The TIPS Liquidity Premium

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Abstract

This paper introduces an arbitrage-free term structure model of nominal and real yields with a liquidity risk factor to account for the liquidity disadvantage of Treasury inflation-protected securities (TIPS) relative to regular Treasury securities. Although latent, the identification of the liquidity factor comes from its unique loading, which mimics the idea that, over time, an increasing amount of the outstanding notional of TIPS gets locked up in buy-and-hold investors' portfolios. This raises their sensitivity to variation in the market-wide liquidity captured by the liquidity factor. The model is estimated using price information for individual TIPS in combination with a standard sample of nominal Treasury yields and delivers unique liquidity premium estimates for each TIPS. In addition, the time-varying value of the deflation protection option embedded in the TIPS contract is explicitly accounted for. This is important when it comes to distinguishing liquidity risks from deflation risks in the pricing of TIPS. The main finding is that TIPS liquidity premiums over the entire sample average 38 basis points, while five- and ten-year on-the-run TIPS liquidity premiums are lower averaging 33 and 30 basis points, respectively.

JEL Classification: E43, E47, G12, G13

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1 Introduction

The U.S. Treasury first issued inflation-indexed bonds, which are now commonly known as Treasury inflation-protected securities (TIPS), in 1997. Since then the U.S. Treasury has steadily expanded the market for TIPS. By the end of 2013, the total outstanding amount of TIPS was $973 billion or 8.2 percent of all total marketable Treasury securities.[1]

Despite the apparent large size of the market for TIPS, there is an overwhelming amount of research that suggests that TIPS are less liquid than regular Treasury securities. Fleming and Krishnan (2012) report market characteristics of TIPS trading that indicate smaller trading volume, longer turnaround time, and wider bid-ask spreads than are normally observed in the nominal Treasury bond market (see also Campbell et al. 2009, Dudley et al. 2009, Gürkaynak et al. 2010, and Sack and Elsasser 2004). Moreover, there is evidence that TIPS yields are elevated for this reason as investors require a premium for assuming this illiquidity risk (see Fleckenstein et al. 2014 for a detailed discussion). However, the degree to which these frictions bias TIPS yields remains a topic of debate because attempts to estimate TIPS liquidity premiums directly have resulted in varying results.[2]

In this paper, we introduce an arbitrage-free term structure model of nominal and real yields with a liquidity risk factor to account for the well-documented liquidity disadvantage of TIPS relative to regular Treasury securities. The model is an extension of the established model of nominal and real yields introduced in Christensen et al. (2010, henceforth CLR) referred to throughout as the CLR model. Within the model, the identification of the latent liquidity factor comes from its unique loading for each TIPS that is supposed to mimic the feature that, over time, an increasing amount of its outstanding notional gets locked up in buy-and-hold investors' portfolios. Thanks to investors' forward-looking behavior, this raises its sensitivity to variation in the market-wide liquidity represented by the liquidity factor. By observing a cross section of TIPS prices over time, the liquidity factor can be distinguished from the fundamental TIPS yield factors in the CLR model.

We estimate the CLR model and its extension using price information for individual TIPS trading from mid-1997 through the end of 2013 in combination with a standard sample of nominal Treasury yields from Gürkaynak et al. (2007). As another novel feature, we account explicitly in the model estimation for the time-varying value of the deflation protection option embedded in the TIPS contract using pricing formulas provided in Christensen et al. (2012). We show that this matters less when it comes to distinguishing liquidity risks from deflation risks in the pricing of TIPS. In addition to delivering unique liquidity premium estimates for each TIPS, the model provides an accurate decomposition of the model-implied frictionless TIPS breakeven inflation into an expected inflation component and a residual inflation risk

[1] The data is available at: http://www.treasurydirect.gov/govt/reports/pd/mspd/2013/opds122013.pdf

[2] Pflueger and Viceira (2013), D'Amico et al. (2014), and Abrahams et al. (2015) are among the studies that estimate TIPS liquidity premiums.

premium component. In particular, the model that accounts for the TIPS liquidity premium outperforms the model that does not when it comes to forecasting CPI inflation. To have more accurate and less biased estimates of bond investor's inflation expectations embedded in nominal and real yields is crucial for portfolio risk management and monetary policy analysis. Finally, we intend to explore the determinants of the time series variation in the TIPS liquidity premium and tie the findings to theories of limits to arbitrage capital as discussed in Hu et al. (2013, henceforth HPW).

In terms of the existing literature, Grishchenko et al. (2013) and Christensen et al. (2015) study the pricing of the deflation protection option in great detail, while Pflueger and Viceira (2013), D'Amico et al. (2014), and Abrahams et al. (2015) are among the studies that attempt to account for the TIPS liquidity premium. We do both. Thus, to the best of our knowledge, this is the first paper to account simultaneously for the liquidity premiums in TIPS prices and for the embedded deflation protection option values within an arbitrage-free model of nominal and real yields. We plan to make a thorough comparison to this existing literature.

Since we get improvement in one-year CPI inflation forecasts from accounting for TIPS liquidity premiums and achieve additional, but smaller improvements from taking the deflation protection option values into account, we are also the first to document that accounting for both of these aspects of TIPS pricing is important.

The main finding of the empirical analysis is that TIPS liquidity premiums over the entire sample average 38 basis points, which is lower than the results reported in existing studies of TIPS liquidity premiums. Furthermore, we note that five- and ten-year on-the-run TIPS liquidity premiums are slightly lower, averaging 33 and 30 basis points, respectively, during our sample. Importantly, whether these results should affect the decision of the U.S. Treasury Department to continue to issue TIPS, requires a comprehensive assessment of the sign and magnitude of the inflation risk premium that represents the gain from issuing TIPS relative to the liquidity disadvantage we document (see Christensen and Gillan 2012 for a discussion and analysis). Thus, we caution against drawing policy implications from our findings without further analysis.

The paper is structured as follows. Section 2 introduces the general theoretical framework for inferring inflation dynamics from nominal and real Treasury yields. It also details our methodology for deriving model-implied values of the deflation protection options embedded in TIPS. Section 3 describes the CLR model and its extension with a liquidity risk factor. Section 4 contains the data description, while Section 5 presents the empirical results with particular emphasis on the role of TIPS liquidity risk. Section 6 is dedicated to an analysis of the risk of deflation, while Section 7 analyzes the model-implied inflation expectations. Section 8 concludes and provides directions for future research. Appendices contain additional technical details and results.

2 Decomposing Breakeven Inflation

In this section, we first demonstrate how to decompose breakeven inflation into inflation expectations and inflation risk premium components within an arbitrage-free model where inflation risk is spanned by nominal and real yields and assuming no frictions to trading. Second, we demonstrate how to value the deflation protection options embedded in individual TIPS within that framework.

2.1 Deriving Market-Implied Inflation Expectations and Risk Premiums

An arbitrage-free term structure model can be used to decompose the difference between nominal and real Treasury yields, also known as the breakeven inflation (BEI) rate, into the sum of inflation expectations and an inflation risk premium. We follow Merton (1974) and assume a continuum of nominal and real zero-coupon Treasury bonds exists with no frictions to their continuous trading. The economic implication of this assumption is that the markets for inflation risk are complete in the limit. Define the nominal and real stochastic discount factors, denoted M_t^N and M_t^R, respectively. The no-arbitrage condition enforces a consistency of pricing for any security over time. Specifically, the price of a nominal bond that pays one dollar in τ years and the price of a real bond that pays one unit of the defined consumption basket in τ years must satisfy the conditions that

$$P_t^N(\tau) = E_t^P\left[\frac{M_{t+\tau}^N}{M_t^N}\right] \quad \text{and} \quad P_t^R(\tau) = E_t^P\left[\frac{M_{t+\tau}^R}{M_t^R}\right],$$

where $P_t^N(\tau)$ and $P_t^R(\tau)$ are the observed prices of the zero-coupon, nominal and real bonds for maturity τ on day t and $E_t^P[.]$ is the conditional expectations operator under the real-world (or P-) probability measure. The no-arbitrage condition also requires a consistency between the prices of real and nominal bonds such that the price of the consumption basket, denoted as the overall price level Π_t, is the ratio of the nominal and real stochastic discount factors:

$$\Pi_t = \frac{M_t^R}{M_t^N}.$$

We assume that the nominal and real stochastic discount factors have the standard dynamics given by

$$\begin{aligned} dM_t^N/M_t^N &= -r_t^N dt - \Gamma_t' dW_t^P, \\ dM_t^R/M_t^R &= -r_t^R dt - \Gamma_t' dW_t^P, \end{aligned}$$

where r_t^N and r_t^R are the instantaneous, risk-free nominal and real rates of return, respectively, and Γ_t is a vector of premiums on the risks represented by W_t^P. By Ito's lemma, the dynamic evolution of Π_t is given by

$$d\Pi_t = (r_t^N - r_t^R)\Pi_t dt.$$

Thus, with the absence of arbitrage, the instantaneous growth rate of the price level is equal to the difference between the instantaneous nominal and real risk-free rates.[3] Correspondingly, we can express the stochastic price level at time t+τ as

$$\Pi_{t+\tau} = \Pi_t e^{\int_t^{t+\tau}(r_s^N - r_s^R)ds}.$$

The relationship between the yields and inflation expectations can be obtained by decomposing the price of the nominal bond as follows:

$$\begin{aligned}
P_t^N(\tau) &= E_t^P\left[\frac{M_{t+\tau}^N}{M_t^N}\right] = E_t^P\left[\frac{M_{t+\tau}^R/\Pi_{t+\tau}}{M_t^R/\Pi_t}\right] = E_t^P\left[\frac{M_{t+\tau}^R}{M_t^R}\frac{\Pi_t}{\Pi_{t+\tau}}\right] \\
&= E_t^P\left[\frac{M_{t+\tau}^R}{M_t^R}\right] \times E_t^P\left[\frac{\Pi_t}{\Pi_{t+\tau}}\right] + cov_t^P\left[\frac{M_{t+\tau}^R}{M_t^R}, \frac{\Pi_t}{\Pi_{t+\tau}}\right] \\
&= P_t^R(\tau) \times E_t^P\left[\frac{\Pi_t}{\Pi_{t+\tau}}\right] \times \left(1 + \frac{cov_t^P\left[\frac{M_{t+\tau}^R}{M_t^R}, \frac{\Pi_t}{\Pi_{t+\tau}}\right]}{E_t^P\left[\frac{M_{t+\tau}^R}{M_t^R}\right] \times E_t^P\left[\frac{\Pi_t}{\Pi_{t+\tau}}\right]}\right).
\end{aligned}$$

