M\beta_1\lambda_1\Delta t_1)\Sigma_0$$

at the first auction for a given value of \(\Sigma_0\) and the second order condition

$$\lambda_n(1 - \theta_n^I\alpha_n\lambda_n) > 0$$

for all auctions \(n = 1, \ldots, N\).

Proof. See Appendix.

As with other Kyle-style models, \(\beta\) corresponds to the measure of intensity of informed investors’ trading strategies. \(\lambda\) relates to the pricing rule of the market maker, and to market depth. \(\Sigma\) is the error variance of the price.

I now introduce a new variable \(q_n = \theta_n^I\alpha_n^I\lambda_n\) to calculate explicit numerical results for the previous proposition. The following proposition characterizes the methods for numerical calculation.

Proposition 2. Let \(q_n = \theta_n^I\alpha_n^I\lambda_n\). The solution of the difference equation system in the previous Proposition given by starting from \(q_N = 0\) and iterating backward for \(q_{N-1}, \ldots, q_1\) by using the unique root of the cubic equation

$$0 = 2M_n\frac{\Delta t_{n-1}}{\Delta t_n} q_{n-1}^3 - (M_n + 1)(\frac{\Delta t_{n-1}}{\Delta t_n})q_{n-1}^2 - 2k_n q_{n-1} + k_n$$

(12)

where

$$k_n = \frac{(\theta_{n-1}^I)^2(1 - q_n)^2}{(1 - 2q_n)(M_n(1 - 2q_n) + 1)^2}$$

(13)

lies in the interval \((0, \frac{1}{2})\). Then starting from the exogenous value \(\Sigma_0\), iterate forward for
each of the following variables in the order listed:

\[
\begin{align*}
\Sigma_n &= \frac{1}{M_n(1 - 2q_n) + 1})\Sigma_{n-1} \\
\lambda_n &= \left(\frac{M_n\Sigma_n(1 - 2q_n)}{\Delta t_n\sigma_n^2[M_n(1 - 2q_n) + 1]}\right)^{1/2} \\
\beta_n &= \left(\frac{(1 - 2q_n)\sigma_n^2}{\Sigma_n\Delta t_n M_n[M_n(1 - 2q_n) + 1]}\right)^{1/2}
\end{align*}
\]

at auctions \( n = 1, \ldots, N \) and calculate \( \alpha_n^I \) and \( \delta_n^I \) using equations (7) and (9), respectively.

Proof. See Appendix.

\( \square \)

1.3 Second-Stage Equilibrium Properties

Note that the number of investors \( M_n \) need not exceed 1. Setting \( M_n = 1 \) and \( \theta_n^I = 1 \) gives the exact Kyle (1985) model, and any \( M_n \) with \( \theta_n^I = 1 \) corresponds to Holden and Subrahmanyam (1992). The flexibility of the model enables us to compare parameters in classic models from the literature.

I compare the case when \( \theta_n^I \in \{1, 8, 5\} \) and \( M_n \in \{1, 2\} \) to be fixed for all \( n \). Figure 3 illustrates the condition during which only one informed investor exists, just as in Kyle (1985). As the probability of losing informational advantage increases, we get similar results when the number of informed investors increases (see Figures 4, 5, and 6 in Holden and Subrahmanyam (1992)). Figure 4 shows parameters for the case during which multiple informed investors exist, as in Holden and Subrahmanyam (1992). Note that \( \theta_n^I \) in this case works similarly to increasing the number of informed investor too. By examining equations (14), (15), and (16) with (3), we can see that \( 1 - 2q_n \) is multiplied by \( M_n \) where \( q_n \) incorporates \( \theta_n^I \). Although they do not work exactly due to how \( q_n \)s are iterated, they do offer similar effects when the informed trader gets lower future expected profits. Increased competition \( (M_n > 1) \) or the uncertainty of future profits \( (\theta_n^I < 1) \) make informed traders trade more aggressively in comparison to the Kyle (1985) model.
1.4 First-Stage Equilibrium

During the first stage of the model, a unique informed trader exists in the market. For simplicity, I assume the number of informed traders after inquiry is public information. The informed investor can predict parameters of future periods even when inquiry disclosure occurs at the end of a period. If $\eta_n = \theta_n = 1$ in the first stage, we have the Kyle (1985) model.

The first-stage total expected profit function considers future profits. Future profits depend on disclosure that might occur at the end of a period, so the informed trader adjusts future profits by weighting possible outcomes. Since the informed trader is risk neutral, the probability of each event is suitable for the weights. Thus, the profit function during the first stage is:

$$
\pi_n(p_n) = \max \Delta \pi_n + \theta_n \eta_n \pi_{n+1} + \theta_n (1 - \eta_n) \pi_{n+1}^I \\
= \max \Delta \pi_n + \theta_n \eta_n \{\alpha_n (v - p_n)^2 + \delta_n \} + \theta_n (1 - \eta_n) \{\alpha_n^I (v - p_n)^2 + \delta_n^I \}.
$$

Superscript $I$ stands for parameters calculated from Proposition 1. I report results for the first stage of this two-stage model.

**Proposition 3.** A unique linear equilibrium exists that is recursive. This equilibrium contains constants $\beta_n$, $\lambda_n$, $\alpha_n$, $\delta_n$, and $\Sigma_n$ such that for

$$
\begin{align*}
\Delta x_n &= \beta_n (v - p_{n-1}) \Delta t_n \quad (17) \\
\Delta p_n &= \lambda_n (\Delta x_n + \Delta u_n) \quad (18) \\
\Sigma_n &= Var(v \mid \Delta x_1 + \Delta u_1, \ldots, \Delta x_n + \Delta u_n) \quad (19) \\
E\{\tilde{\pi}_n \mid p_1, \ldots, p_{n-1}, v\} &= \alpha_{n-1} (v - p_{n-1})^2 + \delta_{n-1} \quad (20)
\end{align*}
$$
Given $\Sigma_0$, the constants are a unique solution to the difference equation system

\begin{align}
\alpha_{n-1} &= \frac{1}{4\lambda_n(1 - \theta_n \eta_n \alpha_n \lambda_n - \theta_n(1 - \eta_n )\alpha_n^{I} \lambda_n)} \\
\beta_n \Delta t_n &= \frac{1 - 2\theta_n \eta_n \alpha_n \lambda_n - 2\theta_n(1 - \eta_n )\alpha_n^{I} \lambda_n}{2\lambda_n(1 - \theta_n \eta_n \alpha_n \lambda_n - \theta_n(1 - \eta_n )\alpha_n^{I} \lambda_n)} \\
\delta_{n-1} &= \theta_n [\eta_n (\alpha_n \lambda_n^2 \sigma_n^2 \Delta t_n + \delta_n) + (1 - \eta_n) (\alpha_n^{I} \lambda_n^2 \sigma_n^2 \Delta t_n + \delta_n^{I})] \\
\lambda_n &= \frac{\beta_n \Sigma_n}{\sigma_n^2} \\
\Sigma_n &= (1 - \beta_n \lambda_n \Delta t_n) \Sigma_{n-1}
\end{align}

subject to $\alpha_N = \delta_N = 0$ and the second-order condition

$$
\lambda_n (1 - \theta_n \eta_n \alpha_n \lambda_n - \theta_n(1 - \eta_n )\alpha_n^{I} \lambda_n) > 0.
$$

Proof. See Appendix.

I report a result that includes a step to calculate numerical results for the previous proposition explicitly.

**Proposition 4.** Let $Q_n \equiv \theta_n [\eta_n \alpha_n + (1 - \eta_n) \alpha_n^{I}]$. Then the solution of the difference equation system in the previous Proposition can be solved by obtaining $\lambda_N$ and iterating backwards for $\lambda_{N-1}, \ldots, \lambda_1$ by using the unique root of the cubic equation

$$
2Q_n \Delta t_n \sigma_n^2 \lambda_n^3 - 2\Delta t_n \sigma_n^2 \lambda_n^2 - 2Q_n \Sigma_n \lambda_n + \Sigma_n = 0
$$

and plugging in the results to (21)-(25) at each period gives

\begin{align}
\Sigma_n &= \frac{1}{2(1 - Q_n)} \Sigma_{n-1} \\
\lambda_n &= \left( \frac{\Sigma_n(1 - 2Q_n)}{2\Delta t_n \sigma_n^2 (1 - Q_n)} \right)^{1/2} \\
\beta_n &= \left( \frac{(1 - 2Q_n) \sigma_n^2}{2\Sigma_n \Delta t_n (1 - Q_n)} \right)^{1/2}
\end{align}

Proof. See Appendix.
1.5 Equilibrium Properties

Figure 5 shows first-stage results of the two-stage switching model when the voluntary disclosure parameter $\theta_n$ differs, with the conditional transition probability from the first to second stage $\eta_n = .95$. Figure 6 shows the difference when transition probability $\eta_n$ changes and voluntary disclosure $\theta_n$ is fixed to 1. As with the previous models, an increase in the probability of early termination of information advantage for informed investors makes them trade aggressively to reap profits during early stages. Similarly, an increase in the probability of competition also makes the initial informed investors act more aggressively during earlier stages.

The line in Figure 5 that indicates $\theta_n = 1$, and all plots in Figure 6 show events during which the informed investors know when the information will be revealed. These are comparable directly with other existing informed-trading models. The work is conducted in Figure 7 by comparing the original models in Kyle (1985), Holden and Subrahmaniam (1992), and Huddart, Hughes, and Levine (2001) (henceforth HHL). Although the first two models differ only by the number of informed investors, comparing the two-stage model that I introduce with Huddart, Hughes, and Levine (2001) demonstrates how each regulation — the disclosure inquiry/response and disclosure of insider trading — influences the market. As the figure shows, all parameters in the two-stage model are between the Kyle (1985) and Holden and Subrahmaniam (1992) models because it takes the similar form of Kyle (1985) during the first stage and Holden and Subrahmaniam (1992) during the second. With $\eta_n = .95$, price converges faster for most of the periods in this model than with HHL, but the initial expected profits for the insiders are greater in the new model. The two-stage model I plot in the figure shows when transition to the second stage does not occur. Thus, it shows that even though the inquiry disclosure does not occur, the probability that it may affects the informed traders to trade more aggressively.

