Financial Market Liquidity: Who Is Acting Strategically? *

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Abstract

In a new environment where liquidity providers as well as liquidity consumers act strategically, understanding how liquidity flows and dries-up is key. We propose a model that specifies the impact of information arrival on market characteristics, in the context of liquidity frictions. We distinguish short-lasting liquidity frictions, which impact intraday prices, from long-lasting liquidity frictions, when information is not fully incorporated into prices within the day. We link the first frictions to the strategic behavior of intraday liquidity providers and the second to the strategic behavior of liquidity consumers, i.e. long-term investors who split up their orders not to be detected. Our results show that amongst 61% of the stocks facing liquidity problems, 57% of them point up liquidity providers as the sole strategic market investor. Another 27% feature long-term investors as the single strategic player, while both liquidity providers and liquidity consumers act strategically in the remaining 16%. This means that 43% of these stocks are actually facing a slow-down in the information propagation in prices, which thus results in a significant decrease of (daily) price efficiency due to long-term investors' strategic behavior.

JEL classification: C51, C52, G12

Key words: High Frequency trading, strategic liquidity trading, market efficiency, mixture of distribution hypothesis, information-based trading, order splitting, Markov regime-switching stochastic volatility model.

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1 Introduction

Financial markets liquidity is a latent characteristic. For decades now, market participants as well as academics have been trying to filter liquidity from market summaries such as prices, volatility and volume. The vast literature on that topic [see e.g. Aitken and Comerton-Forde (2003), and Goyenko et al. (2009)] shows that there was no consensus at that time on how liquidity should be measured. Moreover, are these summaries still able to reflect market liquidity with the significant changes that occurred in the recent years? In fact, while automation was taking over the trading world and spreads were falling, the role of the market marker has changed and high frequency traders (HFTs) have expanded to become one of the most important players in the market. In 2012, high frequency trading was representing half of the US equity average volume and one-third of the European volume. On the one hand, high frequency trading (HFT) has accompanied and even favored the decrease in transaction costs by competing against each other and providing liquidity as a market maker [See for example, Chaboud et al. (2014), Hendershott et al. (2011), and Menkveld (2013)]. On the other hand, they have been accused to provide only illusory liquidity, to weaken financial markets, and even to be responsible for crashes and illiquidity spirals by massively taking liquidity out of the market during turmoils [Kirilenko et al. (2014)]. Jarrow and Protter (2013) show that their profits are at the expense of the other market participants, i.e. the medium to long term investors who must act strategically by splitting their orders in response[1]. As a consequence, the link between market characteristics and liquidity is even more ambiguous now that liquidity providers as well as liquidity consumers are both acting strategically.

In this paper, we propose a model that specifies the impact of information arrival on market characteritics in the context of liquidity frictions. We distinguish between two types of liquidity frictions. On the one hand, the liquidity frictions can be short-lasting and are

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[1]For Jarrow and Protter (2013), these other traders are ordinary traders who submit market orders and sophisticated traders who submit limit orders or trade large quantity splitting up their orders.

represented by the intraday order imbalances that are resorbed within the trading day: it corresponds to the liquidity provision - or arbitrage, provided by the "new market makers" à la Menkveld (2013) who end the trading day flat. They impact the daily and intradaily traded volume but only the intradaily price volatility. These liquidity frictions are a source of trade for the market makers who liquidate their positions once prices are back to the equilibrium level in order to cash the liquidity premium. They can also be assimilated to the cost of immediacy supported by impatient traders. On the other hand, the liquidity frictions can be long-lasting, when information is not completely incorporated into prices within the day. Such a situation can arise when investors are strategically timing their trades, trading at prices that do not fully reflect the information they possess [see Anderson et al. (2013)]. This strategic behavior could explain the increase in the number of daily transactions that comes with the decrease of the average trade size observed in all financial markets in the past 10 years [See for example Lehalle and Laruelle (2013), Fig. 5 p. 17 for the components of the FTSE]. It is also responsible for the autocorrelation of prices that we observe on financial markets [See Anderson et al. (2013), and Toth et al. (2014)]. This situation could also occur when only part of market participants are able to trade for any reason, such as the limited trading capacities of traders in the sense of Brunnermeier and Pedersen (2009), which are due to risk constraints. In this case, their reservation prices reflect only part of the incoming information. These situations are inherent to liquidity risk supported by the market participants who cannot liquidate their positions at the fully revealing price.

Most long-term investors are only considering daily data even if intradaily strategies are impacting their performances. Here, we take the perspective of such investors and explain how intraday strategies, information arrival, and liquidity frictions are affecting intradaily and in turn, daily returns and volume evolution. We extend Darolles et al. (2015) to provide a structural interpretation of the dynamic properties of daily returns and volumes. We exploit the particular structure of our model to build a two-step methodology that extracts the liquidity latent factors using daily time series of returns and volume. We first use a Markov

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regime-switching stochastic volatility model to extract the effect of information shocks from the daily price change time series. Then, conditional on these estimated variables, we use a simple Kalman filter to extract the latent liquidity factor from daily volume observations. This procedure enables us to: (i) capture the impact of long-lasting liquidity frictions on the daily price change and volatility dynamics; (ii) separate out the impact of both, long and short-lasting liquidity frictions, on the serial correlation of the daily volume.

This paper contributes to the literature in several directions. First, we give a theoretical ground to the empirical financial literature dedicated to the measure of illiquidity through the analysis of positive returns autocorrelation [see e.g. Getmansky et al. (2004)]. In our model, these positive serial correlations are directly linked to the lack of liquidity provision at the intraday level. This liquidity deficit generates long-lasting liquidity frictions, and in turn positive returns autocorrelation. The short-term liquidity frictions are responsible for intraday dynamics of return and volatility, while the long-lasting ones result in daily positive serial correlation of stock returns and squared returns.

Second, we extend the analysis of liquidity frictions to the case of information arrival. On no information days, liquidity frictions should push prices away from their equilibrium level (temporary effect) before returning to their previous level. On information days, liquidity frictions should also prevent prices temporary from becoming fully revealing before they revert (permanent effect). In the first case, the inefficiency is considered as over whenever the price revers, while in the second case, the interaction between information and liquidity frictions clearly complicates the issue [see Waelbroeck and Gomes (2013) for a discussion on the permanent effect and information contents of trades]. Working with triangular arbitrage relations is a way to concentrate on frictions without taking care of the information problem [see for example Foucault et al. (2014)]. However, this approach cannot be generalized to single risky assets as it is only relevant for arbitrage relations. Our approach allows to disentangle short versus long-lasting liquidity frictions in the general case where information arrival has an impact on prices.

Third, we propose new liquidity measures that add to the already vast literature [see e.g. Aitken and Comerton-Forde (2003), and Goyenko et al. (2009)]. Just like the new growing literature mostly coming from the trading cost analysis [see for example Almgren and Chriss (2001), Almgren et al. (2005), Criscuolo and Waelbroeck (2013), and Lehalle (2014)], these new measures are dynamic, they separate long-term from short-term effects and are linked to the autocorrelations of returns, volume and volatility. However, conversely to that stream, our measures do not need intradaily data.

Our results show that, over the 61% of stocks from the FTSE100 that are actually facing liquidity problems, the liquidity providers are the only investors acting strategically on a group of stocks accounting for nearly 57%. In another 27% of stocks, the long-term investors are strategic while the liquidity providers are inactive. Finally, for the last 16% group of stocks, the liquidity providers as well as the liquidity consumers are both acting strategically. It means that 43% of these stocks are facing a slow-down of the information propagation to prices and thus a decrease of (daily) price efficiency due to the strategic behavior of the long-term investors.

The implications of our results are threefold. First, they emphasize that liquidity frictions can be different even for assets belonging to the same investment universe. We show that short-term and long-term liquidity problems do not have the same origins and consequences on market quality. Second, from a risk management standpoint, the paper provides a useful tool for understanding the nature of liquidity frictions on a specific risky asset. The short-term liquidity frictions, due to trade asynchronization and resorbed within the trading day, increase the daily traded volume. Their impact is measured by an adjusted volume parameter. This liquidity indicator is useful to market participants who want to detect illiquid individual stocks and thus select the candidates with significant liquidity-based traded volume. On the other hand, the presence of long-lasting liquidity frictions exceeding the intradayly perspective, is captured from the dynamic properties of daily price changes. This provides additional insights on stock liquidity and enables market participants to identify

equities affected by liquidity risk. Third, from an investment perspective, the two liquidity frictions has direct implications for statistical arbitrage strategies. Actually, the statistical arbitrage traders compute the sample serial correlation of stock returns and pick up positive serially correlated stocks to build up momentum strategies. Our results suggest that the sample serial correlation coefficients are not sufficient criteria to select momentum stocks. We propose an alternative selection process derived from our theoretical framework. We show that stocks may have a first order serial correlation not significantly different from zero but still be affected by liquidity friction problems.

The paper is organized as follows. Section 2 introduces our statistic model. Section 3 presents our econometrical methodology to extract information and liquidity latent factors from the daily time series of returns and volume. In Section 4, we apply our econometrical set up to individual stocks belonging to the FTSE100 and discuss the empirical results. Section 5 concludes the article.

2 The statistical model and implications

In this section, we consider a dynamic extension of Darolles et al. (2015), where liquidity frictions can be both short-term and/or long-lasting.[2] We first use this extension to show how the dynamic relationship between daily returns and traded volume as well as the serial correlation of price changes, squared price changes and traded volume depend on these frictions. We then characterize stocks liquidity frictions' types based on the estimated parameters that form the basis of hypothesis testing.