Converting this price into a yield-to-maturity using

$$y_t^N(\tau) = -\frac{1}{\tau}\ln P_t^N(\tau) \quad \text{and} \quad y_t^R(\tau) = -\frac{1}{\tau}\ln P_t^R(\tau),$$

we obtain

$$y_t^N(\tau) = y_t^R(\tau) + \pi_t^e(\tau) + \phi_t(\tau),$$

where the market-implied average rate of inflation expected at time t for the period from t to $t+\tau$ is

$$\pi_t^e(\tau) = -\frac{1}{\tau}\ln E_t^P\left[\frac{\Pi_t}{\Pi_{t+\tau}}\right] = -\frac{1}{\tau}\ln E_t^P\left[e^{-\int_t^{t+\tau}(r_s^N - r_s^R)ds}\right]$$

and the associated inflation risk premium for the same time period is

$$\phi_t(\tau) = -\frac{1}{\tau}\ln\left(1 + \frac{cov_t^P\left[\frac{M_{t+\tau}^R}{M_t^R}, \frac{\Pi_t}{\Pi_{t+\tau}}\right]}{E_t^P\left[\frac{M_{t+\tau}^R}{M_t^R}\right] \times E_t^P\left[\frac{\Pi_t}{\Pi_{t+\tau}}\right]}\right).$$

This last equation highlights that the inflation risk premium can be positive or negative. It is positive if and only if

$$cov_t^P\left[\frac{M_{t+\tau}^R}{M_t^R}, \frac{\Pi_t}{\Pi_{t+\tau}}\right] < 0.$$

[3] We emphasize that the price level, Π_t, is a stochastic process as long as r_t^N and r_t^R are stochastic processes.

That is, the riskiness of nominal bonds relative to real bonds depends on the covariance between the real stochastic discount factor and inflation and is ultimately determined by investor preferences, but for our analysis we will not need to specify those.

Finally, the BEI rate is defined as

$$BEI_t(\tau) \equiv y_t^N(\tau) - y_t^R(\tau) = \pi_t^e(\tau) + \phi_t(\tau). \tag{1}$$

Namely, the BEI rate is the difference between nominal and real yields of the same maturity and can be decomposed into the sum of expected inflation and the inflation risk premium.

Equation (1) highlights that the decomposition of BEI can be corrupted if nominal and real yields are biased by liquidity effects, and the magnitude of the corruption equals the size of the bias. However, the equation also makes clear that it is only the relative liquidity between nominal and real yields that we need to correct BEI rates for any liquidity bias.

In Section 3, we introduce a model that accounts for the liquidity differential of TIPS relative to Treasuries and hence provides estimates of the frictionless nominal and real yields that feature in the expectations above.

2.2 Valuing the TIPS Deflation Protection Option

In this section, we describe how to value the deflation protection option embedded in the TIPS contract assuming no frictions to trading.

To begin, consider a TIPS issued at time t_0 with maturity at time $t+\tau$. By time t its accrued inflation compensation, also known as its index ratio, is given by the change in the price level since issuance, Π_t/Π_0.

To value the embedded deflation protection option, we need to explicitly control for the accrued inflation compensation; that is, the option will only be in the money at maturity provided the change in the price level between t and $t+\tau$ satisfies the following inequality:

$$\frac{\Pi_{t+\tau}}{\Pi_t} \leq \frac{1}{\Pi_t/\Pi_0}.$$

Thus, for the option to be in the money, the deflation experienced over the remaining life of the bond, $\Pi_{t+\tau}/\Pi_t$, has to negate the accumulated inflation experienced since the bond's issuance.

Now, the present value of the principal payment of the TIPS is given by

$$E_t^Q\left[\frac{\Pi_{t+\tau}}{\Pi_t} \cdot e^{-\int_t^{t+\tau} r_s^N ds}\mathbf{1}_{\{\frac{\Pi_{t+\tau}}{\Pi_t} > \frac{1}{\Pi_t/\Pi_0}\}}\right] + E_t^Q\left[1 \cdot e^{-\int_t^{t+\tau} r_s^N ds}\mathbf{1}_{\{\frac{\Pi_{t+\tau}}{\Pi_t} \leq \frac{1}{\Pi_t/\Pi_0}\}}\right].$$

The first term represents the present value of the principal payment conditional on the net change in the price index over the bond's remaining time to maturity not offsetting the accrued inflation compensation as of time t; that is, $\frac{\Pi_{t+\tau}}{\Pi_t} > \frac{1}{\Pi_t/\Pi_0}$. Under this condition, full inflation indexation applies, and the price change adjustment of the principal, $\frac{\Pi_{t+\tau}}{\Pi_t}$, is placed within the expectations operator. The second term represents the present value of the *floored* TIPS principal conditional on the net change in the price level until the bond's maturity eroding the accrued inflation compensation as of time t; that is, the price level change, $\frac{\Pi_{t+\tau}}{\Pi_t}$, is replaced by a value of one to provide the promised deflation protection.

Next, we exploit the fact that absence of arbitrage implies that the price level change is given by

$$\frac{\Pi_{t+\tau}}{\Pi_t} = e^{\int_t^{t+\tau}(r_s^N - r_s^R)ds}.$$

This allows us to rewrite the present value of the principal payment as

$$\begin{aligned} & E_t^Q\Big[e^{-\int_t^{t+\tau} r_s^R ds}\mathbf{1}_{\{\frac{\Pi_{t+\tau}}{\Pi_t} > \frac{1}{\Pi_t/\Pi_0}\}}\Big] + E_t^Q\Big[e^{-\int_t^{t+\tau} r_s^N ds}\mathbf{1}_{\{\frac{\Pi_{t+\tau}}{\Pi_t} \leq \frac{1}{\Pi_t/\Pi_0}\}}\Big] \\ = \; & E_t^Q\Big[e^{-\int_t^{t+\tau} r_s^R ds}\Big] + E_t^Q\Big[e^{-\int_t^{t+\tau} r_s^N ds}\mathbf{1}_{\{\frac{\Pi_{t+\tau}}{\Pi_t} \leq \frac{1}{\Pi_t/\Pi_0}\}}\Big] - E_t^Q\Big[e^{-\int_t^{t+\tau} r_s^R ds}\mathbf{1}_{\{\frac{\Pi_{t+\tau}}{\Pi_t} \leq \frac{1}{\Pi_t/\Pi_0}\}}\Big]. \end{aligned}$$

Here, the first term is the net present value of the TIPS principal payment without any deflation protection, while the two remaining terms equal the net present value of the deflation protection option:

$$DOV_t\Big(\tau; \frac{\Pi_t}{\Pi_0}\Big) \equiv E_t^Q\Big[e^{-\int_t^{t+\tau} r_s^N ds}\mathbf{1}_{\{\frac{\Pi_{t+\tau}}{\Pi_t} \leq \frac{1}{\Pi_t/\Pi_0}\}}\Big] - E_t^Q\Big[e^{-\int_t^{t+\tau} r_s^R ds}\mathbf{1}_{\{\frac{\Pi_{t+\tau}}{\Pi_t} \leq \frac{1}{\Pi_t/\Pi_0}\}}\Big]. \qquad (2)$$

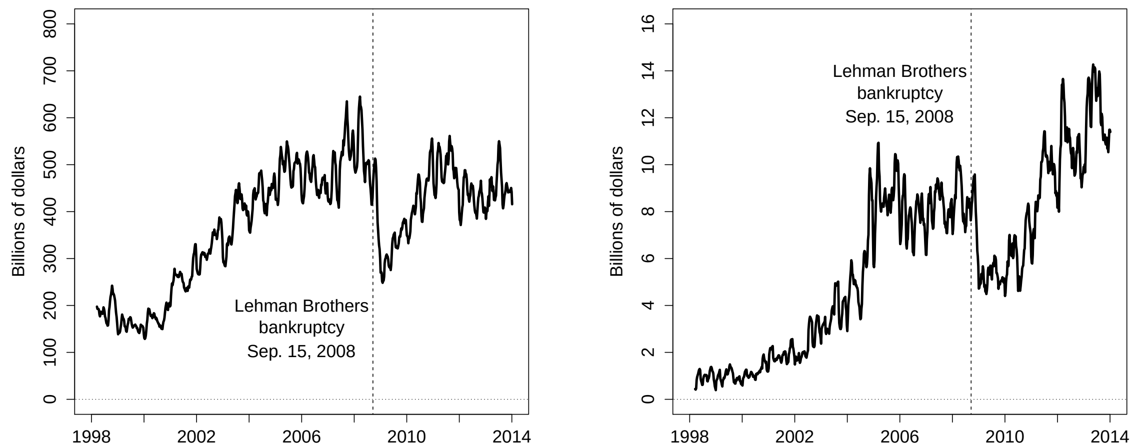
This option value needs to be added to the model-implied TIPS price to match the observed TIPS price.

3 An Arbitrage-Free Model of Nominal and Real Yields with Liquidity Risk

In this section, we first describe how we extend the general framework introduced in the previous section to account for the liquidity risk of a set of real-valued securities relative to a benchmark set of nominal securities. Second, we detail the established CLR model that we subject to this extension and apply in the subsequent empirical analysis.

3.1 The General Model with a Liquidity Risk Factor

Due to the lower liquidity of real-valued securities relative to the benchmark nominal securities, the yields of the former are sensitive to liquidity pressures on a relative basis. As a consequence, the discounting of future cash flows from the real-valued securities is not performed with the frictionless real discount function described in Section 2, but rather with a discount function that also accounts for the liquidity risk. The research by HPW and others suggest that liquidity is indeed a risk that requires a premium. Thus, we choose to represent this by a single liquidity risk factor denoted X_t^{liq}. Furthermore, since liquidity risk is security-



(a) Treasuries.

(b) TIPS.

