Macro Risks and the Term Structure of Interest Rates

November 18, 2015

Abstract

We extract aggregate supply and demand shocks for the US economy from data on inflation and real GDP growth. Imposing minimal theoretical restrictions, we obtain identification by exploiting non-Gaussian features in the data. Novel measures of the risks associated with supply and demand shocks together with expected inflation and economic activity are the key factors in a tractable model of the term structure of interest rates. Despite having non-Gaussian dynamics for the fundamental factors, we obtain closed-form solutions for yields as functions of these state variables. The presence of supply and demand shocks naturally leads to time variation and even sign changes in the covariance between inflation and economic activity. This feature of the model, coupled with non-Gaussian dynamics in the economic shocks, leads to rich patterns in inflation risk premiums and the term structure of interest rates. Expected inflation and expected real economic activity account for the bulk of the variation in the levels of yields. In contrast, demand and supply risks predominantly account for risk and term premiums, with their good and bad components having different effects.

Keywords: macroeconomic volatility, supply and demand shocks, bond markets, inflation risk premium, term premium

JEL codes: E43, E44, G12, G13

1 Introduction

We formulate a no-arbitrage term structure model, which incorporates novel factors capturing the risks associated with aggregate supply and demand shocks for the U.S. economy. We extract these shocks from data on real activity and inflation. Imposing minimal theoretical restrictions, we define aggregate supply shocks as shocks that move inflation and real activity in the opposite direction. Similarly, demand shocks are innovations that move inflation and real activity in the same direction. Blanchard (1989) finds empirically that the joint behavior of output, unemployment, prices, wages and nominal money in the U.S. is approximately consistent with such an interpretation of macroeconomic fluctuations.

In a classic paper, Blanchard and Quah (1989) use a vector-autoregressive dynamic structure to identify demand-like shocks with shocks that affect output temporarily, whereas supply disturbances have a permanent effect on output, with neither having a long-run effect on unemployment. Instead, we use the Bad-Environment Good-Environment (BEGE) model first introduced by Bekaert and Engstrom (2015) to model non-Gaussianities in the macro data, so that identification arises from these non-linearities without further economic assumptions. The macro risks referred to before represent good and bad volatilities of the structural demand and supply shocks. As the good (bad) volatility increases, the shock distribution becomes less (more) negatively skewed.

The inclusion of aggregate supply and demand factors naturally implies that the covariance between inflation and real activity changes through time and can switch sign, a stylized fact we document below.[1] The sign of this covariance affects the sign and magnitude of the inflation risk premium in the model: when supply shocks dominate, real activity and inflation are negatively correlated, and bonds are a poor hedge against macro-economic fluctuations, requiring a high inflation risk premium. The distinction between supply and demand shocks and between good and bad volatility provides more generally an economic interpretation to the time variation in term and risk premiums, important in monetary economics and asset pricing (see Piazzesi and Swanson, 2008; Wright, 2011; Campbell, Sunderam and Viceira, 2013).

The standard approach in the recent finance tem structure literature has been to model this

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[1]Burkhardt and Hasseltoft (2012) document that the correlation between US consumption growth and inflation changes sign.

time variation using Gaussian time-varying prices within a reduced form affine structure (Dai and Singleton (2002), Duffee (2002), Joslin, Singleton, and Zhu (2011)). While these models are empirically successful along many dimensions, they generate interest rates that have (conditional) distributions that are Gaussian, a feature strongly rejected by the data. They also fail to provide an economic interpretation to the time-variation uncovered. It is also possible that failing to accommodate time-varying non-Gaussian risks to fundamentals causes models to exaggerate the degree of movement in prices of risk that are required to fit the data.[2] In our approach, it is straightforward to add a time-varying price of risk factor to investigate how much non-fundamental variation in prices of risk is necessary to fit the data while conditioning on the observed non-Gaussianities in the fundamental macroeconomic variables.

In terms of the macro-finance term structure literature, our model constitutes a useful compromise between pure reduced form models and fully micro-founded models. Shocks to aggregate supply and demand also feature prominently in DSGE macro models, but these models tend to be tightly parameterized and relatively intractable when it comes to the term structure of interest rates (Van Binsbergen, Fernandez-Villaverde, Koijen, and Rubio-Ramirez (2012), Amisano and Tristani (2010), Andreasen (2011)). Rudebusch and Wu (2009) and Bekaert, Cho and Moreno (2010) solve for the term structure in a New Keynesian framework with a standard investment-savings equation defining demand shocks and an aggregate supply equation defining supply shocks, but their framework implies Gaussian and homoscedastic distributions for interest rates, which is at variance with standard data. Instead, our model features non-Gaussian dynamics for macroeconomic variables, but the term structure itself is affine in the factors, and the model is therefore quite tractable. The macro-finance literature has also given rise to a large number of reduced-form term structure models (see Ang and Piazzesi (2003), Dewachter and Lyrio (2006)) but these models have so far not distinguished between aggregate supply and demand shocks or attempted to model macro-risks using the component model of good and bad volatilities.

Other means for incorporating non-Gaussian features in a term structure model have emerged in the literature: for instance, quadratic term structure models (Ahn, Dittmar and Gallant,

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[2]Chernov and Mueller (2012) explicitly try to reduce the variation in risk premiums generated by their parameter estimates to circumvent that problem. Joslin, Priebsch and Singleton (2014) identify a persistence bias that also leads to excessively volatile model-implied risk premiums.

2002) or regime-switching models (for example, Bansal and Zhou (2002); Evans (2003), Dai, Singleton and Yang (2007), Ang, Bekaert and Wei (2008), Bikbov and Chernov (2013)). The model presented here competes well with these alternative models in terms of its ability to fit the relevant non-Gaussianities in the data and in terms of tractability. Technically, our model is related to the general pricing model of Le, Singleton and Dai (2012), which extends earlier work by Gourieroux, Monfort, and Polimenis (2002), and also consider gamma distributed state variables, as we do, to model volatility dynamics, which also admit affine pricing solutions. Our model also fits within the generalized transform framework of Chen and Joslin (2012) who show how to obtain moments for yields within large class of non-linear models. None of these models feature the economic structure we extract from the data however.

Our main findings lie in the areas of macroeconomic and asset pricing. We find that demand shocks exhibit pronounced skewness, driven, for instance, by spikes in bad volatility for demand shocks during the Great Recession. Supply shock variances are high during the seventies and again more recently since 2005. The correlation between GDP growth and inflation is periodically negative, but peaks in positive territory during the Great Recession when demand shocks are important. In the asset pricing application, this time variation in the covariance between inflation and economic activity, coupled with their non-Gaussian dynamics leads to time variation in inflation risk premiums and the term structure. The macroeconomic variables account for over 70 percent of the variation in the yields, mainly attributed to expected GDP growth and inflation. In contrast, macroeconomic risks (supply and demand volatilities) predominantly account for the predictive power of the macro variables for excess holding period returns. We find differentiating between demand and supply volatilities, and further between their good and bad components, important, as they have different implications for return predictability.

The remainder of the paper is organized as follows. In section 1, we set out how we model macro risk factors and extract them from macroeconomic data on inflation, real GDP growth and surveys for the U.S. We envision this section to be of interest to macro-economists not interested in term structure modeling. In Section 2, we produce some preliminary empirical analysis linking the macro-factors to term structure data. We also assess whether they have predictive power for excess bond returns. In section 3, we set out the model and provide general term structure solutions. In Section 4 we report on the estimation of the

term structure model and discuss the economic implications in terms of a decomposition of nominal interest rates in real rates, expected inflation and an inflation risk premium over time. We discuss the importance and role of the macroeconomic factors.

2 Modeling Macro-Risks

2.1 The Empirical Model

2.1.1 The Model

Consider a simple bivariate system in real GDP Growth (g_t) and inflation (π_t):

$$
\begin{aligned}
g_t &= E_{t-1}[g_t] + u_t^g, \\
\pi_t &= E_{t-1}[\pi_t] + u_t^\pi,
\end{aligned}
\tag{1}
$$

In a first departure of standard macro-economic modeling, the shocks to output growth and inflation are a function of two structural shocks, u_t^s and u_t^d:

$$
\begin{aligned}
u_t^\pi &= -\sigma_{\pi s} u_t^s + \sigma_{\pi d} u_t^d, \\
u_t^g &= \sigma_{gs} u_t^s + \sigma_{gd} u_t^d, \\
Cov(u_t^d, u_t^s) &= 0, Var(u_t^d) = Var(u_t^s) = 1.
\end{aligned}
\tag{2}
$$

The first fundamental economic shock, u_t^s, is an aggregate supply shock in that it moves GDP growth and inflation in opposite directions, as happens in stagflations. The second fundamental shock, u_t^d, is an aggregate demand shock as it moves GDP growth and inflation in the same direction as would be typical in a typical economic boom or recession. Both shocks are assumed uncorrelated. Macro-economists may dispute the labelling of these shocks as supply and demand shocks, although it is the "textbook" definition of macro-shocks. These shocks do not necessarily correspond to demand and supply shocks in, say, a New Keynesian framework (see e.g. Woodford, 2003) or identified VARs in the Sims tradition (Sims, 1980). The classic Blanchard and Quah (1989) paper famously identifies "demand like" shocks with shocks that affect output only temporary whereas supply disturbances are the ones that have a permanent effect on output, with neither have a long run effect on unemployment. These restrictions allow them to identify a linear VAR and interpret the shocks economically. However, Blanchard (1989) suggests the short-run and long-run

identification as a temporal implication of the standard "Keynesian" shock definition we employ, because supply shocks include productivity shocks which tend to have a longer run effect on output.

From an empirical perspective, the distinction between supply and demand shocks seems surely relevant. Nobody would dispute that in the 1970s "supply shocks" as defined above were dominant at times or that the Great Recession has been accompanied by a rather large negative aggregate demand shock of the type suggested above (see also Mian and Sufi, 2014). For convenience, we therefore stick to this labeling throughout.

The relevance of such shocks for the term structure and finance is self-evident and was first suggested by Fama (1981). In an environment dominated by supply shocks, bonds and stocks are likely to perform equally well or poorly, whereas in a demand shock environment, their returns should correlate negatively. This should cause inflation risk premiums to be higher in a supply than in a demand shock environment.

Note that the covariance matrix of the empirical shocks only yields three coefficients and we need to identify 4 coefficients in equation (2) to extract the supply and demand shocks. Hence, a Gaussian system would yield under-identification. Fortunately, it is well known that macro-economic data exhibit substantial non-Gaussian features (see e.g. Evans and Wachtel (1993) for inflation, and Hamilton (1989) for GDP growth). Our second departure of standard macroeconomic modeling is to assume that the demand and supply shocks follow a Bad Environment Good Environment (BEGE) structure (see Bekaert and Engstrom, 2014). Let $\omega_{x,t}$ be a time t demeaned Γ-distributed variable with scale parameter equal to 1 and shape parameter x. With the scale normalized to 1, the shape parameter for a de-meaned gamma distributed variable determines where the distribution starts (at minus the value of the shape parameter) and all of its moments.

$$u_t^s = \sigma_p^s \omega_{p,t}^s - \sigma_n^s \omega_{n,t}^s,$$
$$u_t^d = \sigma_p^d \omega_{p,t}^d - \sigma_n^d \omega_{n,t}^d. \tag{3}$$

Note that the σ parameters determine the scale and the shape parameters are indicated by either p (for the ω shock with a positive sign) or n (for the ω shock with the negative sign). This explains the BEGE moniker, which was initially developed for consumption growth or stock returns and therefore is most easily explained in the context of GDP growth. Note that positive supply shocks drive up GDP growth as do the demand shocks. Essentially,

5

the positive ω shock will be associated with good volatility and positive skewness (a good environment variable) whereas the bad ω shock will be associated with bad volatility and negative skewness (a bad environment variable). Of course, this good-bad interpretation is not applicable for the demand shock in the inflation equation.

We repeat the distributional assumptions for the ω-shocks:

$$
\begin{aligned}
\omega_{p,t}^d &\sim \Gamma(p_{t-1}^d, 1) - p_{t-1}^d, \\
\omega_{p,t}^s &\sim \Gamma(p_{t-1}^s, 1) - p_{t-1}^s, \\
\omega_{n,t}^d &\sim \Gamma(n_{t-1}^d, 1) - n_{t-1}^d, \\
\omega_{n,t}^s &\sim \Gamma(n_{t-1}^s, 1) - n_{t-1}^s.
\end{aligned} \tag{4}
$$

We denote the shape parameters by p for the good environment variable and by n for the bad environment variables. These shape parameters are assumed to vary through time in a simple autoregressive fashion:

$$
\begin{aligned}
p_t^d &= \bar{p}^d(1 - \phi_p^d) + \phi_p^d p_{t-1}^d + \sigma_p^d \omega_{p,t}^d, \\
p_t^s &= \bar{p}^d(1 - \phi_p^s) + \phi_p^s p_{t-1}^s + \sigma_p^s \omega_{p,t}^s, \\
n_t^d &= \bar{n}^d(1 - \phi_n^d) + \phi_n^d n_{t-1}^d + \sigma_n^d \omega_{n,t}^d, \\
n_t^s &= \bar{s}^d(1 - \phi_n^s) + \phi_n^s p_{t-1}^s + \sigma_n^s \omega_{n,t}^s.
\end{aligned} \tag{5}
$$

Note that by imposing suitable restrictions on the the volatilities of the shocks, the various shape parameter processes can be kept strictly positive even in discrete time.[3]

2.1.2 Macro risks

At this point, we have set out 4 shock economy with potentially 4 state variables, which we collect in $X_t^{mr} = [p_t^s, n_t^s, p_t^d, n_t^d]'$. These 4 state variables summarize the macro-economic risks in the economy. Using the properties of the demeaned gamma distribution, we have, for example:

$$
\begin{aligned}
E_{t-1}[u_t^s] &= 0, \\
E_{t-1}[u_t^{s^2}] &= \sigma_p^{s^2} p_t^s + \sigma_n^{s^2} n_t^s, \\
E_{t-1}[u_t^{s^3}] &= 2\sigma_p^{s^3} p_t^s - 2\sigma_n^{s^3} n_t^s, \\
E_{t-1}[u_t^{s^4}] - 3E_{t-1}[u_t^{s^2}] &= 6\sigma_p^{s^4} p_t^s + 6\sigma_n^{s^4} n_t^s.
\end{aligned} \tag{6}
$$

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[3]The conditions are $\phi_p^d > \sigma_p^d$, $\phi_p^s > \sigma_p^s$, $\phi_n^d > \sigma_n^d$, $\phi_n^s > \sigma_n^s$. The model is essentially the autoregressive conditional variance model explored in Gourieroux and Jasiak (2006) paper.

And analogously for u_t^d.

Thus, the BEGE structure implies that the conditional variance of inflation and output varies through time, with the time-variation potentially coming from either demand or supply shocks, and either "bad" or "good" volatility. In addition, the distribution of inflation and output shocks is conditionally non-Gaussian, with time variation in the higher order moments driven by variation in X_t^{mr}. It is the factor structure on higher order moments that keeps the model parsimonious despite having a very rich non-Gaussian structure.

The model also implies that the conditional variance between inflation and GDP growth shocks is time-varying and can switch signs:

$$Cov_{t-1}[u_t^g, u_t^\pi] = -\sigma_{\pi s}\sigma_{gs}Var_{t-1}u_t^s + \sigma_{\pi d}\sigma_{gd}Var_{t-1}u_t^d. \tag{7}$$

Thus, when demand shocks dominate the covariance is positive but when supply shocks dominate it is negative.

2.1.3 Expected inflation and expected GDP growth

While we empirically estimate the conditional means for inflation and GDP growth, our model is closed by assuming that these expectations follow a first-order VAR:

$$
\begin{aligned}
E_t[g_{t+1}] &= \bar{g} + \rho_{gg}E_{t-1}[g_t] + \rho_{g\pi}E_{t-1}[\pi_t] + \sigma_g^e u_t^g, \\
E_t[\pi_{t+1}] &= \bar{\pi} + \rho_{\pi g}E_{t-1}[g_t] + \rho_{\pi\pi}E_{t-1}[\pi_t] + \sigma_\pi^e u_t^\pi.
\end{aligned}
\tag{8}
$$

Note that for now we assume that the macro-shocks and the shocks to expectations are perfectly correlated. However, as we explain below, this assumption is innocuous for the term-structure model we develop below. The full model in equations (1)-(8) therefore has a total of 6 macroeconomic state variables, $X_t = [X_t^{mr}, E_t[g_{t+1}], E_t[\pi_{t+1}]]$.