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[2]Based on the theoretical framework of Grossman and Miller (1988), Darolles et al. (2015) extend the standard MDH model of Tauchen and Pitts (1983) in order to account for the impact of both information and short-term liquidity frictions on daily returns and volumes.

2.1 The model

The economy is defined by a sequence of trading on a single risky asset by two types of market participants: the active traders who react to information flow and the market makers who trade in response to liquidity frictions. Within the trading day, the market passes through a sequence of distinct equilibria due to the arrival of new pieces of information. These information arrivals generate trades that impact the intradaily price increments and transaction volumes, and in turn, the daily price changes and volumes.

In the absence of liquidity frictions, the intradaily as well as daily price changes and volumes fully reflect the incoming information instantaneously. In this case, we get the standard Mixture of Distribution Hypothesis (MDH) framework of Tauchen and Pitts (1983). This model suggests that a bivariate mixture of distributions, in reduced form, can represent daily price changes and volumes with information being the only mixing variable.[3]

Because trading might, and usually is, asynchronous, short-term liquidity frictions exist creating at least temporary order imbalances at the intradaily frequency. These order imbalances result in price deviations from their fully revealing information levels and are resorbed by the market within the trading day thanks to the intervention of liquidity providers who act as market makers. These "new market makers" à la Menkveld (2013) who end the trading day flat are strategic. As discussed in Darolles et al. (2015), market makers trade on short-term liquidity frictions and increase the daily traded volume by doing so. However, they do not impact the daily price changes; at the end of the trading day prices reflect all the incoming information during that day. The Mixture of Distribution Hypothesis with Liquidity (MDHL) model of Darolles et al. (2015) suggests that the daily price changes and volumes can be represented by a bivariate mixture of distributions, in reduced form, with two latent variables, I_t and L_t supposed to be i.i.d.

We now go further in the analysis and allow for the presence of long-lasting liquidity

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[3]Note that in the frictionless economy of Tauchen and Pitts (1983), there is no need for the intervention of the market makers.

frictions reflecting situations where information is not completely incorporated into prices within the trading day. This can arise when only part of market participants are able to trade for any reason, or more likely, when long-term investors are slicing efficiently in order not to be spotted by the strategic liquidity providers. By doing so, they disseminate slowly, over several days or even weeks, the information they possess to the market.

Let ΔP_t and V_t be the daily price change and traded volume, respectively. The statistical model proposed here is a bivariate mixture of distributions, in reduced form, conditioned by two latent variables, I_t and L_t, which are supposed to be time-persistent:

$$\Delta P_t = \alpha + \mu_p I_t + \sigma_p \sqrt{I_t} Z_{1t}, \tag{2.1}$$

$$V_t = \mu_{v1} I_t + \mu_{v2} L_t + \sigma_v \sqrt{I_t} Z_{2t}, \tag{2.2}$$

where

$$\ln I_t = \beta \ln I_{t-1} + \eta_t, \tag{2.3}$$

$$L_t = a L_{t-1} + \omega_t, \tag{2.4}$$

with η_t and ω_t being two i.i.d. mutually independent variables. Note that I_t and L_t capture the effects of long-term and short-term liquidity frictions, respectively. Indeed, the first latent variable I_t represents the portion of the incoming information which is absorbed by the market at day t. We allow I_t to be time-persistent in order to capture the effect of long-lasting liquidity frictions on daily price change and traded volume. The more important the slicing of orders, the smaller each bit of I_t and the longer the persistence time. The second latent variable L_t accounts for liquidity providers' intervention in response to short-term liquidity frictions. Contrary to Darolles et al. (2015), the L_t process is assumed to be also time-persistent to reflect stock-specific or market-wide liquidity conditions.

Finally, note that Andersen (1996) propose an *ad hoc* dynamic version of his static modified MDH model in order to account for the time-persistence of daily squared price

change and traded volume series. In Andersen's modified MDH version, daily price change and traded volume series are generated by a unique latent variable, the information flow process. In contrast, in our framework, we account for both latent factors, information and liquidity, and suggest that long-lasting liquidity problems as well as the persistence of short-term liquidity frictions are responsible for daily return and volume dynamics. In addition, Andersen's MDH version does not account for a drift effect of information process on the daily price change. As it will be discussed below, we account for a drift effect of information flow on daily returns in order to deal with serial correlation of daily returns which is a well known stylized fact. Richardson and Smith (1994) include a drift effect of information flow process in the price change equation. However, since they consider that information process is i.i.d., their framework does not explain the serial correlation of daily returns.

2.2 Implications in terms of price change and volume autocorrelations

Equations (2.1) and (2.3) represent a stochastic volatility model, henceforth SV model. The presence of long-lasting liquidity frictions is responsible for the presence of stochastic volatility in the daily price change time series (as captured by I_t), with β measuring its persistence.[4] The μ_p parameter is a mean-in-variance parameter related to the presence of drift effects of I_t on price increments at the daily frequency. It determines how strongly daily price changes fluctuate in response to new information.[5] The α is a constant term

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[4]The disturbance of the volatility equation η_t is assumed to be a gaussian white noise process with mean zero and variance σ_η^2, independent of Z_{1t} and Z_{2t}. Although the assumption of normality of η_t can seem *ad hoc* at first sight, Andersen et al. (2001) and Andersen et al. (2003) show that the log-volatility process can be well approximated by a normal distribution. This is coherent with the hypothesis of log-normality of information flow I_t made in the MDHL model, which is not new in the literature; Tauchen and Pitts (1983) assume a log-normal distribution for the mixing variable I_t in order to ensure its positiveness. Log-normality has also been suggested by several authors, such as Clark (1973) as well as Foster and Viswanathan (1993). Richardson and Smith (1994) tested several distribution functions for the information inflow and concluded that the data reject the log-normal distribution less frequently than the other distribution candidates, such as inverted gamma and Poisson distributions. These results motivate us to retain a log-normal distribution for I_t.

[5]The SV model considered here corresponds to the stochastic volatility-in-mean model initially developed by Koopman and Upensky (2002) in order to capture the positive relationship between the expected returns

representing the mean parameter of daily price change unrelated to I_t. Estimating (2.1)-(2.3) for individual stocks allows us to infer the presence of long-lasting liquidity frictions; a stock affected by long-term liquidity frictions should present statistically significant μ_p. In the absence of long-lasting liquidity frictions, μ_p should equal to zero and (2.1)-(2.3) reduces to the daily price change equation of Darolles et al. (2015), with I_t representing an i.i.d. information process.[6]

Equations (2.1) and (2.3) imply that daily returns and squared returns are serially correlated. Their respective autocovariances are given by:

$$Cov(\Delta P_t, \Delta P_{t+1}) = \mu_p^2 Cov(I_t, I_{t+1}), \tag{2.5}$$

$$\begin{aligned} Cov(\Delta P_t^2, \Delta P_{t+1}^2) &= \sigma_p^4 Cov(I_t, I_{t+1}) + \mu_p^4 Cov(I_t^2, I_{t+1}^2) \\ &\quad + \mu_p^2 \sigma_p^2 Cov(I_t, I_{t+1}^2) + \mu_p^2 \sigma_p^2 Cov(I_t^2, I_{t+1}) \\ &\quad + (4\alpha\mu_p\sigma_p^2 + 4\alpha^2\mu_p^2) Cov(I_t, I_{t+1}) \\ &\quad + 2\alpha\mu_p^3 [Cov(I_t, I_{t+1}^2) + Cov(I_t^2, I_{t+1})]. \end{aligned} \tag{2.6}$$

Equation (2.5) shows that the presence of serial correlation in daily price changes results from the interaction of the drift effect of I_t on prices (μ_p parameter) with the serial correlation pattern of I_t process ($Cov(I_t, I_{t+1})$). As for equation (2.6), the dynamics of daily volatility is entirely due to the presence of long-lasting liquidity frictions; it results from a combination of information intensity μ_p and precision σ_p as well as the constant term α with the time-persistence of I_t process. Note also that, the short-term liquidity frictions have no impact neither on daily price change, nor on squared price change dynamics.

The μ_{v2} parameter captures the impact of short-term liquidity provision activity, on average, on daily traded volume.[7] In contrast, μ_{v1} parameter represents the impact of

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and the the volatility level. In our framework, we explain this positive correlation by the presence of long-lasting liquidity frictions.

[6] Recall that in Darolles et al. (2015), I_t represents the information shock process which is completely incorporated into daily price changes within the trading day, because there is no long-lasting liquidity frictions.

[7] Note also, that, in our framework, the long-lasting liquidity frictions are supposed to be independent

information process on daily traded volume. Note that μ_{v2} parameter corresponds to the static liquidity measure proposed by Darolles et al. (2015). However, contrary to Darolles et al. (2015), the L_t process is assumed to be time-persistent, as characterized by equation (2.4) to reflect stock specific or market-wide liquidity conditions. Estimating (2.2)-(2.4) conditionally on I_t, gives the effect of short-term liquidity frictions on daily traded volume; a stock affected by short-term liquidity frictions should present statistically significant μ_{v2}.

Since the daily traded volume depends on both I_t and L_t processes, its dynamics will be impacted by both the occurrence of the long-lasting liquidity frictions (through μ_{v1}) and the persistence of short-term liquidity frictions (through μ_{v2}). More precisely, equations (2.2) and (2.4) imply that:

$$Cov(V_t, V_{t+1}) \quad = \quad (\mu_{v1})^2 Cov(I_t, I_{t+1}) + (\mu_{v2})^2 Cov(L_t, L_{t+1}). \qquad (2.7)$$

This equation shows that the unconditional autocovariance of daily traded volume results from the interaction of long-lasting and short-term liquidity frictions. Based on our model, it is possible to disentangle the two types of liquidity frictions, which cannot be assessed by simply computing the serial correlation coefficients from daily traded volume time-series. In this context, being able to estimate the parameters of our model allows us to separate the impacts of both types of liquidity frictions on volume serial correlation.