Figure 1: **Trading Volume in the Treasury and TIPS Markets.**
Weekly average of daily trading volume in the secondary market for Treasuries and TIPS. The shown series are the 8-week moving average of the data, which are from the Federal Reserve Bank of New York and cover the period from January 15, 1998, to December 31, 2013.

specific in nature, the discount function used to discount the cash flow of a given real-valued security indexed i is assumed to be unique and take the following form:

$$\widehat{r}_t^{R,i} = r_t^R + \beta^i (1 - e^{-\lambda^{L,i}(t - t_0^i)}) X_t^{liq}, \tag{3}$$

where r_t^R is the frictionless real instantaneous rate as before, t_0^i denotes the date of issuance of the security, and β^i is its sensitivity to the variation in the liquidity risk factor. While we could expect the sensitivities to be identical across securities, the results from our subsequent empirical application shows that it is important to allow for the possibility that the sensitivities differ across securities. Furthermore, we allow the decay parameter $\lambda^{L,i}$ to vary across securities as well. Since β^i and $\lambda^{L,i}$ have a nonlinear relationship in the bond pricing formula, it is possible to identify both empirically.

The inclusion of the issuance date t_0^i in the pricing formula is a proxy for the phenomenon that, as time passes, it is typically the case that an increasing fraction of each security is held by buy-and-hold investors. This limits the amount of the security available for trading and drives up the liquidity premium. Rational, forward looking investors will take this dynamic pattern into consideration when they determine what they are willing to pay for a security at any given point in time between the date of issuance and the maturity of the bond. This dynamic pattern is built into the model structure.

In the specific case of liquidity risk in the pricing of TIPS, we justify both the notion of a liquidity disadvantage of TIPS relative to nominal Treasuries and the assumption of a single liquidity risk factor by pointing to the properties of the trading volume series for both types of securities. Figure 1 shows the 8-week moving average of the weekly average of daily trading volumes in the secondary market for Treasuries and TIPS.[4] As these two time series exhibit a very similar pattern,[5] the liquidity across the two markets appear highly correlated.[6] This suggests that a single factor is adequate to capture the variation in the relative liquidity across these two markets as in the general model described above.

3.2 The CLR Model

Building on the insights from the general theoretical discussion in Section 2, we need an accurate model of the instantaneous nominal and real rate, r_t^N and r_t^R, in order to measure the market-implied inflation expectations precisely. With that goal in mind we choose to focus on the established and tractable affine dynamic term structure model of nominal and real yields introduced in CLR and briefly summarized in the following.

The CLR model of nominal and real yields is a direct extension of the three-factor, arbitrage-free Nelson-Siegel (AFNS) model developed by Christensen et al. (2011, henceforth CDR) for nominal yields. In the CLR model, the state vector is denoted by $X_t = (L_t^N, S_t, C_t, L_t^R)$, where L_t^N is the level factor for nominal yields, S_t and C_t represent slope and curvature factors common to both nominal and real yields, and L_t^R is the level factor for real yields.[7] The instantaneous nominal and real risk-free rates are defined as:

$$r_t^N = L_t^N + S_t, \tag{4}$$

$$r_t^R = L_t^R + \alpha^R S_t. \tag{5}$$

Note that the differential scaling of the real rates to the common slope factor is captured by the parameter α^R. To preserve the Nelson-Siegel factor loading structure in the yield functions, the risk-neutral (or Q-) dynamics of the state variables are given by the stochastic

[4] The trading volume data are available at: http://www.newyorkfed.org/markets/statrel.html.

[5] At a weekly frequency, the correlation between the trading volume in the secondary market for TIPS and that for nominal Treasury securities was 82.4% from 1998 until the end of 2013.

[6] Abrahams et al. (2015) use the ratio of primary dealers Treasury transaction volumes relative to TIPS transaction volumes as an input in their construction of a TIPS illiquidity risk factor. D'Amico et al. (2014) also only allow for a single TIPS liquidity factor.

[7] Chernov and Mueller (2012) provide evidence of a hidden factor in the nominal yield curve that is observable from real yields and inflation expectations. The CLR model accommodates this stylized fact via the L_t^R factor.

differential equations:[8]

$$\begin{pmatrix} dL_t^N \\ dS_t \\ dC_t \\ dL_t^R \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -\lambda & \lambda & 0 \\ 0 & 0 & -\lambda & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} L_t^N \\ S_t \\ C_t \\ L_t^R \end{pmatrix} dt + \Sigma \begin{pmatrix} dW_t^{L^N,Q} \\ dW_t^{S,Q} \\ dW_t^{C,Q} \\ dW_t^{L^R,Q} \end{pmatrix}, \tag{6}$$

where Σ is the constant covariance (or volatility) matrix.[9] Based on this specification of the Q-dynamics, nominal zero-coupon bond yields preserve the Nelson-Siegel factor loading structure as

$$y_t^N(\tau) = L_t^N + \left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right) S_t + \left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right) C_t - \frac{A^N(\tau)}{\tau}, \tag{7}$$

where the nominal yield-adjustment term is given by

$$\begin{aligned} \frac{A^N(\tau)}{\tau} &= \frac{\sigma_{11}^2}{6}\tau^2 + \sigma_{22}^2\Big[\frac{1}{2\lambda^2} - \frac{1}{\lambda^3}\frac{1-e^{-\lambda\tau}}{\tau} + \frac{1}{4\lambda^3}\frac{1-e^{-2\lambda\tau}}{\tau}\Big] \\ &+\sigma_{33}^2\Big[\frac{1}{2\lambda^2} + \frac{1}{\lambda^2}e^{-\lambda\tau} - \frac{1}{4\lambda}\tau e^{-2\lambda\tau} - \frac{3}{4\lambda^2}e^{-2\lambda\tau} + \frac{5}{8\lambda^3}\frac{1-e^{-2\lambda\tau}}{\tau} - \frac{2}{\lambda^3}\frac{1-e^{-\lambda\tau}}{\tau}\Big]. \end{aligned}$$

Similarly, real zero-coupon bond yields have a Nelson-Siegel factor loading structure expressed as

$$y_t^R(\tau) = L_t^R + \alpha^R\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right) S_t + \alpha^R\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right) C_t - \frac{A^R(\tau)}{\tau}, \tag{8}$$

where the real yield-adjustment term is given by

$$\begin{aligned} \frac{A^R(\tau)}{\tau} &= \frac{\sigma_{44}^2}{6}\tau^2 + \sigma_{22}^2(\alpha_S^R)^2\Big[\frac{1}{2\lambda^2} - \frac{1}{\lambda^3}\frac{1-e^{-\lambda\tau}}{\tau} + \frac{1}{4\lambda^3}\frac{1-e^{-2\lambda\tau}}{\tau}\Big] \\ &+\sigma_{33}^2(\alpha_S^R)^2\Big[\frac{1}{2\lambda^2} + \frac{1}{\lambda^2}e^{-\lambda\tau} - \frac{1}{4\lambda}\tau e^{-2\lambda\tau} - \frac{3}{4\lambda^2}e^{-2\lambda\tau} + \frac{5}{8\lambda^3}\frac{1-e^{-2\lambda\tau}}{\tau} - \frac{2}{\lambda^3}\frac{1-e^{-\lambda\tau}}{\tau}\Big]. \end{aligned}$$

3.3 The CLR-L Model

In this section, we augment the CLR model with a liquidity risk factor to account for the liquidity risk in the pricing of TIPS relative to Treasuries, throughout referred to as the CLR-L model.

To begin, let $X_t = (L_t^N, S_t, C_t, L_t^R, X_t^{liq})$ denote the state vector of this five-factor model. As before, L_t^N and L_t^R denote the level factor unique to the nominal and real yield curve, respectively, while S_t and C_t represent slope and curvature factors common to both yield curves. Finally, X_t^{liq} represents the added liquidity factor.

[8]As discussed in CDR, with unit roots in the two level factors, the model is not arbitrage-free with an unbounded horizon; therefore, as is often done in theoretical discussions, we impose an arbitrary maximum horizon.

[9]As per CDR, Σ is a diagonal matrix, and θ^Q is set to zero without loss of generality.

As in the CLR model, we let the frictionless instantaneous nominal and real risk-free rates be defined by equations (4) and (5), respectively, while the risk-neutral dynamics of the state variables used for pricing are given by

$$\begin{pmatrix} dL_t^N \\ dS_t \\ dC_t \\ dL_t^R \\ X_t^{liq} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda & -\lambda & 0 & 0 \\ 0 & 0 & \lambda & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \kappa_{liq}^Q \end{pmatrix} \left[\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \theta_{liq}^Q \end{pmatrix} - \begin{pmatrix} L_t^N \\ S_t \\ C_t \\ L_t^R \\ X_t^{liq} \end{pmatrix} \right] dt + \Sigma \begin{pmatrix} dW_t^{L^N,Q} \\ dW_t^{S,Q} \\ dW_t^{C,Q} \\ dW_t^{L^R,Q} \\ dW_t^{liq,Q} \end{pmatrix},$$

where Σ continues to be a diagonal matrix.

Based on the Q-dynamics above, nominal Treasury zero-coupon bond yields preserve the Nelson-Siegel factor loading structure in equation (7). On the other hand, due to the lower liquidity in the TIPS market relative to the market for nominal Treasuries, TIPS yields are sensitive to liquidity pressures. As detailed in Section 3.1, pricing of TIPS is not performed with the frictionless real discount function, but rather with a discount function that accounts for the liquidity risk:

$$\widehat{r}_t^{R,i} = r_t^R + \beta^i(1 - e^{-\lambda^{L,i}(t-t_0^i)})X_t^{liq} = L_t^R + \alpha^R S_t + \beta^i(1 - e^{-\lambda^{L,i}(t-t_0^i)})X_t^{liq}, \qquad (9)$$

where t_0^i denotes the date of issuance of the specific TIPS and β^i is its sensitivity to the variation in the liquidity factor. Furthermore, the decay parameter $\lambda^{L,i}$ is assumed to vary across securities as well.