3 Identifying macro-risks in the US economy

While there are multiple ways to estimate the system in Equations (1)-(8), the presence of the gamma distributed shocks makes the exercise non-trivial. We therefore split up the problem in various manageable steps. First, we use linear projections on a set of information variables to come up with empirical estimates of the conditional mean of GDP growth and

inflation. Second, we filter the demand and supply shocks from the system in Equation (2) by essentially estimating a GMM system that includes higher-order moments. The use of higher-order moments is essential to achieve identification. Third, once the demand and supply shocks are filtered, we can estimate a univariate BEGE system using approximate maximum likelihood as in Bates (2006). This section ends by discussing the empirical properties of the identified macro-risks.

3.1 Conditional means

The data consists of the quarterly time series of US real GDP growth and inflation, survey forecasts of real GDP growth and inflation and the Anxious index. Inflation is defined as percentage changes in consumer price index for all urban consumers for all items. Real GDP growth is defined as the percentage change in the seasonally and inflation adjusted value of the goods and services produced by labor and property in the United States. The Anxious index estimates the probability that real GDP will decline in the next quarter; the survey forecasts for inflation and GDP growth are also for the next quarter. For all three forecast measures, we take the median forecast across panelists. Growth rates are in natural logs. The CPI data is obtained from the Federal Bank of St. Louis website. The GDP data is obtained from the NIPA website. The survey and the Anxious index data is from the Survey of Professional Forecasters. Due to the availability of the survey forecasts, the data is quarterly from 1969Q1 to 2012Q3.

Our approach is to simply consider all possible combinations up to three lags in linear projections and use the Bayesian information criterion (BIC) to select the optimal combination. Appendix A reports the top 10 models for each equation. The resulting model is:

$$
\begin{aligned}
g_{t+1} &= \underset{(0.0014)}{0.0064^{***}} + \underset{(0.0951)}{0.3401^{***}} g_t + \underset{(0.0783)}{-0.1721^{**}} \pi_t \\
\pi_{t+1} &= \underset{(0.0010)}{-0.0002} + \underset{(0.1772)}{0.9055^{***}} \pi_{t,t+1}^e + \underset{(0.1202)}{0.2355^{**}} \pi_t
\end{aligned}
\tag{9}
$$

Note that the optimal model for GDP growth would be implied by a first-order VAR, whereas for inflation the median survey forecast enters as an important predictor with a 0.9055 weight, but past inflation still enters significantly too. Not surprisingly, the inflation conditional mean is highly but not perfectly correlated with the corresponding survey

8

forecast, the correlation being 0.8608. For GDP growth, the correlation between the survey forecast and our conditional mean is only 0.42.

Interestingly, when we use these estimates in equation (10) to derive the residuals for inflation and GDP expectations, we find these residuals to be highly correlated with the actual GDP growth and inflation residuals of equation (1). The correlation is 0.9035 for GDP growth and expected GDP growth residuals and 0.8487 for inflation and expected inflation residuals. This high correlation rationalizes the implicit assumption in equation (8) of having the shocks to the macro variables and their expectations identical. However, economically, we identify the residual coefficients in equation (8) by projecting expected GDP growth and expected inflation onto the supply and demand shocks we identify next, leaving room for uncorrelated measurement error that will not enter the term structure model we develop below.

3.2   Identifying supply and demand shocks

The estimation of the conditional means delivers time series observations on u_t^g and u_t^π. Theoretically, it is of course possible to estimate the system (2)-(5) in one step, but computationally this is a very tall order. There are 4 unobserved state variables (the X_t^{mr} vector) which have non-Gaussian innovations. However, note that we if somehow can identify the 4 σ-coefficients in (2), we can actually filter the supply and demand shocks from the original GDP growth and inflation shocks. Note that the covariance matrix does not suffice to achieve identification of these shocks as it only delivers three moments but there are 4 parameters to identify. In standard macro, additional identification restrictions would be imposed on VAR; for example, the famous long-run restrictions imposed by Blanchard and Quah (1989) to identify demand shocks. In our model, identification can simply be achieved using higher-order moments, which are constrained by the BEGE system.

The BEGE system implies that the following 7 moments can be written as a function of 6

parameters $(\sigma_{gs}, \sigma_{gd}, \sigma_{\pi s}, \sigma_{\pi d}, E(u_t^{d^3}), E(u_t^{s^3}))$:

$$E(u_t^{g^2}) = \sigma_{gs}^2 + \sigma_{gd}^2,$$

$$E(u_t^{g^3}) = 2\sigma_{gs}^3 E(u_t^{s^3}) + 2\sigma_{gd}^3 E(u_t^{d^3}),$$

$$E(u_t^{\pi^2}) = \sigma_{\pi s}^2 + \sigma_{\pi d}^2,$$

$$E(u_t^{\pi^3}) = -2\sigma_{\pi s}^3 E(u_t^{s^3}) + 2\sigma_{\pi d}^3 E(u_t^{d^3}), \tag{10}$$

$$E(u_t^g u_t^\pi) = -\sigma_{gs}\sigma_{\pi s} + \sigma_{gd}\sigma_{\pi d},$$

$$E(u_t^g u_t^{\pi^2}) = 2\sigma_{gs}\sigma_{\pi s}^2 E(u_t^{s^3}) + 2\sigma_{gd}\sigma_{\pi d}^2 E(u_t^{d^3}),$$

$$E(u_t^{g^2} u_t^\pi) = -2\sigma_{gs}^2 \sigma_{\pi s} E(u_t^{s^3}) + 2\sigma_{gd}^2 \sigma_{\pi d} E(u_t^{d^3}).$$

We use the generalized method of moments (GMM) to estimate this simple moments system. As the weighting matrix for the GMM optimization, we use the inverse of the covariance matrix of the moment conditions. We compute this matrix by bootstrapping 10,000 time series of historical length consisting of u_t^g and u_t^π, computing the moments in (10) for each of the time series and calculate the covariance matrix over the 10,000 replications.

Details on the estimation are reported in Table 1. Panel A reports the moments used in the estimation. Note that unconditionally GDP growth and inflation shocks are negatively correlated, as would be the case in a stagflation environment. Inflation shocks exhibit negative, not positive skewness, whereas GDP growth shocks exhibit positive skewness, but it is not statistically significantly different from zero. The parameter estimates for the σ-coefficients are reported in Panel B; each being very precisely estimated. The test for the over-identifying restrictions fails to reject. Panel C summarizes the unconditional properties of extracted demand and supply shocks. The non-Gaussianity is stronger for demand shocks which are very leptokurtic and exhibit negative skewness. Much of this behavior turns out to be driven by the Great Recession, as we will show below. Supply shocks show little skewness but are leptokurtic. The Jarque-Bera normality test rejects Gaussianity for both shocks.

3.3   Maximum likelihood estimation

The details of the estimation are in Appendix B and here only the informal intuition is presented. Only the demand shock estimation is considered, as the supply shock estimation

is identical. The system to estimate is:

$$u_{t+1}^d = \sigma_p^d \omega_{p,t+1}^d - \sigma_n^d \omega_{n,t+1}^d,$$

$$\omega_{p,t+1}^d \sim \Gamma(p_t^d, 1) - p_t^d,$$

$$\omega_{n,t+1}^d \sim \Gamma(n_t^d, 1) - n_t^d, \tag{11}$$

$$p_{t+1}^d = \bar{p}^d + \rho_p^d(p_t^d - \bar{p}^d) + \sigma_{pp}^d \omega_{p,t+1}^d,$$

$$n_{t+1}^d = \bar{n}^d + \rho_n^d(n_t^d - \bar{n}^d) + \sigma_{nn}^d \omega_{n,t+1}^d,$$

where only the time series of demand shock realizations, $\{u_t^d\}_{t=1}^T$ is observed. The estimation consists of three stages, following the approach in Bates (2006):

Stage 0. Initialization. At time 0, the distributions of p_0^d and n_0^d are initialized with the unconditional distributions of p_t^d and n_t^d.

Stage 1. Computing the likelihood. The likelihood of the observation u_1^d given the distributions of p_0^d and n_0^d is computed as in lines 1-3 of (11).

Stage 2. Bayesian updating of the p_0^d and n_0^d distributions given the value of u_1^d. Note from lines 1-3 of (11), that for some values of p_0^d and n_0^d the likelihood of observing u_1^d will be higher than for others. The prior distributions of p_0^d and n_0^d are updated so that these values yielding higher likelihoods get more probability weight. For instance, if u_1^d is very negative, the expected value of n_0^d is likely to go up and the expected value of p_0^d is likely to go down.

Stage 3. Computing the conditional distributions of p_1^d and n_1^d given the value of u_1^d. This is done using the evolution processes in lines 4 and 5 of (11). The distributions of n_0^d and p_0^d are available from the previous stage, but also the distributions of $\omega_{p,1}^d$ and $\omega_{n,1}^d$ are needed. To compute these distributions, note from the first line of (11) that u_1^d is a linear combination of $\omega_{p,1}^d$ and $\omega_{n,1}^d$. From lines 2 and 3 of (11), the distributions of $\omega_{p,1}^d$ and $\omega_{n,1}^d$ depend on p_0^d and n_0^d, respectively. Thus, given the distributions of p_0^d and n_0^d, some of the $\omega_{p,1}^d$ and $\omega_{n,1}^d$ combinations[4] will be more likely than others, yielding the distributions of $\omega_{p,1}^d$ and $\omega_{n,1}^d$.

Stages 1-3 are then repeated for all the following time points. The total likelihood is computed as the sum of individual likelihoods from stage 1. Note that from stage 2 the estimation also yields the expected values of the state variables, p_t^d and n_t^d. The estimation is conducted under the restrictions that $\sigma_{pp}^d < \rho_p^d$, $\sigma_{pp}^s < \rho_p^s$, $\sigma_{nn}^d < \rho_n^d$, $\sigma_{nn}^s < \rho_n^s$, preventing

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[4]It is appropriate to speak about the combinations, because, given u_1^d, from the first line of (11) the value of $\omega_{p,1}^d$ defines the value of $\omega_{n,1}^d$ and vice versa.

shape parameters from going negative, which ensures the accuracy of the closed form solutions for the term structure model developed in Section 5. Standard errors are computed using a parametric bootstrap, where 250 time series of historical length are simulated using the estimated model parameters and parameters are reestimated for each time series.

The parameter estimates are reported in Table 2. We report the various parameter estimates including the unconditional values of the p_t and n_t variables, which determine the extent of non-Gaussianity in the shock distributions. For the demand shocks, we find that, unconditionally, the good demand variable is relatively Gaussian, but the bad environment variable is very non-Gaussian. This variable is also far less persistent than the good environment variable, therefore capturing rather short-lived recessionary bursts. For the supply shocks, the estimation failed to discover any time-variation (σ_{pp}^s is hitting its 0 lower bound) in the good variance, so p_t^s was set to a constant.

Yet, the (time invariant) distribution of the good environment shock is relatively non-Gaussian with skewness equal to 1.55 and excess kurtosis equal to 3.26. The bad-environment distribution is relatively more Gaussian with the unconditional mean of the shape parameter equal to 4.38. The bad-environment shape parameter is highly persistent with autoregressive coefficient equal to 0.99. This could imply, for example, that supply driven recessions are longer-lived than demand driven ones.

In Table 3 we document that the BEGE system fits the higher order moments of inflation and GDP growth rather well. We show the fit with the volatility, skewness, excess kurtosis and left and right tail probabilities and the correlation between inflation and GDP shocks. With the exception of the kurtosis of inflation, which is estimated to be too high, all model moments are well within a two standard error band of the empirical point estimate. This is also true for the skewness of GDP growth which is, surprisingly, positive in the data but negative in the BEGE model.

3.4 Macro-risks in the US economy

Having filtered demand and supply shocks, and their "good" and "bad" volatility, the "macro-risks", we can now re-interpret the history of real activity and inflation over the 1970-2012 period.

In Figure 1, we show the extracted demand shocks, and the conditional good (p_t^d) and bad

(n_t^d) variances. The Great Recession demand shock, occurring in the third quarter of 2008, is very apparent in the shock panel. For all variance statistics, recall that while the shocks are normalized to have a unit unconditional variance, the good and bad components may of course have different relative variances, which in turn may vary through time. The good demand variance was relatively high in the 70s and the early 80s, and then decreased to low levels consistent with the Great Moderation. It has increased slightly since, roughly, 2005. However, the bad demand variance is almost constant over time with just a few peaks, namely one in the mid-80s and 8 quarters during the Great Recession. This pattern is consistent with the relatively low autocorrelation we estimated for the n_t^d process.

Figure 2 performs the same exercise for supply shocks. The shock graph already shows the higher volatility prevalent during the stagflationary 70s, with volatility recurring over the last decade. The good variance is time-invariant at a relatively low level. The bad variance remains high until the early 80s, and then decreases to quite low levels, before rising again after 2005. Together with a similar pattern uncovered for the demand variance, it appears that the BEGE system captures the Great Moderation period of lower inflation and GDP growth variances rather well. The results in Bekaert et.al. (2015) suggest that the period roughly lasted from 1980 to 2007. The results here suggest a somewhat shorter period from 1985 to 2005.

Figure 3 super-imposes the conditional demand and supply variances. While most of the time the contribution of supply and demand variances to the conditional variance is about the same, the demand variance distribution peaks in the Great Recession (at 500%) and is also relatively high in the early 70:s and right after 1985.

In Figure 4, we plot the conditional variances of inflation and GDP growth. The inflation variance is relatively elevated in the 70:s, before declining to quite low levels in the 90:s. By the mid-2000:s, it starts to rise again and then peaks in the Great Recession. For GDP growth, the conditional variance follows a similar pattern, but the recent rise in volatility is more subdued with the volatility remaining high during most of the Great Recession till the end of the sample. Unconditionally, if we decompose the variance of GDP growth (inflation) shocks attributable to demand shocks, we find it to be 7.75% (84.91%). So, as in RBC models real output fluctuations are mostly supply driven and inflation is mostly demand driven. This does not mean that GDP growth shocks are always mostly supply driven as the demand shock is very skewed and can be quite large at times. For example,

in Figure 5, we use our filtered estimates to show the demand component to both inflation shocks (bottom panel) and GDP growth shocks (top panel). Whole it is obvious that most of the demand shock identification comes from inflation shocks, the Great Recession does coincide with large negative demand driven GDP growth shocks. What helps account for the dominance of supply shocks in explaining GDP growth variation are the seventies, where demand shocks are tiny.

Figure 6 (top panel) plots the conditional correlation between GDP growth and inflation. The covariance is predominantly negative, and on average it is -0.1084. In the bottom panel, we provide some economic interpretation by splitting the conditional covariance in demand and supply components. Consistent with our main intuition, the demand component of the covariance is always positive, but is mostly dominated by the negative supply component. Only in the 2000:s, when demand shocks become relatively more important, and particularly during the Great Recession do demand shocks increase the covariance between inflation and GDP growth, making it occasionally positive with a peak at the height of the Great Recession.

4 Macro-Risks and the Term Structure: An Initial Look

Having identified the macro-risks and expected inflation and expected GDP growth from macro-data, we now explore their dynamic correlations with the term structure. To this end, we use the zero coupon data on nominal US Treasury bonds from Gürkaynak, Sack and Wright (2010), yielding interest rates with a maturity of 1 quarter, and 1 to 15 years. We denote these yields as $y_{n,t}$ with n the maturity in quarters. We address two basic questions. First, how are our macro-economic factors related to the yield curve? Second, do the macro-shocks capture predictable components in (excess) bond returns?

4.1 Macro Risks and the Yield Curve

We start by computing the classic yield curve factors. The level factor is the equally weighted average of all yields (from the one year to the 15 year maturity); the slope factor is the

difference between the 10 year yield and one quarter yield; and finally, the curvature factor subtracts twice the two-year rate from the sum of the one quarter rate and the 10 year yield. Table 4 shows regressions of these factors and, in addition, of the 1 quarter, 1 year and 10 year yields on the 5 macroeconomic state variables. We also show regressions using the one quarter and the 10 year yield. Lets start with examining our findings regarding the level factor. First, the explanatory power of these pure macro-economic factors for the level factor is large with the R^2:s exceeding 70%. Figure 7 shows that the overall explanatory power does not vary much with maturity and is highly statistically significant. Second, the macro-risks contribute in a statistically significant fashion to this explanatory power, but their contribution to R^2 is far less than that of expected GDP growth and inflation, with the R^2 increasing from about 65% (59%) to about 71% (73%) at the 1 quarter (10 year) maturity. Thus, the explanatory power of the macro risks is more apparent for longer maturities. Figure 7 shows this clearly by also graphing the R^2 while excluding the macro-risk factors. While the R^2 increase from including the macro-risk factors is overall perhaps modest, it is statistically significant at the 1% level as shown by the p-values of the joint F-test in Table 4. Individually, the coefficients are mostly not statistically significant, although the signs are mostly consistent across equations Good demand and bad supply variance risk have a negative effect on yields whereas bad demand risk has a positive effect. In contrast, the regression coefficients for expected GDP growth and expected inflation are highly statistically significant. These coefficients are consistent with a simple Taylor with a weight on expected inflation larger than 1, and the weight on output growth much smaller (see e.g. Bekaert, Cho and Moreno (2010); Clarida, Gali and Gertler (2009)). Of course, in standard New-Keynesian models, the inflation target is subtracted from expected inflation; the output measure is typical the output gap and the monetary policy equation features interest rate smoothing and a discretionary shock. Yet, these coefficients seem economically reasonable.