Generally speaking, our model encompasses some competing MDH versions proposed in the literature. In particular, the MDHL model of Darolles et al. (2015), the modified MDH model of Andersen (1996) and the standard MDH model of Tauchen and Pitts (1983) are particular cases of our model.

(i) Recall that the standard MDHL model of Darolles et al. (2015) accounts only for short-term liquidity frictions which are absorbed by the market within the trading day, and which are assumed to be i.i.d. Our model specification reduces to the standard MDHL model for

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from the short-lasting ones which are almost instantaneously absorbed by the market when the HFTs act as exogenous liquidity providers in order to exploit non toxic arbitrage opportunities in the sense of Foucault et al. (2014). We thus assume that $Cov(I_t, L_t) = 0$.

$\mu_p = 0$, and $\beta = 0$. These restrictions imply:

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\begin{aligned}
Cov(\Delta P_t, \Delta P_{t+1}) &= 0, \\
Cov(\Delta P_t^2, \Delta P_{t+1}^2) &= 0, \\
Cov(V_t, V_{t+1}) &= 0.
\end{aligned}
$$

(ii) Andersen (1996) proposed an *ad hoc* dynamic version of his static modified MDH model in order to account for the time-persistence of daily squared price change and traded volume series. In Andersen's modified MDH version, daily price change and traded volume series are generated by a unique latent variable, the information flow process. Andersen's dynamic MDH model explains the time-persistence of daily squared returns and traded volumes by the serial correlation of information flow process. However, the author does not account for drift effects of I_t on daily price change. Our model reduces to Andersen's dynamic MDH version for $\mu_p = 0$, and $\mu_{v2} = 0$. In particular, Andersen's framework implies, in our notations, that:

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\begin{aligned}
Cov(\Delta P_t, \Delta P_{t+1}) &= 0, \\
Cov(\Delta P_t^2, \Delta P_{t+1}^2) &= \sigma_p^4 Cov(I_t, I_{t+1}), \\
Cov(V_t, V_{t+1}) &= \mu_{v1}^2 Cov(I_t, I_{t+1}).
\end{aligned}
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As compared to our model, Andersen's dynamic MDH version does not account for time-persistence of daily price changes while this stylized fact is empirically well-known. In addition, the intuition behind the presence of stochastic volatility is not the same in both models. In Andersen (1996), the information flow time-persistence is responsible for the presence of stochastic volatility. In our framework, although information process is considered to be i.i.d., the presence of long-lasting liquidity frictions modifies the way news are incorporated into daily price changes. Thus, the stochastic volatility is the consequence of the presence of long-lasting liquidity problems. Finally, in Andersen's model, the serial

correlation of daily traded volume is due to the time-persistence of information flow process. In our framework, volume serial correlation is due to the presence of both long-lasting and short-term liquidity frictions.

(iii) If we disregard the effect of both types of liquidity frictions on daily trading character-istics by setting $\mu_p = 0$, and $\mu_{v2} = 0$, we get the standard MDH model of Tauchen and Pitts (1983) implying that:

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\begin{aligned}
Cov(\Delta P_t, \Delta P_{t+1}) &= 0, \\
Cov(\Delta P_t^2, \Delta P_{t+1}^2) &= 0, \\
Cov(V_t, V_{t+1}) &= 0.
\end{aligned}
$$

Our model given in (2.1)-(2.2) presents a triangular structure which can be exploited to build a two-step methodology in order to extract I_t and L_t latent processes from daily time series of returns and volume. As discussed in the next section, we first use a Markov regime-switching stochastic volatility model to extract I_t from the daily price changes. Then, conditionally on I_t, we extract L_t by applying a simple Kalman filter procedure on daily volume observations.

2.3   Implications in terms of liquidity frictions

From this procedure, we can picture four different stock environment types:

(i) The first type is made of stocks which do not suffer from liquidity frictions of any kind. They are characterized by $\mu_p = 0$ and $\mu_{v2} = 0$, which corresponds to the Tauchen and Pitts (1983) case. Without any liquidity frictions, information propagates smoothly to the prices. In that case, there is no need for intermediaries.

(ii) The second category consists in stocks with short-term liquidity frictions only. They are characterized by $\mu_p = 0$, and $\mu_{v2} \neq 0$. We call this stock category, the pure short-term liquidity friction case. Note that, this short-term liquidity friction case differs from that of

Darolles et al. (2015) since in our framework, the L_t process is supposed to be time-persistent. For these stocks, liquidity providers are strategic while liquidity consummers are not. The latters are trading at once or at least withing the day and the formers are providing the missing liquidity. By the end of the day, there is no more liquidity frictions in the market and the prices contain all the information possessed by the market participants.

(iii) The third category concerns stocks that are facing long-term liquidity frictions. We have $\mu_p \neq 0$, and $\mu_{v2} = 0$. For these stocks, the liquidity providers are inactive. Maybe because, liquidity consummers are strategic and efficient in hiding from the liquidity arbitragers. As a consequence, information does not disseminate to the prices accurately.

(iv) Stocks belonging to the last category are affected by both long-lasting and short-term liquidity frictions (the mixed liquidity friction case). For these stocks, $\mu_p \neq 0$, and $\mu_{v2} \neq 0$. The short-term liquidity providers' activity (μ_{v2}) is not enough to ease the trading so that the incoming information is not completely revealed on the day of its arrival. It means that both are acting strategically but non of them is efficient in doing so. The short-term liquidity frictions disminish because liquidity providers are trading. However, because the liquidity consumers are splitting up their orders, information propagates slowly to the prices over several days.

Finally, estimating the parameters of our model enables us to test, for each stock during a given period of time, the four hypotheses presented above in order to characterize its liquidity profile. Our model tells us whether a stock has been facing liquidity frictions during the test period. If this is the case, we can go further in the analysis by specifying the type of liquidity frictions (long-lasting, short-term liquidity frictions or both at the same time) affecting the stock trading characteristics.

3 Estimation

We exploit the triangular structure of our model to perform time-varying analysis of liquidity problems affecting stock markets. We first use Markov regime-switching stochastic volatility framework to filter the latent variable I_t from equation (2.1). Then, a simple Kalman filter applied to volume equation (2.2) enables us to extract the L_t latent process conditional on I_t.

3.1 A stochastic volatility formulation of the price change equation

Equations (2.1) and (2.3) represent the general form of a stochastic volatility-in-mean model. We use a two step iterative procedure to estimate the model parameters and extract the latent variable I_t simultaneously. In the first step, we suppose that $\alpha = 0$ and $\mu_p = 0$; equations (2.1)-(2.3) reduce to a standard stochastic volatility (SV) model:

$$\Delta P_t = \sigma_p \sqrt{I_t} Z_{1t}, \tag{3.1}$$

$$\ln I_t = \beta \ln I_{t-1} + \eta_t. \tag{3.2}$$

We use a Markov regime-switching version of (3.1)-(3.2), henceforth SVMRS model, in order to estimate the parameters and extract the latent factor I_t, denoted by $\tilde{I}_t$. In the second step, we regress the observed ΔP_t on $\tilde{I}_t$ to get $\tilde{\alpha}$ and $\tilde{\mu}_p$, the estimated α and μ_p. We compute the weighted least squares (WLS) where the weighting vector W contains the inverse variances of the residuals estimated in the first step.[8] We then compute $\widetilde{\Delta P_t} = \Delta P_t - \tilde{\alpha} - \tilde{\mu}_p \tilde{I}_t$ for all $t = 1, ..., T$, that we inject in the SVMRS model estimation of the first step. We obtain a new estimation of I_t and σ_{pt}, that we use to determine the weighting vector in the WLS estimation of the second step. We iterate this procedure K times until the change in $\tilde{\mu}_p$ parameter value becomes negligible. Finally, we use the estimated $\tilde{I}_t$ latent factor of the last

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[8]Note that, the residual variance at a given point in time can be expressed as a function of $\tilde{\sigma}_{pt}$ and $\tilde{I}_t$ estimated in the first step: $W_t = \frac{1}{\tilde{\sigma}_{pt}^2 \tilde{I}_t}$. The σ_{pt} parameter is also indexed by t to indicate that, in the SVMRS framework, it can take two different values depending on the state of the nature at time t.

iteration as a conditioning variable in the volume equation (2.2) to estimate L_t.

Note that the standard SV model (3.1)-(3.2) implies extremely high levels of persistence. This is useful in practice since the estimation or forecasting becomes easier and estimation errors are smaller. However, the high persistence of volatility may actually arise because of limitations inherent to the AR(1) process assumed for the (log) stochastic volatility in (3.2). This assumption may be too restrictive if there are structural breaks. For example, based on weekly S&P500 index volatility, So et al. (1998) find that volatility is less persistent than that of SV models. Smith (2002) and Kalimipalli and Susmel (2004) find that, when applied to short-term interest rates, their regime-switching models perform better than the GARCH or standard SV models. The motivation behind regime changes in the level of volatility is similar to the jump diffusion stochastic volatility process. Different levels of volatility in stock markets could be attributed, for example, to various political and/or macroeconomic events.