Since the model is affine, the net present value of one unit of the consumption basket paid by TIPS i at time $t + \tau$ is given by[10]

$$\begin{aligned} P_t^i(t_0^i, \tau) &= E^Q\left[e^{-\int_t^{t+\tau} \widehat{r}^{R,i}(s,t_0^i)ds}\right] \\ &= \exp\left(B_1^i(t_0^i,\tau)L_t^N + B_2^i(t_0^i,\tau)S_t + B_3^i(t_0^i,\tau)C_t + B_4^i(t_0^i,\tau)L_t^R + B_5^i(t_0^i,\tau)X_t^{Liq} + A^i(t_0^i,\tau)\right). \end{aligned}$$

Now, consider the whole value of the TIPS issued at time t_0 with maturity at $t+\tau$ that pays an annual coupon C semi-annually and has accrued inflation compensation equal to Π_t/Π_0. Its price is given by

$$P_t^i(t_0^i,\tau) = \frac{C}{2}\frac{(t_1-t)}{1/2}E^Q\left[e^{-\int_t^{t_1} \widehat{r}^{R,i}(s,t_0^i)ds}\right] + \sum_{j=2}^{N}\frac{C}{2}E^Q\left[e^{-\int_t^{t_j} \widehat{r}^{R,i}(s,t_0^i)ds}\right] + E^Q\left[e^{-\int_t^{t+\tau} \widehat{r}^{R,i}(s,t_0^i)ds}\right] + DOV_t\left(\tau; \frac{\Pi_t}{\Pi_0^i}\right),$$

where the last term is the value of the deflation protection option provided in equation (2) and calculated using the dynamics for frictionless nominal and real instantaneous short rates, r_t^N and r_t^R.

[10]The analytical formulas needed to calculate this expression are provided in Appendix A.

Finally, the model is completed by describing the connection to the real-world P-dynamics. Using the essentially affine risk premium specification introduced in Duffee (2002), the implied measure change is given by

$$dW_t^Q = dW_t^P + \Gamma_t dt,$$

where $\Gamma_t = \gamma^0 + \gamma^1 X_t$, $\gamma^0 \in \mathbf{R}^5$, and $\gamma^1 \in \mathbf{R}^{5\times 5}$. The resulting unrestricted five-factor CLR-L model has P-dynamics given by

$$\begin{pmatrix} dL_t^N \\ dS_t \\ dC_t \\ dL_t^R \\ X_t^{liq} \end{pmatrix} = \begin{pmatrix} \kappa_{11}^P & \kappa_{12}^P & \kappa_{13}^P & \kappa_{14}^P & \kappa_{15}^P \\ \kappa_{21}^P & \kappa_{22}^P & \kappa_{23}^P & \kappa_{24}^P & \kappa_{25}^P \\ \kappa_{31}^P & \kappa_{32}^P & \kappa_{33}^P & \kappa_{34}^P & \kappa_{35}^P \\ \kappa_{41}^P & \kappa_{42}^P & \kappa_{43}^P & \kappa_{44}^P & \kappa_{45}^P \\ \kappa_{51}^P & \kappa_{52}^P & \kappa_{53}^P & \kappa_{54}^P & \kappa_{55}^P \end{pmatrix} \left(\begin{pmatrix} \theta_1^P \\ \theta_2^P \\ \theta_3^P \\ \theta_4^P \\ \theta_5^P \end{pmatrix} - \begin{pmatrix} L_t^N \\ S_t \\ C_t \\ L_t^R \\ X_t^{liq} \end{pmatrix} \right) dt + \Sigma \begin{pmatrix} dW_t^{L^N,P} \\ dW_t^{S,P} \\ dW_t^{C,P} \\ dW_t^{L^R,P} \\ dW_t^{liq,P} \end{pmatrix}.$$

This is the transition equation in the Kalman filter estimation.

3.4 Model Estimation and Econometric Identification

Two things are worth highlighting regarding the TIPS pricing formula provided in the previous section. First, it neglects the lag in the inflation indexation for pricing, which we argue is a mild omission since we only track each TIPS as long as it has more than two years to maturity. Second, due to the nonlinear relationship between the state variables and the TIPS prices, the model cannot be estimated with the standard Kalman filter, instead we use the extended Kalman filter as in Christensen and Rudebusch (2015). Furthermore, to make the fitted errors comparable across TIPS of various maturities, we scale each TIPS price by its duration. Thus, the measurement equation for the TIPS prices take the following form:

$$\frac{P_t^i(t_0^i,\tau)}{D_t^i(t_0^i,\tau)} = \frac{\widehat{P}_t^i(t_0^i,\tau)}{D_t^i(t_0^i,\tau)} + \varepsilon_t^i,$$

where $\widehat{P}_t^i(t_0^i,\tau)$ is the model-implied price of TIPS i and $D_t^i(t_0^i,\tau)$ is its duration.[11]

From the five-factor model structure above it follows that we will be fitting TIPS yields with two separate factors, the real level factor, L_t^R, and the TIPS liquidity factor, X_t^{liq}, in addition to the common slope and curvature factor that can be identified from the nominal yields. Thus, for reasons of identification, we need to have at least two TIPS securities trading at each observation date. This requirement implies that the earliest starting point for the model estimation coincides with the issuance date of the second TIPS in mid-July 1997.

Furthermore, since the liquidity factor is a latent factor that we do not observe, its level is

[11] For robustness, we repeated the estimations using the mid-market yield-to-maturities for each TIPS downloaded from Bloomberg instead and got very similar results. However, we note that those estimations are extremely time consuming since yield-to-maturity is only defined implicitly as a fix point that needs to be calculated for each observation. Hence, we advise against that approach.

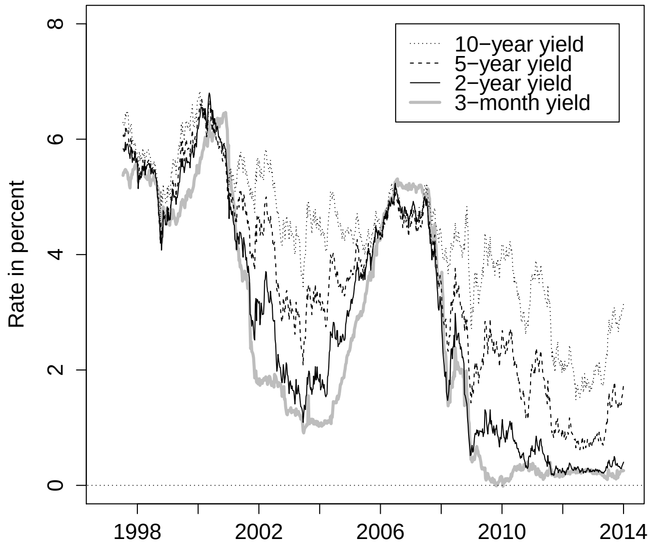


Figure 2: **Nominal Treasury Yields.**
Illustration of the weekly nominal Treasury zero-coupon bond yields covering the period from July 11, 1997, to December 27, 2013. The yields shown have maturities in three months, two years, five years, and ten years, respectively.

not identified without additional restrictions. As a consequence, we let the first TIPS issued, that is, the ten-year TIPS with 3.375% coupon issued in January 1997 with maturity on January 15, 2007, have a unit loading on the liquidity factor, that is, $\beta^i = 1$ for this security. This choice implies that the β^i sensitivity parameters measure liquidity sensitivity relative to that of the ten-year 2007 TIPS.

Finally, we note that the $\lambda^{L,i}$-parameters can be hard to identify if their values are too large or too small. As a consequence, we impose the restriction that they fall within the range from 0.01 to 10, which is without practical consequences. Also, for numerical stability during model optimization, we impose the restriction that the β^i-parameters fall within the range from 0 to 40, which turns out not to be a binding constraint in optimum.

4 Data

This section briefly describes the data we use in the model estimation.

Maturity in months	Mean in %	St. dev. in %	Skewness	Kurtosis
3	2.59	2.14	0.27	1.48
6	2.61	2.15	0.27	1.49
12	2.68	2.13	0.25	1.53
24	2.89	2.02	0.15	1.60
36	3.10	1.89	0.06	1.70
48	3.33	1.76	-0.03	1.82
60	3.54	1.64	-0.11	1.96
72	3.74	1.54	-0.19	2.11
84	3.93	1.45	-0.27	2.27
96	4.10	1.37	-0.34	2.41
108	4.25	1.30	-0.40	2.53
120	4.38	1.25	-0.45	2.63

Table 1: **Summary Statistics for the Nominal Treasury Yields.**
Summary statistics for the sample of weekly nominal Treasury zero-coupon bond yields covering the period from July 11, 1997, to December 27, 2013, a total of 860 observations.

4.1 Nominal Treasury Yields

The specific nominal Treasury yields we use are zero-coupon yields constructed by the method described in Gürkaynak, Sack, and Wright (2007) and briefly detailed here.[12] For each business day a zero-coupon yield curve of the Svensson (1995)-type

$$y_t(\tau) = \beta_0 + \frac{1 - e^{-\lambda_1 \tau}}{\lambda_1 \tau}\beta_1 + \Big[\frac{1 - e^{-\lambda_1 \tau}}{\lambda_1 \tau} - e^{-\lambda_1 \tau}\Big]\beta_2 + \Big[\frac{1 - e^{-\lambda_2 \tau}}{\lambda_2 \tau} - e^{-\lambda_2 \tau}\Big]\beta_3$$

is fitted to price a large pool of underlying off-the-run Treasury bonds. Thus, for each business day, we have the fitted values of the four factors $(\beta_0(t), \beta_1(t), \beta_2(t), \beta_3(t))$ and the two parameters $(\lambda_1(t), \lambda_2(t))$. From this data set zero-coupon yields for any relevant maturity can be calculated as long as the maturity is within the range of maturities used in the fitting process. As demonstrated by Gürkaynak, Sack, and Wright (2007), this model fits the underlying pool of bonds extremely well. By implication, the zero-coupon yields derived from this approach constitute a very good approximation to the true underlying Treasury zero-coupon yield curve. We construct nominal Treasury zero-coupon bond yields with the following maturities: 3-month, 6-month, 1-year, 2-year, 3-year, 4-year, 5-year, 6-year, 7-year, 8-year, 9-year, and 10-year. We use weekly data and limit our sample to the period from July 11, 1997, to December 27, 2013. The summary statistics are provided in Table 1, while Figure 2 illustrates the constructed time series of the 3-month, 2-year, 5-year, and 10-year Treasury zero-coupon yields.