Two plots in Figure 8 show a comparison with HHL by varying the $\eta_n$ in the two-stage model when $\theta_n$ is fixed to 1. Although not shown in the figure, even when $\eta_n = .98$ for all periods, the price is closer to the true value than HHL for the first 13 periods over the 20-period horizon, even if transition to the second stage does not occur. When $\eta_n = .99$ for all periods, the expected price, which is the weighted average of the price over the probability that the inquiry is made, is closer to the true value than HHL for the first 14 periods. The top right panel shows a comparison of the expected cumulative profits from each period. HHL
is better at lowering insiders’ profits in comparison to the two-stage model with $\eta_n > .9$. I conclude that HHL and the two-stage model offer different advantages that depend on either acceleration in price discovery or lessening insider profits.

2 Empirical Strategy

2.1 Data and Data Treatment

I use two types of data that range from July 2009 to June 2011: 1) the intraday stock data acquired from KOSCOM, which provides all trades executed in KOSPI$^3$; and 2) disclosure data, which were collected manually from KRX’s E-Disclosure system. The disclosure system is available publicly, and provides all disclosures that relate to firms listed on the KRX stock market (KOSPI market and KOSDAQ market), including other regulated firms that must provide certain disclosures. All disclosures are available immediately, and a time stamp was placed on all disclosures. I collect all rumor inquiry disclosures and matched them to a response disclosure related to the inquiry. I also use daily data, but only to estimate when information trading occurred prior to a inquiry disclosure.

Matching stock data yielded 204 cases, excluding cases in which multiple rumor inquiries had been made within 5 trading days. Whereas the KOSPI market trades publicly from 9AM to 3PM, disclosures can occur any time during the day. Although the opening- and closing-time trading behavior might differ from other times, I disregard any disparities when dealing with intraday data, and assume trading behavior does not differ by clock time. Doing so allows me to focus on only the periods prior to and after disclosures.

Although we may trace all trades transaction by transaction, I use hourly observations of stock prices, allowing me to use the theory discussed in the previous section which is in discrete time.$^4$ Each trading hour is translated to each auction period. The second stage of the two-stage model begins when an inquiry disclosure occurs, and ends when a response disclosure is announced. If I let time $T$ be the time stamp on an inquiry disclosure, I use intraday data from hour $T - 30hr$ until a response was made, which led to $T + khr$, where $k$ is some positive but small number. The hours (periods) are all in trading hours. The basic

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$^3$Unlike the NYSE TAQ data, it does not provide quote data.

$^4$The reason that the model should not be used in continuous time version is discussed in Section 4.
Statistics for the disclosure samples are provided in Table I.

Stocks listed on the KOSPI market have price limits in which stock price movements cannot exceed $\pm 15\%$ of the previous trading day’s closing price. When the stock price hits this limit during trading and does not fall below the limit, I replace the stock price with the next trading day’s opening price to adjust for the price it would have been without the price constraint.

2.2 Estimating the Time of Initial Informed Trading via Non-Linear Least Squares

I estimate when initial informed trading occurred for each case in the sample. This is crucial since the Kyle-style model begins the period with informed trading, testing whether the model should be restricted to the timeframe when informed trading existed. Otherwise, we would be testing the asymmetric information model when there was no asymmetry.

From equations (18) and (17),

$$\Delta p_n = \gamma_n (v - p_{n-1}) + \epsilon_n,$$

(31)

where $\gamma_n = \lambda_n M_n \beta_n \Delta t_n$ and $\epsilon_n = \lambda_n \Delta u_n$. Equation (31) shows the price moves towards the true value because $\gamma > 0$. The true value $v$ is calculated by using the hourly stock price at the response disclosure.

Although the rumor becomes widespread during the inquiry disclosure period $T$, I assume the initial insider becomes informed at some $t_0 \leq T$. So for any trading period $n \geq t_0$, informed trading occurs. I use logistic function $f(n, t_0)$ as a continuous approximation of an indicator function for when informed trading occurs. With this specification, three regimes exist: before informed trading occurs (regime 0), a small number of informed traders before an inquiry disclosure (regime 1), and a large number of informed traders after an inquiry disclosure (regime 2). The latter two combined coincide with the two-stage model I introduced in the previous section. Using the specification as in Ellison and Mullin (2008), I set the price sensitivity to increase over time as $\gamma_n = \gamma_1 + \gamma_2 n$. This feature is supported by Figure 7. Applying this equation, we can run a non-linear least squares (NLLS):
\[ \Delta p_n = \alpha + \gamma_1 (v - p_{n-1}) f(n, t_0) + \gamma_2 (v - p_{n-1}) \left( \frac{n - t_0}{T - t_0} \right) f(n, t_0), \]  
where \( f(n, t_0) = \frac{e^{n-t_0}}{1+e^{n-t_0}}. \)

Sample periods for each disclosure case are from 40 days prior to an inquiry disclosure. Table II shows results using a subsample of data in which a response disclosure did not deny a rumor fully and with the entire 204 sample. The reason for estimating two samples is that one might argue that in the false rumor case, informed trading cannot occur before it is widespread (i.e., after a inquiry disclosure) since the rumor is false. However, if an insider who acknowledges the (false) rumor before an inquiry disclosure, she may still trade for profit if she can correctly value the true stock price. For each sample, I run a constrained model that does not consider whether \( \gamma \) increases over time, having \( \gamma_2 = 0 \) in the regression equation (32) and the unconstrained full model. In both models, daily stock prices depend positively on \( (v - p_{n-1}) \). Based on the estimations of the model, an insider obtaining the information and trading in the market occurs on average \( t_0 - 1 = 23 \) days prior to an inquiry disclosure. From this estimate, I assume there exists an informed trader at least five days prior to an inquiry disclosure in each case at least in the non-false rumor samples in the estimations that follow.

2.3 Empirical Estimates of Noise Trading, Number of Informed Traders, and other Model Parameters

Parameters that require estimation include \( \Sigma_0, \eta, N, \theta^I, \sigma^2_n \), and \( M_n \). To estimate the parameters, adjustments and interpretations are necessary. As in the previous section, I let an auction period occur each trading hour. Since the KOSPI market operates for six hours per trading day, each trading day has six periods. I use non-false rumor data in the two-stage model when estimating parameters. The false rumor case would be estimated separately for policy implications only during the second stage.

To keep calculations simple, I choose \( \eta \) to be constant over all periods prior to an inquiry disclosure. Theoretically, the information can be lived infinitely, allowing us to calculate \( \eta \) easily, which is the probability parameter in the negative binomial model. However, the rumor content of an inquired disclosure is largely important information that a firm must
disclose timely. Thus, I restrict the informational advantage time to not exceed 25 trading
days, or \( N = 150 \) trading hours. I choose \( \eta = .99925 \) which gives about 142 average trading
hours of monopolistic advantage for the initial informed traders.\(^5\) This corresponds with
the average of about 23.6 days of monopolistic informed trading from Table II. The \( \theta^I \)s
are calculated directly from Table I.C. Since a response disclosure occurs within six trading
periods after inquiry, if we set \( T \) to be the period of inquired disclosure, \( \theta^I_{T+n} \) for \( n = 1, \ldots, 6 \)
are only needed because the information will be revealed surely during \( T + 7 \).

Measuring the noise traders’ variance parameter \( \sigma^2_n \) is difficult since we cannot distinguish
noise trading and informed trading from the data. However, with the assumption from (1)
and with our model, we can estimate the variance of noise trading. First, noise traders and
informed traders might each submit either buy or sell orders, or both. Then the market
makers clear the market of what remains, net of all submitted orders. This allows the total
trading volume to be

\[
TV = \max\{\text{buy order, sell order}\} = \max\{\text{noise buy+informed buy, noise sell+informed sell}\}
\]

An informed trader sees the same signal and will take only one side of buy or sell. Without
loss of generality, suppose \( v > p_{n-1} \) at the beginning of period \( n \) so the informed selling
quantity is zero. If the informed traders submit \( \Delta X_n \),

\[
\begin{align*}
\text{noise buy+informed buy} & \sim N(\mu + \Delta X_n, \frac{1}{2}\sigma^2_n) \\
\text{noise sell} & \sim N(\mu, \frac{1}{2}\sigma^2_n)
\end{align*}
\]

Then by 33

\[
E[TV_n] = (\mu + \Delta X_n)\Phi\left(\frac{\Delta X_n}{\sigma_n}\right) + \mu \Phi\left(-\frac{\Delta X_n}{\sigma_n}\right) + \sigma_n \phi\left(\frac{\Delta X_n}{\sigma_n}\right)
\]

\(^5\)If we assume information to live infinitely, we get \( \eta = .9996 \).
where \( \Phi \) and \( \phi \) represent the CDF and PDF of standard normal distribution, respectively.\(^6\)

Note that \( \Delta X_n + \Delta u_n \) is \( \max\{\text{buy order}, \text{sell order}\} - \min\{\text{buy order}, \text{sell order}\} \) so

\[
\Delta p_n = \lambda_n (\Delta X_n + \Delta u_n)
\]

\[
= \lambda_n \left[ \Delta X_n \left( \Phi \left( \frac{\Delta X_n}{\sigma_n} \right) - \Phi \left( -\frac{\Delta X_n}{\sigma_n} \right) \right) + 2 \sigma_n \phi \left( \frac{\Delta X_n}{\sigma_n} \right) \right]
\]

Also,

\[
\Delta p_n^2 = \lambda_n^2 (\Delta X_n + \Delta u_n)^2
\]

\[
= \lambda_n^2 \left( \Delta X_n^2 + \Delta u_n^2 + 2 \Delta X_n \Delta u_n \right)
\]

\[
E[\Delta p_n^2] = \lambda_n^2 (\Delta X_n^2 + \sigma_n^2)
\]