The bottom two panels in Table 4 focus on the slope and curvature. For the slope (10 year rate minus 1 quarter rate), the R^2 of all macro factors amounts to 42%. Interestingly, expected GDP growth and inflation do not have a significant effect on the slope, but two of three macro risks do. Again, an F test rejects the null of the coefficients on the macro risk factors jointly being zero, and the R^2 drops to 18% when they are eliminated. Panel B in Figure 7 shows the total R^2 and the R^2 when the macro risk factors are dropped for the

slope over the whole maturity spectrum, that is, we use all yields between the 1 year and the 15 year yield as the long maturity in the slope computation. The total R^2 increases with maturity, but now the relative importance of the macro-risk factors decreases with maturity.

The final panel in Table shows that the curvature factor is significantly negatively correlated with expected GDP growth and inflation, but none of the macro risk factor is significant. The decrease in R^2 when they are dropped from the regression is economically small and only significant at the 10% level.

4.2 Macro Risks and Bond Return Predictability

The literature on bond return predictability is quite voluminous, but mostly focuses on using information extracted from the yield curve to predict future holding period returns (e.g. Cochrane and Piazzesi (2005)). Ludvigson and Ng (2009) find that "real" and "inflation" factors, extracted from a large number of macro-economic tomes series, have important forecasting power for future excess returns on U.S. government bonds. Moreover, this forecastability is above and beyond the predictive power contained in forward rates and yield spreads. Also, the bond risk premia have a marked countercyclical component. Cieslak and Pavola (2015) uncover short-lived predictability in bond returns by controlling for a persistent component in inflation expectations.

In Table 5, we explore the link between bond returns and our macro factors. We focus on excess holding period returns relative to the one year yield (so the shortest maturity we consider is two years). Because we use overlapping quarterly data, all standard errors use 40 Newey-West (1987) lags.[5] In the most general regression, we use the 5 state variables as predictive variables but we also include the actual macro shocks as regressors. If the predictability reflects risk premiums, it would be logical to see opposite signs on the predictive and contemporaneous coefficients.

Excluding the contemporaneous variables, the predictive R^2 increases from about 5% for two year maturity bonds to about 10% for 10 year maturity bonds. The bulk of this predictability is coming from the macrorisk factors and not from expected GDP growth and inflation. As can also be seen from Figure 10, expected GDP growth is only statistically significant at

---

[5]This is the (rounded) optimal number of lags according to procedures discussed in Newey and West (1994) given the autocorrelation structure and the length of our data.

low horizons and expected inflation never is. Figure 9 shows that the predictive R^2 raises until about maturity 8 and then stays relatively flat. Adding the innovations increases the R^2 to about 14% at the 2 and 24% at the 10 year horizon.

As Table 5 and Figure 10 show, the coefficients on the macro risks have consistent signs across maturity but increase and become more statistically significant the longer the maturity of the bond considered in the predictive regressions. The coefficients on the demand risk (p_t^d and n_t^d) are negative whereas the coefficient on the n_t^s is positive. For the bad environment variables, the coefficients on the macro risk innovations are opposite to that of the predictive coefficients. Hence, if bad demand (supply) volatility increases, bond prices increase (decrease) and expected returns decrease (increase). This is consistent with the intuition that in demand (supply) environments bonds are good (poor) hedges against general macro-economic risks. For the good demand volatility the signs of the predictive and innovation coefficients, however, are the same.

5 A Term Structure Model with Macro Risks

This section develops a reduced-form term structure model, which allows to decompose the fluctuations in nominal yields into the fluctuations in real yields, expected inflation and inflation risk premium. The real log risk-free rate is assumed to be:

$$y_{1,t} = a_0 + a_g E_t g_{t+1} + a_\pi E_t \pi_{t+1} + a_{p^d} p_t^d + a_{n^d} n_t^d + a_{n^s} n_t^s + z_t, \tag{12}$$

where a_0, a_g, a_π, a_{p^d}, a_{n^d}, and a_{n^s} are constants and z_t is a latent factor, which follows an AR(1) process:

$$z_t = \rho_z z_{t-1} + \sigma_z \epsilon_{t+1}^z, \tag{13}$$

where ρ_z and σ_z are constants and ϵ_{t+1}^z is assumed to be a unit variance zero-mean Gaussian shock.

The implied real log stochastic discount factor is:

$$
\begin{aligned}
m_{t+1} =&(-a_0 + f(\lambda_{p^s})\bar{p}^s) \\
&-a_g E_t g_{t+1} - a_\pi E_t \pi_{t+1} - z_t + (-a_{p^d} + f(\lambda_{p^d}))p_t^d + \\
&(-a_{n^d} + f(\lambda_{n^d}))n_t^d + (-a_{n^s} + f(\lambda_{n^s}))n_t^s + \\
&\lambda_{p^d}\omega_{p,t+1}^d + \lambda_{n^d}\omega_{n,t+1}^d + \lambda_{p^s}\omega_{p,t+1}^s + \lambda_{n^s}\omega_{n,t+1}^s + \lambda_z \epsilon_{t+1}^z = \\
&b_0 - a_g E_t g_{t+1} - a_\pi E_t \pi_{t+1} - z_t + b_{p^d}p_t^d + b_{n^d}n_t^d + b_{n^s}n_t^s + \\
&\lambda_{p^d}\omega_{p,t+1}^d + \lambda_{n^d}\omega_{n,t+1}^d + \lambda_{p^s}\omega_{p,t+1}^s + \lambda_{n^s}\omega_{n,t+1}^s + \lambda_z \epsilon_{t+1}^z,
\end{aligned}
\tag{14}
$$

where $f(x) = x + ln(1-x)$, λ_{p^d}, λ_{n^d}, λ_{p^s}, λ_{n^s}, and λ_z are constants, and b_0, b_{p^d}, b_{n^d}, b_{p^s}, b_{n^s} are implicitly defined.

By recursively taking the expectations of the stochastic discount factor, it follows that the price of a real risk-free zero-coupon n period bond at time t, $P_{t,n}$ is:

$$
P_{t,n} = exp(C_n + G_n E_t g_{t+1} + \Pi_n E_t \pi_{t+1} + P_n^d p_t^d + N_n^d n_t^d + N_n^s n_t^s + Z_n z_t),
$$

$$
C_1 = -a_0,
$$

$$
G_1 = -a_g,
$$

$$
\Pi_1 = -a_\pi,
$$

$$
P_1^d = -a_{p^d},
$$

$$
N_1^d = -a_{n^d},
$$

$$
N_1^s = -a_{n^s},
$$

$$
Z_1 = -1,
$$

$$
C_n = C_{n-1} + b_0 + G_{n-1}\bar{g} + \Pi_{n-1}\bar{\pi} + P_{n-1}^d \bar{p}^d (1 - \rho_p^d) +
\tag{15}
$$

$$
N_{n-1}^d \bar{n}^d (1 - \rho_n^d) + N_{n-1}^s \bar{n}^s (1 - \rho_n^s) -
$$

$$
f(\lambda_{p^s} + G_{n-1}\sigma_g^e \sigma_{gs}\sigma_p^s - \Pi_{n-1}\sigma_\pi^e \sigma_{\pi s}\sigma_p^s)\bar{p}^s + \frac{1}{2}(\lambda_z + Z_{n-1}\sigma_z)^2,
$$

$$
G_n = -a_g + G_{n-1}\rho_{gg} + \Pi_{n-1}\rho_{\pi g},
$$

$$
\Pi_n = -a_\pi + G_{n-1}\rho_{g\pi} + \Pi_{n-1}\rho_{\pi\pi},
$$

$$
P_n^d = b_{p^d} + \rho_p^d P_{n-1}^d - f(\lambda_{p^d} + P_{n-1}^d \sigma_{pp}^d + G_{n-1}\sigma_g^e \sigma_{gd}\sigma_p^d + \Pi_{n-1}\sigma_\pi^e \sigma_{\pi d}\sigma_p^d),
$$

$$
N_n^d = b_{n^d} + \rho_n^d N_{n-1}^d - f(\lambda_{n^d} + N_{n-1}^d \sigma_{nn}^d - G_{n-1}\sigma_g^e \sigma_{gd}\sigma_n^d - \Pi_{n-1}\sigma_\pi^e \sigma_{\pi d}\sigma_n^d),
$$

$$
N_n^s = b_{n^s} + \rho_n^s N_{n-1}^s - f(\lambda_{n^s} + N_{n-1}^s \sigma_{nn}^s - G_{n-1}\sigma_g^e \sigma_{gs}\sigma_n^s + \Pi_{n-1}\sigma_\pi^e \sigma_{\pi s}\sigma_n^s),
$$

$$
Z_n = -1 + \rho_z Z_{n-1}.
\tag{16}
$$

Instead of bond prices, logarithmic yields, defined as $y_{t,n} = -\frac{1}{n} \ln P_{t,n}$, will be used.

As usual, the nominal stochastic discount factor is $m_{t+1}^\$ = m_{t+1} - \pi_{t+1}$. Again, the price of a nominal risk-free zero-coupon n period bond at time t, $P_{t,n}^\$$, can be obtained by recursively taking the expectations of the nominal stochastic discount factor:

$$P_{t,n}^\$ = exp(C_n^\$ + G_n^\$ E_t g_{t+1} + \Pi_n^\$ E_t \pi_{t+1} + P_n^{d\$} p_t^d + N_n^{d\$} n_t^d + N_n^{s\$} n_t^s + Z_n^\$ z_t),$$

$$C_1^\$ = b_0 - f(\lambda_{p^s} + \sigma_{\pi s} \sigma_p^s)\bar{p}^s,$$

$$G_1^\$ = -a_g,$$

$$\Pi_1^\$ = -a_\pi - 1,$$

$$P_1^{d\$} = b_{p^d} - f(\lambda_{p^d} - \sigma_{\pi d} \sigma_p^d),$$

$$N_1^{d\$} = b_{n^d} - f(\lambda_{n^d} + \sigma_{\pi d} \sigma_n^d),$$

$$N_1^{s\$} = b_{n^s} - f(\lambda_{n^s} - \sigma_{\pi s} \sigma_n^s),$$

$$Z_1^\$ = -1,$$

$$C_n^\$ = C_{n-1}^\$ + b_0 + G_{n-1}^\$ \bar{g} + \Pi_{n-1}^\$ \bar{\pi} + P_{n-1}^{d\$} \bar{p}^d (1 - \rho_p^d) + \qquad (17)$$

$$N_{n-1}^{d\$} \bar{n}^d (1 - \rho_n^d) + N_{n-1}^{s\$} \bar{n}^s (1 - \rho_n^s) -$$

$$f(\lambda_{p^s} + G_{n-1}^\$ \sigma_g^e \sigma_{gs} \sigma_p^s - \Pi_{n-1}^\$ \sigma_\pi^e \sigma_{\pi s} \sigma_p^s + \sigma_{\pi s} \sigma_p^s)\bar{p}^s + \frac{1}{2}(\lambda_z + Z_{n-1}\sigma_z)^2,$$

$$G_n^\$ = -a_g + G_{n-1}^\$ \rho_{gg} + \Pi_{n-1}^\$ \rho_{\pi g},$$

$$\Pi_n^\$ = -a_\pi - 1 + G_{n-1}^\$ \rho_{g\pi} + \Pi_{n-1}^\$ \rho_{\pi\pi},$$

$$P_n^{d\$} = b_{p^d} + \rho_p^d P_{n-1}^{d\$} - f(\lambda_{p^d} + P_{n-1}^{d\$} \sigma_{pp}^d + G_{n-1}^\$ \sigma_g^e \sigma_{gd} \sigma_p^d + \Pi_{n-1}^\$ \sigma_\pi^e \sigma_{\pi d} \sigma_p^d - \sigma_{\pi d} \sigma_p^d),$$

$$N_n^{d\$} = b_{n^d} + \rho_n^d N_{n-1}^{d\$} - f(\lambda_{n^d} + N_{n-1}^{d\$} \sigma_{nn}^d - G_{n-1}^\$ \sigma_g^e \sigma_{gd} \sigma_n^d - \Pi_{n-1}^\$ \sigma_\pi^e \sigma_{\pi d} \sigma_n^d + \sigma_\pi^d \sigma_n^d),$$

$$N_n^{s\$} = b_{n^s} + \rho_n^s N_{n-1}^{s\$} - f(\lambda_{n^s} + N_{n-1}^{s\$} \sigma_{nn}^s - G_{n-1}^\$ \sigma_g^e \sigma_{gs} \sigma_n^s + \Pi_{n-1}^\$ \sigma_\pi^e \sigma_{\pi s} \sigma_n^s - \sigma_{\pi s} \sigma_n^s),$$

$$Z_n^\$ = -1 + \rho_z Z_{n-1}^\$.$$

The estimation is conducted via maximizing the likelihood of observed bond yields assuming that all bond yields are observed with independent zero-mean Gaussian errors with bond specific variances, which are constant over time. This is implemented using standard Kalmant filtering where z_t is a latent state variable and bond yields are noisy observations.

The data consists of quarterly observations of 1, 5, and 10 year nominal zero-coupon Treasury yields from the online appendix of Gürkaynak, Sack and Wright (2007) from 1969Q1 to 2012Q3 and 2, 5, and 10 year real zero-coupon yields constructed using data from the online appendix of Gürkaynak, Sack and Wright (2010) from 2004Q1 to 2012Q3, as there were

serious liquidity issues in the TIPS markets before that.

The use of the real yields is essential for the identification, because we are interested in decomposing the difference between the nominal rate and the expected inflation into the real rate and the inflation risk premium. Both of these components are unobserved and depend on the stochastic discount factor parameters which we need to estimate. Empirically, this leads to the situation where the likelihood function becomes flat as it is possible to fit the time-varying difference between the nominal yields and the expected inflation (two observables) either through the time-varying real rates or time-varying inflation risk premium, depending on the parameters of the stochastic discount factor. Thus, without using the real rates a number of maxima could be identified.

The TIPS real yields reported in Gürkaynak, Sack and Wright (2010) must be adjusted for liquidity. We use the approach similar to Gürkaynak, Sack and Wright (2010) for this purpose. In particular, we regress the "inflation compensation" variable reported in Gürkaynak, Sack and Wright (2010) on a constant and three measures of TIPS liquidity. The first measure is the relative trading volume of TIPS compared to nominal Treasuries. This is obtained from the FR-2004 forms available on the Federal Reserve Bank of New York website. The second and third measures are the Pastor and Stambaugh (2003) aggregate and traded liquidity factors. Although these are equity liquidity factors, the large literature starting from Chordia, Sarkar, and Subrahmanyam (2005) suggests that there is a strong commonality in liquidity across markets. The regressions of the "inflation compensation" on liquidity factors give the time-variation in liquidity but not its absolute levels. To pin down the absolute levels, we follow Chen, Lesmond, and Wei (2007) and Gürkaynak, Sack and Wright (2010) and set the liquidity premium to 0 at the time point it achieved its lowest value. Figure 11 illustrates the results. We can see that the liquidity premium patterns and levels are very similar to the ones reported in Gürkaynak, Sack and Wright (2010): the liquidity premium for all bonds have been relatively low and spiked during the Great Recession (over 200 basis points for the 2 year real bonds) and the liquidity premium is higher for the bonds with shorter maturities (for instance, for the 10 year bond the spike during the Great Recession had been less than 100 basis points). After the Great Recession, the liquidity premium in TIPS has returned to and below the pre-crisis period, consistently with the evidence in D'Amico, Kim, and Wei (2014).

Table 6 reports the maximum likelihood parameter estimates. Estimates of a_g (around

0.5) and a_π (above 0) are in line with the Taylor rule dynamics. a_{n^s} is negative, which might be corresponding to the precautionary savings effect. a_{p^d} and a_{n^d} are positive but insignificant. As n_t^d behaves like a rare-disaster type variable, the positive coefficient on a_{n^d} might be attributed to the intertemporary smoothing effect. λ_{n^d} and λ_{n^s} are positive and significant, which might be attributed to negative shocks increasing marginal utility. λ_{p^d} and λ_{p^s} are positive, but insignificant. Positive λ_{p^d} might be related to positive inflation shocks decreasing marginal utility (as, e.g., in Ang and Piazzesi, 2003, and Moneta and Baludzzi, 2012).