Researchers have typically found that three factors are sufficient to model the time-variation in the cross section of nominal Treasury bond yields (e.g., Litterman and Scheinkman,

[12] The Board of Governors in Washington DC frequently updates the factors and parameters of this method, see the related website http://www.federalreserve.gov/pubs/feds/2006/index.html

Maturity in months	Loading on		
	First P.C.	Second P.C.	Third P.C.
3	0.35	0.42	-0.50
6	0.35	0.40	-0.25
12	0.35	0.31	0.12
24	0.34	0.12	0.42
36	0.32	-0.01	0.41
48	0.30	-0.12	0.30
60	0.27	-0.19	0.15
72	0.25	-0.25	0.02
84	0.24	-0.29	-0.10
96	0.22	-0.32	-0.19
108	0.21	-0.35	-0.26
120	0.19	-0.36	-0.32
% explained	95.26	4.51	0.21

Table 2: **Eigenvectors of the First Three Principal Components in Nominal Treasury Yields.**
The loadings of yields of various maturities on the first three principal components are shown. The final row shows the proportion of all bond yield variability accounted for by each principal component. The data consist of weekly nominal zero-coupon U.S. Treasury bond yields from July 11, 1997, to December 27, 2013.

1991). Indeed, for our weekly nominal Treasury bond yield data, 99.98% of the total variation is accounted for by three factors. Table 2 reports the eigenvectors that correspond to the first three principal components of our data. The first principal component accounts for 95.3% of the variation in the nominal Treasury bond yields, and its loading across maturities is uniformly negative. Thus, like a level factor, a shock to this component changes all yields in the same direction irrespective of maturity. The second principal component accounts for 4.5% of the variation in these data and has sizable negative loadings for the shorter maturities and sizable positive loadings for the long maturities. Thus, like a slope factor, a shock to this component steepens or flattens the yield curve. Finally, the third component, which accounts for only 0.2% of the variation, has a hump shaped factor loading as a function of maturity, which is naturally interpreted as a curvature factor. This motivates our use of the AFNS model with its level, slope, and curvature structure for the nominal yields even though we emphasize that the estimated state variables are *not* identical to the principal component factors discussed here.[13]

4.2 TIPS Data

The U.S. Treasury started issuing TIPS in 1997. The first TIPS was issued on February 6, 1997, with maturity on January 15, 2007, and a coupon rate of 3.375%.[14] Since then the U.S. Treasury has issued five-, ten-, twenty-, and thirty-year TIPS. However, only ten-year TIPS

[13]A number of recent papers use principal components as state variables. Joslin et al. (2011) is an example.

[14]TIPS are issued with a minimum coupon of 0.125%. Since April 2011 this has been a binding constraint for five-year TIPS and occasionally also for ten-year TIPS.

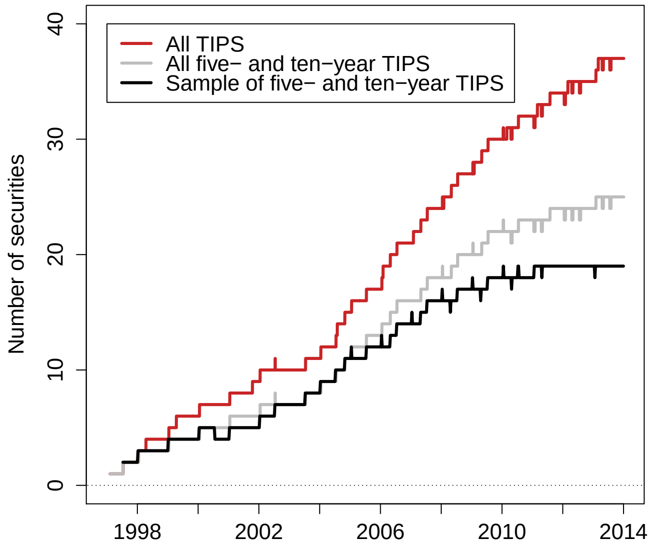


Figure 3: **Number of TIPS Outstanding.**
Illustration of the number of TIPS outstanding. The sample covers the period from February 6, 1997, to December 31, 2013.

have been regularly issued since the start of the TIPS program. As of the end of 2013, a total of 50 TIPS had been issued, while the total number of TIPS outstanding at any point in time since the start of the TIPS program is shown with a solid red line in Figure 3. At the end of our sample period there was a total of 37 TIPS outstanding.

To facilitate the empirical implementation and improve model fit, we limit our focus to five- and ten-year TIPS.[15] This reduces the total number of TIPS to 38, while the number of those TIPS outstanding at any point in time is shown with a solid grey line in Figure 3. At the end of 2013, this subset included 25 TIPS. Furthermore, as TIPS prices near maturity tend to exhibit erratic behavior due to seasonal variation in CPI, we drop TIPS from our sample when they have less than two years to maturity.[16] Thus, our analysis is centered around the two- to ten-year maturity range that is the most widely used for both bond risk management and monetary policy analysis. Using this cutoff, the number of TIPS in the sample is further reduced and shown with a solid black line in Figure 3. As of the end of 2013, it included 19 securities. Relevant statistics for our sample of 38 TIPS are reported in Table 3.

To estimate the CLR-L model introduced in Section 3.3, we need prices for each TIPS since issuance. To that end, we use the mid-market clean prices for each TIPS downloaded from Bloomberg. Since the model has two TIPS specific factors, we start the model estimation

[15] As a robustness check, we estimated the model with all available TIPS in combination with nominal yields with maturities up to thirty years. This produced qualitatively similar, but less accurate results.
[16] Gürkaynak et al. (2010) only use TIPS with more than two years to maturity for similar reasons.

TIPS security	No. obs.	Issuance		First reopen		Second reopen	
		Date	Amount	Date	Amount	Date	Amount
(1) 3.375% 1/15/2007 TIPS	393	2/6/97	7,353	4/15/97	8,403	n.a.	n.a.
(2) 3.625% 7/15/2002 TIPS*	158	7/15/97	8,401	10/15/97	8,412	n.a.	n.a.
(3) 3.625% 1/15/2008 TIPS	419	1/15/98	8,409	10/15/98	8,401	n.a.	n.a.
(4) 3.875% 1/15/2009 TIPS	419	1/15/99	8,531	7/15/99	7,368	n.a.	n.a.
(5) 4.25% 1/15/2010 TIPS	418	1/18/00	6,317	7/17/00	5,002	n.a.	n.a.
(6) 3.5% 1/15/2011 TIPS	418	1/16/01	6,000	7/16/01	5,000	n.a.	n.a.
(7) 3.375% 1/15/2012 TIPS	419	1/15/02	6,000	n.a.	n.a.	n.a.	n.a.
(8) 3% 7/15/2012 TIPS	419	7/15/02	10,010	10/15/02	7,000	1/15/03	6,000
(9) 1.875% 7/15/2013 TIPS	419	7/15/03	11,000	10/15/03	9,000	n.a.	n.a.
(10) 2% 1/15/2014 TIPS	419	1/15/04	12,000	4/15/04	9,000	n.a.	n.a.
(11) 2% 7/15/2014 TIPS	419	7/15/04	10,000	10/15/04	9,000	n.a.	n.a.
(12) 0.875% 4/15/2010 TIPS*	181	10/29/04	12,000	4/29/05	9,000	10/28/05	7,000
(13) 1.625% 1/15/2015 TIPS	418	1/18/05	10,000	4/15/05	9,000	n.a.	n.a.
(14) 1.875% 7/15/2015 TIPS	418	7/15/05	9,000	10/17/05	8,000	n.a.	n.a.
(15) 2% 1/15/2016 TIPS	416	1/17/06	9,000	4/17/06	8,000	n.a.	n.a.
(16) 2.375% 4/15/2011 TIPS*	155	4/28/06	11,000	10/31/06	9,181	n.a.	n.a.
(17) 2.5% 7/15/2016 TIPS	390	7/17/06	10,588	10/16/06	9,412	n.a.	n.a.
(18) 2.375% 1/15/2017 TIPS	364	1/16/07	11,250	4/16/07	6,000	n.a.	n.a.
(19) 2% 4/15/2012 TIPS*	156	4/30/07	10,123	10/31/07	7,158	n.a.	n.a.
(20) 2.625% 7/15/2017 TIPS	338	7/16/07	8,000	10/15/07	6,000	n.a.	n.a.
(21) 1.625% 1/15/2018 TIPS	312	1/15/08	10,412	4/15/08	6,000	n.a.	n.a.
(22) 0.625% 4/15/2013 TIPS*	156	4/30/08	8,734	10/31/08	6,266	n.a.	n.a.
(23) 1.375% 7/15/2018 TIPS	286	7/15/08	8,000	10/15/08	6,974	n.a.	n.a.
(24) 2.125% 1/15/2019 TIPS	260	1/15/09	8,662	4/15/09	6,096	n.a.	n.a.
(25) 1.25% 4/15/2014 TIPS*	156	4/30/09	8,277	10/30/09	7,000	n.a.	n.a.
(26) 1.875% 7/15/2019 TIPS	234	7/15/09	8,135	10/15/09	7,055	n.a.	n.a.
(27) 1.375% 1/15/2020 TIPS	207	1/15/10	10,388	4/15/10	8,586	n.a.	n.a.
(28) 0.5% 4/15/2015 TIPS*	155	4/30/10	11,235	10/29/10	10,000	n.a.	n.a.
(29) 1.25% 7/15/2020 TIPS	182	7/15/10	12,003	9/15/10	10,108	11/15/10	10,268
(30) 1.125% 1/15/2021 TIPS	154	1/31/11	13,259	3/31/11	11,493	5/31/11	11,926
(31) 0.125% 4/15/2016 TIPS*	140	4/29/11	14,000	8/31/11	12,367	12/30/11	12,000
(32) 0.625% 7/15/2021 TIPS	128	7/29/11	13,000	9/30/11	11,342	11/30/11	11,498
(33) 0.125% 1/15/2022 TIPS	102	1/31/12	15,282	3/30/12	13,000	5/31/12	13,000
(34) 0.125% 4/15/2017 TIPS*	89	4/30/12	16,430	8/31/12	14,000	12/31/12	14,000
(35) 0.125% 7/15/2022 TIPS	76	7/31/12	15,000	9/28/12	13,000	11/30/12	13,000
(36) 0.125% 1/15/2023 TIPS	49	1/31/13	15,000	3/28/13	13,000	5/31/13	13,000
(37) 0.125% 4/15/2018 TIPS*	37	4/30/13	18,000	8/30/13	16,000	12/31/13	16,000
(38) 0.375% 7/15/2023 TIPS	24	7/31/13	15,000	9/30/13	13,000	11/29/13	13,000

Table 3: **Sample of TIPS.**
The table reports the characteristics, issuance dates, and issuance amounts in millions of dollars for the 38 TIPS used in the analysis. Also reported are the number of weekly observation dates for each TIPS during the sample period from July 11, 1997, to December 27, 2013. Asterisk * indicates five-year TIPS.

on Friday July 11, 1997, when prices become available for the second ever TIPS, the five-year TIPS with maturity on July 15, 2002. We end the sample on December 27, 2013, with 19 TIPS trading. The number of weekly observations for each of our 38 TIPS is also reported in Table 3.