I further simplify the estimation in a way that \( \sigma_n^2 \) changes only at the inquiry disclosure. Then the moment restrictions are as follows for six trading periods:

\[
\begin{pmatrix}
E[TV_1 | v, p_0] \\
\vdots \\
E[TV_6 | v, p_5] \\
E[\Delta p_1 | v, p_0] \\
\vdots \\
E[\Delta p_6 | v, p_5] \\
E[\Delta p_1^2 | v, p_0] \\
\vdots \\
E[\Delta p_6^2 | v, p_5]
\end{pmatrix}
= \begin{pmatrix}
(\mu + \Delta X_1) \Phi \left( \frac{\Delta X_1}{\sigma_n} \right) + \mu \Phi \left( -\frac{\Delta X_1}{\sigma_n} \right) + \sigma_n \phi \left( \frac{\Delta X_1}{\sigma_n} \right) \\
\vdots \\
(\mu + \Delta X_6) \Phi \left( \frac{\Delta X_6}{\sigma_n} \right) + \mu \Phi \left( -\frac{\Delta X_6}{\sigma_n} \right) + \sigma_n \phi \left( \frac{\Delta X_6}{\sigma_n} \right) \\
\lambda_1 \left[ \Delta X_1 \left( \Phi \left( \frac{\Delta X_1}{\sigma_n} \right) - \Phi \left( -\frac{\Delta X_1}{\sigma_n} \right) \right) + 2 \sigma_n \phi \left( \frac{\Delta X_1}{\sigma_n} \right) \right] \\
\vdots \\
\lambda_6 \left[ \Delta X_6 \left( \Phi \left( \frac{\Delta X_6}{\sigma_n} \right) - \Phi \left( -\frac{\Delta X_6}{\sigma_n} \right) \right) + 2 \sigma_n \phi \left( \frac{\Delta X_6}{\sigma_n} \right) \right] \\
\lambda_1^2 \left( \Delta X_1^2 + \sigma_n^2 \right) \\
\vdots \\
\lambda_6^2 \left( \Delta X_6^2 + \sigma_n^2 \right)
\end{pmatrix}
\]

The unknowns are \( \mu, \sigma_n, \lambda_n \) and \( \Delta X_n \) where \( \lambda_n \) and \( \Delta X_n \) are for all \( n = 1, \ldots, 6 \) so we

\(^6\)Suppose \( \begin{bmatrix} X \\ Y \end{bmatrix} \sim N \left( \begin{bmatrix} \mu_x \\ \mu_y \end{bmatrix}, \begin{bmatrix} \sigma_x^2 & \rho \sigma_x \sigma_y \\ \rho \sigma_x \sigma_y & \sigma_y^2 \end{bmatrix} \right) \). Then, by Cain (1994),

\[
E[\max(X, Y)] = \mu_x \Phi \left( \frac{\mu_x - \mu_y}{\theta} \right) + \mu_y \Phi \left( \frac{\mu_y - \mu_x}{\theta} \right) + \theta \phi \left( \frac{\mu_x - \mu_y}{\theta} \right)
\]

\[
E[\min(X, Y)] = \mu_x \Phi \left( \frac{\mu_y - \mu_x}{\theta} \right) + \mu_y \Phi \left( \frac{\mu_x - \mu_y}{\theta} \right) - \theta \phi \left( \frac{\mu_x - \mu_y}{\theta} \right)
\]
have 14 unknown parameters to estimate from 18 moment conditions. I estimate using these moment conditions in two samples: 1) six trading periods prior to an inquiry disclosure and 2) trading periods before a response and after an inquiry disclosure, which is a maximum of 6 trading periods in each rumor case. Normalization for trading volume was calculated by subtracting the mean and dividing by the standard deviation of the sample from $-120$ to $-61$ hours before inquiry of each stock. Price normalization is calculated by dividing by the post response stock price, value $v_i$.

Results of the GMM estimation are shown in Table III. The net noise trading variance $\sigma_n^2$ increased after the inquired disclosure by more than four times compared to before inquiry disclosure. In all six periods before and after the inquiry, informed trading volume is not statistically different from zero. This result may reflect the fact that the parameters are estimated with wide standard errors or, related to this possibility, that informed traders successfully hide amongst the order flow of the uninformed traders. Market makers and econometricians alike would both struggle to identify noise trading with trading volume.

The average number of informed investors $M_n$ is estimated by matching price movements roughly to the data after an inquiry but before a response. I find the number of informed traders $M$ that minimizes the price gap between the model and data for each period:

$$\min_M \sum_{i=1}^{S} \sum_{n=1}^{N} \left( \frac{(M\beta_n(M)(v_i-p_{in-1})-\Delta p_{in}}{\lambda_n(M)v_i})^2 \right).$$

(36)

where the above equation inside the bracket comes from (31) divided by $v_i$ for normalization. $p_{in}$ is the actual price of stock (rumor case) $i$ at at $n$. The expression $v_i - p_{in-1}$ shows the disparity in price changes with real data. Since I used the sample between the two inquiries, maximum number of periods for the above equation is $N = 6$. $S = 204$ is number of rumor cases. I run the non-linear least squares regression dropping the samples if $v_i - p_{in-1} = 0$ since the price movement should only come from the noise trading activity. The regression gives $M = 1.1711$ where we see more than one informed trader after an inquiry, and significantly different from 1 (standard error on $M$ is .0014). While the number may seem to be not far from 1, the economic significance is important since it allows competition among informed traders. For periods prior to inquiry disclosure, I assume a monopolistic informed trader

$^7$The scale affects only the degree of $\lambda$. 

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because if competition exists before an inquiry, price convergence is likely to occur before
the rumor becomes public, as in Holden and Subrahmanyam (1992).

To estimate $\Sigma_0$, first, the price of initial trading period is normalized to $p_0 = 10$. Then
the variance of the normalized value $\bar{v}_i = \frac{v_i}{p_0}$ is calculated. All of the estimated parameters
are shown in column (1) of Table IV.

3 Estimation Results and Policy Implications

From estimation of $\sigma^2$ and $M$, two opposite effects of inquired disclosure to informed traders’
profit are observed. An increase in noise traders’ variance ($\sigma^2_n$) makes informed traders more
aggressive, who realize higher profits, as in Back and Pederson (1998) and Collin-Dufresne
and Fos (2012). However, an increase in competition among informed traders (higher $M_n$)
lowers informed traders’ profit, a result known well from Holden and Subrahmanyam (1992).
We can compare profits earned by informed traders before and after the regulation to find
out if it benefits or worsens the informed traders.

Columns (1)-(4) show various models I test in this section. Columns (1)-(2) are the 2-
stage models that incorporate a structural difference before and after an inquiry disclosure.
Column (3) is the basic Kyle (1985) model for the baseline and column (4) is the Holden and
Subrahmanyam (1992) models. I first compare the fit of two-stage model to data using (36)
with parameters of each model in Table IV. I use non-false data starting from $-6$ trading
hours prior to inquiry disclosure until a response disclosure. Results show that the model
with increase in the number of informed traders (column (1)) fits price data better, even when
we include sample periods before an inquiry disclosure. However, with 10000 bootstrap data
samples, the fit is statistically indifferent with the model of unique informed trader (column
(2)).

Informed traders’ profits are worth noting. Consider the case of a non-false rumor in
which there exists a monopolistic informed trader prior to inquiry. Note that $\pi$ from (20)
is the initial informed trader’s expected profit. To compare results from other models, we
must include the newly informed trader’s expected profits after an inquiry as follows:

\[
\pi^{NF} = E[\tilde{\pi}_1] + \left\{ \sum_{n=1}^{N} (M_n - 1)E[\tilde{\pi}_{n+1}^I](1 - \eta_n)(\prod_{i=1}^{n} \theta_i^I)(\prod_{i=1}^{n-1} \eta_i) \right\}
\]

\[
= E[\tilde{\pi}_1] + \left\{ \sum_{n=1}^{N} (M_n - 1)E[\tilde{\pi}_{n+1}^I](1 - \eta_n)(\theta_1^I)^n\eta^n-1 \right\},
\]

(37)

where the second equality assumes \(\theta^I\) and \(\eta\) are constant before an inquiry. \(E[\tilde{\pi}_1]\) is the expected profit of an initial informed trader, and the second term captures the profits earned by the newly informed trader after an inquiry disclosure. \((M_n - 1)E[\tilde{\pi}_{n+1}^I]\) is the expected profit of additional informed traders when inquiry disclosure is made during period \(n + 1\) and \(\prod_{i=1}^{n} \theta_i^I(1 - \eta_i)\) is the probability of inquiry disclosure during \(n + 1\).

In the case of a false rumor, I assume that there does not exist any informed trader before the inquiry. Inquiry disclosure produces informed traders, which has a negative effect if the policy decreases insiders’ profits. We can calculate this by using equation (6). Incorporating the false rumor too, total expected profit per inquiry is:

\[
\pi = \kappa \pi^{NF} + (1 - \kappa)\pi^f,
\]

(38)

where \(\kappa\) is the probability that an inquired rumor turns out to be non-false and \(\pi^f\) is the profit after the inquiry when the rumor turned out to be false. Comparing profits with other models, we must compare the results above with \(\kappa \hat{\pi}\) where \(\hat{\pi}\) is profit per informed event.

Informed traders’ expected profits for each model are shown in Table V. Comparing the first row of columns (1) and (3) show that the disclosure regulation lowers initial informed traders’ profits. The main source of this affect is from the competition among informed traders after the inquiry. However, when false rumors are inquired, newly informed traders can reap sufficient benefits which is unavailable without the inquiry disclosure. Comparing the total average profits by informed traders, the calibrated model shows that the disclosure regulation increases overall profits made by the informed traders.

Making false alerts is not the only problem the regulation faces. When there exists a possibility of early termination of informational advantage due to inquiry disclosure from a response following within six trading hours, increase in noise trading provide greater benefits for informed traders. This becomes a downside of the inquiry/response disclosure regulation.
that it increases informed profits when the variance of noise trading grows.

When implementing the inquiry/response policy, one may think about profits that informed traders can gain. Although price efficiency is enhanced, the fairness measure, in terms of lowering insiders’ profits, may increase. To prevent higher profits for informed traders, market regulators need discretion when making a disclosure. Public inquiry should be used when there is high belief that a rumor is true. Although the variance of noise trading might not be controlled, one might require a firm to respond more quickly to an inquiry. Since the firm should know what the response should be when it receives an inquiry, it is puzzling why most firms take six trading hours before responding.