Figure 12 shows that the model fits the 5 year yields accurately: the correlation between the data and the model is 0.99, implying the R^2 of over 98%. Table 7 confirms this fit for other yields. The fit for the real yields is slightly worse than for nominal yields because, as can also be seen from Figure 12, 2000:s seem to be a slightly more difficult period to fit. Short yields are also more difficult to fit than long-term yields as they are more volatile.

Figure 13 decomposes the 5 year nominal yield into real yields, expected inflation, and inflation risk premium. From the top panel, we can see that the real yields had been around 1-2% in the 1970:s, went up to 7-8% in the first half of the 80:s and had been 3-4% until the early 2000:s when they started to decline hitting 0 in 2004 and during and post the Great Recession. The expected inflation has been 6-8% throughout the 70:s and the early 80:s and stayed at around 2-3% after that, plummetting during the Great Recession. The inflation risk-premium has been over 1.5% in the 70:s and early 80:s, but has remained around 0 after that. This declining pattern is consistent with the economy shifting from the supply environment into the demand environment documented in previous sections.

Table 8 summarizes the unconditional properties of the three term structure components. In terms of both levels and volatility, expected inflation and real yields are most important. Although on average the inflation risk premium is less important, as the bottom panel of Figure 13 shows, its contribution to the yield is strongly time-varying. Our estimates of inflation risk premium are in line with the recent studies using TIPS, such as Grischenko and Huang (2013) and Fleckenstein, Longstaff and Lustig (2014) and are somewhat lower than in studies which do not use real yields such as Buraschi and Jiltsov (2005) and Ang, Bekaert, and Wei (2010). However, as described above, using the real yields is essential for accurate decomposition of the difference between nominal yields and expected inflation into real yields and inflation risk premium.

Figure 14 illustrates that model implied nominal and real yield curves are upward sloping. The top panel shows that the model accurately replicates the nominal yield curve, albeit the model implied curve has slightly higher slope. The real yield curve slope is also positive but smaller than the nominal yield curve slope (around 1 percentage point versus around 2 percentage points) indicating the the term structure of inflation risk premium is upward sloping as well. The estimated real yield curve is consistent with other recent estimates in the literature, such as Ang and Ulrich (2012) and Chernov and Mueller (2012).

Figure 15 plots the time series of the latent factor expected values, which is instructive about episodes difficult to explain using the purely macroeconomic model. In particular, the figure suggests that the high interest rates observed during the Volcker experiment (the latent variable achieves its highest values during that period) and the low interest rates during the Great Recession (the latent variable achieves its lowest values during that period) are difficult for purely macroeconomy driven model, indicating that economically the latent factor might be related to unconventional monetary policy.

6   Conclusion

In this article, we provide three main contributions. First, we develop a new dynamic model for real economic activity and inflation, where the shocks are drawn from a Bad Environment-Good Environment model, which accommodates time-varying non-Gaussian features with "good" and "bad" volatility. The shocks are decomposed into "demand" shocks which move inflation and GDP growth in the same direction and "supply shocks" which move inflation and GDP growth in opposite direction. We find the demand shocks to exhibit marked skewness, driven, inter alia, by peaks in "bad demand volatility" during the Great Recession. Supply shock variances are high during the seventies and again more recently since 2005. The correlation between GDP growth and inflation is mostly negative but peaks during the Great Recession when demand shocks are important. Second, we link the macro factors extracted from the dynamic macro model, expected GDP growth and inflation and the macro risk variables represented by the conditional variances (shape parameters) of the shocks, to the term structure. The macro variables explain a little over 70% of the variation in the levels of yields, but the proportion of explanatory power accounted for by the macro risk variables rises from 8.4% at the one quarter maturity to

19.2% at the 10 year maturity. When we run predictive regressions of excess holding period returns onto the macro variables, the R^2 rises from 5% for 2 maturity bonds to 10% for 10 year maturity bonds with the bulk of it accounted for by the macro-risk variables. Third, we build a term structure model in which the macro factors feature as state variables in addition to one latent variable. Despite the non-Gaussianities in the state variables, the term structure model is affine in the state variables.

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Appendix A - Models of GDP growth and inflation expectations

This appendix compares the optimal linear models of expected GDP growth and inflation. The predictors used are the lagged GDP growth and inflation values, the median survey forecasts of GDP growth and inflation over 1, 2, and 3 quarters ahead and the Federal Reserve Bank of Philadelphia anxious index over 1, 2 and 3 quarters ahead. The table below reports the top 15 optimal linear models in terms of the Bayesian information criterion (BIC).

Optimal Linear Models for GDP Growth, Inflation and Inflation Expectations. The data are quarterly US inflation and real GDP growth. Inflation is defined as percentage changes in consumer price index for all urban consumers for all items. Real GDP growth is defined as the percentage change in seasonally and inflation adjusted value of the goods and services produced by labor and property in the United States. The data is logarithmised. The time span is from 1969Q1 to 2012Q3. BIC is Bayesian information criterion.

Panel A: GDP growth	
Predictors	BIC
GDP 1 lag, inflation 1 lag	-1646
GDP 1 lag	-1644
GDP lag 1, inflation lag 3	-1644
GDP lags 1,2	-1644
GDP 1 lag, anxious 1	-1643
GDP forecast 1, inflation 1 lag	-1642
GDP lag 1, inflation lag 1,3	-1642
GDP lag 1, anxious 2	-1642
anxious 1	-1640
GDP lags 1,3	-1640

Panel B: Inflation	
Predictors	BIC
Inflation lag 1, inflation forecast 1	-1723
Inflation lag 1,2,3	-1718
Inflation lag 1, inflation forecast 1,2	-1715
Inflation lag 1,3, inflation forecast 1	-1714
Inflation forecast 1,2	-1714
Inflation lag 1, GDP growth lag 1, inflation forecast 1	-1710
Inflation forecast 1	-1709
Inflation lag 1,2,3, inflation forecast 1	-1709
Inflation lag 1, GDP growth lag 2, inflation forecast 1	-1709
Inflation lag 1, GDP growth lag 3, inflation forecast 1	-1709

Appendix B - Maximum likelihood estimation of demand and supply shock dynamics

The estimation procedure is a modification of Bates (2006)'s algorithm for the component model of two gamma distributed variables. Below the step-by-step estimation strategy for the demand shock is described. The estimation for the supply shock is identical.

The methodology below is an approximation, because, in order to facilitate the computation, at each time point the conditional distribution (conditioned on the previous realizations of u_t^d of state variables p_t^d and n_t^d is assumed to be gamma, although the distribution does not have a closed form solution. The choice of the approximating distributions is discussed in details in section 1.3 of Bates (2006). Here the gamma distributions are used, because they are bounded from the left at 0, which ensures that the shape parameters of the gamma distribution in the model (p_t^d and n_t^d) will always stay positive, like they should.

The system to estimate is:

$$u_{t+1}^d = \sigma_p^d \omega_{p,t+1}^d - \sigma_n^d \omega_{n,t+1}^d,$$

$$\omega_{p,t+1}^d \sim \Gamma(p_t^d, 1) - p_t^d,$$

$$\omega_{n,t+1}^d \sim \Gamma(n_t^d, 1) - n_t^d,$$

$$p_{t+1}^d = \bar{p}^d + \rho_p^d(p_t^d - \bar{p}^d) + \sigma_{pp}^d \omega_{p,t+1}^d,$$

$$n_{t+1}^d = \bar{n}^d + \rho_n^d(n_t^d - \bar{n}^d) + \sigma_{nn}^d \omega_{n,t+1}^d.$$

The following notation is defined:

$U_t^d \equiv \{u_1^d, ..., u_t^d\}$ is the sequence of observations up to time t.

$F(i\phi, i\psi^1, i\psi^2 | U_t^d) \equiv E(e^{i\phi u_{t+1}^d + i\psi^1 p_{t+1}^d + i\psi^2 n_{t+1}^d} | U_t^d)$ is the next period's joint conditional characteristic function of the observation and the state variables.

$G_{t|s}(i\psi^1, i\psi^2) \equiv E(e^{i\psi^1 p_t^d + i\psi^2 n_t^d} | U_s^d)$ is the characteristic function of the time t state variables conditioned on observing data up to time s.

At time 0, the characteristic function of the state variables $G_{0|0}(i\psi^1, i\psi^2)$ is initialized. As mentioned above, the distribution of p_0^d and n_0^d is approximated with gamma distributions. Note that the unconditional mean and variance of p_t^d are $E(p_t^d) = \bar{p}^d$ and $Var(p_t^d) = \frac{\sigma_{pp}^2}{1-\rho_p^{d2}}\bar{p}^d$, respectively. The approximation by the gamma distribution with the shape parameter k_0 and the scale parameter σ_0^p is done by matching the first two unconditional moments.

Using the properties of the gamma distribution, $k_0^p = \frac{E^2 p_t^d}{Var(p_t^d)}$ and $\theta_0^p = \frac{Var(p_t^d)}{E(p_t^d)}$. Thus, p_0^d is assumed to follow $\Gamma(k_0^p, \theta_0^p)$ and n_0^d is assumed to follow $\Gamma(k_0^n, \theta_0^n)$, where k_0^n and θ_0^n are computed in the same way. Using the properties of the expectations of the gamma variables, $G_{0|0}(i\psi^1, i\psi^2) = e^{-k_0^p \ln(1 - \theta_0^p i\psi^1) - k_0^n \ln(1 - \theta_0^n i\psi^2)}$. Given $G_{0|0}(i\psi^1, i\psi^2)$, computing the likelihood of U_T^d is performed by repeating the steps 1-3 below for all subsequent values of t.

Step 1. Computing the next period's joint conditional characteristic function of the observation and the state variables:

$$F(i\Phi, i\psi^1, i\psi^2 | U_t^d) = E(E(e^{i\Phi(\sigma_p^d \omega_{p,t+1}^d - \sigma_n^d \omega_{n,t+1}^d) + i\psi^1(\bar{p}^d + \rho_p^d p_t^d + \sigma_{pp}^d \omega_{p,t+1}^d) + i\psi^2(\bar{n}^d(1 - \rho_n^d) + \rho_n^d n_t^d + \sigma_{nn}^d \omega_{n,t+1}^d)} | U_t^d)$$

$$= E(e^{i\psi^1 \bar{p}^d(1 - \rho_p^d) + i\psi^2 \bar{n}^d(1 - \rho_n^d) + (i\psi^1 \rho_p^d - \ln(1 - i\Phi\sigma_p^d - i\psi^1 \sigma_{pp}^d) - i\Phi\sigma_p^d - i\psi^1 \sigma_{pp}^d)p_t^d + (i\psi^2 \rho_n^d - \ln(1 + i\Phi\sigma_n^d - i\psi^2 \sigma_{nn}^d) + i\Phi\sigma_n^d - i\psi^2 \sigma_{nn}^d)n_t^d} | U_t^d)$$

$$= e^{i\psi^1 \bar{p}^d(1 - \rho_p^d) + i\psi^2 \bar{n}^d(1 - \rho_n^d)} G_{t|t}(i\psi^1 \rho_p^d - \ln(1 - i\Phi\sigma_p^d - i\psi^1 \sigma_{pp}^d) - i\Phi\sigma_p^d - i\psi^1 \sigma_{pp}^d, i\psi^2 \rho_n^d - \ln(1 + i\Phi\sigma_n^d - i\psi^2 \sigma_{nn}^d) + i\Phi\sigma_n^d - i\psi^2 \sigma_{nn}^d).$$

Step 2. Evaluating the conditional likelihood of the time $t + 1$ observation:

$$p(u_{t+1}^d | U_t^d) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(i\Phi, 0, 0 | U_t^d) e^{-i\Phi u_{t+1}^d}) d\Phi,$$

where the function F is defined in step 1 and the integral is evaluated numerically.

Step 3. Computing the conditional characteristic function for the next period, $G_{t+1|t+1}(i\psi^1, i\psi^2)$:

$$G_{t+1|t+1}(i\psi^1, i\psi^2) = \frac{\frac{1}{2\pi} \int_{-\infty}^{\infty} F(i\Phi, i\psi^1, i\psi^2 | U_t^d) e^{-i\Phi u_{t+1}^d} d\Phi}{p(u_{t+1}^d | U_t^d)}.$$

As above, the function $G_{t+1|t+1}(i\psi^1, i\psi^2)$ is also approximated with the gamma distribution via matching the first two moments of the distribution. The moments are obtained by taking the first and second partial derivatives of the joint characteristic function:

$$E_{t+1}p_{t+1}^d = \frac{1}{2\pi p(u_{t+1}^d | U_t^d)} \int_{-\infty}^{\infty} F_{\psi^1}(i\Phi, 0, 0 | U_t^d) e^{-i\Phi u_{t+1}^d} d\Phi,$$

$$Var_{t+1}p_{t+1}^d = \frac{1}{2\pi p(u_{t+1}^d | U_t^d)} \int_{-\infty}^{\infty} F_{\psi^1\psi^1}(i\Phi, 0, 0 | U_t^d) e^{-i\Phi u_{t+1}^d} d\Phi - E_{t+1}^2 p_{t+1}^d,$$

$$E_{t+1}n_{t+1}^d = \frac{1}{2\pi p(u_{t+1}^d | U_t^d)} \int_{-\infty}^{\infty} F_{\psi^2}(i\Phi, 0, 0 | U_t^d) e^{-i\Phi u_{t+1}^d} d\Phi,$$

$$Var_{t+1}n_{t+1}^d = \frac{1}{2\pi p(u_{t+1}^d | U_t^d)} \int_{-\infty}^{\infty} F_{\psi^2\psi^2}(i\Phi, 0, 0 | U_t^d) e^{-i\Phi u_{t+1}^d} d\Phi - E_{t+1}^2 n_{t+1}^d,$$

where F_{ψ^i} denotes the derivative of F with respect to ψ^i. The expressions inside the integral are obtained in closed form by derivating the function $F(i\Phi, i\psi^1, i\psi^2 | U_t^d)$ in step 1, and integrals are evaluated numerically. Using the properties of the gamma distribution, the values of the shape and the scale parameters are $k_{t+1}^p = \frac{E_{t+1}^2 p_{t+1}^d}{Var_{t+1}p_{t+1}^d}$ and $\theta_{t+1}^p = \frac{Var_{t+1}p_{t+1}^d}{E_{t+1}p_{t+1}^d}$, respectively. The expressions for k_{t+1}^n and θ_{t+1}^n are similar.