Hwang et al. (2007) generalize the standard SV model in order to account for changes in regime of the level, the variance and the persistence of volatility. They show that the SV models with Markov regime change outperforms the standard SV and GARCH models. In the first step of our iterative procedure, we apply the Hwang et al. (2007) framework to (3.1)-(3.2) in order to extract the latent variable I_t from the observed time series of price changes.

First, linearizing the standard SV model (3.1)-(3.2) by taking logarithms of squared returns yields:

$$\ln \Delta P_t^2 \;=\; \ln Z_{1t}^2 + \ln \sigma_p^2 + h_t = \mu + h_t + \phi_t, \tag{3.3}$$

$$h_t \;=\; \beta h_{t-1} + \eta_t,$$

where $h_t = \ln I_t$, $\mu = E[\ln Z_{1t}^2] + \ln \sigma_p^2$ and $\phi_t = \ln Z_{1t}^2 - E[\ln Z_{1t}^2]$ is a martingale difference, but not normal. Replacing $\ln \Delta P_t^2$ with z_t and $\mu + h_t$ with x_t, equation (3.3) can be written

as:

$$z_t = x_t + \phi_t, \tag{3.4}$$

$$x_t - \mu = \beta(x_{t-1} - \mu) + \eta_t. \tag{3.5}$$

Then, supposing that there is a state variable s_t that can assume only $\{1, 2\}$ values, the SV model with Markov regime-switching equations becomes:

$$z_t = x_t + \phi_t, \tag{3.6}$$

$$x_t = \begin{cases} \mu_0 + \beta(x_{t-1} - \mu_0) + \eta_{0,t}, & when \quad s_t = 0, s_{t-1} = 0, \\\\ \mu_0 + \beta(x_{t-1} - \mu_1) + \eta_{0,t}, & when \quad s_t = 0, s_{t-1} = 1, \\\\ \mu_1 + \beta(x_{t-1} - \mu_0) + \eta_{1,t}, & when \quad s_t = 1, s_{t-1} = 0, \\\\ \mu_1 + \beta(x_{t-1} - \mu_1) + \eta_{1,t}, & when \quad s_t = 1, s_{t-1} = 1, \end{cases} \tag{3.7}$$

where $\eta_{i,t} \sim N(0, \sigma_{\eta_i}^2)$ for $i = 0, 1$ and s_t follows a Markov chain with transition matrix given by:

$$\mathbf{p} = \begin{bmatrix} p^{(0,0)} & 1 - p^{(1,1)} \\ 1 - p^{(0,0)} & p^{(1,1)} \end{bmatrix}, \tag{3.8}$$

with $p^{(i,j)} = Pr(s_t = i | s_{t-1} = j)$.

Finally, equations (3.6)-(3.7) represent the SVMRS model.[9] We apply Quasi-Maximum Likelihood (QML) estimation method as in Hwang et al. (2007) in order to estimate the SVMRS model parameters as well as the unobserved process x_t from (3.6)-(3.7). A detailed description of the QML estimation procedure applied to SVMRS model is given in the Appendix A.

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[9]This generalized form comprises the standard SV model as a particular case when $\mu_0 = \mu_1$, and $\sigma_{\eta_1}^2 = \sigma_{\eta_2}^2$.

3.2 Extracting the intraday liquidity frictions

We now estimate the volume equation parameters and extract the latent process L_t conditional on I_t obtained in the previous subsection.

In particular, equations (2.2) and (2.4) represent a state space form:

$$V_t = \mu_{v1}I_t + \mu_{v2}L_t + \sigma_v\sqrt{I_t}Z_{2t}, \tag{3.9}$$

$$L_t = aL_{t-1} + \omega_t. \tag{3.10}$$

The observed daily traded volume V_t is related to the latent factor L_t, known as the state vector, via the measurement equation (3.9). The latent variable L_t is supposed to be generated by a first-order Markov process given in (3.10), called the state equation. The disturbance ω_t is a white noise process with mean zero and variance σ_ω^2, independent of Z_{1t}, Z_{2t} and η_t.

Replacing I_t in (3.9) by its estimated counterpart obtained in the previous subsection and applying the standard Kalman filter to the state space form (3.9)-(3.10) enables us to estimate the volume parameters and extract the latent variable L_t, simultaneously.[10]

4 Empirical applications

4.1 The data

Our sample consists in all FTSE100 stocks listed on March 27, 2014. We consider the period from January 1st, 2010 to December 31, 2013, i.e., 1043 observation dates. We exclude stocks with missing observations ending up with 92 stocks. Daily returns and transaction volumes are extracted from Bloomberg databases. Following Bialkowski et al. (2008), we retain the

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[10]We provide in Appendix B a brief summary of the Kalman filter procedure applied to the state space form (3.9)-(3.10) in order to filter L_t conditional on I_t and estimate the model parameters accordingly.

	Returns				Volume			
	Average	Dispersion	Min	Max	Average	Dispersion	Min	Max
Mean	0.0007	0.0005	-0.0006	0.0024	0.0033	0.0011	0.0008	0.0066
Volatility	0.0166	0.0045	0.0102	0.0282	0.0023	0.0013	0.0004	0.0079
Skewness	-0.1808	0.7734	-4,3628	0.6959	5,2824	4,9046	1,1783	28,0834
Kurtosis	8,4518	10.1659	3,7169	66,6601	82,1967	138,7313	5,2143	864,9621
$(\text{Return})^2$ with Volume Correlation	0.4051	0.1302	0.1177	0.7307	-	-	-	-
1st Order Serial Correlation	0.0882	0.0164	0.0612	0.1227	0.4476	0.0992	0.1911	0.7711
1st Order Serial Correlation $(\text{Return})^2$	0.1369	0.0595	0.0633	0.3559	-	-	-	-

Table 1: Summary statistics for return and turnover time series from January 1st, 2010 to December 31, 2013.

turnover ratio[11] as a measure of volume which controls for dependency between the traded volume and the float. The float represents the difference between annual common shares outstanding and closely held shares for any given fiscal year. Common and closely held shares are also extracted from Bloomberg databases.

For each of the 92 stocks, we compute: (i) the empirical four first moments (mean, volatility, skewness and kurtosis) of volume and returns; (ii) the correlation between squared returns and volume; (iii) the first-order serial correlation of returns, volume and squared returns. The distributions of these statistics are summarized in Table 1. The first row reports the average, the dispersion, the minimum, and the maximum of the means of returns and volume across the 92 stocks. The second row gives the same cross-section statistics (average, dispersion, minimum and maximum) of the volatilities of returns and volume, and

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[11]Let q_{kt} be the number of shares traded for asset k, $k = 1, ..., K$ on day t, $t = 1, ..., T$, and N_{kt} the float for asset k on day t. The individual stock turnover for asset k on day t is $V_{kt} = \frac{q_{kt}}{N_{kt}}$.

so on for the skewness, kurtosis, the correlation between squared returns and volume as well as the serial correlation coefficients. Finally, note that we perform a Pearson test to check the significance of the correlation coefficients computed in Table 1. The statistics reported in the three last rows are computed using the statistically significant coefficients only.

The results reported in Table 1 are consistent with the empirical implications of the standard mixture of distribution hypothesis, predicting that (i) volume should be positive and should present a positive skewness; (ii) the return and volume distributions should be characterized by fat tails; (iii) the correlation between the squared returns and volume should be positive. Indeed, Table 1 shows that the average and minimum statistics of volume skewness are strictly positive. In addition, the fat tail hypothesis is empirically supported by the data: the average and minimum statistics of return and volume kurtosis are greater then 3. Finally, the correlation coefficients between squared returns and volume are strictly positive; the correlation coefficients across stocks evolve between 0.1177 and 0.7307, with an average value of 0.41 and a dispersion of 0.1302. Moreover, the Pearson test shows that the correlation of squared return with volume are statistically significant for 90 over 92 stocks. Note that, the standard MDHL model of Darolles et al. (2015) explains this positive correlation by the effects of both information flow and short-term liquidity friction processes, while in other competing MDH versions [such as that of Tauchen and Pitts (1983) or Andersen (1996)] this positive correlation is strictly due to the information flow process, disregarding the presence of liquidity frictions.

Our framework generalizes the standard MDHL model by assuming that both information flow and short-term liquidity friction processes are time-persistent.[12] Our modelling implies that, in the presence of long-lasting and short-term liquidity frictions, the daily return, squared return and volume time-series should exhibit significant serial correlation.

The results reported in the last three rows of Table 1 support these conclusions. In particular, at the 95% confidence level, the first order serial correlations of returns, volumes

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[12]Recall that we suggest that the information flow process is serially correlated because of the presence of long-lasting liquidity frictions.

and squared returns are significant for 34, 90 and 72 over 92 stocks, respectively. For daily returns, these serial correlations range from 0.0612 to 0.1227 indicating that long-lasting liquidity frictions might be at play. For the daily traded volume, we get larger values lying in between 0.1911 and 0.7711 that could be due to the occurrence of (short-lasting and/or long-lasting) liquidity frictions. The cross-section average values of first-order serial correlation coefficients is 0.089 for returns (with a dispersion of 0.016) and 0.45 for volume (with a dispersion of 0.099). On the other hand, the first order serial correlation coefficients of squared returns evolve between 0.063 and 0.356, with a cross-sectional average of 0.14 and a dispersion of 0.059.

These descriptive statistics motivate our framework suggesting that the time-series dynamics of daily returns and volume result from the interaction of long-lasting and short-term liquidity frictions. However, the empirical serial correlation coefficients are not sufficient to separate the effects of both types of liquidity frictions on daily returns and volumes. In this context, our structural framework improves the comprehension of daily time-series dynamics by separating the impacts of both types of liquidity frictions on daily trading characteristics. Finally, in subsection 4.3, we confront the implications of our modified MDHL model with the sample autocorrelation coefficients, and show that the parameters of the model improve the naive analysis based on sample autocorrelation coefficients.