Huddart, Hughes, and Levine (2001) outperforms all other models in terms of lowering insiders’ profits. However, the HHL model is extreme in the sense that it requires an insider to disclose his/her trade at the conclusion of each period. In the context we follow throughout this paper, an insider must report every trading hour. If there is a 24-hour lag in disclosure and the insider acts immediately after an inquiry, market participants cannot observe their trades before a response disclosure. Not all informed traders are subject to the insider disclosure regulation. Insider disclosure regulation dominates the original Kyle (1985) model both in efficiency and fairness, but it works only when an informed trader is subject to the regulation. The inquiry/response regulation might have a downside in terms of fairness, but it leads all informed traders to react which enhances market efficiency.

4 Discussions

4.1 Theoretical Discussions

As the number of auction periods $N$ goes to infinity during the second stage, the results are the same as those in Holden and Subrahmanyam (1992) limiting results. After inquiry disclosure when the number of informed investors is $M \geq 2$, for any auction period $\tau \in (0, N]$, we can find an infinite number of auctions in the range of $[0, \tau)$. Thus, competition among informed investors causes information to reveal at any $\tau > 0$ after the inquiry disclosure.

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8Estimated profits of insiders in Huddart, Hughes, and Levine (2001) are .5835 using parameters of Kyle (1985) model in Table IV.

9Setting $N \to \infty$ is equivalent to $\Delta t \to 0$. 

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For the first stage, results are not obvious. For example, let \( \theta_n^I = 1 - \frac{k}{N} \) and \( \eta_n = 1 - \frac{l}{N} \) for some fixed \( k, l > 0 \), for all \( n \) so that the expected number of auction periods before the response disclosure increases as \( N \) increases. When \( N \to \infty \), the solution to the equation limits to the original Kyle (1985) model. However, if we allow the probabilities to differ from the example, we might achieve different results. Caldentey and Stacchetti (2012) shows the limiting results of a special case when the parameters are \( \eta_n = 1 \), and \( \theta_n^I \) following a geometric distribution.

The number of informed investors is set to be exogenous during the second stage. The cost of verifying the rumor conditioning on the acknowledgment of its existence is not present in the model, but one can introduce cost to obtain true information and solve for the endogenous number of informed investors during equilibrium. In this case, the future expected profit informed investors can earn depends on the price when an inquiry is made. The number of informed investors varies depending on the period when the uncertain information is released to the public. Intuitively, the expected number of additional informed traders decreases as the period continues without inquiry disclosure due to less expected profits that the informed investors can reap because of price convergence. Some cutoff period after that period might exist during which inquiry disclosure does not induce additional informed traders. This would make the last few periods after the cutoff during the first stage of the two-stage model similar to the Kyle model.

The model can be generalized to a multi-stage model in which the number of informed investors increases (changes) multiple times. This is shown in the Appendix B.5. This generalization can be seen as a diffusion of information in a sequential auction setting, so that the idea of Hong and Stein (1999) is incorporated in Kyle (1985) world.

4.2 Liquidity Measure Around Disclosures

Here I show how market liquidity changes during a rumor event. Although the bid-ask spread is one representative measure, the intraday data obtained do not provide information about bid and ask figures. However, using method provides estimates of bid-ask spreads. To be consistent with the trading period described in other parts of this section, I estimate the measures per trading hour. Since each stock has different prices and ticks, I normalize trading volume and estimate bid-ask spreads by dividing average estimates for 60 trading
hours\textsuperscript{10} of data.

Figure 9 shows normalized estimated bid-ask spreads regarding inquiry/response disclosures. It was normalized by dividing estimates by average value of each hour from $t - 120$ to $t - 61$ trading hours prior to an inquiry disclosure. If a trading period was below 50 minutes, data were removed for that period. $ratio_1$ and $ratio_2$ differs when a bid-ask spread estimate was negative. $ratio_1$ converts all negative estimates to zero, and $ratio_2$ does not consider negative estimates. High peaks after two disclosures show that market participants reacted to disclosure events, and trading volumes were above normal between two disclosures. Estimated spreads peaked after two disclosure events, which demonstrates greater information asymmetry among traders. However, other than the periods immediately after two disclosures, no great differences appeared in other periods.

I extend the Amihud (2002) measure to an intraday context. Although the original Amihud measure is applied to low-frequency samples, I calculate the Amihud price impact measure each day by dividing each day into trading hours.

$$ILLIQ_{id} = \frac{1}{P_{id}} \sum_{i=1}^{P_{id}} \frac{|R_{idh}|}{VOLW_{idh}}$$

where $ILLIQ_{id}$ is the daily Amihud measure for stock $i$ during day $d$, $P_{id}$ is the number of periods (i.e., trading hours) where trading occurred for stock $i$ at day $d$, $R_{idh}$ is the return on stock $i$ during hour $h$, and $VOLW_{idh}$ is the respective hour trading volume in Wons (KRW). Day zero is the time between an inquiry and response disclosure. Measure $ILLIQ_{id}$ is multiplied by 1 billion, and averaged across 204 samples. Results are shown in Figure 10. Illiquidity increase a day prior to an inquiry disclosure, but the market is very liquid between two disclosures when information asymmetry might have been greater. When the information was resolved, the measure increased abnormally for the following two days and then fell to its regular state. This implies that after a response disclosure, prices are very volatile for a few periods.

\textsuperscript{10}−120 to −61 trading hours before inquiry
5 Conclusion

Understanding how information gets reflected in price is important. I derive a sequential trading model in which the market regulator inquires with some probability about a rumor that can affect the fundamental value of an asset. The model captures the feature of the inquiry/response disclosure and explains why the stock price converge faster to the true value of the asset. Possible revelation of the rumor causes informed traders to trade more aggressively. The form of public disclosure alerts the prospective informed traders which causes competition. These features increases speed of convergence not only after the inquiry, but before inquiry as well since initial informed traders foresee this shortened informational advantage and increased competition.

Informed traders’ profits are calculated by taking the model to the data. Using data of rumor disclosure in the KRX, I show that informed traders profit actually increase with the regulation. I estimate the variance of net trading of noise trading quadruples after the inquiry disclosure due to increase in investors’ attention, thus allowing informed traders to trade more quantity and earn more profits.

Optimal regulation must take into the account how uninformed market participants respond to regulation. The inquiry disclosure alerts the uninformed traders to be aware of the rumor and forces to resolve the asymmetry within a short period of time. However, this attention causes uninformed traders to trade more causing higher profits for the informed traders. Although the inquiry/response disclosure is effective in terms of market efficiency which is intended by the market regulators, it increases the informed traders profits. Regulators should be aware of unintended consequences when implementing and analyzing a regulation.

Trading volume and Amihud (2002) measure around the disclosures show that there exists some trading behavior prior to inquiry disclosure and after response disclosure. Although this may be explained by information leakage and noise trading due to market attention, respectively, more analyses can be done. This analyses of this paper can be extended to analyze the events where similar inquiry/response disclosure exists in some markets when the security price or trading volume show extreme behavior. In the case of extreme behavior, market participants does not observe any new information about the underlying asset at the time of the inquiry. However, it still creates attention to all the market participants. The
problem of the extreme behavior disclosure is that the mechanism that triggers inquiry is unrevealed and the market regulator can make subjective decision to make an inquiry disclosure.
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A Background: Regulations on Rumors and News in the KRX market

The Korean Exchange set rules for market rumors in February 1976. When the exchange is notified of a rumor, the market is entitled to a special post that contains rumors whose contents might lead to stock price changes. Due to expansion of the market and preparation for opening of security markets to worldwide investment, a securities and exchange law was passed on the last day of 1991. The law enforced regulations regarding insider trading and public disclosures to make the market fairer. The law is the backbone of current disclosure requirements.

KRX market disclosure regulates both KOSPI and KOSDAQ markets, requiring listed firms to disclose information relevant to a firm’s material business, validate inquiries to securities market headquarters concerning rumors and news, and explain extreme price or trading volume issues. I focus on the KOSPI market. On the KOSPI Market Disclosure Regulation (2012), several sections deal with this issue, but Chapter II of the regulation does so explicitly. Paragraph (1) of “Section 2: Disclosure obligation, Article 12: Inquired disclosure” states, “...the Exchange may request a KOSPI-listed corporation to make an inquired disclosure in order to verify the rumors and news...” Paragraph (2) states, “...even though there are no rumors, etc., the Exchange may, when the price or trading volume of stock issued by a KOSPI-listed corporation falls under the criteria separately established by the Exchange, request the concerned corporation to make an inquired disclosure to confirm the presence of material information...”

A request for inquired disclosure consists of 1. details of request, 2. deadline for disclosure, and 3. other references useful for making investment decisions. For example, an inquired disclosure of STX group included details as 'Bond with Warrants (BW) issue and capital budgeting issue related report,' and the deadline as October 24, 2011 at 18:00 (the inquiry was made on the same date at 08:49). Other references are usually related disclosures, if any, but in this case, none existed.

When a firm receives a request to make an inquired disclosure, it should respond during the afternoon if the request was made in the morning, and the next morning if made during the afternoon. For the case of extreme price or trading volumes, the response should be made by the afternoon the next day.
Responses to requests for inquired disclosure consist of 1. title and 2. details. A title is nearly as exact as the details of request of the inquired disclosure. Details of a response consists of whether 1) the rumor is confirmed, 2) the rumor is false and no relevant information exists about the inquiry, or 3) a relevant decision has not been made and the firm cannot confirm the rumor.

After an interview with KRX officials, they informed that KRX did not record specific sources of rumors or news, nor did they store information related to the issue. Sources are varied, and examples include talking to individuals who work at a security company, visiting in person or phone calls, searching the Internet for investment websites or messengers and SNSs, and sometimes exchanging information with other agencies. If a stock price or trading volume falls below a criterion established by the exchange, the securities market headquarters requests undisclosed information to a point at which it might cause an event. KRX does not reveal these criteria.