The total likelihood of the time series is the sum of individual likelihoods from step 2:

$$L(Y_T) = \ln p(u_1^d | k_0^p, \theta_0^p) + \sum_{t=2}^{T} \ln p(u_{t+1}^d | U_t^d).$$

Figure 1: Demand Shock. The graph is quarterly. The demand shock is defined as a shock which moves GDP growth and inflation in the same direction.
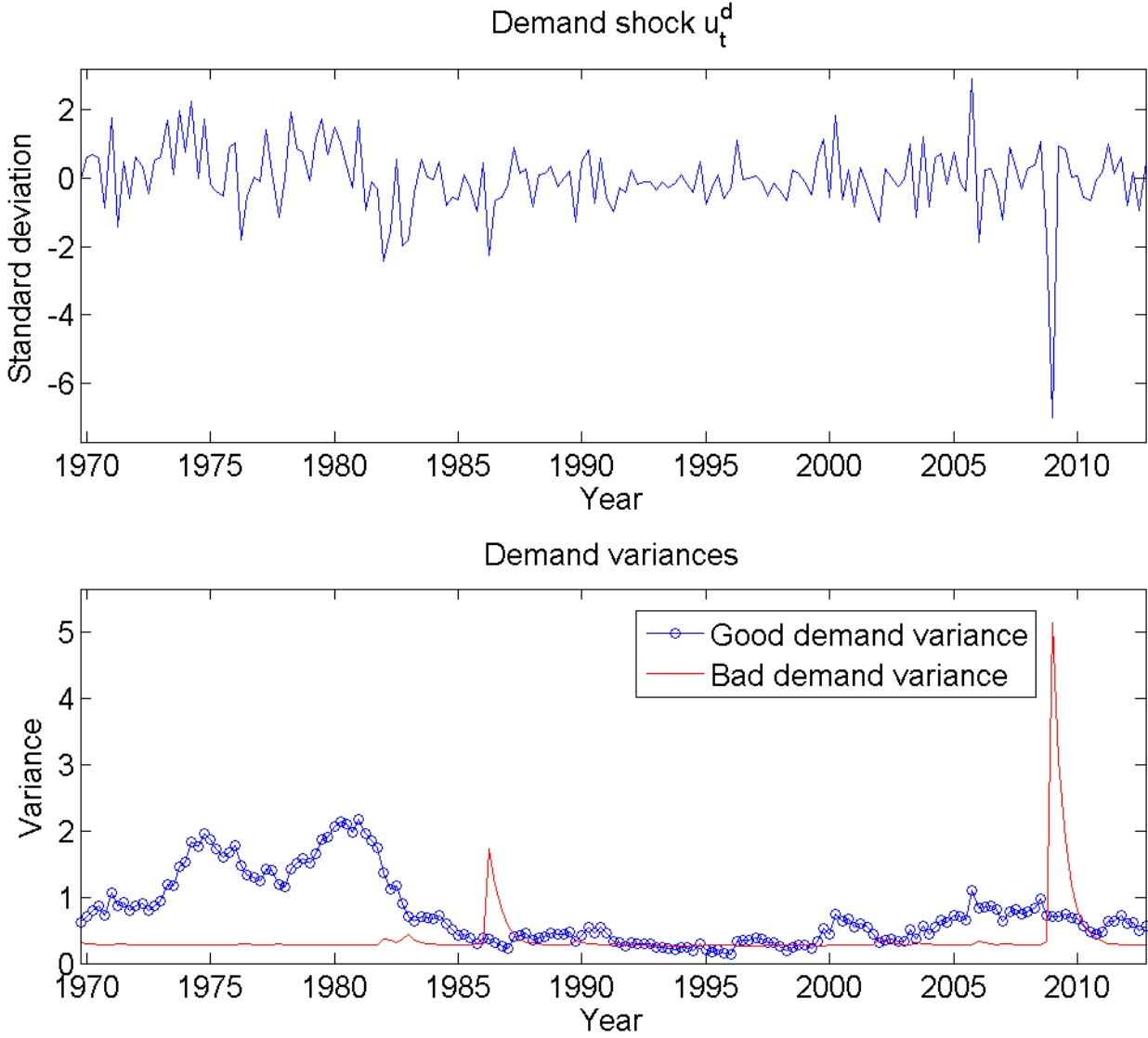
Figure 2: Supply Shock. The data are quarterly US observations from 1969Q1 to 2012Q3. The supply shock is defined as a shock which moves GDP growth and inflation in opposite directions.
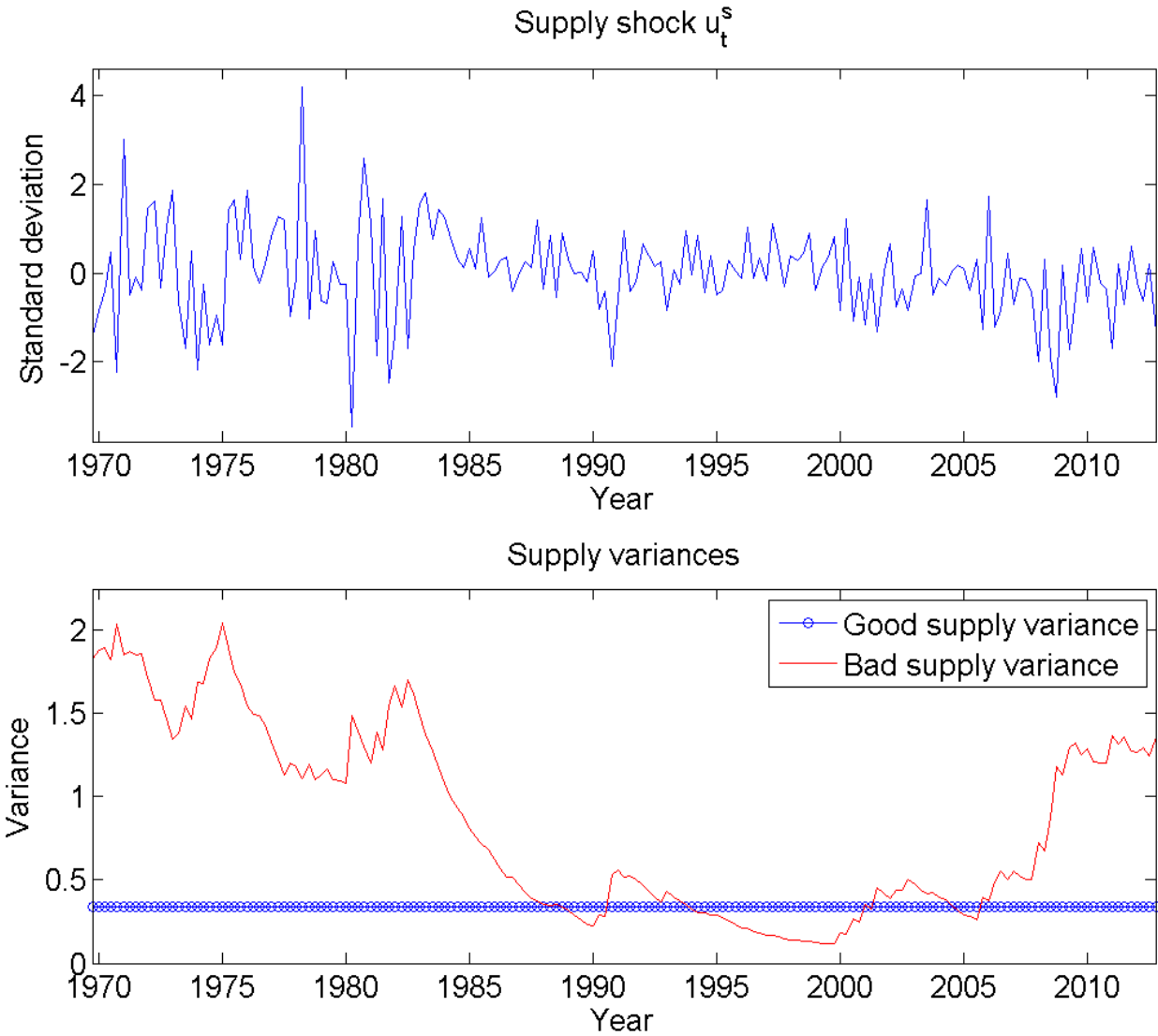
Figure 3: Demand and Supply Shock Variances. The data are quarterly US observations from 1969Q1 to 2012Q3. The demand shock is defined as a shock which moves GDP growth and inflation in the same direction. The supply shock is defined as a shock which moves GDP growth and inflation in opposite directions.
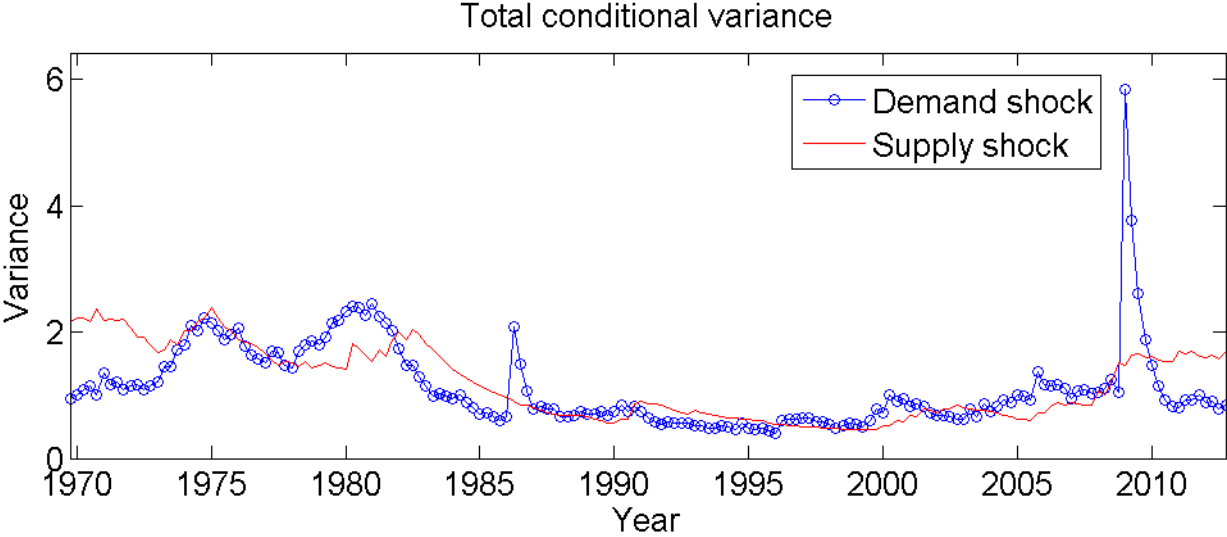
Figure 4: Conditional Inflation and GDP Growth Variances. The data are quarterly US observations from 1969Q1 to 2012Q3.
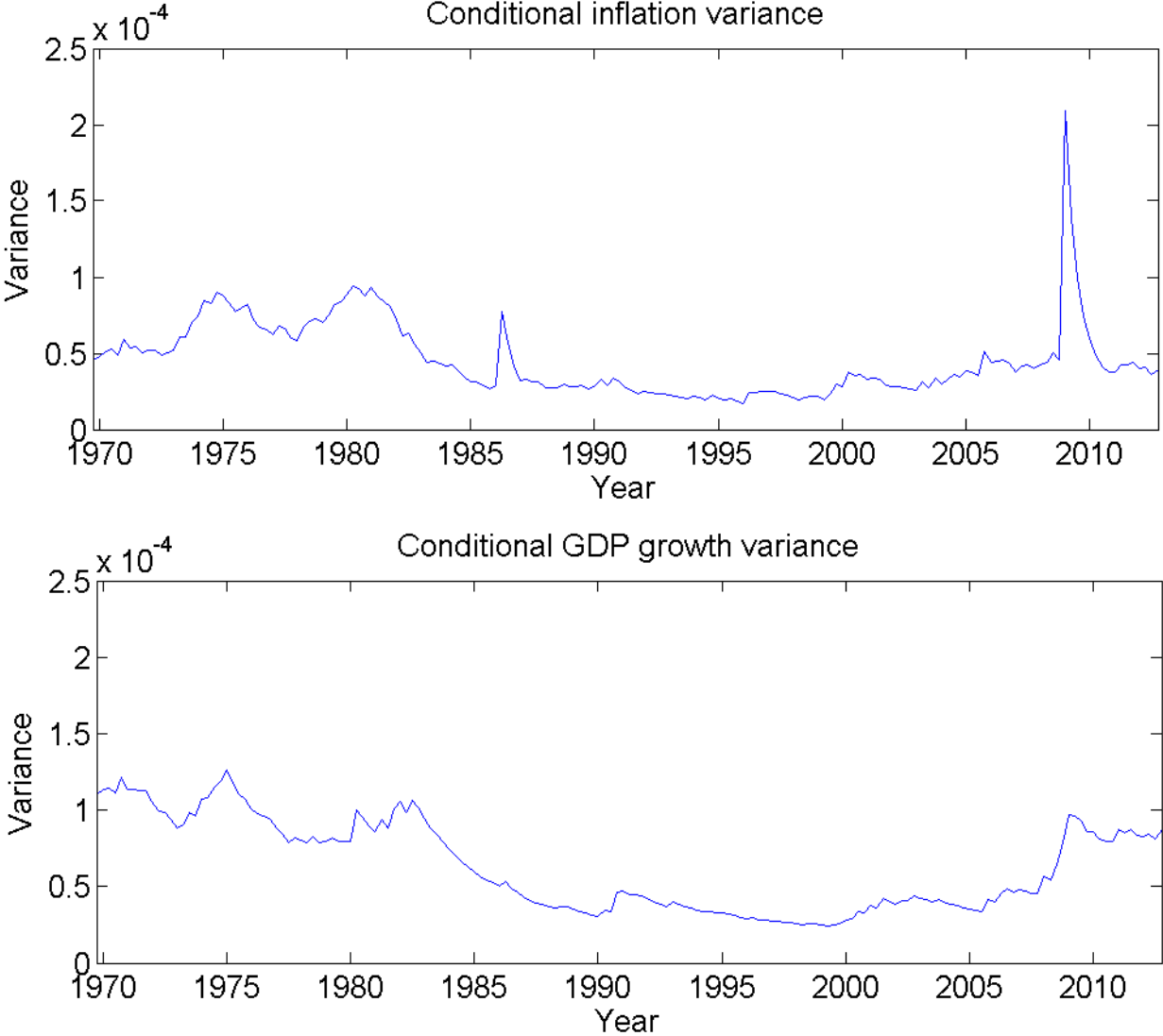
Figure 5: Demand and Supply Shock Components of GDP Growth and Inflation Shocks. The data are quarterly US observations from 1969Q1 to 2012Q3. The demand shock is defined as a shock which moves GDP growth and