CLR-L model								
K^P	$K^P_{\cdot,1}$	$K^P_{\cdot,2}$	$K^P_{\cdot,3}$	$K^P_{\cdot,4}$	$K^P_{\cdot,5}$	θ^P		Σ
$K^P_{1,\cdot}$	0.2408	0	0	0	0	0.0610	σ_{11}	0.0059
	(0.1904)					(0.0082)		(0.0001)
$K^P_{2,\cdot}$	0	0.0939	0	0	0	-0.0301	σ_{22}	0.0099
		(0.1458)				(0.0276)		(0.0002)
$K^P_{3,\cdot}$	0	0	0.4602	0	0	-0.0316	σ_{33}	0.0250
			(0.2934)			(0.0145)		(0.0005)
$K^P_{4,\cdot}$	0	0	0	0.3139	0	0.0334	σ_{44}	0.0075
				(0.3169)		(0.0099)		(0.0002)
$K^P_{5,\cdot}$	0	0	0	0	0.6987	0.0087	σ_{55}	0.0124
					(0.3953)	(0.0076)		(0.0007)

CLR-L model, option adjusted								
K^P	$K^P_{\cdot,1}$	$K^P_{\cdot,2}$	$K^P_{\cdot,3}$	$K^P_{\cdot,4}$	$K^P_{\cdot,5}$	θ^P		Σ
$K^P_{1,\cdot}$	0.2348	0	0	0	0	0.0612	σ_{11}	0.0060
	(0.1907)					(0.0083)		(0.0001)
$K^P_{2,\cdot}$	0	0.0873	0	0	0	-0.0294	σ_{22}	0.0099
		(0.1461)				(0.0297)		(0.0002)
$K^P_{3,\cdot}$	0	0	0.4069	0	0	-0.0323	σ_{33}	0.0249
			(0.2941)			(0.0158)		(0.0005)
$K^P_{4,\cdot}$	0	0	0	0.2585	0	0.0342	σ_{44}	0.0072
				(0.2782)		(0.0102)		(0.0002)
$K^P_{5,\cdot}$	0	0	0	0	0.7244	0.0074	σ_{55}	0.0124
					(0.3936)	(0.0075)		(0.0007)

Table 4: **Estimated Dynamic Parameters.**
The top panel shows the estimated parameters of the K^P matrix, θ^P vector, and diagonal Σ matrix for the CLR-L model. The estimated value of λ is 0.4473 (0.0019), while $\alpha^R = 0.7620$ (0.0077), $\kappa^Q_{liq} = 0.8257$ (0.0436), and $\theta^Q_{liq} = 0.0016$ (0.0001). The bottom panel shows the corresponding estimates for the CLR-L model with deflation option adjustment. In this case, the estimated value of λ is 0.4442 (0.0019), while $\alpha^R = 0.7584$ (0.0072), $\kappa^Q_{liq} = 0.9004$ (0.0598), and $\theta^Q_{liq} = 0.0014$ (0.0001). The numbers in parentheses are the estimated parameter standard deviations.

5 Estimation Results

In this section, we first describe the results from the CLR-L model estimated with and without adjustment for the the value of the deflation protection option embedded in the TIPS contract. Second, we compare the estimated state variables and model fit to those obtained from standard AFNS and CLR models. Finally, we analyze in detail the estimated TIPS liquidity premium. In particular, we follow Christensen and Gillan (2015, henceforth CG) and study the effects on TIPS liquidity premiums from the Fed's TIPS purchases during its second large-scale asset purchases program—frequently referred to as QE2—that operated from November 2010 through June 2011. Up front we note that, for each model class, we limit the focus to the most parsimonious independent-factor specification to make the results as comparable as possible.[17]

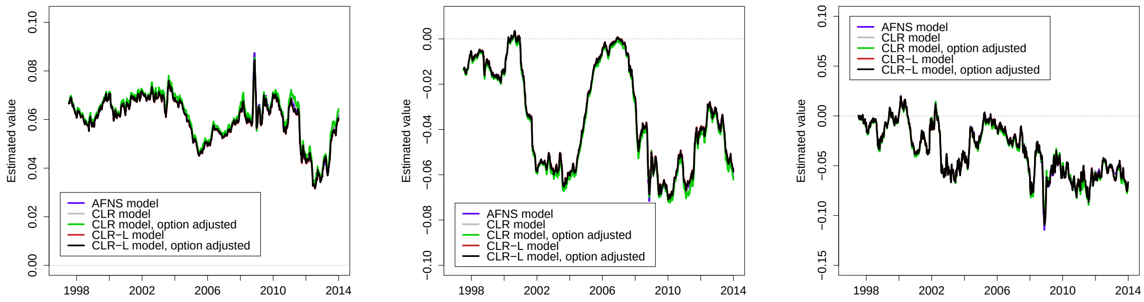
[17] Since the model fit and the estimated factors are insensitive to the specification of the mean-reversion matrix K^P, this limitation comes at practically no loss of generality for the results presented in this section.

TIPS security	CLR-L model				CLR-L model, option adjusted			
	β^i	Std	$\lambda^{L,i}$	Std	β^i	Std	$\lambda^{L,i}$	Std
(1) 3.375% 1/15/2007 TIPS	1	n.a.	0.7047	0.3563	1	n.a.	0.7892	0.5808
(2) 3.625% 7/15/2002 TIPS*	0.8260	0.1333	8.9414	2.1275	0.8397	0.1402	7.9001	1.9886
(3) 3.625% 1/15/2008 TIPS	2.1317	0.4677	0.1320	0.0493	2.7179	0.5367	0.0954	0.0481
(4) 3.875% 1/15/2009 TIPS	3.0805	0.8843	0.0988	0.0402	3.9884	1.0288	0.0735	0.0404
(5) 4.25% 1/15/2010 TIPS	2.0739	0.1818	0.2360	0.0415	2.2469	0.1968	0.2206	0.0399
(6) 3.5% 1/15/2011 TIPS	2.3928	0.1943	0.2143	0.0320	2.5637	0.2187	0.2098	0.0313
(7) 3.375% 1/15/2012 TIPS	2.4185	0.1833	0.2384	0.0364	2.5856	0.2126	0.2395	0.0341
(8) 3% 7/15/2012 TIPS	2.3956	0.1686	0.2604	0.0412	2.5697	0.1895	0.2597	0.0403
(9) 1.875% 7/15/2013 TIPS	3.0073	0.3761	0.1781	0.0434	3.6307	0.5301	0.1439	0.0397
(10) 2% 1/15/2014 TIPS	5.3622	1.2267	0.0838	0.0264	7.3189	1.6844	0.0620	0.0187
(11) 2% 7/15/2014 TIPS	2.5352	0.1962	0.3410	0.0657	2.8097	0.2341	0.3095	0.0561
(12) 0.875% 4/15/2010 TIPS*	1.9994	0.0802	9.9988	2.2979	2.1262	0.0875	9.9994	2.1352
(13) 1.625% 1/15/2015 TIPS	3.4597	0.3973	0.1871	0.0371	3.8698	0.5574	0.1763	0.0369
(14) 1.875% 7/15/2015 TIPS	2.1376	0.1231	0.9495	0.3939	2.3197	0.1356	0.8955	0.2641
(15) 2% 1/15/2016 TIPS	2.5252	0.1953	0.3848	0.0677	2.7908	0.2285	0.3771	0.0687
(16) 2.375% 4/15/2011 TIPS*	1.9328	0.0798	5.2441	2.0697	2.0565	0.0867	4.6394	2.1190
(17) 2.5% 7/15/2016 TIPS	1.8922	0.1077	6.2509	2.1911	2.0867	0.1202	9.8130	2.1433
(18) 2.375% 1/15/2017 TIPS	1.9108	0.1052	9.9794	2.0559	2.1207	0.1193	9.9189	2.0233
(19) 2% 4/15/2012 TIPS*	1.8565	0.0826	9.9975	1.9748	1.9795	0.0892	10.0000	2.1203
(20) 2.625% 7/15/2017 TIPS	1.5773	0.0929	9.9943	2.3952	1.7740	0.1074	9.9991	2.2029
(21) 1.625% 1/15/2018 TIPS	1.9053	0.1815	0.4481	0.1237	2.1567	0.1953	0.4756	0.1342
(22) 0.625% 4/15/2013 TIPS*	5.3129	1.1659	0.1526	0.0450	4.3600	0.5288	0.2246	0.0536
(23) 1.375% 7/15/2018 TIPS	1.2974	0.1566	0.8968	0.3108	1.5303	0.1734	0.8955	0.3362
(24) 2.125% 1/15/2019 TIPS	28.0660	4.1001	0.0100	0.0018	31.9838	4.1442	0.0105	0.0018
(25) 1.25% 4/15/2014 TIPS*	38.6112	4.1291	0.0230	0.0030	15.1962	3.7701	0.0701	0.0449
(26) 1.875% 7/15/2019 TIPS	1.4761	0.3116	0.4666	0.3420	1.7695	0.3098	0.4706	0.3960
(27) 1.375% 1/15/2020 TIPS	29.9390	4.7859	0.0100	0.0018	35.6542	4.6957	0.0104	0.0017
(28) 0.5% 4/15/2015 TIPS*	23.8180	4.8797	0.0372	0.0084	9.5635	3.6681	0.1165	0.0765
(29) 1.25% 7/15/2020 TIPS	1.8997	1.0819	0.3470	0.5347	2.2639	0.7916	0.4111	0.6262
(30) 1.125% 1/15/2021 TIPS	3.0499	1.6842	0.2461	0.3036	3.6223	1.5942	0.2821	0.3255
(31) 0.125% 4/15/2016 TIPS*	9.8655	6.2960	0.0990	0.0757	6.8538	4.0340	0.1820	0.1326
(32) 0.625% 7/15/2021 TIPS	2.1138	0.7052	0.6286	0.9480	2.8131	0.6675	0.5685	1.1072
(33) 0.125% 1/15/2022 TIPS	3.1227	1.6588	0.4219	0.6766	4.3189	1.8193	0.3685	0.6817
(34) 0.125% 4/15/2017 TIPS*	17.4890	8.2002	0.0530	0.0272	15.7646	8.2915	0.0712	0.0445
(35) 0.125% 7/15/2022 TIPS	2.2077	0.2444	5.4838	8.7416	2.8941	0.3308	4.7134	7.8943
(36) 0.125% 1/15/2023 TIPS	2.7209	0.2336	9.9980	9.7693	3.6017	0.3226	9.9793	8.9224
(37) 0.125% 4/15/2018 TIPS*	24.6981	10.9722	0.0416	0.0199	3.4604	0.9269	0.9891	1.2716
(38) 0.375% 7/15/2023 TIPS	1.8409	0.3227	9.9575	16.7198	2.5613	0.4752	9.9949	13.9168

Table 5: **Estimated Liquidity Sensitivity Parameters.**
The estimated β^i sensitivity and $\lambda^{L,i}$ decay parameters for each TIPS from the CLR-L model with and without deflation option adjustment are shown. Also reported are the estimated parameter standard deviations. Asterisk * indicates five-year TIPS. The sample used in each model estimation is weekly covering the period from July 11, 1997, to December 27, 2013. All numbers are measured in basis points.