Three types of disclosure insincerities relate to this issue: 1) disclosures not made by a deadline, 2) retracting disclosed statement, and 3) changes to disclosed statement. The first relates to the speed of a disclosure, and the latter two to a disclosure’s accuracy. During these events, the market is able to penalize a company by issuing penalty points, or by one or more of the following: 1) suspension of trade, 2) designate the firm as ‘invest with care’ and publicize the insincere disclosure, 3) enforce market surveillance and notify the Financial Supervision Service\(^{11}\), and 4) delist from the market. If penalty points exceed a criterion, the firm is delisted automatically.

\(^{11}\)This regulatory body is similar to the SEC in the United States.
B Proofs

B.1 PROOF OF PROPOSITION 1:

\[ E\{\pi_{n+1} \mid p_1, \ldots, p_n, v\} = \alpha^I_n(v - p_n)^2 + \delta^I_n \]

We then have

\[ E\{\pi_n \mid p_1, \ldots, p_n, v\} = \max_x E[(v - p_n)\Delta x + \theta^I_n(\alpha^I_n(v - p_n)^2 + \delta^I_n) \mid p_1, \ldots, p_{n-1}, v]. \tag{39} \]

A linear equilibrium \( p_n \) is given by,

\[ p_n = p_{n-1} + \lambda_n(\Delta X_n + \Delta u_n) + h, \tag{40} \]

where \( h \) is some linear function of \( \Delta X_1 + \Delta u_1, \ldots, \Delta X_{n-1} + \Delta u_{n-1} \). Note that \( \Delta X_n \) can be written as \( \Delta x_n + (M_n - 1)\Delta \bar{x}_n \), where \( \Delta \bar{x}_n \) represents the particular informed trader’s conjecture of the average of the other informed traders’ strategies. Substituting (40) into (39) and evaluating the conditional expectation,

\[ E\{\pi_n \mid p_1, \ldots, p_{n-1}, v\} = \max_{\Delta x_n} [(v - \Delta \bar{x}_n - \lambda_n(M_n - 1)\Delta \bar{x}_n - h)\Delta x_n \tag{41} \]

\[ + \theta^I_n(\alpha^I_n(v - p_{n-1} - \lambda_n(\Delta x_n + (M_n - 1)\Delta \bar{x}_n) - h)^2 \]

\[ + \theta^I_n(\alpha^I_n\lambda_n^2\sigma_n^2\Delta t_n + \delta^I_n)]. \]

Solving the above maximization problem, we get

\[ \Delta x_n = \frac{(v - p_{n-1} - h - \lambda_n(M_n - 1)\Delta \bar{x}_n)(1 - 2\theta^I_n\alpha^I_n\lambda_n)}{2\lambda_n(1 - \theta^I_n\alpha^I_n\lambda_n)} \]

To solve for the equilibrium, we set \( \Delta \bar{x}_n = \Delta x_n \) and solve for \( \Delta x_n \). We then have,

\[ \Delta x_n = (v - p_{n-1} - h)\frac{1 - 2\theta^I_n\alpha^I_n\lambda_n}{\lambda_n[M_n(1 - 2\theta^I_n\alpha^I_n\lambda_n) + 1]} \tag{42} \]

To prove that \( h = 0 \), note that

\[ E\{\Delta p_n \mid \Delta X_1 + \Delta u_1, \ldots, \Delta X_n + \Delta u_n\} = 0. \]

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From (40), we have

\[
E\{\Delta p_n \mid \Delta X_1 + \Delta u_1, \ldots, \Delta X_n + \Delta u_n\} = \frac{h}{M_n(1 - 2\theta_n^I \alpha_n^I \lambda_n) + 1},
\]

thus \( h = 0 \). Hence, (8) follows from (42), and (7) and (9) from (41).

With the market efficiency condition and the normality condition, projection theorem gives

\[
\lambda_n = \frac{M_n \beta_n \Sigma_{n-1}}{M_n^2 \beta_n^2 \Sigma_{n-1} \Delta t_n + \sigma_n^2}
\]

and

\[
\Sigma_n = \frac{\sigma_n^2 \Sigma_{n-1}}{M_n^2 \beta_n^2 \Sigma_{n-1} \Delta t_n + \sigma_n^2}.
\]

(43) and (44) gives (10) and (11). Given \( \Sigma_0 \), the boundary conditions \( \alpha_N = \delta_N = 0 \) and the equations (3)-(11) are necessary and sufficient for a linear equilibrium. The solution to the problem is also unique, which is analogous to the proof in Kyle (1985).

**B.2 PROOF OF PROPOSITION 2:**

Define \( q_n = \theta_n^I \alpha_n^I \lambda_n \). From (7), we have

\[
\alpha_{n-1} = \frac{1 - q_n}{\lambda_n[M_n(1 - 2q_n) + 1]^2}
\]

or

\[
\frac{q_{n-1}}{\theta_{n-1}^I} = \frac{\lambda_{n-1}(1 - q_n)}{\lambda_n[M_n(1 - 2q_n) + 1]^2}
\]

implying

\[
\frac{\lambda_n}{\lambda_{n-1}} = \frac{\theta_{n-1}^I(1 - q_n)}{q_{n-1}[M_n(1 - 2q_n) + 1]^2}
\]

from (10) we also have

\[
\frac{\lambda_n}{\lambda_{n-1}} = \frac{\beta_n}{\beta_{n-1} \Sigma_{n-1}}.
\]
and from (11), this is equivalent to

$$\frac{\lambda_n}{\lambda_{n-1}} = \frac{\beta_n}{\beta_{n-1}}[1 - M_n \beta_n \lambda_n \Delta t_n].$$  \hfill (46)

From (8),

$$\beta_n \Delta t_n = \frac{1 - 2q_n}{\lambda_n[M_n(1 - 2q_n) + 1]}.$$  \hfill (47)

Substituting $\beta_n$ and $\beta_{n-1}$ from (47) to (46), we have

$$\frac{\lambda_n}{\lambda_{n-1}} = \left(\frac{1 - 2q_n}{[M_n(1 - 2q_n) + 1]^2}\right)\left(\frac{M_n(1 - 2q_{n-1}) + 1}{1 - 2q_{n-1}}\right)\left(\frac{\lambda_{n-1} \Delta t_{n-1}}{\lambda_n \Delta t_n}\right) \hfill (48)

$$

Squaring (45) and comparing with (48),

$$\frac{(\theta_{n-1}^I)^2(1 - q_n)^2}{q_{n-1}^2[M_n(1 - 2q_n) + 1]^4} = \left(\frac{1 - 2q_n}{[M_n(1 - 2q_n) + 1]^2}\right)\left(\frac{M_n(1 - 2q_{n-1}) + 1}{1 - 2q_{n-1}}\right)\left(\frac{\Delta t_{n-1}}{\Delta t_n}\right)$$

$$\frac{(\theta_{n-1}^I)^2(1 - q_n)^2}{(1 - 2q_n)[M_n(1 - 2q_n) + 1]^2} = q_{n-1}^2\left(\frac{M_n(1 - 2q_{n-1}) + 1}{1 - 2q_{n-1}}\right)\left(\frac{\Delta t_{n-1}}{\Delta t_n}\right)$$

$$k_n = (M_n q_{n-1}^2 + \frac{q_{n-1}^2}{1 - 2q_{n-1}})\left(\frac{\Delta t_{n-1}}{\Delta t_n}\right)$$

$$= \frac{1}{1 - 2q_{n-1}}(M_n q_{n-1}^2 - 2M_n q_{n-1}^2 + q_{n-1}^2)\left(\frac{\Delta t_{n-1}}{\Delta t_n}\right)$$

$$k_n - 2q_{n-1}k_n = -2M_n\left(\frac{\Delta t_{n-1}}{\Delta t_n}\right)q_{n-1}^2 + (M_n + 1)(\frac{\Delta t_{n-1}}{\Delta t_n})q_{n-1}^2.$$  

Thus we have (12) with (13). Note that $\alpha_{N}^I = 0$ implies $q_N = 0$. The other parts of the theorem are exactly same as in Holden and Subrahmanyam (1992).

### B.3 PROOF OF PROPOSITION 3:

$$E\{\pi_{n+1}(X, P) \mid p_1, \ldots, p_n, v\} = \alpha_n(v - p_n)^2 + \delta_n$$
Since $\tilde{\pi}_n$ is recursive by

$$\pi_n = (v - p_n)\Delta x_n + \theta_n \eta \pi_{n+1} + \theta_n (1 - \eta) \pi'_{n+1},$$

$$E\{\pi_n(X, P) \mid p_1, \ldots, p_n, v\} = \max_{x} E\{x\}$$

$$= \max_{x} \left[ (v - p_n)\Delta x_n + \theta_n \eta \{\alpha_n (v - p_n)^2 + \delta_n \mid p_1, \ldots, p_{n-1}, v\} \right.$$  

$$+ \theta_n (1 - \eta) \{\alpha_n' (v - p_n)^2 + \delta_n' \mid p_1, \ldots, p_{n-1}, v\} \right].$$

In a linear equilibrium, $p_n$ is given by

$$p_n = p_{n-1} + \lambda_n (\Delta x_n + \Delta u_n) + h,$$  

where $h$ is some linear function of $\Delta x_1 + \Delta u_1, \ldots, \Delta x_n + \Delta u_n$. Plugging this to (49) and solving the expectation,

$$E\{\tilde{\pi}_{n+1}(X, P) \mid p_1, \ldots, p_n, v\} = \max_{x} \left[ (v - \tilde{p}_{n-1} - \lambda_n \Delta x_n - h)\Delta x_n \right.$$  

$$+ \theta_n \eta \alpha_n (v - p_{n-1} - \lambda_n \Delta x_n - h)^2$$  

$$+ \theta_n (1 - \eta) \alpha_n' (v - p_{n-1} - \lambda_n \Delta x_n - h)^2$$  

$$+ \theta_n \eta \alpha_n (\lambda_n^2 \sigma_n^2 \Delta t_n + \delta_n)$$  

$$+ \theta_n (1 - \eta) (\alpha_n' \lambda_n^2 \sigma_n^2 \Delta t_n + \delta_n') \right].$$