inflation in the same direction. The supply shock is defined as a shock which moves GDP growth and inflation in opposite directions. The supply shock component is equal to GDP growth/inflation shock - demand component of GDP growth/inflation shock.
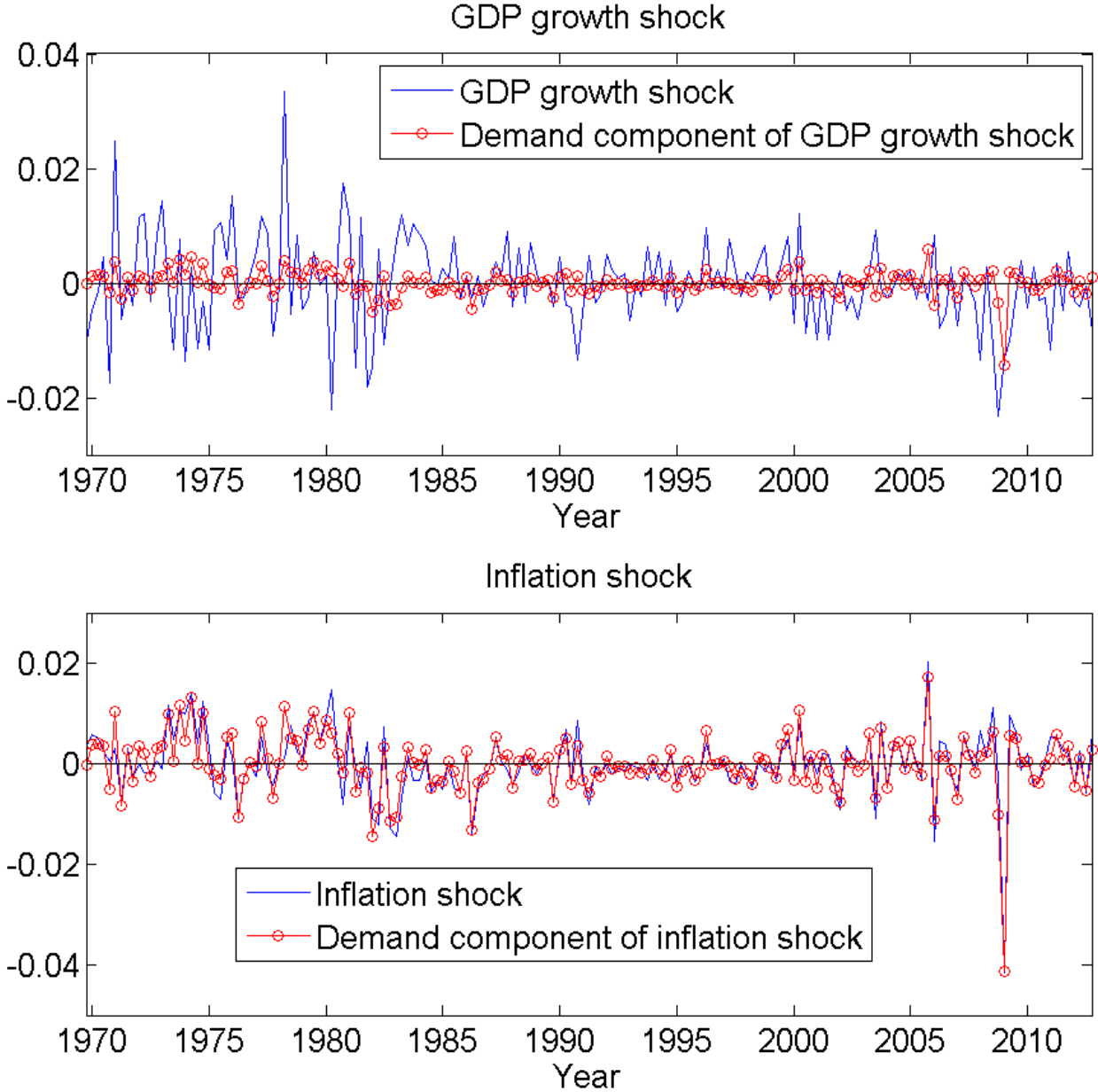
Figure 6: Conditional Correlation between GDP Growth and Inflation. The data are quarterly US observations from 1969Q1 to 2012Q3.
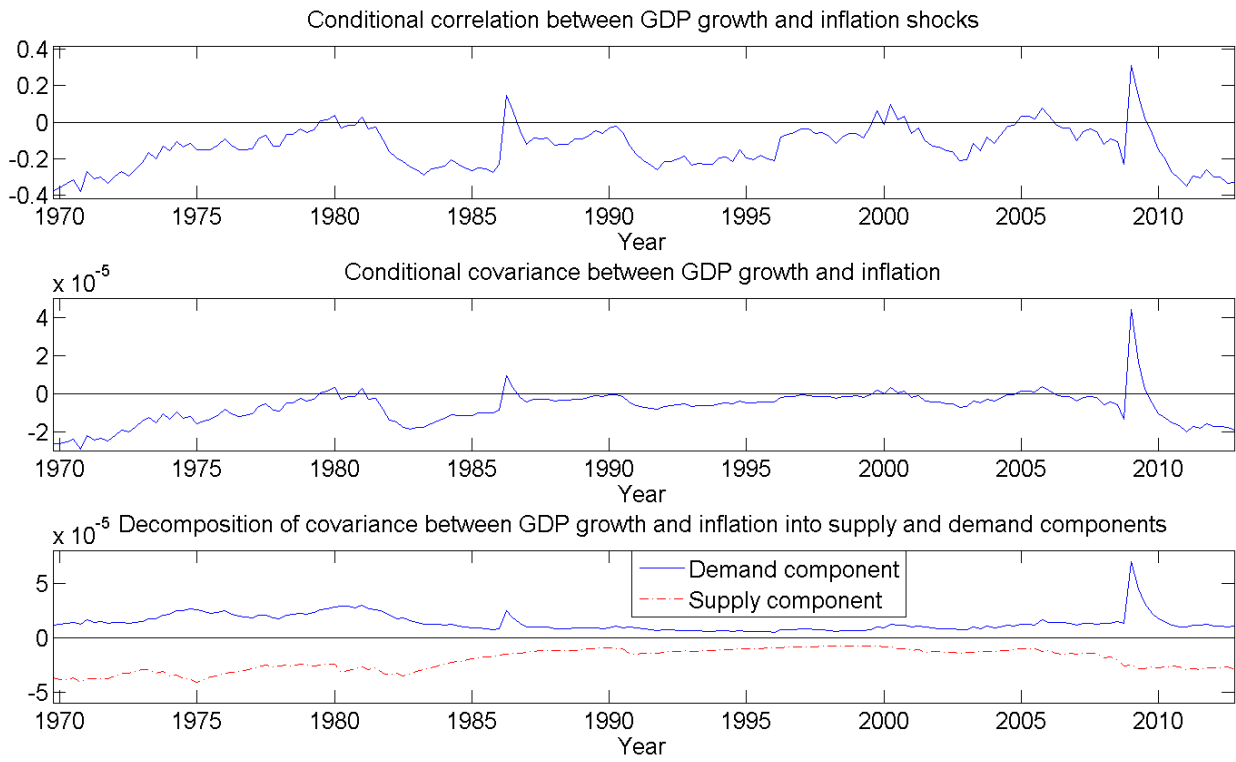
Figure 7: Explanatory Power of State Variables on Yields. Adjusted R^2 statistics are for the ordinary least squares regressions. The data are quarterly US observations from 1969Q1 to 2012Q3. The slope is defined with respect to the 1 quarter nominal bond. The confidence intervals are bootstrap confidence intervals.
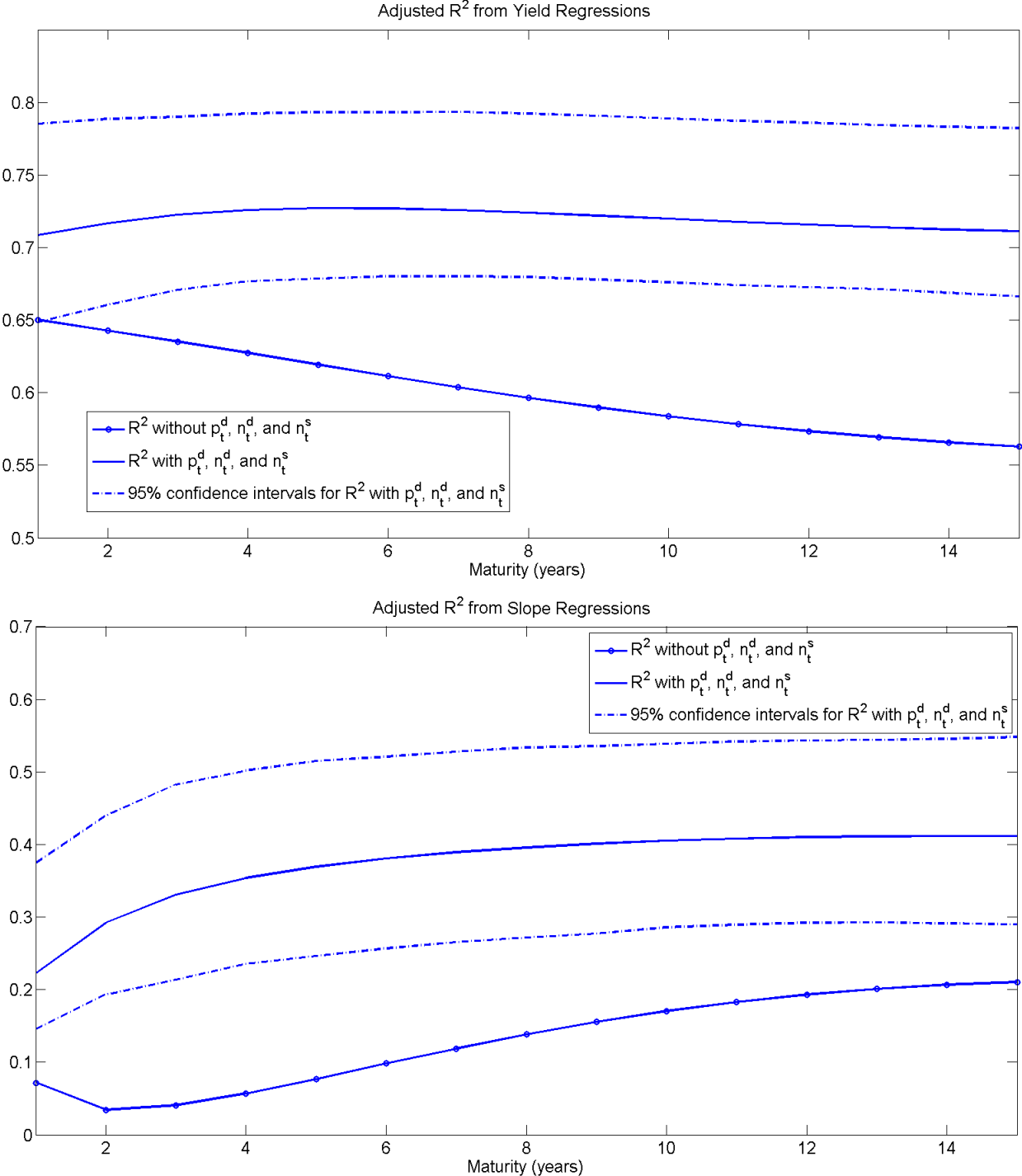
Figure 8: Impact of State Variables on Yields. The coefficients (solid lines) are ordinary least squares coefficients from the multivariate regression of the yields on a constant and state variables. The 95% confidence intervals (dotted lines) are based on the Newey-West standard errors with 40 lags. The data are quarterly US observations from 1969Q1 to 2012Q3. The slope is defined with respect to the 1 quarter nominal bond. The confidence intervals are bootstrap confidence intervals.