4.2   Characterizing stock liquidity profile

The two-step estimation methodology presented in the previous section exploits the triangular structure of our statistic model (2.1)-(2.2) in order to separate the impact of both types of liquidity frictions (long-lasting versus short-term ones) on daily trading characteristics.

First, applying Markov regime-switching stochastic volatility framework to the price change equation, we filter the I_t process and estimate its related parameters. Note that the dynamics of the I_t process captures the effect of long-term liquidity frictions on daily price changes. In the presence of long-lasting liquidity frictions, I_t is serially correlated,

which explains the time-series dynamics of daily price change through the μ_p parameter. Stocks presenting statistically significant μ_p are affected by long-lasting liquidity frictions.

Second, the Kalman filter procedure allows us to estimate stock parameters related to L_t process, based on the volume equation, and conditionally on I_t process filtered in the first step. The parameter of interest here is μ_{v2} which measures the effect of liquidity provision on daily volume. In the presence of short-term liquidity frictions, the trading activity of liquidity providers acting as market makers increases the daily traded volume. Stocks having statistically significant μ_{v2} parameter are affected by the presence of short-term liquidity frictions.

We report in columns 2 and 3 of Tables 2 and 3 in Appendix C, the estimated μ_p and μ_{v2} parameters for 92 stocks of our sample.[13] Tables 2 and 3 show that 61% (56 out of 92) of the stocks present significant μ_p, μ_{v2}, or both, meaning that they are facing either short-term, long-term liquidity frictions or both.

Recall that, as listed in section 2.3, based on the estimated μ_p and μ_{v2} parameters, we can characterize the liquidity profile of each stock. In particular, we can split our universe of 92 stocks into four categories: the TP case ($\mu_p = 0$ and $\mu_{v2} = 0$), the pure short-term liquidity friction case ($\mu_p = 0$ and $\mu_{v2} \neq 0$), the pure long-lasting liquidity friction case ($\mu_p \neq 0$ and $\mu_{v2} = 0$), and the mixed liquidity friction case ($\mu_p \neq 0$ and $\mu_{v2} \neq 0$). We focus on the last three categories to concentrate on stocks with liquidity problems, i.e. 56 stocks.

Table 4 contains the thirty two stocks (57%) belonging to the pure short-term liquidity friction case. For these stocks, the liquidity consumers are not trading strategically meaning that they do not split their orders. The liquidity providers act strategically; they enter the market to provide liquidity and cash the liquidity premium supported by non-strategic (impatient) liquidity consumers. In this case, the intervention of liquidity providers increases the average traded volume, as captured by μ_{v2} and at the end of the day, the information possessed by the market participants is completely incorporated into the prices.

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[13]The remaining parameters of price change and volume equations, not reported here, are available upon request.

Table 5 reports the estimated parameters for the fifteen stocks (27%) of the pure long-term liquidity friction category. For these stocks, liquidity consumers act strategically by splitting their trades in order to hide from the liquidity providers. It may be because the strategic liquidity consumers are efficient in remaining out of the radar of the strategic liquidity providers that the latter are not trading. A by-side effect of their splitting is a slowdown of the propagation of information in the prices that can last for days and even weeks inducing a deterioration of the market (price) efficiency.

The remaining nine stocks (16%) are of the mixed liquidity friction type and their results are reported in Table 6. As for these stocks, liquidity providers as well as liquidity consumers act strategically without being fully efficient. In fact, liquidity consumers are splitting but their splitting scheme is detected by the strategic liquidity providers who lower the impact of short-term liquidity frictions on price changes. However, they are not able to detect, or to correct these short-term frictions completely, and the end-of-the-day prices are revealing only part of the information possessed by market participants.

Generally speaking, estimating μ_p and μ_{v2} parameters for individual stocks provides a better understanding of their liquidity profile, which is strongly related to the behavior of liquidity consumers and providers. In particular, our results show that liquidity profiles can be different even for stocks belonging to the same universe. But more important, we show that short-term liquidity providers do not accelerate the propagation of information in the prices because liquidity consumers are strategic. For 43% of the stocks facing long-lasting liquidity problems, end-of-the-day prices are not reflecting all the incoming information because the informed traders are splitting their orders not to be detected.

4.3 Long-lasting liquidity frictions and momentum strategies

We now discuss the implications of our results in statistical arbitrage strategies. Actually, the statistical arbitrage traders observe the empirical serial correlation of stock returns and pick up positive serially correlated stocks to build up momentum strategies. Based on our statistic

model, we suggest that the empirical first order serial correlation is not a sufficient criteria to select stocks to be included in these strategies. In particular the tests of significance of the sample autocorrelation coefficients are not appropriate when the observed variables are heteroscedastic. In addition, the sample serial correlation coefficients measure, on average during a time-interval, the dependence of returns. A stock may have sample autocorrelations close to zero and may still be facing significant long-term liquidity frictions. The impact of those frictions on stock returns may be blurred out during a given test period. Therefore, a stock may have a first-order serial correlation not significantly different from zero when performing classical test statistics and still be affected by long-term liquidity frictions whose presence will be empirically detected using the parameters of our model.

Tables 2 and 3 in Appendix C report, in columns 4 to 6, the serial correlation parameters for daily returns, squared returns and traded volume, respectively. The statistically significant coefficients at the 95% level of confidence have bolded values.

Based on the values of the correlation coefficients of returns and squared returns, we distinguish three types of stocks. The first one contains stocks whose return autocorrelation is not statistically different from zero but whose squared returns are significantly serially correlated. The second category consists of stocks with apparently independent returns (return as well as squared return serial correlations not statistically different from zero). The third one includes stocks with positive autocorrelation coefficients for daily returns. In the absence of any structural model, an *ad hoc* momentum strategy would consist in selecting only the third group of stocks.

According to our framework, the presence of long-term liquidity frictions is captured by the estimated μ_p parameter which can be considered as long-lasting liquidity frictions indicator.[14]

Stocks belonging to the first group (such as stocks BAB LN, BARC LN, ITV LN and

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[14]Recall that equation 2.5 shows that the autocovariance of daily returns depends on the absolute value of μ_p and on the autocovariance of I_t process. Since $Cov(I_t, I_{t-1})$ can be expressed as a function of the persistence parameter β, a statistically positive β and a μ_p parameter statistically different from zero ensure the positiveness of the (theoretical) autocovariance of daily returns for a given stock.

RB LN among others) may have μ_p parameters statistically significant, while their sample return autocorrelations do not differ from zero significantly. As opposed to the *ad hoc* analysis consisting of selecting only stocks with a positive (empirical) serial correlation of returns, our framework suggests that stocks like BAB LN, BARC LN, ITV LN and RB LN should be included in the momentum strategies even if their first-order (empirical) serial correlation of returns does not differ significantly from zero, because their μ_p parameter is statistically significant. The *ad hoc* stock picking strategy should be at least completed by the dynamic properties of the squared returns. In particular, the presence of a positive serial correlation of squared returns conveys a favorable signal concerning the presence of long-term liquidity frictions, whose presence should be then verified based on our model parameters. Note also that, the parameter μ_p helps identifying appropriate stocks to be included in the momentum strategies. The variance parameter σ_p^2 of return equation (2.1) provides un idea of risk related to the position held on these stocks.

Let now consider firms (such as for example, ADN LN, CRH LN and MKS LN stocks) belonging to the second category. The *ad hoc* selection strategy based on the sample auto-correlations of returns and even squared returns would not include these three stocks in the momentum strategies. According to our structural approach, stock ADN LN appears to be affected by long-term liquidity frictions since its μ_p parameter (and also its β parameter not reported here) differ significantly from zero. We suggest that this stock exhibits dynamic properties as captured by μ_p and β. As for stocks CRH LN and MKS LN, their μ_p parameters are not significant (and also their β parameters − not reported here − are not statistically different from zero). In this case, our analysis confirms the *ad hoc* approach results.

Stocks belonging to the third group (such as stocks ADM LN, AMEC LN, BP LN, BG LN, LSE LN and MRO LN among others) present positive sample autocorrelations for daily returns. According to the *ad hoc* selection procedure, these candidates should be included in the momentum strategies. However, BP LN and BG LN are the only stocks to present statistically significant μ_p parameters.

More generally, our framework provides additional insights concerning the dynamic properties of stock returns. First, by imposing a theoretical structure on the data, we explain the dynamic properties of returns by the presence of long-term liquidity frictions. Second, our estimation procedure enables us to infer the presence of the long-lasting liquidity frictions using the daily time series of equity returns. The long-term liquidity indicator, μ_p, represents an alternative criteria in selecting the appropriate stocks in the momentum strategies.

4.4 Short-term liquidity frictions and contrarian arbitrage strategies

In our framework, the μ_{v2} parameter represents the effect of the strategic liquidity arbitrage trading on daily traded volume. In contrast with Darolles et al. (2015), this indicator is more robust in our modified framework since it is based on a less restrictive serially correlated L_t process.

The results reported in column 3 of Tables 2 and 3 in Appendix C, suggest that for stocks such as BATS LN, CNA LN, GKN LN and HSBA LN among others, the daily traded volume is significantly impacted by liquidity arbitragers who trade in response to short-term liquidity frictions. These stocks represent liquidity arbitrage opportunities at the intradaily frequency. These trading opportunities are a source of trade for the HFT who enter the market to provide the missing liquidity and then liquidate their positions in order to cash the liquidity premium. Our methodology is thus interesting since it allows: (i) the liquidity arbitragers to detect stocks presenting significant short-term liquidity arbitrage opportunities; (ii) the uninformed traders to detect stocks whose prices deviate from their fully revealing information levels at the intradaily periodicity.