Solving the FOC for the above maximization problem with respect to $\Delta x_n$,

$$\Delta x_n = \frac{1 - 2\theta_n \eta \alpha_n \lambda_n - 2\theta_n (1 - \eta) \alpha_n' \lambda_n}{2\lambda_n (1 - \theta_n \eta \alpha_n \lambda_n - \theta_n (1 - \eta) \alpha_n' \lambda_n)} (v - p_{n-1} - h)$$  

$$= \beta_n (v - p_{n-1} - h)\Delta t_n,$$  

where $\beta_n \Delta t_n$ is as in (22). The SOC is (26).
See that

\[
\begin{align*}
E\{p_n - p_{n-1} \mid \Delta x_1 + \Delta u_1, \ldots, \Delta x_{n-1} + \Delta u_{n-1}\} &= E\{\lambda_n \Delta x_n + \lambda_n \Delta u_n + h \mid \Delta x_1 + \Delta u_1, \ldots, \Delta x_{n-1} + \Delta u_{n-1}\} \\
&= E\{\frac{1 - 2\theta_n \eta \alpha_n \lambda_n - 2\theta_n (1 - \eta_n) \alpha_n^I \lambda_n}{2(1 - \theta_n \eta \alpha_n \lambda_n - \theta_n (1 - \eta_n) \alpha_n^I \lambda_n)} \\
&\quad \times (v - p_{n-1} - h) + h \mid \Delta x_1 + \Delta u_1, \ldots, \Delta x_{n-1} + \Delta u_{n-1}\} \\
&= \frac{h}{2(1 - \theta_n \alpha_n \lambda_n - \theta_n (1 - \eta_n) \alpha_n^I \lambda_n)} + \beta_n \Delta t_n E\{v - p_{n-1} \mid \Delta x_1 + \Delta u_1, \ldots, \Delta x_{n-1} + \Delta u_{n-1}\} \\
&= \frac{h}{2(1 - \theta_n \alpha_n \lambda_n - \theta_n (1 - \eta_n) \alpha_n^I \lambda_n)},
\end{align*}
\]

where the last equality comes from the market efficient condition. Thus, \( h = 0 \) with probability 1. Hence we see that \( \Delta p_n \) and \( \Delta x_n \) have the recursive form given by (50) and (17).

Now plugging in (52) to (51), we have (21) and (23).

Other parts of the proofs are omitted since it is analogous to Proposition 1.

**B.4 PROOF OF PROPOSITION 4:**

From (22),

\[
\beta_n = \frac{1 - 2\lambda_n Q_n}{2\lambda_n (1 - \lambda_n Q_n) \Delta t_n}
\]

Plugging this to (24) gives,

\[
\lambda_n = \frac{1 - 2\lambda_n Q_n}{2\lambda_n (1 - \lambda_n Q_n) \Delta t_n} \times \frac{\Sigma_n}{\sigma_u^2}
\]

so we have (27).

Applying the second order condition (26) with \( \lambda_n > 0 \), gives \( \lambda_n < \frac{1}{Q_n} \). Rewriting (27) gives,

\[
\lambda_n^2 (\lambda_n - \frac{1}{Q_n}) = \frac{\Sigma_n}{\Delta t_n \sigma_u^2} (\lambda_n - \frac{1}{2Q_n})
\]

(53)

Note that \( \lambda_n^2, \frac{\Sigma_n}{\Delta t_n \sigma_u^2} > 0 \). Thus, by the SOC \( \lambda_n < \frac{1}{Q_n} \), we also have \( \lambda_n < \frac{1}{2Q_n} \) since both sides must have same signs. Now let \( a, b, \) and \( c \) be the three roots to the cubit root equation.
We know from the cubic root equation that

\[ a + b + c = \frac{1}{Q_n} \]

\[ ab + bc + ca = -\frac{\Sigma_n}{\Delta t_n \sigma_n^2} \]

\[ abc = -\frac{\Sigma_n}{2Q_n \Delta t_n \sigma_n^2}. \]

WLOG, let \( a > b > 0 > c \) by (54) and (56). Note that both sides of (53) is increasing in \( \lambda_n \).

Since LHS is positive only when \( \lambda_n > \frac{1}{Q_n} \) and the slope increment is \( 3\lambda_n^2 - \frac{2\lambda_n}{Q_n} > 0 \) while the RHS is positive at \( \lambda_n > \frac{1}{Q_n} \) and linearly increasing in \( \lambda_n \). So we have \( a > \frac{1}{Q_n} \). Note that the RHS of (53) is always positive when \( \lambda_n > \frac{1}{Q_n} \) and the LHS is always negative where \( \frac{1}{2Q_n} \leq \lambda_n < \frac{1}{Q_n} \). The root \( b \) exists and positive, where RHS and LHS equates with negative values. Thus, \( b < \frac{1}{2Q_n} \) and the uniqueness and existence is proved.

To work the proof using backward induction, first at period \( N \), \( \alpha_N = \delta_N = 0 \) is known, and \( \alpha_n, \delta_n \) are known for all \( n = 1, \ldots, N \). Fix some \( \Sigma_N > 0 \). Now, we can solve (27) for some \( \lambda_N > 0 \). Then, we are able to get \( \alpha_{N-1}, \delta_{N-1}, \beta_N, \) and \( \Sigma_{N-1} \) as well by (21)-(25). Now, for any period \( n \), notice that the same analogy holds. Thus we can iterate backwards to obtain \( \alpha_n, \beta_n, \delta_n, \lambda_n, \) and \( \Sigma_n \) for all \( n = 1, \ldots, N \). Note that as in Kyle (1985) and Proposition 3, the solution is unique for any initial value \( \Sigma_0 \). Thus, if we know that \( \Sigma_0 = x \), notice that the uniqueness of our solution derives us the correct \( \Sigma_N \) that induces \( \Sigma_0 = x \).

\(^{12}\)For the case when \( n = N \), the problem becomes a quadratic equation since \( \alpha_N = \alpha_N^H = 0 \) which leads to \( Q_n = 0 \). The problem reduces to where the roots have different signs with the same magnitude. Then I just take the positive root to be the solution to the equation. The positive root automatically satisfies the SOC as well.
B.5 Generalizing to a Multi-stage Model

Proposition 5. A unique linear equilibrium exists in our model, in which there are constants \( \alpha^a_n, \beta_n, \delta^a_n, \lambda_n, \) and \( \Sigma_n \) characterized by the following:

\[
\Delta X_n = M_n \beta_n (v - p_{n-1}) \Delta t_n \\
\Delta p_n = \lambda_n (\Delta X_n + \Delta u_n) \\
\Sigma_n = \text{Var}(v \mid \Delta X_1 + \Delta u_1, \ldots, \Delta X_n + \Delta u_n) \\
E(\pi^a_n \mid p_1, \ldots, p_{n-1}, v) = \alpha^a_{n-1} (v - p_{n-1})^2 + \delta^a_{n-1}
\]

for all auctions \( n = 1, \ldots, N \) and for informed traders \( i = 1, \ldots, M_n \).

Let \( Q_n = \theta_n [\eta_n \alpha^a_n + (1 - \eta_n) \alpha^a_n] \) and the superscript \( \alpha' \) denotes the case where the number of informed investors increase next period. The constants \( \alpha^a_n, \beta_n, \delta^a_n, \lambda_n, \) and \( \Sigma_n \) are the unique solutions to the difference equation system

\[
\alpha^a_{n-1} = \frac{1 - \lambda_n Q_n}{\lambda_n [M_n (1 - 2 \lambda_n Q_n) + 1]^2} \\
\beta_n \Delta t_n = \frac{1 - 2 \lambda_n Q_n}{\lambda_n [M_n (1 - 2 \lambda_n Q_n) + 1]} \\
\delta^a_{n-1} = Q_n \lambda_n^2 \sigma^2_u \Delta t_n + \theta_n \eta_n \delta^a_n + \theta_n (1 - \eta_n) \delta^a_n' \\
\lambda_n = \frac{M_n \beta_n \Sigma_n}{\sigma^2_u} \\
\Sigma_n = (1 - M_n \beta_n \lambda_n \Delta t_n) \Sigma_{n-1}
\]

for auctions \( n = 1, \ldots, N - 1 \), subject to the boundary condition

\[
\lambda_n (1 - \lambda_n Q_n) > 0
\]

Proof of Proposition 5:

Notice that

\[
\pi^a_n(p_n) = \max \Delta \pi_n + \theta_n \eta_n \pi^a_{n+1} + \theta_n (1 - \eta_n) \pi^a_{n+1}' \\
= \max \Delta \pi_n + \theta_n \eta_n \{ \alpha^a_n (v - p_n)^2 + \delta^a_n \} + \theta_n (1 - \eta_n) \{ \alpha^a_n' (v - p_n)^2 + \delta^a_n' \}.
\]
Then following the steps from Proposition 3 gives the result as desired.
C Figures and Tables

Table I: Basic Statistics

This paper uses rumor data from July 2009 through June 2011. Panel A summarizes rumor sources and initial responses to rumors. The inquiry-disclosure request explicitly identifies the source of rumor as being from a private source ('Personal') or as originating from news and reports ('Media'). One inquiry-disclosure request during the sample period did not state the source and it is classified as 'Other'. The responses tabulated in this table are the initial responses from the related firm. These initial responses may be succeeded by other responses if the initial response is 'undecided'. Panel B reports the distributions of firms’ response times. The response time is measured as the difference between the response disclosure time and the inquiry disclosure time that are indicated by the E-Disclosure system’s time stamp. “Response in trading time” is difference between the two time stamps in trading time. Trading in the KOSPI market takes place from 9:00 AM through 3:00 PM during weekdays. Panel C tabulates the number of response disclosures depending on how long the response took from inquiry disclosure (in trading hours) and on what content the response disclosure had during the sample period.