Table 4 contains the estimated dynamic parameters for the CLR-L model estimated with and without accounting for the deflation option values, while Table 5 reports the corresponding estimated β^i and $\lambda^{L,i}$ parameters. Overall, the model parameters, the dynamic parameters in particular, are relatively insensitive to adjusting for the deflation option values.

As for the state variables, Figure 4 shows the estimated paths for (L_t^N, S_t, C_t), which are primarily determined from nominal yields. We note that the estimated paths of these three



(a) L_t^N. (b) S_t. (c) C_t.

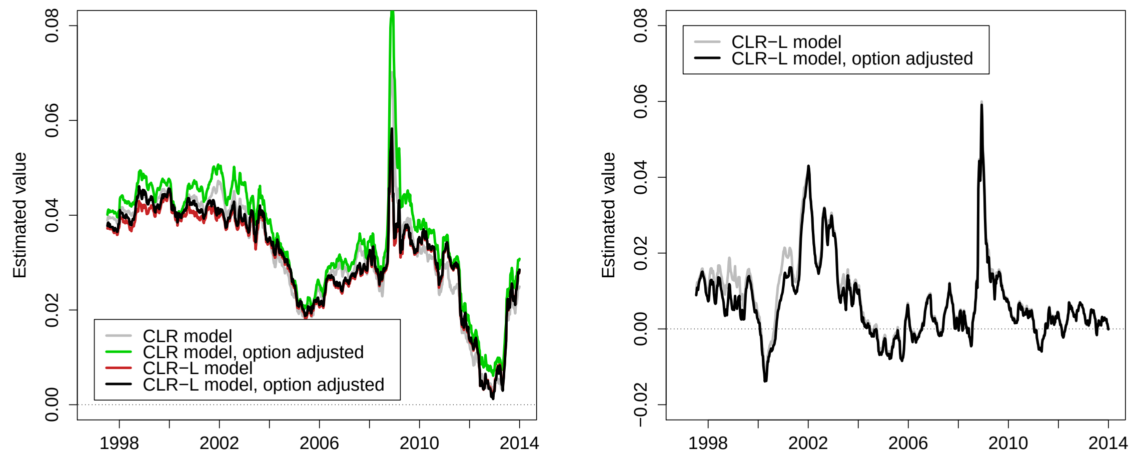
Figure 4: **Estimated State Variables.**
Illustration of the estimated state variables that affect nominal yields from the AFNS model, the CLR model, the CLR model with deflation option adjustment, the CLR-L model, and the CLR-L model with deflation option adjustment. The data are weekly covering the period from July 11, 1997, to December 27, 2013.

Maturity in months	AFNS model		CLR model				CLR-L model			
			Base model		Option adjusted		Base model		Option adjusted	
	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE
3	-0.99	7.27	-0.99	7.36	-0.97	7.52	-1.15	7.40	-1.14	7.44
6	-0.88	2.64	-0.90	2.66	-0.94	2.67	-0.85	2.70	-0.87	2.70
12	0.95	6.98	0.92	7.02	0.82	7.13	1.19	7.03	1.14	7.04
24	2.54	5.74	2.54	5.98	2.48	6.30	2.78	6.16	2.75	6.17
36	1.32	3.04	1.35	3.43	1.36	3.73	1.35	3.62	1.38	3.63
48	-0.48	2.78	-0.42	2.95	-0.37	2.98	-0.62	2.98	-0.56	2.96
60	-1.72	3.71	-1.65	3.69	-1.61	3.64	-1.95	3.68	-1.88	3.65
72	-2.09	3.83	-2.03	3.80	-2.02	3.83	-2.32	3.82	-2.25	3.80
84	-1.64	3.04	-1.61	3.12	-1.63	3.28	-1.79	3.17	-1.76	3.16
96	-0.56	1.97	-0.58	2.27	-0.61	2.56	-0.59	2.37	-0.60	2.36
108	0.92	2.70	0.85	2.92	0.82	3.10	1.03	3.07	0.98	3.03
120	2.60	5.15	2.47	5.13	2.50	5.16	2.86	5.31	2.76	5.26
All yields	0.00	4.41	0.00	4.52	-0.01	4.64	0.00	4.59	0.00	4.59

Table 6: **Summary Statistics of Fitted Errors of Nominal Yields.**
The mean fitted errors and the root mean squared fitted errors (RMSE) of nominal U.S. Treasury yields from five model estimations are shown. The full sample used in each model estimation is weekly covering the period from July 11, 1997, to December 27, 2013. All numbers are measured in basis points.

factors are practically indistinguishable from the estimated paths obtained with a stand-alone AFNS model of nominal yields only.

This is also reflected in the summary statistics for the fitted errors of nominal yields reported in Table 6. The results show that the CLR and CLR-L models, with and without deflation option adjustment, provide a very close fit to the entire cross section of nominal yields. Importantly, the fit is as good as that obtained with a stand-alone AFNS model for the nominal yields. Thus, allowing for a joint modeling of nominal and real yields based on the CLR model framework comes at effectively no cost in terms of fit to the nominal yields



(a) L_t^R.

(b) X_t^{liq}.

Figure 5: **Estimated State Variables.**
Illustration of the estimated state variables that affect real yields from the CLR-L model and the CLR-L model with deflation option adjustment. The data are weekly covering the period from July 11, 1997, to December 27, 2013.

as also emphasized by CLR.

The estimated level factor for real yields shown in Figure 5(a) is obviously sensitive to whether a liquidity risk factor is included or not. However, it only seems to be periodically that the differences are notable. Finally, Figure 5(b) shows the estimated liquidity risk factor that is unique to the CLR-L model.

To evaluate the fit of the models to the TIPS price data, we calculate the model-implied time series of the yield-to-maturity for each TIPS and compare that to the mid-market TIPS yield-to-maturity available from Bloomberg. The summary statistics of the fitted yield errors calculated this way are reported in Table 7. First, we note that the CLR model tends to provide less than ideal fit as measured by RMSEs and with notable bias for some TIPS. Second, accounting for the value of the deflation protection option improves the fit of the model, but this modification is not sufficient for the model to deliver satisfactory fit (as measured by RMSEs) or to eliminate the bias for select TIPS. For all TIPS yields combined, the RMSEs remain close to 15 basis points. Third, incorporating the liquidity factor into the CLR model leads to a significant improvement in model fit for practically all TIPS in the sample. Also, and equally important, there is no material bias for any of the TIPS. With the extension, the fit to the TIPS data is about as good as the fit to the nominal Treasury yields. Finally, accounting for the deflation option values in addition to incorporating the liquidity factor provides a further modest improvement in model fit.

To give a visual representation of the fit from the CLR-L model with deflation option value adjustment, Figure 6 shows the fitted errors of the TIPS yield-to-maturities from Bloomberg