5   Concluding remarks

In this article, we distinguish between two types of liquidity problems: short-lasting and long-lasting liquidity frictions. The former correspond to the liquidity provision and impact stock returns at the intradaily frequency while affecting the traded volume at the daily periodicity. The later are responsible for the dynamics of daily return and volume time-series because information is not completely incorporated into prices within the day. These frictions come from the strategic behavior of liquidity consumers, i.e. the long-term investors. We propose a statistical model that specifies the impact of information arrival on market characteristics in the context of liquidity frictions. It accounts for both the long-lasting liquidity frictions and the time-persistence of short-term liquidity frictions whose presence can be inferred from the daily return dynamics. We exploit the triangular structure of our model to disentangle both types of liquidity frictions on stock return and volume dynamics. Our approach is made of two steps. In the first step, we estimate the long-lasting liquidity frictions from the daily return equation. In particular, we use a Markov regime-switching stochastic volatility framework to filter the It process whose dynamics is due to the presence of long-lasting liquidity frictions. In the second step, we apply the Kalman filter procedure to the daily volume equation to infer the presence of short-term liquidity frictions conditionally on the latent variable I_t estimated in the first step. Our results show that amongst 61% of the stocks facing liquidity problems, 57% of them point up liquidity providers as the sole strategic market investor. Another 27% feature long-term investors as the single strategic player, while both liquidity providers and liquidity consumers act strategically in the remaining 16%. If the strategic liquidity providers give the missing liquidity on an intradaily basis, and as a consequence accelerate the propagation of information to prices, the adaptation of the liquidity consumers, i.e. the long-term investors, destroys part of that benefit. In fact, 43% of these stocks are actually facing a slow-down in the information's coalesce with prices, which thus results in a significant decrease of (daily) price efficiency due to long-term investors' strategic behavior.

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Appendices

A The QML estimation procedure applied to SVMRS models

The QML approach for estimating SV models, proposed independently by Nelson (1988) and Harvey et al. (1994) and generalized by Smith (2002) to account for Markov regime-switching, is based on the Kalman filter. This is applied to the logarithm of the observed squared returns z_t to obtain one-step-ahead errors and their variances. These are then used to construct a quasi-likelihood fonction. Because z_t is not Gaussian, the Kalman filter yields minimum mean square linear estimators (MMSLE) of x_t and future observations rather than minimum mean square estimators (MMSE). Although the estimates obtained from the QML may not be as efficient as those obtained by Bayesian methods, such as Markov Chain Monte Carlo (MCMC), or the Efficient Method of Moments (EMM), Broto and Ruiz (2004) show that for large sample sizes the QML estimator behaves similarly to Bayesian estimators.

Consider the state space model given in (3.6)-(3.7). Let $x_{t-1|t-1}^{(j)}$ be the optimal[15] estimator of x_{t-1} based on the observations up to and including z_{t-1} and for $s_{t-1} = j$ ($j = 0.1$). Let $P_{t-1|t-1}^{(j)}$ denote the variance of the prediction error. Given $x_{t-1|t-1}^{(i,j)}$, $P_{t-1|t-1}^{(i,j)}$, and conditional on $s_t = i$ and $s_{t-1} = j$, ($i, j = 0.1$), the optimal estimator of x_t, $x_{t|t-1}^{(i,j)}$, and its mean squared error, $P_{t|t-1}^{(i,j)}$ are obtained using the prediction equations as follows:

$$x_{t|t-1}^{(i,j)} = E[x_t|s_t = i, s_{t-1} = j, \Im_{t-1}] = (\mu_i - \mu_j\beta) + \beta x_{t-1|t-1}^{(j)}, \tag{A.1}$$

$$P_{t|t-1}^{(i,j)} = E[(x_t - x_{t|t-1}^{(i,j)})^2|s_t = i, s_{t-1} = j, \Im_{t-1}] = \beta^2 P_{t-1|t-1}^{(j)} + \sigma_{\eta_i}^2. \tag{A.2}$$

Once the new observation z_t becomes available, we can calculate the prediction error $v_t^{(i,j)}$,

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[15] Because z_t is not Gaussian, the Kalman filter yields minimum mean square linear estimators (MMSLE) of x_t and future observations rather than minimum mean square estimators (MMSE).

the prediction variance $F_t^{(i,j)}$, and then update $x_{t|t-1}^{(i,j)}$ and $P_{t|t-1}^{(i,j)}$ accordingly:

$$v_t^{(i,j)} = z_t - x_{t|t-1}^{(i,j)} = x_t - x_{t|t-1}^{(i,j)} + \phi_t, \tag{A.3}$$

$$F_t^{(i,j)} = P_{t|t-1}^{(i,j)} + \sigma_\phi^2, \tag{A.4}$$

$$x_{t|t}^{(i,j)} = x_{t|t-1}^{(i,j)} + P_{t|t-1}^{(i,j)} \left(F_t^{(i,j)} \right)^{-1} v_t^{(i,j)}, \tag{A.5}$$

$$P_{t|t}^{(i,j)} = P_{t|t-1}^{(i,j)} - P_{t|t-1}^{(i,j)} \left(F_t^{(i,j)} \right)^{-1} P_{t|t-1}^{(i,j)}. \tag{A.6}$$

Let $\xi_{t|t-1}^{(i,j)}$ be the filtered transition probability indicating how likely the process is to be in regime i ($i = 1, 2$) in period t given $s_{t-1} = j$ ($j = 1, 2$) and information available through date $t - 1$ ($\xi_{t|t-1}^{(i,j)} = Pr(s_t = i, s_{t-1} = j | \Im_{t-1})$). Following Hamilton (1989), it can be obtained using the transition probability $p^{(i,j)} = Pr(s_t = i | s_{t-1} = j)$ and the conditional $\xi_{t-1|t-1}$ which infers the possibility that the $t - 1$th observation was generated by regime j ($\xi_{t-1|t-1}^{(j)} = Pr(s_{t-1} = j | \Im_{t-1})$):

$$\xi_{t|t-1}^{(i,j)} = p^{(i,j)} \xi_{t-1|t-1}^{(j)}. \tag{A.7}$$

The conditional probability can be updated with information available at date t ($\xi_{t|t}^{(i)} = Pr(s_t = i | \Im_t)$) as follows:

$$\xi_{t|t}^{(i)} = \sum_{j=0}^{1} \xi_{t|t}^{(i,j)}, \tag{A.8}$$

where

$$\begin{aligned} \xi_{t|t}^{(i,j)} &= \frac{f(z_t, s_t = i, s_t = j | \Im_{t-1})}{f(z_t | \Im_{t-1})}, \\ &= \frac{f(z_t | s_t = i, s_t = j, \Im_{t-1}) \xi_{t|t-1}^{(i,j)}}{\sum_{i=0}^{1} \sum_{j=0}^{1} f(z_t | s_t = i, s_t = j, \Im_{t-1}) \xi_{t|t-1}^{(i,j)}}. \end{aligned} \tag{A.9}$$

The log-likelihood function of the SVMRS model (3.6)-(3.7) is:

$$\pounds(z|\theta_p) = \sum_{t=1}^{T} log[f(z_t|\Im_{t-1})], \tag{A.10}$$

where $\theta_p = \{\mu_0, \mu_1, \beta, \sigma_{\eta_0}, \sigma_{\eta_1}, p^{(0,0)}, p^{(1,1)}, \sigma_\phi^2\}$, $\Im_{t-1}$ includes all available information at time $t-1$, and $f(z_t|\Im_{t-1})$ is the likelihood function of z_t given by:

$$f(z_t|\Im_{t-1}) = \sum_{i=0}^{1} \sum_{j=0}^{1} f(z_t|s_t = i, s_{t-1} = j, \Im_{t-1})\xi_{t|t-1}^{(i,j)}. \tag{A.11}$$

In this equation, $f(z_t|s_t = i, s_{t-1} = j, \Im_{t-1})$ is the density of z_t conditional on s_t, s_{t-1} and $\Im_{t-1}$; under the normality assumption, it is written as:

$$f(z_t|s_t = i, s_{t-1} = j, \Im_{t-1}) = \frac{1}{\sqrt{2\pi F_t^{(i,j)}}} exp\left\{-\frac{(v_t^{(i,j)})^2}{2F_t^{(i,j)}}\right\}. \tag{A.12}$$

Given $x_{t|t}^{(i,j)}$, $P_{t|t}^{(i,j)}$ and equation (A.8), we get:

$$x_{t|t}^{(i)} = E[x_t|s_t = i, \Im_t] = \frac{\sum_{j=0}^{1} \xi_{t|t}^{(i,j)} x_{t|t}^{(i,j)}}{\xi_{t|t}^{(i)}}, \tag{A.13}$$

$$P_{t|t}^{(i)} = E[(x_t - x_{t|t}^{(j)})^2|s_t = i, \Im_t] = \frac{\sum_{j=0}^{1} \xi_{t|t}^{(i,j)} P_{t|t}^{(i,j)}}{\xi_{t|t}^{(i)}}, \tag{A.14}$$

Finally, note that from (A.9) and (A.8) we have:

$$\sum_{i=0}^{1} \xi_{t|t}^{(i)} = 1. \tag{A.15}$$

It follows that:

$$x_{t|t} = E[x_t|\Im_t] = \sum_{i=0}^{1} \xi_{t|t}^{(i)} x_{t|t}^{(i)}, \tag{A.16}$$

$$P_{t|t} = E[(x_t - x_{t|t}^{(i)})^2|\Im_t] = \sum_{i=0}^{1} \xi_{t|t}^{(i)} P_{t|t}^{(i)}. \tag{A.17}$$

B The Kalman filter procedure applied to the volume equation

Replacing I_t in (3.9) by its estimated counterpart obtained in the first step of our methodology as described in subsection 3.1 and applying the standard Kalman filter to the state space form (3.9)-(3.10) enables us to estimate the volume parameters and extract the latent variable L_t, simultaneously.