Panel A: Rumor Source and Response

<table>
<thead>
<tr>
<th>Source of Rumor Obtained by Regulators</th>
<th>Personal</th>
<th>Media</th>
<th>Other</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Confirmed</td>
<td>2</td>
<td>12</td>
<td>1</td>
<td>15</td>
</tr>
<tr>
<td>Initial Undecided</td>
<td>42</td>
<td>105</td>
<td>0</td>
<td>147</td>
</tr>
<tr>
<td>Response Denied</td>
<td>21</td>
<td>21</td>
<td>0</td>
<td>42</td>
</tr>
<tr>
<td>Total</td>
<td>65</td>
<td>138</td>
<td>1</td>
<td>204</td>
</tr>
</tbody>
</table>

Panel B: Minutes to Respond after the Inquiry

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>SD</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Response in Clock Time</td>
<td>631.108</td>
<td>810.973</td>
<td>19</td>
<td>4125</td>
</tr>
<tr>
<td>Response in Trading Time</td>
<td>250.809</td>
<td>102.966</td>
<td>8</td>
<td>360</td>
</tr>
</tbody>
</table>
Panel C: Response by Hours and Response

<table>
<thead>
<tr>
<th>Response Time (Hours)</th>
<th>Confirmed</th>
<th>Undecided</th>
<th>Denied</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t \leq 1$</td>
<td>1</td>
<td>4</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>$1 &lt; t \leq 2$</td>
<td>1</td>
<td>11</td>
<td>3</td>
<td>15</td>
</tr>
<tr>
<td>$2 &lt; t \leq 3$</td>
<td>1</td>
<td>20</td>
<td>10</td>
<td>31</td>
</tr>
<tr>
<td>$3 &lt; t \leq 4$</td>
<td>2</td>
<td>24</td>
<td>9</td>
<td>35</td>
</tr>
<tr>
<td>$4 &lt; t \leq 5$</td>
<td>0</td>
<td>18</td>
<td>4</td>
<td>22</td>
</tr>
<tr>
<td>$5 &lt; t \leq 6$</td>
<td>10</td>
<td>70</td>
<td>11</td>
<td>91</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>15</td>
<td>147</td>
<td>42</td>
<td>204</td>
</tr>
</tbody>
</table>
Table II: Estimation of Initial Informed Trading Period via Non-Linear Least Squares

This table reports parameter estimates from a Non-Linear Least Squares Model (NLLS) model to measure the length of the pre-disclosure request period using 204 inquiry-disclosure events from July 2009 through June 2011. The NLLS model is $\Delta p_n = \alpha + \gamma_1(v - p_{n-1})f(n, t_0) + \gamma_2(v - p_{n-1})\left(\frac{n-t_0}{T-t_0}\right)f(n, t_0) + \varepsilon_n$ where $f(n, t_0) = \frac{e^{n-t_0}}{1+e^{n-t_0}}$ so that it represents a continuous approximation of an indicator function. The function $f(n, t_0)$ distinguishes between regimes 1 (no informed trading) and 2 (exists informed trading). $T = 41$ indicates the inquiry disclosure date. $\gamma_1$ and $\gamma_2$ are the price convergence ratios and $t_0$ is the number of days where informed trading exists prior to response disclosure that resolves the information asymmetry. The constrained model differs from the full model by setting $\gamma_2 = 0$ so that it does not allow price sensitive to change over time. Standard errors are reported in parentheses. ** denotes parameter estimates that are statistically significantly different from zero at the 1% level.

<table>
<thead>
<tr>
<th></th>
<th>Non-False Rumor ($n = 162$)</th>
<th>Full Sample ($n = 204$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Constrained</td>
<td>Unconstrained</td>
</tr>
<tr>
<td>Intercept $\alpha$</td>
<td>-13.186</td>
<td>97.696</td>
</tr>
<tr>
<td></td>
<td>(62.767)</td>
<td>(60.966)</td>
</tr>
<tr>
<td>Slope 1 $\gamma_1$</td>
<td>0.070**</td>
<td>0.022</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td>(0.064)</td>
</tr>
<tr>
<td>Slope 2 $\gamma_2$</td>
<td>0.317**</td>
<td>0.318**</td>
</tr>
<tr>
<td></td>
<td>(0.077)</td>
<td>(0.077)</td>
</tr>
<tr>
<td>Mean Informed Trading Days $t_0$</td>
<td>9.671**</td>
<td>24.648**</td>
</tr>
<tr>
<td></td>
<td>(0.553)</td>
<td>(2.445)</td>
</tr>
</tbody>
</table>
This table reports estimates of noise trading variance that are estimated using trading volume data. The data consist of 204 inquiry-disclosure events in the KOSPI market from July 2009 through June 2011. The model is estimated using the Generalized Method of Moments (GMM). The GMM equations are given in (35). I use six trading hours of data before and after the inquiry-disclosure request. If a firm responds to the request within six hours, I exclude the data after this response. Standard errors are reported in parentheses. ** denotes parameter estimates that are statistically significantly different from zero at the 1% level.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Before Inquiry</th>
<th>After Inquiry</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average of Net Noise Trading</td>
<td>μ</td>
<td>.543</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(5.305)</td>
</tr>
<tr>
<td>Variance of Net Noise Trading</td>
<td>σ₂ₙ</td>
<td>1.244**</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(.398)</td>
</tr>
<tr>
<td>Liquidity Parameter</td>
<td>λ₁</td>
<td>.863</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(3.589)</td>
</tr>
<tr>
<td></td>
<td>λ₂</td>
<td>.588</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1.224)</td>
</tr>
<tr>
<td></td>
<td>λ₃</td>
<td>.446</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1.407)</td>
</tr>
<tr>
<td></td>
<td>λ₄</td>
<td>.447</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(2.185)</td>
</tr>
<tr>
<td></td>
<td>λ₅</td>
<td>.719</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(.400)</td>
</tr>
<tr>
<td></td>
<td>λ₆</td>
<td>.444</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(.818)</td>
</tr>
<tr>
<td>Informed Trading Volume</td>
<td>ΔX₁</td>
<td>−.841</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(9.956)</td>
</tr>
<tr>
<td></td>
<td>ΔX₂</td>
<td>−.513</td>
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<td>(7.930)</td>
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<td>ΔX₃</td>
<td>−.714</td>
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<td>(9.175)</td>
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<td>ΔX₄</td>
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<td>ΔX₅</td>
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</tr>
<tr>
<td></td>
<td>ΔX₆</td>
<td>2.029</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(5.414)</td>
</tr>
</tbody>
</table>
Table IV: Estimated Parameters and Model Comparison

This table reports the parameters for each model that I analyze in this paper. Parameters in Column (1) are estimated using data from the KOSPI market during from July 2009 through June 2011. Parameter $\eta$ is estimated to match the informed trading period in Table II. Parameter $\theta$ after inquiry disclosure is calculated from Table I Panel C. The number of informed traders $M$ after inquiry is estimated using equation (36). Noise trading volatility estimates are from Table III. Asset volatility is estimated as the variance of the price one hour after the response disclosure after normalizing initial price to 10. Columns (1) and (2) are from two-stage models that differ by the number of informed traders after the inquiry. “Sum of Squares” is calculated from equation (36) using six trading hours of data before and after the inquiry-disclosure request. Line “Bootstrap SS SD” reports the standard deviations of the sum-of-squares estimates. These standard deviations are computed by bootstrapping the 204 inquiry-response events 10,000 times.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum Period of Informed Trading</td>
<td>$N$</td>
<td>150</td>
<td>150</td>
<td>150</td>
</tr>
<tr>
<td>1−Probability of Inquiry Disclosure</td>
<td>$\eta$</td>
<td>.99925</td>
<td>.99925</td>
<td>1</td>
</tr>
<tr>
<td>1−Probability of Voluntary Disclosure</td>
<td>$\theta</td>
<td>Before Inquiry</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1−Probability of Response Disclosure</td>
<td>$\theta</td>
<td>After Inquiry</td>
<td>#</td>
<td>#</td>
</tr>
<tr>
<td>Number of Informed Traders</td>
<td>$M</td>
<td>Before Inquiry</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>$M</td>
<td>After Inquiry</td>
<td>1.1711</td>
<td>1</td>
</tr>
<tr>
<td>Noise Trading Volatility</td>
<td>$\sigma^2</td>
<td>Before Inquiry</td>
<td>1.244</td>
<td>1.244</td>
</tr>
<tr>
<td></td>
<td>$\sigma^2</td>
<td>After Inquiry</td>
<td>5.734</td>
<td>5.734</td>
</tr>
<tr>
<td>Asset Volatility</td>
<td>$\Sigma_0$</td>
<td>1.736</td>
<td>1.736</td>
<td>1.736</td>
</tr>
<tr>
<td>Sum of Squares from (36)</td>
<td>129.6398</td>
<td>129.6552</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>Bootstrap SS SD (10000 replications)</td>
<td>20.2219</td>
<td>19.6725</td>
<td>N/A</td>
<td>N/A</td>
</tr>
</tbody>
</table>

# for inquired period $T$, $\theta_{T+1} = 1 - \frac{10}{204}$, $\theta_{T+2} = 1 - \frac{15}{194}$, $\theta_{T+3} = 1 - \frac{31}{179}$, $\theta_{T+4} = 1 - \frac{35}{148}$, $\theta_{T+5} = 1 - \frac{22}{113}$, and $\theta_{T+6} = 0$. 
Table V: Expected Profits by Informed Traders by Models

This table reports the expected profits of informed traders for each model described in Table IV. I split the profits of the initial informed trader and the additional informed trader who enters after the inquiry for the non-false rumor inquired. For the false rumors, I assume no informed trading prior to the inquiry. The profits are calculated using equations (37) and (38) for each models in Table IV. The total expected profits per inquiry is calculated by \( \frac{167}{204}(\pi_1 | NF + \pi_{n>1} | NF) + \frac{42}{204}(\pi_{n>1} | F) \). For models in columns (3)-(4), there does not exist inquiry-response disclosures in Kyle (1985) and Holden and Subrahmanyam (1992), so that the profits induced by inquiry disclosure does not exist.