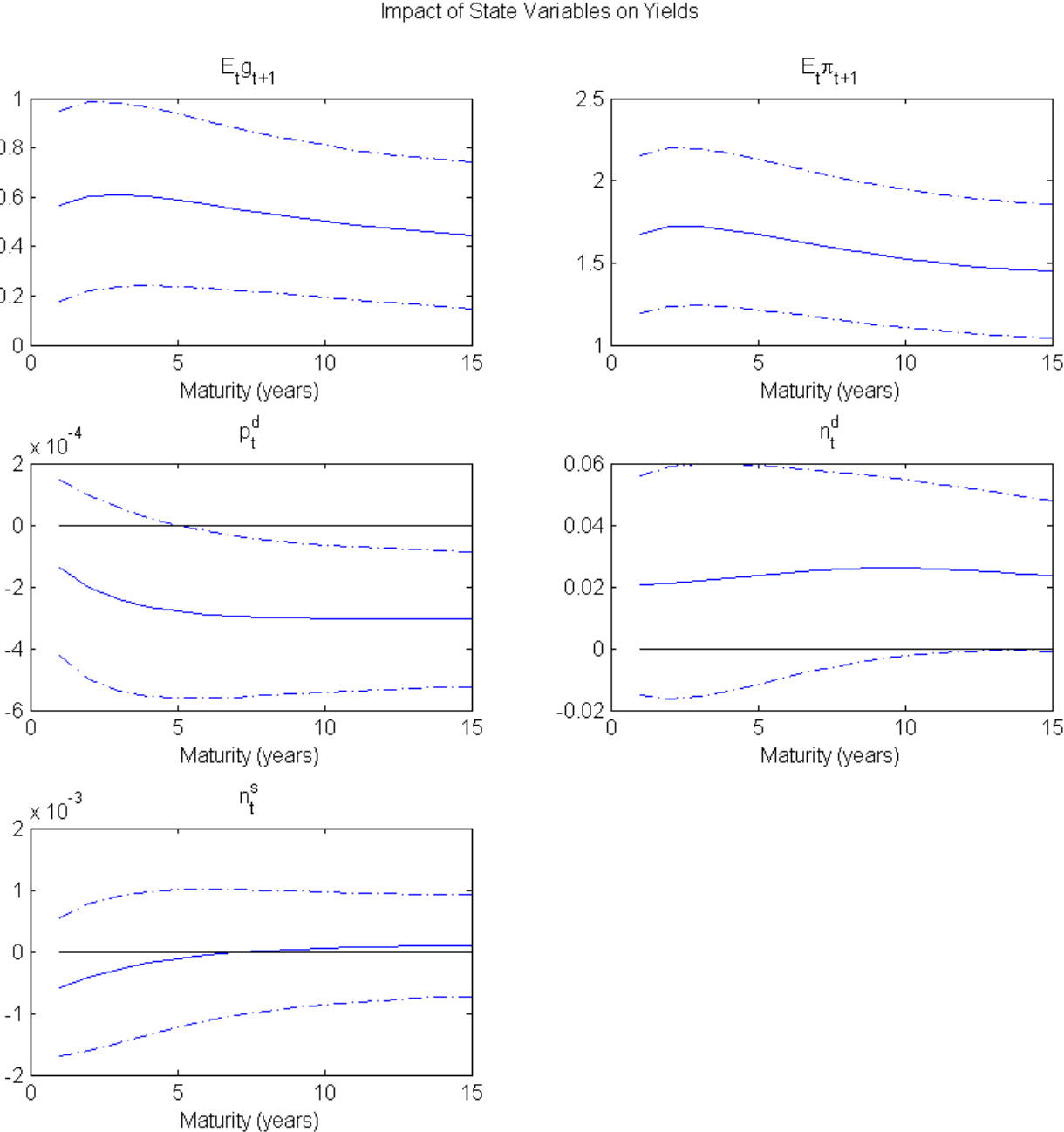Impact of State Variables on Yields

Figure 9: Explanatory Power of State Variables and Macroeconomic Shocks on 1 Quarter Excess Bond Returns. The data are quarterly US observations from 1969Q1 to 2012Q3. Adjusted R^2 statistics are for the ordinary least squares regressions. The confidence intervals are bootstrap confidence intervals.
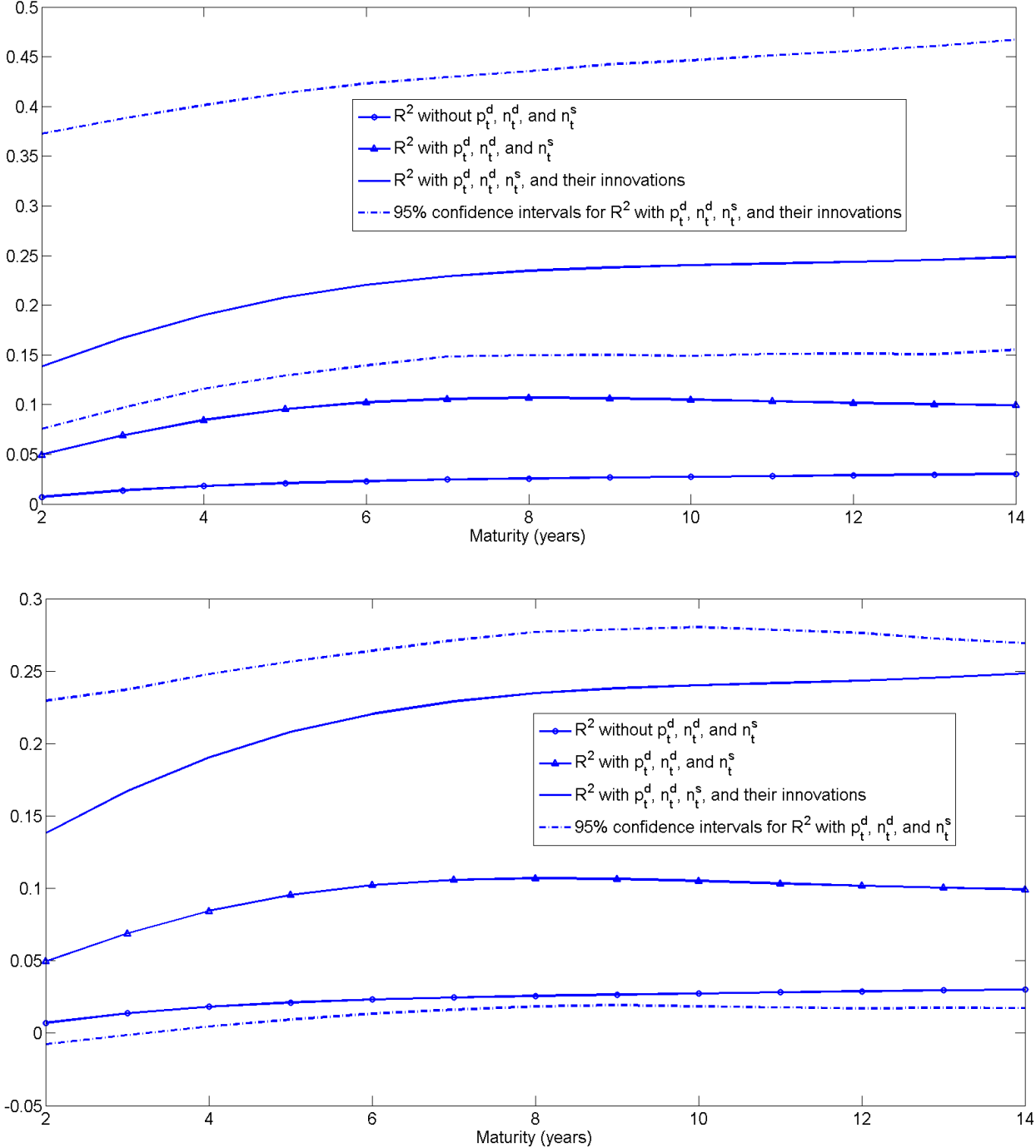
Figure 10: Impact of State Variables on 1 Quarter Excess Holding Returns. The coefficients (solid lines) are ordinary least squares coefficients from the multivariate regression of the excess holding return on a constant, state variables and macroeconomic innovations. The 95% confidence intervals (dotted lines) are based on the Newey-West standard errors with 40 lags. The data are quarterly US observations from 1969Q1 to 2012Q3. The confidence intervals are bootstrap confidence intervals.
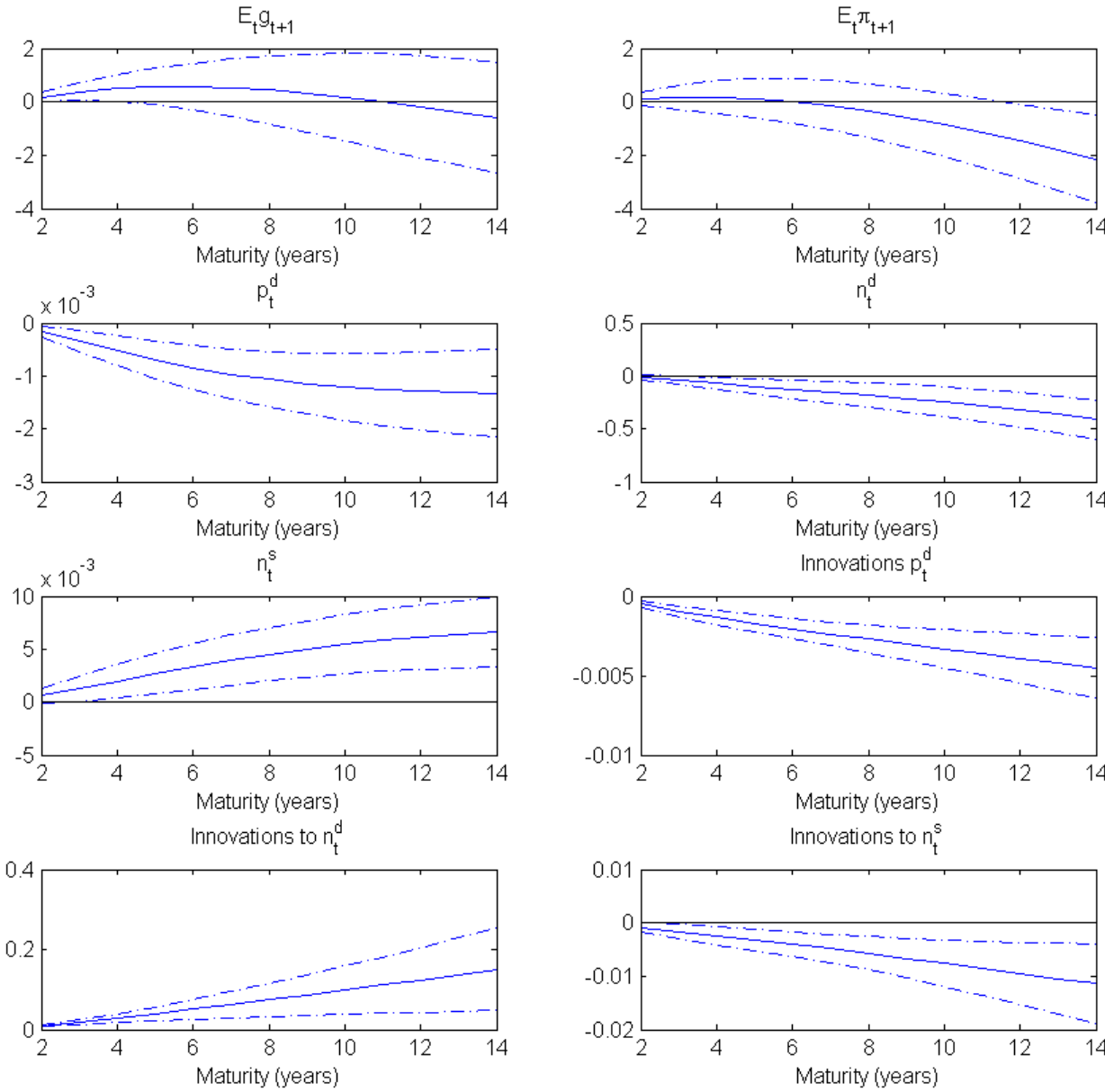
Figure 11: Real Yields. The figure plots annualized zero-coupon interpolated TIPS yields from Gürkaynak, Sack and Wright (2010) and their liquidity adjusted counterparts.
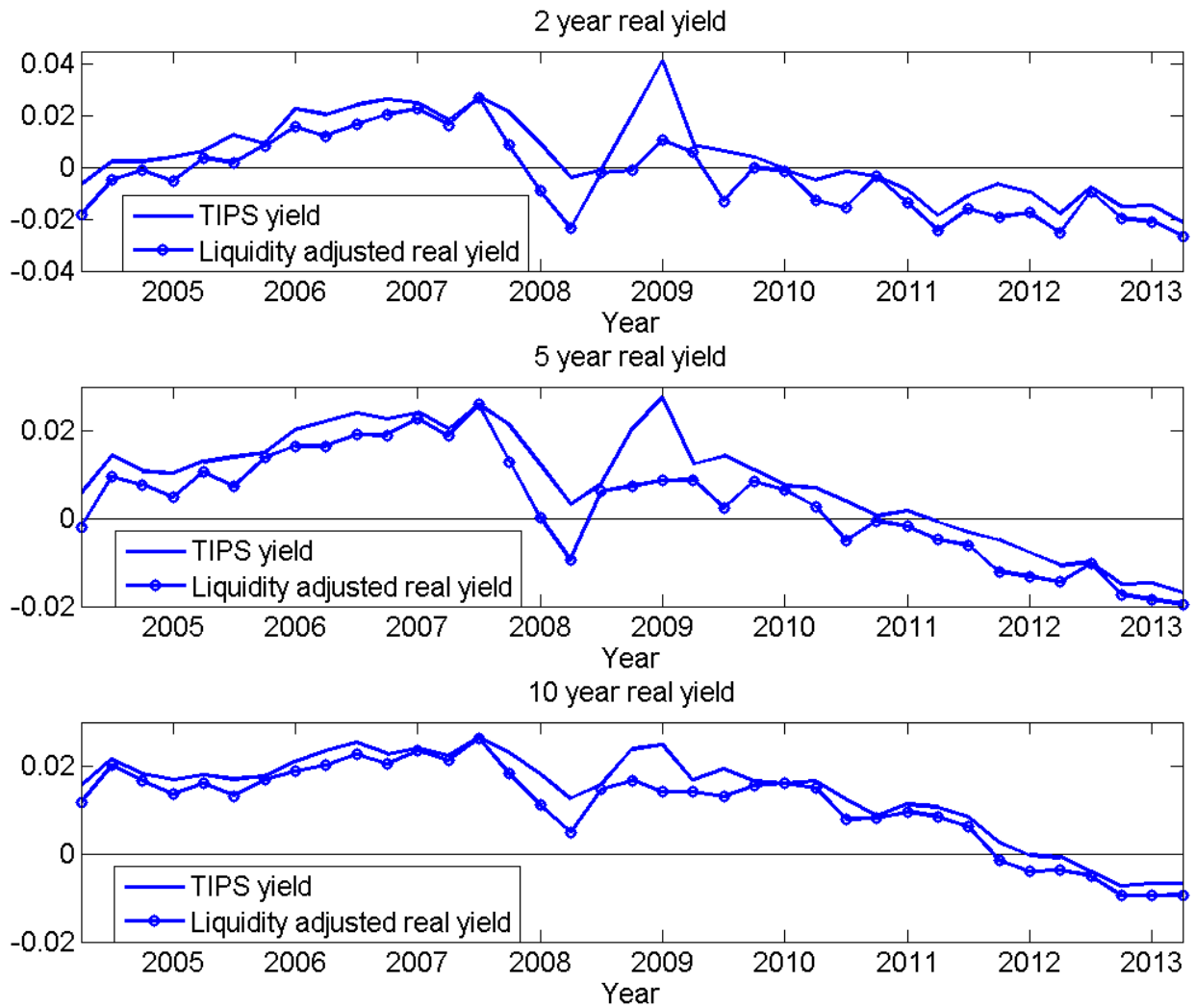
Figure 12: Historical 5 Year Nominal Yield Fit. The yield is a zero-coupon annualized yield.

5 year nominal yield

Corr(Model, Data)=0.99
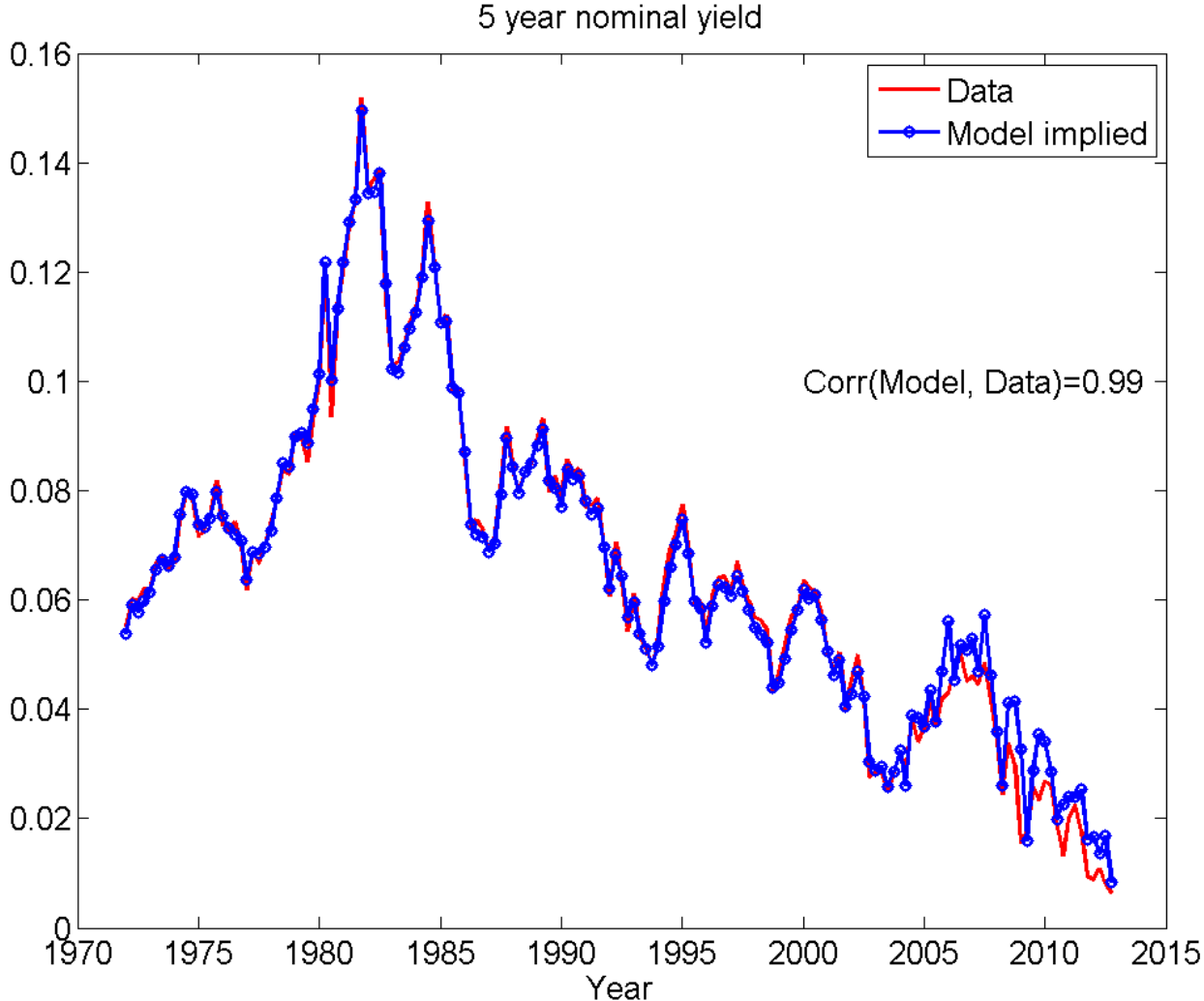
Figure 13: Decomposition of the Historical 5 Year Nominal Yield. The yield is decomposed into the real yield, expected inflation and the inflation risk premium. All yields are zero-coupon annualized yields.

Figure 14: Zero-coupon Real and Nominal Yield Curves. All yields are annualized.

Figure 15: The Historical Dynamics of the Latent Variable. The graph is the expected value of the latent factor filtered using Kalman filtering.

Latent factor z_t
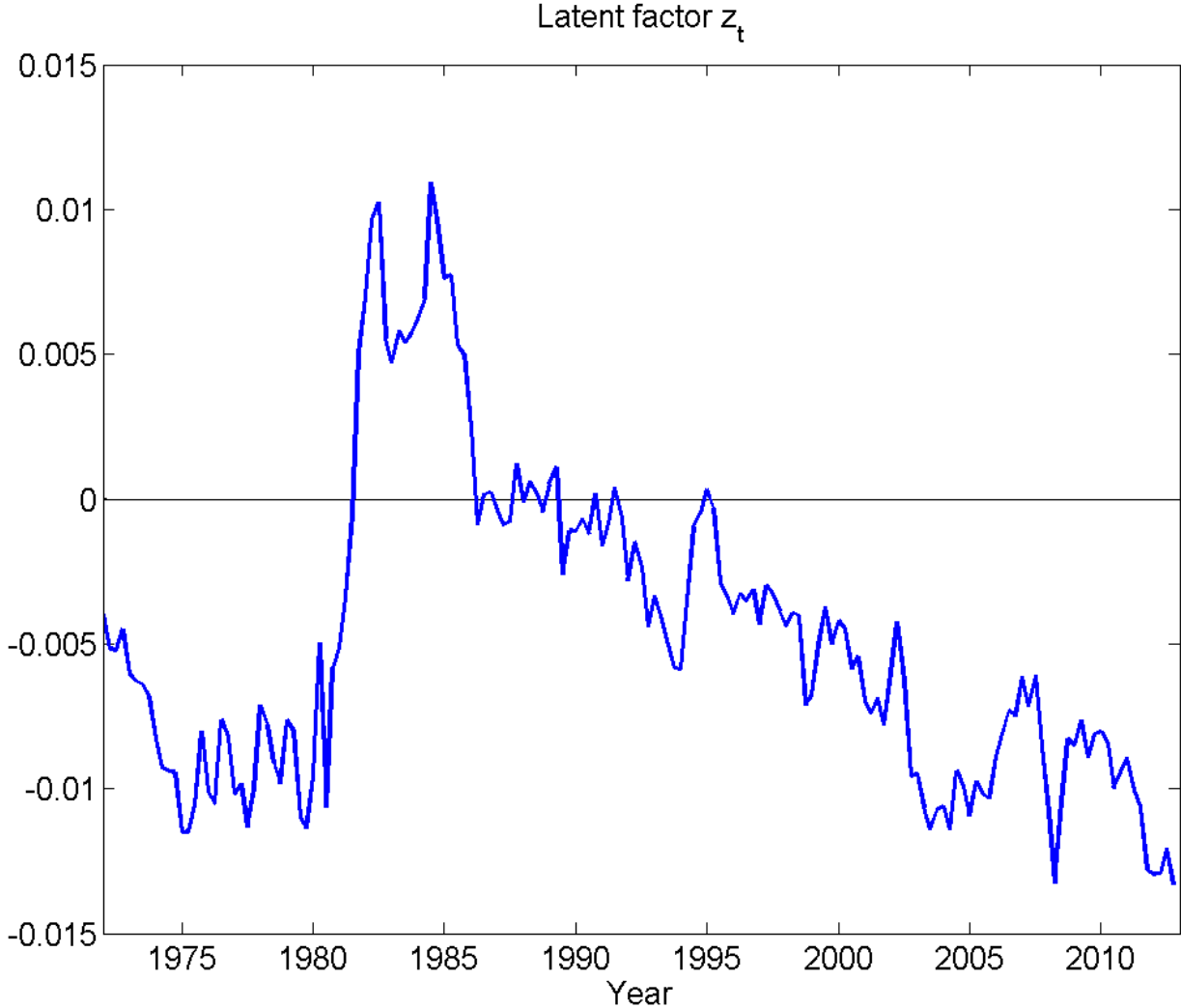
Table 1: Extracting Supply and Demand Shocks from GDP and Inflation Shocks. The data are quarterly GDP growth and inflation data from 1969Q1 to 2012Q3. In Panel A, standard errors in parentheses are bootstrap standard errors. In Panel B, standard errors in parentheses are GMM standard errors. *** indicates the statistical significance at the 1% level.