It is important to distinguish between the hyperparameters $\theta_v = \{\mu_{v2}, \sigma_v, a\}$ which determine the stochastic properties of the model and the μ_{v1} parameter associated to I_t which is known. In this case, the term $\mu_{v1}I_t$ only affects the expected value of the state variable L_t and observations in a deterministic way. Since $\mu_{v1}I_t$ is a linear function of the unknown μ_{v1}, this parameter can be treated as a state variable. Let $l_t = [L_t \ \mu_{v1}]'$ be the state vector. The state space form becomes:

$$V_t = Z_t l_t + \epsilon_t, \tag{B.1}$$

$$l_t = T l_{t-1} + R \omega_t, \tag{B.2}$$

where $Z_t = [\mu_{v2} \ I_t]$, $\epsilon \sim N(0, H_t)$ with $H_t = I_t \sigma_v^2$, $R = [1 \ 0]'$, and T is a 2×2 matrix given by:

$$T = \begin{bmatrix} a & 0 \\ 0 & 1 \end{bmatrix}. \tag{B.3}$$

Let $l_{t-1|t-1}$ denote the optimal estimator of l_{t-1} based on the observations up to and including V_{t-1}. Let $P_{t-1|t-1}$ denote the 2×2 covariance matrix of the estimation error:

$$P_{t-1|t-1} = E[(l_{t-1} - l_{t-1|t-1})(l_{t-1} - l_{t-1|t-1})' | \Im_{t-1}], \tag{B.4}$$

where $\Im_{t-1}$ denotes the information set available up to date $(t-1)$. Given $l_{t-1|t-1}$ and

$P_{t-1|t-1}$, the optimal estimator of l_t is given by:

$$l_{t|t-1} \;=\; E[l_t|\Im_{t-1}] = Tl_{t-1|t-1}, \tag{B.5}$$

while the covariance matrix of the estimation error is:

$$P_{t|t-1} \;=\; E[(l_t - l_{t|t-1})(l_t - l_{t|t-1})'|\Im_{t-1}] = TP_{t-1|t-1}T' + R\sigma_w^2 R', \tag{B.6}$$

with σ_w^2 being the variance of the disturbance w_t. Equations (B.5)-(B.6) represent the prediction equations.

Once the new observation V_t becomes available, $l_{t|t-1}$ can be updated using the updating equations as follows:

$$l_{t|t} \;=\; l_{t|t-1} + P_{t|t-1}Z_t'F_t^{-1}(V_t - Z_t l_{t|t-1}), \tag{B.7}$$

and

$$P_{t|t} \;=\; P_{t|t-1} - P_{t|t-1}Z_t'F_t^{-1}Z_t P_{t|t-1}, \tag{B.8}$$

where[16]

$$F_t \;=\; Z_t P_{t|t-1}Z_t' + H. \tag{B.9}$$

Taken together (B.5)-(B.8) make up the Kalman filter whose recursive application yields the time series of the state variables l_t conditional on θ_v. We use the QML method to estimate the model parameters θ_v. Applying the Kalman filter (B.5)-(B.8) to the observed daily traded volume, we first obtain one-step-ahead errors $(V_t - V_{t|t-1})$ and their variances

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[16]It is assumed that the inverse of F_t exists. It can can be replaced by a pseudo-inverse.

F_t, which are then used to construct the likelihood function:

$$\log L = -\frac{NT}{2}log2\pi - \frac{1}{2}\sum_{t=1}^{T}log|F_t| - \frac{1}{2}\sum_{t=1}^{T}(V_t - V_{t|t-1})F_t^{-1}(V_t - V_{t|t-1}), \qquad \text{(B.10)}$$

where $V_{t|t-1} = Z_t l_{t|t-1}$ is the conditional mean of the observable V_t. Finally, numerical methods can be used to maximize the likelihood function with respect to the unknown parameters θ_v.

C   The estimation results

Bloomberg Code	μ_p	μ_{v2}	$\rho_{(R_t, R_{t-1})}$	$\rho_{(R_t^2, R_{t-1}^2)}$	$\rho_{(V_t, V_{t-1})}$
AAL LN	0,00007	**0,00353**	0,033	0,032	**0,515**
ABF LN	**-0,00004**	0,00371	-0,036	**0,117**	**0,477**
ADM LN	-0,00028	0,00280	**0,083**	**0,072**	**0,510**
ADN LN	**0,00073**	0,00428	0,056	0,045	0,060
AGK LN	0,00000	0,00354	**0,091**	0,009	**0,338**
AHT LN	0,00053	0,00239	**0,077**	**0,102**	**0,371**
AMEC LN	0,00037	**0,00384**	**0,070**	**0,186**	**0,445**
ANTO LN	-0,00011	0,00845	0,016	**0,112**	**0,538**
ARM LN	0,00018	**0,00308**	0,038	**0,096**	**0,497**
AV LN	0,00016	**0,00362**	0,036	**0,121**	**0,476**
AZN LN	0,00012	**0,00248**	0,022	0,054	**0,444**
BA LN	0,00017	**0,00269**	-0,022	**0,269**	**0,558**
BAB LN	**0,00034**	**0,00379**	0,001	**0,254**	**0,405**
BARC LN	**-0,00445**	0,00449	0,058	**0,097**	**0,426**
BATS LN	0,00008	**0,00149**	-0,035	**0,082**	**0,417**
BG LN	0,00025	**0,00187**	**0,063**	**0,095**	**0,382**
BLND LN	0,00100	0,00348	0,038	**0,245**	**0,455**
BLT LN	**-0,00066**	0,00476	**0,082**	**0,129**	**0,447**
BNZL LN	**-0,00070**	0,00185	**0,066**	**0,161**	**0,296**
BP LN	**0,00130**	0,00131	**0,109**	**0,356**	**0,670**
BRBY LN	0,00000	**0,00392**	**0,109**	0,005	**0,491**
BSY LN	0,00000	**0,00369**	0,049	0,036	**0,425**
BT/A LN	**0,00395**	**0,00150**	-0,010	**0,074**	**0,563**
CCL LN	0,00007	0,00785	**0,111**	0,027	**0,443**
CNA LN	0,00022	**0,00215**	-0,024	**0,102**	**0,433**
CPG LN	0,00063	**0,00222**	-0,055	**0,129**	**0,464**
CPI LN	0,00081	**0,00284**	0,032	**0,159**	**0,449**
CRH LN	0,00010	**0,00364**	0,050	0,046	**0,276**
DGE LN	**0,00030**	0,00319	-0,025	0,039	**0,370**
EXPN LN	**-0,00041**	0,00301	0,024	0,031	**0,410**
EZJ LN	0,00006	0,01025	**0,075**	**0,090**	**0,367**
GFS LN	**-0,00087**	0,00262	0,012	**0,154**	**0,488**
GKN LN	0,00070	**0,00421**	**0,105**	**0,101**	**0,363**
GSK LN	**-0,00052**	0,00184	-0,026	**0,120**	**0,398**
HMSO LN	0,00005	0,00000	**0,089**	**0,184**	0,051
HSBA LN	0,00022	**0,00145**	-0,005	**0,066**	**0,528**
IAG LN	-0,00007	0,01082	**0,106**	**0,086**	**0,597**
IHG LN	0,00000	0,00780	0,050	**0,175**	**0,429**
IMI LN	-0,00008	0,00335	**0,123**	**0,267**	**0,279**
IMT LN	-0,00014	**0,00332**	0,047	**0,078**	**0,425**
ITRK LN	**0,00397**	**0,00245**	0,031	**0,080**	**0,191**
ITV LN	**0,00102**	0,00671	0,041	**0,072**	**0,421**
JMAT LN	-0,00005	**0,00398**	0,026	**0,163**	**0,477**
KGF LN	**0,00169**	0,00306	0,051	**0,117**	**0,543**
LAND LN	0,00000	0,00314	**0,080**	**0,176**	**0,484**
LGEN LN	**0,00346**	0,00225	0,083	**0,225**	**0,593**
LLOY LN	0,00012	0,00673	**0,072**	**0,198**	**0,532**
LSE LN	0,00053	0,00368	**0,083**	0,041	**0,368**

Table 2: Columns 2 and 3 report the estimated μ_p and μ_{v2} parameters for the first 48 FTSE 100 stocks. The bolded parameters are statistically significant at the 95% level of confidence. Columns 4 to 6 report the estimated first order serial correlation coefficients of returns, squared returns and volume time series, respectively. We perform Pearson test to assess the statistic significance of the correlation coefficients. The bolded coefficients are statistically significant at the 95% level of confidence.