<table>
<thead>
<tr>
<th>Models</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-Stage</td>
<td>2-Stage</td>
<td>Kyle</td>
<td>HS</td>
<td></td>
</tr>
<tr>
<td>Model 1</td>
<td>Model 2</td>
<td>Model</td>
<td>Model</td>
<td></td>
</tr>
</tbody>
</table>

| Expected Profits by Initial Informed Traders | \( \pi_1 | NF \) | 13.1970 | 13.4684 | 13.7671 | 10.0745 |
| (per non-false rumor inquired) | | | | | |
| Expected Profits by Additional Informed Trader After Inquiry | \( \pi_{n>1} | NF \) | .0023 | N/A | N/A | N/A |
| (per non-false rumor inquired) | | | | | |
| Expected Profits by Informed Traders After Inquiry | \( \pi_{n>1} | F \) | 3.0978 | 4.7378 | N/A | N/A |
| (per false rumor inquired) | | | | | |
| Total Expected Profits by Informed Traders per Inquiry | \( \pi \) | 11.1195 | 11.6710 | 10.9327 | 8.0004 |
Figure 1: Price Movements and Trading Volume Around Inquiry/Response Disclosure

This figure illustrates the price movements and trading volume changes around the inquiry/response disclosure. Top two panels show the normalized average stock price (left) and trading volume (right) movements around the rumor related disclosures when the response was not denying the rumor content. Bottom two rows shows the case when the rumor was denied (false). For the stock price, the price is normalized so that the 1 hour after the response price is 100, and the price before is below 100. The horizontal axis is in trading hours where 0 is the inquired period for the stock price. Trading volumes are normalized using $-40 \sim -11$ trading days prior to inquiry as a window for normalization and calculate $\frac{Vol_t - mean(Vol_{t-40 \sim t-11})}{\sigma(Vol_{t-40 \sim t-11})}$. Horizontal axis is in trading days for trading volume to remove trading volume behavior at the market open/close period.
Panels (a) and (b) illustrates the sequence of events before and after the inquiry disclosure, respectively. Orders $\Delta x_n$ and $\Delta u_n$ are submitted by informed trader(s) and noise traders, respectively. $p_n$ is set by market makers after observing $\Delta x_n$ and $\Delta u_n$. Then the firm may make a voluntary disclosure if inquiry has not been made or response disclosure if inquiry has been made with some probability. If the voluntary disclosure has not been made in the first stage, inquiry disclosure may be made by the market regulators with some probability. Panel (c) illustrates the probabilities of each event and corresponding profits when each event occurs. Dotted line are events that eliminates the information asymmetry. Dashed lines are events that changes the state from stage 1 (before inquiry) to stage 2 (after inquiry).

(a) Sequence of Events Before Inquiry Disclosure (Stage 1)

(b) Sequence of Events After Inquiry Disclosure (Stage 2)

(c) Payoffs and Probabilities
Figure 3: Liquidity, Volatility, Asset Prices, and Informed Trading by Varying the Probability of Resolving Asymmetry (One Informed Trader)

This figure illustrates how liquidity, volatility, asset prices, and informed trading varies over time when the probability of response disclosure \((1 - \theta)\), that resolves the information asymmetry, differs after the inquiry disclosure. The number of informed traders is fixed at \(M_n = 1\) for all periods \(n = 1, \ldots, 20\) after the inquiry disclosure. True value of the stock is set at \(v = 21\), the ex-ante stock price at the inquiry \(t = 0\) is \(p_0 = 20\). The variance of noise trading per unit time \(\sigma_u^2 = 1\) and the ex-ante variance of the terminal value is \(\Sigma_0 = 1\). The four figures each show the values of Liquidity parameter \(\lambda_n\) (upper left), Error variance of the price \(\Sigma_n\) (upper right), Total expected quantity of informed trading \(\Delta X_n\) (lower left), and Stock price \(p_n\) (lower right) over time. \(x\)-axis is the trading period after the inquiry.
This figure illustrates how liquidity, volatility, asset prices, and informed trading varies over time when the probability of response disclosure \((1 - \theta)\), that resolves the information asymmetry, differs after the inquiry disclosure. The number of informed traders is fixed at \(M_n = 2\) for all periods \(n = 1, \ldots, 20\) after the inquiry disclosure. True value of the stock is set at \(v = 21\), the ex-ante stock price at the inquiry \(t = 0\) is \(p_0 = 20\). The variance of noise trading per unit time \(\sigma^2_u = 1\) and the ex-ante variance of the terminal value is \(\Sigma_0 = 1\). The four figures each show the values of Liquidity parameter \(\lambda\) (upper left), Error variance of the price \(\Sigma\) (upper right), Total expected quantity of informed trading \(\Delta X_n\) (lower left), and Stock price \(p_n\) (lower right). \(x\)-axis is the trading period after the inquiry.
Figure 5: Liquidity, Volatility, Asset Prices, and Informed Trading by Varying the Probability of Resolving Asymmetry (Before Inquiry)

This figure illustrates how liquidity, volatility, asset prices, and informed trading varies over time when the probability of response disclosure \((1 - \theta)\), that resolves the information asymmetry, differs before the inquiry disclosure. The probability of inquiry disclosure if fixed at \(\eta = .95\) for all periods. The initial number of informed trader is fixed at \(M_n = 1\) and after the inquiry, number of insiders increase to \(M_n = 2\). True value of the stock is set at \(v = 21\), the ex-ante stock price at the inquiry \(t = 0\) is \(p_0 = 20\). The variance of noise trading per unit time \(\sigma_u^2 = 1\) and the ex-ante variance of the terminal value is \(\Sigma_0 = 1\). The four figures each show the values of Liquidity parameter \(\lambda_n\) (upper left), Error variance of the price \(\Sigma_n\) (upper right), Total expected quantity of informed trading \(\Delta X_n\) (lower left), and Stock price \(p_n\) (lower right). \(x\)-axis is the trading period after the inquiry. The following are the results when at the 1st stage of the two-stage model.
Figure 6: Liquidity, Volatility, Asset Prices, and Informed Trading by Varying the Probability of Anticipated Competition

This figure illustrates the how liquidity, volatility, asset prices, and informed trading varies over time when the probability of inquiry disclosure \((1 - \eta)\), that increases the number of informed traders next period, varies before the inquiry disclosure. The probability that asymmetry will be resolved next period is \(1 - \theta = 0\) for all periods. The initial number of informed trader is fixed at \(M_n = 1\) and after the inquiry, number of insiders increase to \(M_n = 2\). True value of the stock is set at \(v = 21\), the ex-ante stock price at the inquiry \(t = 0\) is \(p_0 = 20\). The variance of noise trading per unit time \(\sigma_u^2 = 1\) and the ex-ante variance of the terminal value is \(\Sigma_0 = 1\). The four figures each show the values of Liquidity parameter \(\lambda_n\) (upper left), Error variance of the price \(\Sigma_n\) (upper right), Total expected quantity of informed trading \(\Delta X_n\) (lower left), and Stock price \(p_n\) (lower right). \(x\)-axis is the trading period after the inquiry. The following are the results when at the 1st stage of the two-stage model.
This figure illustrates how liquidity, volatility, asset prices, and informed trading varies over time for different Kyle-class models. The initial number of informed traders is fixed at $M_n = 2$ for the Holden and Subrahmanyam (1992) and $M_n = 1$ for Kyle (1985), Huddart, Hughes, and Levine (2001), and the two-stage model. After the inquiry in the two-stage model, $M_n$ increases to 2. True value of the stock is set at $v = 21$, the ex-ante stock price at the inquiry $t = 0$ is $p_0 = 20$. The variance of noise trading per unit time $\sigma_u^2 = 1$ and the ex-ante variance of the terminal value is $\Sigma_0 = 1$. The information is revealed at $N = 20$ so that $\theta = 1$ for all periods. The inquiry disclosure probability is $\eta = .95$ for the two-stage model. The six figures show the values of Liquidity parameter $\lambda_n$ (upper left), Error variance of the price $\Sigma_n$ (upper right), Total expected quantity of informed trading $\Delta X_n$ (lower left), and Stock price $p_n$ (lower right), expected insider profits $\pi_n$ (next page left), and price sensitivity $\gamma_n = \lambda_n M_n \beta_n$ over time for different models. $x$-axis is the auction period.
Figure 7 (cont’d)
This figure illustrates the comparison of price efficiency and informed traders’ expected profits under the inquiry/response disclosure and public disclosure of insider trading. The values under inquiry/response disclosure are plotted for different probability of inquired disclosure $\eta_n \in \{.9, .95, .99\}$. The values under insider trading disclosure are from Huddart, Hughes, and Levine (2001). The initial number of informed traders is fixed at $M_n = 1$ and after the inquiry, number of insiders increase to $M_n = 2$. True value of the stock is set at $v = 21$, the ex-ante stock price at $t = 0$ is $p_0 = 20$, the variance of noise trading per unit time $\sigma_u^2 = 1$, the ex-ante variance of the terminal value is $\Sigma_0 = 1$. Asymmetry is resolved at the final trading period $N = 20$ so that $\theta = 1$ for all trading periods. For the top two figures, stock price $p_n$ (left) and expected insider profits $\pi_n$ (right) are shown over time.
Figure 9: Estimated Bid-Ask Ratios Around Disclosure

This figure illustrates estimated bid-ask spreads using Corwin and Schultz (2012) around inquiry/response disclosure. Then the spread ratios are normalized by dividing the average spread ratio of each firm from \( t - 120 \) to \( t - 61 \) trading hours before inquiry. \( \text{ratio}_1 \) and \( \text{ratio}_2 \) differs when the bid-ask spread estimate is negative. \( \text{ratio}_1 \) converts all negative estimates as 0 and \( \text{ratio}_2 \) do not take into account the negative estimates. The plots show the mean of 204 samples.
This figure illustrates Amihud price impact measure for each day around inquiry/response disclosures. Daily Amihud measure is calculated by

\[ ILLIQ_{id} = \frac{1}{P_{id}} \sum_{i=1}^{P_{id}} \frac{|R_{idh}|}{VOLW_{idh}} \]

where \( ILLIQ_{id} \) is the daily Amihud (2002) measure for stock \( i \) at day \( d \), \( P_{id} \) is the number of periods (trading hours) where trading has occurred for stock \( i \) at day \( d \), \( R_{idh} \) is the return on stock \( i \) on hour \( h \), and \( VOLW_{idh} \) is the respective hour trading volume in Wons (KRW). Day 0 is the time between the inquiry and response disclosure has been made. The measure \( ILLIQ_{id} \) is multiplied by 1 billion, and averaged across 204 samples.