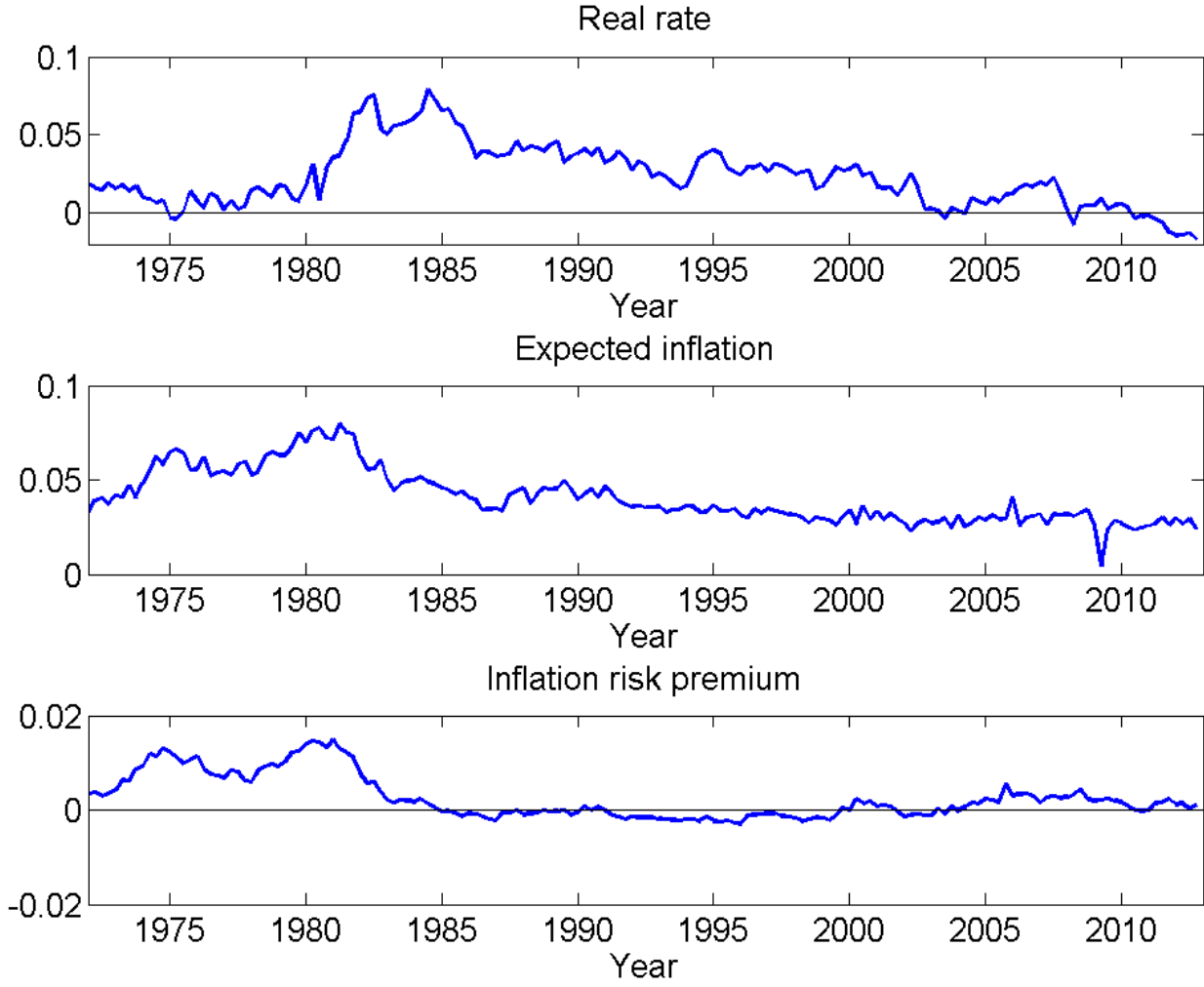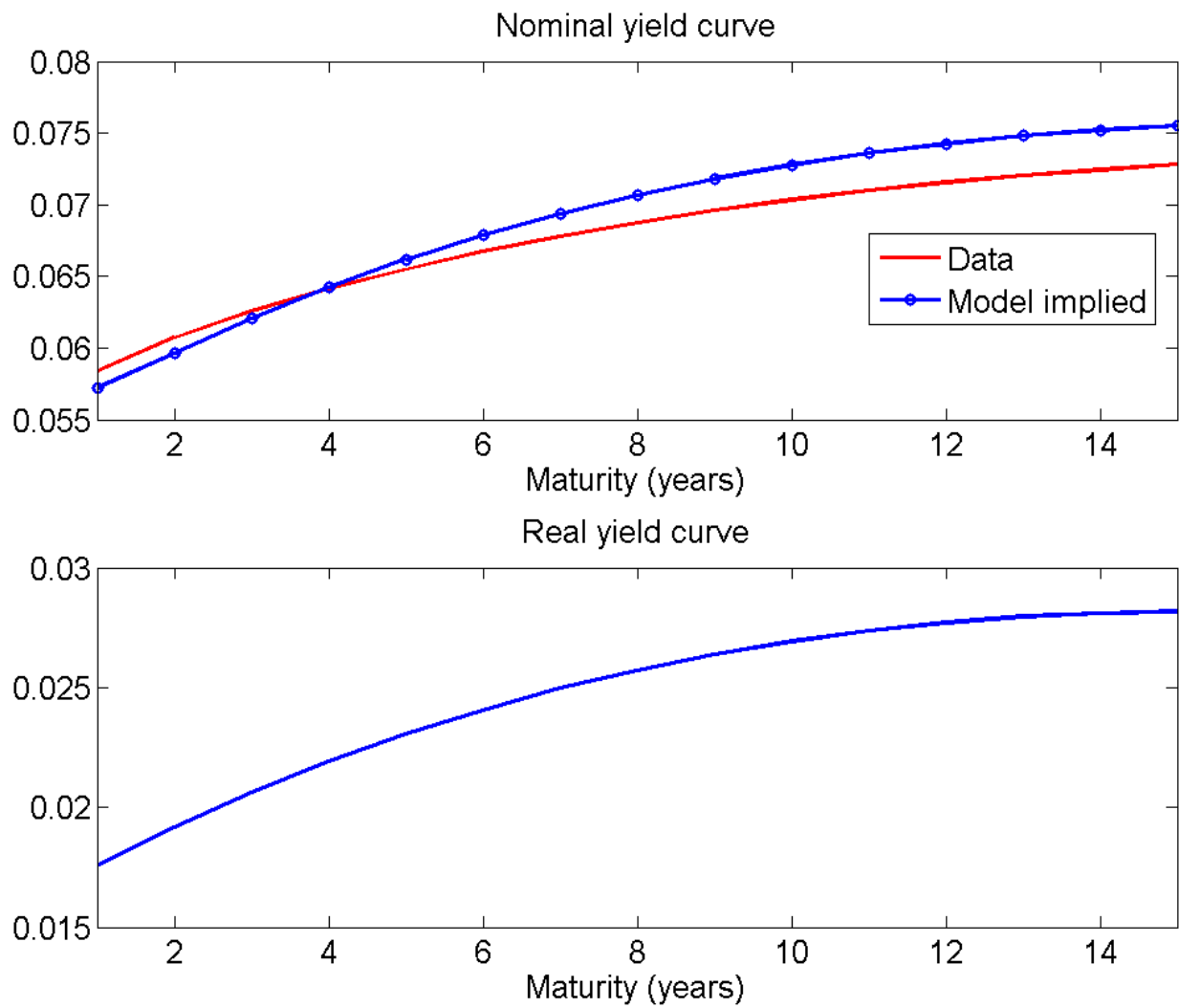| Panel A: Moments used to invert supply and demand shocks from GDP and inflation shocks		
Moment	Historical value	GMM fitted value
$u_t^{\pi 2}$	4.14E-05	4.14E-05
	(1.08E-05)	
u_t^{g2}	5.62E-05	5.06E-05
	(0.89E-05)	
$u_t^g u_t^\pi$	-5.88E-06	-5.86E-06
	(4.70E-06)	
$u_t^{\pi 3}$	-3.70E-07	-3.73E-07
	(4.29E-07)	
u_t^{g3}	6.48E-08	-14.38E-08
	(25.86E-08)	
$u_t^{\pi 2} u_t^g$	-1.55E-07	-1.44E-07
	(1.28E-07)	
$u_t^\pi u_t^{g2}$	3.00E-08	0.65E-08
	(6.77E-08)	
Panel B: GMM parameter estimates		
Parameter	Estimate	
σ_{gd}	0.0020	
	(0.0005)	
σ_{gs}	0.0070	
	(0.0005)	
$\sigma_{\pi d}$	0.0059	
	(0.0009)	
$\sigma_{\pi s}$	0.0025	
	(0.0008)	
p-value of the overidentification test	0.2281	
Panel C: Properties of demand and supply shocks		
	Demand shock	Supply shock
Variance	1.0284	1.1198
Skewness	-1.7792	0.0724
Excess kurtosis	12.4529	1.5959
Jarque-Berra normality test p-value	0.0001***	0.0034***

Table 2: Maximum Likelihood Parameter Estimates For Demand and Supply Shocks. The standard errors in parentheses are parametric bootstrap standard errors and computed by simulating the sampled paths for the shocks of historical length under the optimal maximum likelihood parameters 250 times and re-estimating the parameters for each of the simulated time series.

Parameters	Demand shock	Supply shock
σ_p	0.1925	0.4501
	(0.0744)	(0.0825)
$\bar{p}$	20.2846	1.6567
	(1.9108)	(0.2942)
σ_n	5.1541	0.4534
	(1.3974)	(0.0944)
$\bar{n}$	0.0137	4.3830
	(0.0192)	(0.5636)
ρ_p	0.9493	
	(0.0550)	
σ_{pp}	0.7752	
	(0.1688)	
ρ_n	0.6844	0.9912
	(0.2000)	(0.1508)
σ_{nn}	0.1332	0.2327
	(0.0717)	(0.1875)

Table 3: Unconditional Moments of GDP Growth and Inflation Shocks. The data are quarterly GDP growth and inflation data from 1969Q1 to 2012Q3. Bootstrap standard errors are in parentheses. Std is standard deviation, Skw is skewness, $Ekur$ is excess kurtosis, and Pr is probability.

Moment	Data	Model
$Std(u_t^g)$	0.0078	0.0078
	(0.0006)	
$Skw(u_t^g)$	0.2485	-0.4443
	(0.3999)	
$Ekur(u_t^g)$	2.0482	1.2953
	(1.0337)	
$Pr(u_t^g < 2Std(u_t^g))$	0.0231	0.0323
	(0.0113)	
$Pr(u_t^g > 2Std(u_t^g))$	0.0173	0.0216
	(0.0089)	
$Std(u_t^\pi)$	0.0064	0.0070
	(0.0008)	
$Skw(u_t^\pi)$	-1.4355	-1.9976
	(1.0274)	
$Ekur(u_t^\pi)$	9.9054	24.9804
	(5.4129)	
$Pr(u_t^\pi < 2Std(u_t^\pi))$	0.0289	0.0159
	(0.0117)	
$Pr(u_t^\pi > 2Std(u_t^\pi))$	0.0173	0.0202
	(0.0128)	
$Corr(u_t^g, u_t^\pi)$	-0.1084	-0.0849
	(0.1128)	

Table 4: Relationship between State Variables and Yield Properties. The coefficients are ordinary least squares regression estimates. The left-hand side variables in the regression are quarterly yield properties and the right-hand side variables are state variables. The Newey-West (40 lags) 95% - standard errors are in parentheses. ***, **, and * indicate the statistical significance at 10%, 5%, and 1% level. For the R^2:s the statistical significance is for the difference between the models with and without macroeconomic shocks state variables (p_t^d, n_t^d, and n_t^s) which is determined based on the F-statistic for the joint significance of the macroeconomic shocks state variables.

			1 quarter nominal interest rate				
Constant	$E_t g_{t+1}$	$E_t \pi_{t+1}$	p_t^d	n_t^d	n_t^s	R^2	$R^2_{\text{no macro factors}}$
-0.0022	0.4944***	1.5208***	-0.0001	0.0193	-0.0008*	0.7074	0.6483***
(0.0027)	(0.1849)	(0.2205)	(0.0001)	(0.0149)	(0.0004)		

			1 year nominal interest rate				
Constant	$E_t g_{t+1}$	$E_t \pi_{t+1}$	p_t^d	n_t^d	n_t^s	R^2	$R^2_{\text{no macro factors}}$
-0.0022	0.5645***	1.6767***	-0.0001	0.0206	-0.0006	0.7174	0.6544***
(0.0028)	(0.1936)	(0.2393)	(0.0001)	(0.0178)	(0.0006)		

			10 year nominal interest rate				
Constant	$E_t g_{t+1}$	$E_t \pi_{t+1}$	p_t^d	n_t^d	n_t^s	R^2	$R^2_{\text{no macro factors}}$
0.0036*	0.5011***	1.5623***	-0.0003**	0.0261*	0.0001	0.7284	0.5888***
(0.0022)	(0.1538)	(0.2100)	(0.0001)	(0.0143)	(0.0004)		

			Level (average over 1-15 year yields)				
Constant	$E_t g_{t+1}$	$E_t \pi_{t+1}$	p_t^d	n_t^d	n_t^s	R^2	$R^2_{\text{no macro factors}}$
0.0021	0.5313***	1.5844***	-0.0003**	0.0242	-0.00013	0.7353	0.6180***
(0.0023)	(0.1645)	(0.2176)	(0.0001)	(0.0153)	(0.0005)		

			Slope (10 year yield - 1 quarter yield)				
Constant	$E_t g_{t+1}$	$E_t \pi_{t+1}$	p_t^d	n_t^d	n_t^s	R^2	$R^2_{\text{no macro factors}}$
0.0059***	0.0068	0.0055	-0.0002***	0.0068	0.0008***	0.4232	0.1805***
(0.0008)	(0.0495)	(0.0777)	(0.0000)	(0.0104)	(0.0001)		

			Curvature (10 year yield + 1 quarter yield-2×2 year yield)				
Constant	$E_t g_{t+1}$	$E_t \pi_{t+1}$	p_t^d	n_t^d	n_t^s	R^2	$R^2_{\text{no macro factors}}$
0.0048***	-0.2101**	-0.3928***	0.0000	0.028	0.0001	0.3934	0.3603*
(0.0011)	(0.0942)	(0.0735)	(0.0001)	(0.0118)	(0.0003)		

Table 5: Relationship between State Variables and 1 Quarter Excess Bond Returns. The coefficients are ordinary least squares coefficients from the multivariate regression of the 1 quarter excess holding return on a constant, state variables and macroeconomic innovations. The Newey-West (40 lags) 95% - standard errors are in parentheses. ***, **, and * indicate the statistical significance at 10%, 5%, and 1% level. For the R^2:s the statistical significance is with respect to the R^2 on the line above it determined based on the F-statistic for the joint significance of the regressors not included in the model but included in the model above it.

	Excess return on 2 year bond	Excess return on 10 year bond
Constant	-0.0003	0.0228***
	(0.0012)	(0.0081)
$E_t g_{t+1}$	0.1810**	0.1797
	(0.0846)	(0.8231)
$E_t \pi_{t+1}$	0.0967	-0.8669
	(0.1252)	(0.5974)
p_t^d	-0.0002***	-0.0012***
	(0.0001)	(0.0003)
n_t^d	-0.0150	-0.2448***
	(0.0103)	(0.0719)
n_t^s	0.0006*	0.0055***
	(0.0003)	(0.0014)
$\omega_{p,t+1}^d$	-0.0005***	-0.0033***
	(0.0001)	(0.0006)
$\omega_{n,t+1}^d$	0.0092***	0.0984***
	(0.0012)	(0.0300)
$\omega_{n,t+1}^s$	-0.0009**	-0.0076***
	(0.0004)	(0.0022)
R^2	0.1380	0.2402
$R^2_{\text{no } \omega\text{:s}}$	0.0496*	0.1051***
$R^2_{\text{no macro factors and } \omega\text{:s}}$	0.0071***	0.0274***

Table 6: Maxium Likelihood Parameter Estimates. Parameters are estimated using Kalman filtering. The sample is quarterly from 1969Q1 to 2012Q3 for nominal yields and from 2004Q1 to 2012Q3 for real yields. The nominal yields are 1 year, 5 year and 10 year yields. The real yields are 2 year, 5 year, and 10 year yields. Standard errors in parentheses are inverse information matrix standard errors.

Parameter	Estimate
Real interest rate	
a_0	0.0039
	(0.0036)
a_g	0.5433
	(0.2694)
a_π	0.0376
	(0.0390)
a_{p^d}	0.0002
	(0.0002)
a_{n^d}	0.0074
	(0.1685)
a_{n^s}	-0.0006
	(0.0003)
Latent variable	
ρ_z	0.9956
	(0.0094)
σ_z	0.0023
	(0.0009)
Prices of risk	
λ_{p^d}	0.0310
	(2.2437)
λ_{n^d}	21.7673
	(9.9818)
λ_{p^s}	0.4052
	(4.7303)
λ_{n^s}	4.7812
	(2.3985)
λ_z	0.0594
	(1.0828)

Table 7: Model Fit. The correlations for the nominal yields are from quarterly observations from 1969Q1 to 2012Q3 and for the real yields from 2004Q1 to 2012Q3.

Yield	Correlation(Model, Data)
1 year nominal	0.9660
5 year nominal	0.9946
10 year nominal	0.9873
2 year real	0.8974
5 year real	0.9910
10 year real	0.9700

Table 8: Decomposition of the Historical 5 Year Nominal Yield. The yield is decomposed into the real yield, expected inflation and the inflation risk premium. All yields are zero-coupon annualized yields.

Component	Average level	Standard deviation
Real rate	2.31%	2.03%
Expected inflation	4.06%	1.40%
Inflation risk premium	0.25%	0.46%