Bloomberg Code	μ_p	μ_{v2}	$\rho_{(R_t, R_{t-1})}$	$\rho_{(R_t^2, R_{t-1}^2)}$	$\rho_{(V_t, V_{t-1})}$
MGGT LN	0,00042	**0,00310**	0,055	**0,115**	**0,350**
MKS LN	-0,00011	0,00528	0,018	0,025	**0,513**
MRO LN	0,00051	0,00157	**0,080**	**0,164**	**0,321**
MRW LN	-0,00007	0,00696	0,050	**0,094**	**0,440**
NG LN	0,00031	**0,00225**	0,057	**0,075**	**0,771**
NXT LN	0,00005	**0,00561**	0,029	0,029	**0,567**
OML LN	0,00035	0,00570	0,030	**0,109**	**0,462**
PFC LN	-0,00005	0,00540	0,021	0,054	**0,482**
PRU LN	0,00007	0,00172	**0,061**	**0,184**	**0,718**
PSN LN	-0,00027	0,00778	**0,097**	**0,129**	**0,431**
PSON LN	-0,00026	0,00244	0,006	**0,094**	**0,405**
RB LN	**0,00020**	0,00233	-0,059	**0,166**	**0,418**
RBS LN	-0,00040	0,00404	**0,111**	**0,183**	**0,505**
RDSA LN	**0,00136**	**0,00087**	0,086	**0,155**	**0,235**
RDSB LN	-0,00004	0,00361	**0,072**	**0,134**	**0,395**
REL LN	0,00139	0,00330	**0,086**	**0,070**	**0,531**
REX LN	0,00144	**0,00296**	**0,104**	**0,076**	**0,449**
RIO LN	-0,00005	0,00805	**0,096**	**0,148**	**0,484**
RR LN	0,00000	**0,00290**	-0,008	0,046	**0,516**
RRS LN	0,00015	**0,00608**	0,005	0,078	**0,523**
RSA LN	0,00004	**0,00296**	0,014	0,027	**0,395**
SAB LN	0,00060	**0,00263**	0,016	**0,117**	**0,353**
SBRY LN	0,00027	0,00433	0,041	**0,150**	**0,394**
SDR LN	0,00003	0,00835	0,054	**0,063**	**0,381**
SGE LN	**0,00028**	**0,00309**	-0,008	**0,153**	**0,519**
SHP LN	0,00003	**0,00540**	0,004	0,047	**0,399**
SL LN	0,00087	**0,00194**	**0,094**	**0,116**	**0,425**
SMIN LN	0,00099	0,00238	**0,104**	**0,183**	**0,261**
SN LN	0,00087	0,00263	0,024	**0,241**	**0,599**
SSE LN	**-0,00202**	0,00220	0,032	**0,072**	**0,430**
STAN LN	-0,00011	**0,00350**	-0,002	**0,251**	**0,411**
SVT LN	0,00124	0,00766	0,046	0,004	**0,667**
TATE LN	-0,00019	**0,00352**	0,032	0,049	**0,270**
TLW LN	-0,00005	**0,00503**	0,043	**0,112**	**0,453**
TPK LN	0,00008	0,00190	0,056	**0,153**	**0,395**
TSCO LN	-0,00003	**0,00399**	**0,070**	0,016	**0,390**
ULVR LN	**-0,00359**	0,00211	0,010	**0,067**	**0,423**
UU LN	**0,00049**	0,00628	0,059	**0,096**	**0,471**
VOD LN	**0,00098**	0,00524	-0,010	**0,095**	**0,405**
WEIR LN	0,00057	0,00378	**0,108**	**0,190**	**0,431**
WMH LN	-0,00004	0,00506	0,058	**0,117**	**0,306**
WOS LN	0,00098	0,00300	**0,065**	**0,116**	**0,478**
WPP LN	0,00004	0,00870	0,014	**0,099**	**0,543**
WTB LN	**0,00088**	**0,00355**	0,089	**0,179**	**0,492**

Table 3: Columns 2 and 3 report the estimated μ_p and μ_{v2} parameters for the last 44 FTSE 100 stocks. The bolded parameters are statistically significant at the 95% level of confidence. Columns 4 to 6 report the estimated first order serial correlation coefficients of returns, squared returns and volume time series, respectively. We perform Pearson test to assess the statistic significance of the correlation coefficients. The bolded coefficients are statistically significant at the 95% level of confidence.

Stock	μ_p	μ_{v2}	$\rho_{(R_t,R_{t-1})}$	$\rho_{(R_t^2,R_{t-1}^2)}$	$\rho_{(V_t,V_{t-1})}$
AAL LN	0,00007	**0,00353**	0,033	0,032	**0,515**
AMEC LN	0,00037	**0,00384**	**0,070**	**0,186**	**0,445**
ARM LN	0,00018	**0,00308**	0,038	**0,096**	**0,497**
AV LN	0,00016	**0,00362**	0,036	**0,121**	**0,476**
AZN LN	0,00012	**0,00248**	0,022	0,054	**0,444**
BA LN	0,00017	**0,00269**	-0,022	**0,269**	**0,558**
BATS LN	0,00008	**0,00149**	-0,035	**0,082**	**0,417**
BG LN	0,00025	**0,00187**	**0,063**	**0,095**	**0,382**
BRBY LN	0,00000	**0,00392**	**0,109**	0,005	**0,491**
BSY LN	0,00000	**0,00369**	0,049	0,036	**0,425**
CNA LN	0,00022	**0,00215**	-0,024	**0,102**	**0,433**
CPG LN	0,00063	**0,00222**	-0,055	**0,129**	**0,464**
CPI LN	0,00081	**0,00284**	0,032	**0,159**	**0,449**
CRH LN	0,00010	**0,00364**	0,050	0,046	**0,276**
GKN LN	0,00070	**0,00421**	**0,105**	**0,101**	**0,363**
HSBA LN	0,00022	**0,00145**	-0,005	**0,066**	**0,528**
IMT LN	*-0,00014*	**0,00332**	0,047	**0,078**	**0,425**
JMAT LN	-0,00005	**0,00398**	0,026	**0,163**	**0,477**
MGGT LN	0,00042	**0,00310**	0,055	**0,115**	**0,350**
NG LN	0,00031	**0,00225**	0,057	**0,075**	**0,771**
NXT LN	0,00005	**0,00561**	0,029	0,029	**0,567**
REX LN	0,00144	**0,00296**	**0,104**	**0,076**	**0,449**
RR LN	0,00000	**0,00290**	-0,008	0,046	**0,516**
RRS LN	0,00015	**0,00608**	0,005	0,078	**0,523**
RSA LN	0,00004	**0,00296**	0,014	0,027	**0,395**
SAB LN	0,00060	**0,00263**	0,016	**0,117**	**0,353**
SHP LN	0,00003	**0,00540**	0,004	0,047	**0,399**
SL LN	0,00087	**0,00194**	**0,094**	**0,116**	**0,425**
STAN LN	-0,00011	**0,00350**	-0,002	**0,251**	**0,411**
TATE LN	-0,00019	**0,00352**	0,032	0,049	**0,270**
TLW LN	-0,00005	**0,00503**	0,043	**0,112**	**0,453**
TSCO LN	-0,00003	**0,00399**	**0,070**	0,016	**0,390**

Table 4: 2nd stock category: the pure short-term liquidity friction case

Stock	μ_p	μ_{v2}	$\rho_{(R_t, R_{t-1})}$	$\rho_{(R_t^2, R_{t-1}^2)}$	$\rho_{(V_t, V_{t-1})}$
ABF LN	**-0,00004**	0,00371	-0,036	**0,117**	**0,477**
ADN LN	**0,00073**	0,00428	0,056	0,045	0,060
BARC LN	**-0,00445**	0,00449	0,058	**0,097**	**0,426**
BLT LN	**-0,00066**	0,00476	**0,082**	**0,129**	**0,447**
BNZL LN	**-0,00070**	0,00185	**0,066**	**0,161**	**0,296**
BP LN	**0,00130**	0,00131	**0,109**	**0,356**	**0,670**
DGE LN	**0,00030**	0,00319	-0,025	0,039	**0,370**
EXPN LN	**-0,00041**	0,00301	0,024	0,031	**0,410**
GFS LN	**-0,00087**	0,00262	0,012	**0,154**	**0,488**
ITV LN	**0,00102**	0,00671	0,041	**0,072**	**0,421**
KGF LN	**0,00169**	0,00306	0,051	**0,117**	**0,543**
LGEN LN	**0,00346**	0,00225	0,083	**0,225**	**0,593**
ULVR LN	**-0,00359**	0,00211	0,010	**0,067**	**0,423**
UU LN	**0,00049**	0,00628	0,059	**0,096**	**0,471**
VOD LN	**0,00098**	0,00524	-0,010	**0,095**	**0,405**

Table 5: 3rd stock category: the pure long-term liquidity friction case

Stock	μ_p	μ_{v2}	$\rho_{(R_t, R_{t-1})}$	$\rho_{(R_t^2, R_{t-1}^2)}$	$\rho_{(V_t, V_{t-1})}$
BAB LN	**0,00034**	**0,00379**	0,001	**0,254**	**0,405**
BT/A LN	**0,00395**	**0,00150**	-0,010	**0,074**	**0,563**
GSK LN	**-0,00052**	**0,00184**	-0,026	**0,120**	**0,398**
ITRK LN	**0,00397**	**0,00245**	0,031	**0,080**	**0,191**
RB LN	**0,00020**	**0,00233**	-0,059	**0,166**	**0,418**
RDSA LN	**0,00136**	**0,00087**	0,086	**0,155**	**0,235**
SGE LN	**0,00028**	**0,00309**	-0,008	**0,153**	**0,519**
SSE LN	**-0,00202**	**0,00220**	0,032	**0,072**	**0,430**
WTB LN	**0,00088**	**0,00355**	0,089	**0,179**	**0,492**

Table 6: 4^{rth} stock category: the mixed liquidity friction case