TIPS security	CLR model				CLR-L model			
	Base model		Option adjusted		Base model		Option adjusted	
	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE
(1) 3.375% 1/15/2007 TIPS	-4.98	13.81	-3.33	10.17	2.53	4.93	2.43	4.80
(2) 3.625% 7/15/2002 TIPS*	5.92	12.56	1.04	10.58	3.25	4.01	3.26	4.04
(3) 3.625% 1/15/2008 TIPS	-2.92	12.10	-1.33	10.44	2.14	4.48	2.15	4.38
(4) 3.875% 1/15/2009 TIPS	0.38	9.77	1.12	9.40	1.29	2.67	1.36	2.69
(5) 4.25% 1/15/2010 TIPS	1.85	8.87	2.41	9.38	0.77	2.94	0.86	3.04
(6) 3.5% 1/15/2011 TIPS	5.52	36.37	3.50	21.00	-0.09	4.33	-0.22	4.27
(7) 3.375% 1/15/2012 TIPS	7.33	23.93	3.79	11.33	0.12	5.16	-0.10	5.19
(8) 3% 7/15/2012 TIPS	5.00	18.30	1.47	10.12	-0.18	4.98	-0.36	4.99
(9) 1.875% 7/15/2013 TIPS	1.54	17.14	-0.48	14.66	-0.76	6.63	-0.98	6.52
(10) 2% 1/15/2014 TIPS	6.61	14.03	5.54	12.25	0.53	3.79	0.39	3.72
(11) 2% 7/15/2014 TIPS	2.65	12.73	3.41	13.80	-0.14	4.43	-0.09	4.49
(12) 0.875% 4/15/2010 TIPS*	1.39	11.43	1.46	9.79	1.86	4.33	2.36	4.46
(13) 1.625% 1/15/2015 TIPS	6.52	12.97	8.33	14.99	0.77	4.36	0.90	4.34
(14) 1.875% 7/15/2015 TIPS	-1.11	10.84	1.66	11.20	0.07	4.38	0.22	4.50
(15) 2% 1/15/2016 TIPS	0.75	11.54	4.27	9.40	1.06	4.67	1.11	4.77
(16) 2.375% 4/15/2011 TIPS*	20.44	48.66	15.05	33.31	4.67	12.20	4.76	12.08
(17) 2.5% 7/15/2016 TIPS	-6.95	15.64	-3.51	10.13	-0.50	5.44	-0.47	5.55
(18) 2.375% 1/15/2017 TIPS	-3.08	17.17	-0.72	8.21	1.92	4.39	1.95	4.46
(19) 2% 4/15/2012 TIPS*	8.94	18.46	19.46	37.86	5.62	11.25	5.59	11.19
(20) 2.625% 7/15/2017 TIPS	-10.77	24.31	-8.43	14.60	0.54	3.70	0.55	3.76
(21) 1.625% 1/15/2018 TIPS	-9.45	29.99	-7.33	18.05	0.45	3.73	0.48	3.73
(22) 0.625% 4/15/2013 TIPS*	-19.22	34.49	-0.14	16.03	0.32	11.55	0.32	11.35
(23) 1.375% 7/15/2018 TIPS	-18.34	38.75	-15.52	26.57	0.27	4.49	0.29	4.58
(24) 2.125% 1/15/2019 TIPS	-6.89	23.27	-7.80	20.35	-0.08	3.14	-0.09	3.23
(25) 1.25% 4/15/2014 TIPS*	-5.91	14.09	0.73	10.98	0.10	4.19	0.25	4.27
(26) 1.875% 7/15/2019 TIPS	-5.35	13.90	-7.96	14.01	0.00	2.32	0.00	2.29
(27) 1.375% 1/15/2020 TIPS	3.68	10.21	0.93	8.14	-0.62	3.71	-0.65	3.62
(28) 0.5% 4/15/2015 TIPS*	3.73	14.41	8.73	15.06	0.38	3.31	0.54	3.23
(29) 1.25% 7/15/2020 TIPS	2.72	10.71	0.13	8.27	-0.32	2.67	-0.28	2.67
(30) 1.125% 1/15/2021 TIPS	12.20	15.49	9.23	11.61	-0.56	3.85	-0.50	3.79
(31) 0.125% 4/15/2016 TIPS*	1.31	7.86	5.53	8.70	-0.22	3.67	-0.14	3.53
(32) 0.625% 7/15/2021 TIPS	6.87	10.12	5.77	8.33	-0.17	2.72	0.11	2.58
(33) 0.125% 1/15/2022 TIPS	15.79	17.85	14.26	15.58	0.11	2.47	0.12	2.32
(34) 0.125% 4/15/2017 TIPS*	-2.66	5.39	1.51	5.20	-0.01	2.58	-0.01	2.53
(35) 0.125% 7/15/2022 TIPS	12.94	14.41	10.88	11.87	0.15	3.84	0.34	3.40
(36) 0.125% 1/15/2023 TIPS	23.53	24.30	19.59	20.46	-0.03	5.93	0.12	5.30
(37) 0.125% 4/15/2018 TIPS*	0.82	3.00	5.47	6.28	-0.68	3.47	-0.21	3.08
(38) 0.375% 7/15/2023 TIPS	15.23	15.60	9.80	10.19	0.44	2.77	0.54	2.62
All TIPS yields	0.42	19.57	1.20	14.58	0.65	4.89	0.66	4.87
Max log L	107,249.3		109,593.5		118,915.3		118,945.5	

Table 7: **Summary Statistics of Fitted Errors of TIPS Yields.**
The mean fitted errors and the root mean squared fitted errors (RMSE) of the yield-to-maturity for individual TIPS according to the CLR-L model with and without deflation option adjustment. Asterisk * indicates five-year TIPS. The sample used in each model estimation is weekly covering the period from July 11, 1997, to December 27, 2013. All numbers are measured in basis points.

for all 38 TIPS. For most of the 17-year period the fitted errors are less than 10 basis points in size and clearly satisfactory. Only the period around the peak of the financial crisis is characterized by outsized fitted errors.

Figure 7 shows the time series of the average yield difference between the fitted yield-to-maturity of individual TIPS and the corresponding frictionless yield-to-maturity with the

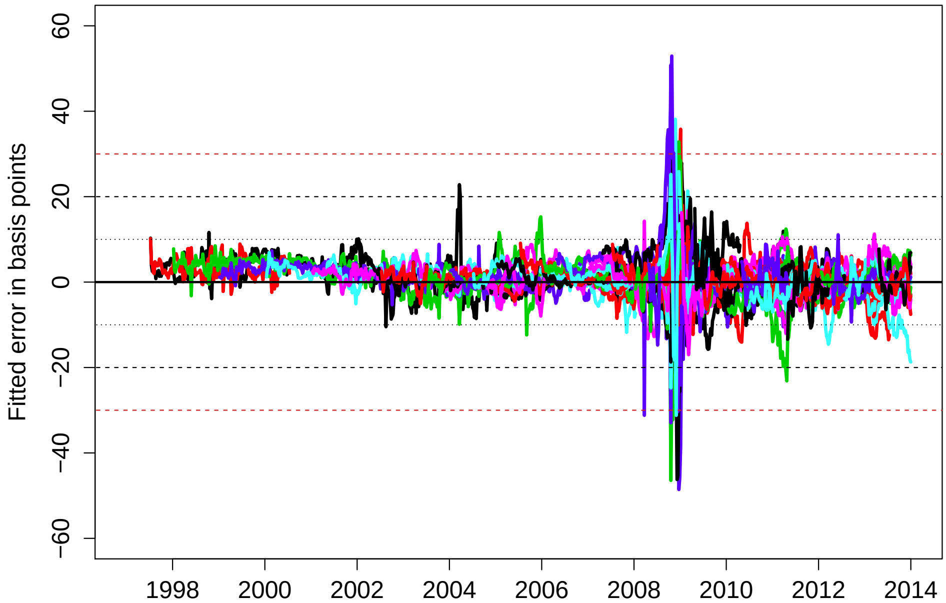


Figure 6: **Fitted Errors of TIPS Yields.**
Illustration of the fitted errors of TIPS yield-to-maturities implied by the estimated CLR-L model with deflation option adjustment. The data cover the period from July 11, 1997, to December 27, 2013.

liquidity risk factor turned off. This represents the average TIPS liquidity premium for each observation date in our sample. According to the CLR-L model, the sample average of the weekly average liquidity premium is 42.2 basis points. Once we account for the value of the deflation option in the model estimation, the sample average drops to 37.9 basis points. Thus, the assessment of the average TIPS liquidity premium is sensitive to the inclusion of the deflation protection option. Furthermore, we note that over the main sample period analyzed in CLR (January 3, 2003 to March 28, 2008) and again from early 2010 through the end of 2013, this measure of the TIPS liquidity premium has averaged lower, 27.3 basis points and 34.8 basis points, respectively. Thus, omitting to account for the TIPS liquidity premium as in CLR, may be a reasonable approximation during normal times. However, from Figure 7 it is also clear that, in parts of the early years of TIPS trading as well as during the financial crisis, TIPS liquidity premiums were of nontrivial magnitudes with significant time variation.

As an alternative, we analyze the estimated liquidity premium of the most recently issued (on-the-run) five- and ten-year TIPS, which are shown in Figure 8. According to the CLR-L model with option adjustment, the five- and ten-year on-the-run TIPS liquidity premium has averaged 33.2 basis points and 29.8 basis points, respectively. For the CLR-L model without option adjustment, the corresponding averages are slightly higher at 37.5 basis points and

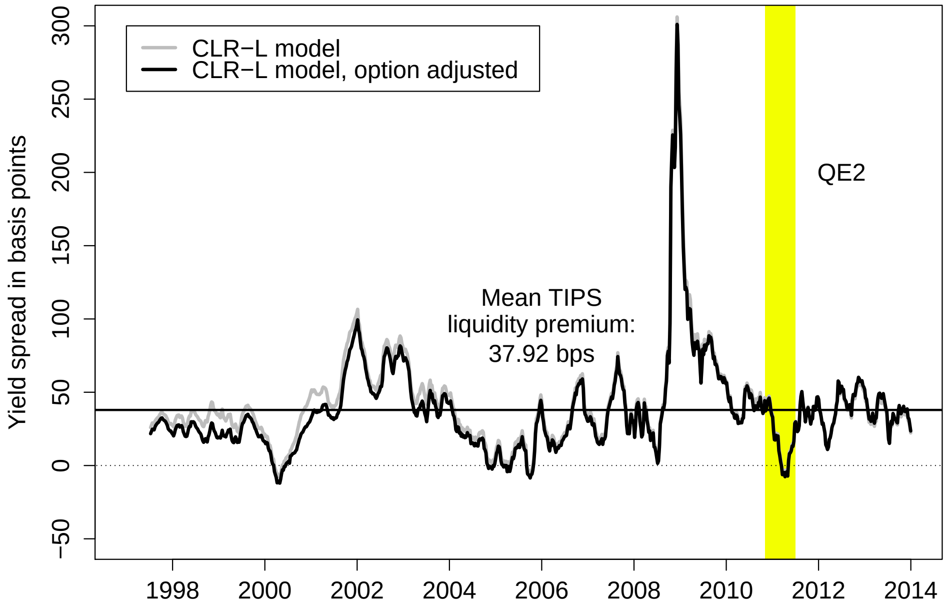


Figure 7: **Average Estimated TIPS Liquidity Premium.**
Illustration of the average estimated TIPS liquidity premium for each observation date implied by the CLR-L model with and without deflation option adjustment. TIPS liquidity premiums are measured as the estimated yield difference between the fitted yield-to-maturity of individual TIPS and the corresponding frictionless yield-to-maturity with the liquidity risk factor turned off. The average TIPS liquidity premium according to the option-adjusted CLR-L model is shown with a solid black horizontal line. The data cover the period from July 11, 1997, to December 27, 2013.

31.8 basis points, respectively.

Finally, we note that our estimate of the TIPS liquidity premium exhibits a significant temporary decline during the Fed's QE2 program, which included a significant amount of TIPS purchases. CG argue that this decline is evidence of a liquidity channel as a transmission mechanism for QE to affect long-term interest rates. In the following section, we repeat parts of their analysis to assess the robustness of their findings.

5.1 The Fed's TIPS Purchases during QE2

To begin, we note that CG argue that central bank large-scale asset purchases—commonly known as quantitative easing (QE)—can reduce priced frictions to trading through a liquidity channel that operates by changing the *shape* of the price distribution of the targeted securities. For evidence of this liquidity channel, they analyze how the Federal Reserve's second QE program (QE2) that included repeated purchases of TIPS affected a measure of priced frictions in TIPS yields and inflation swap rates. They find that, for the duration of the program,

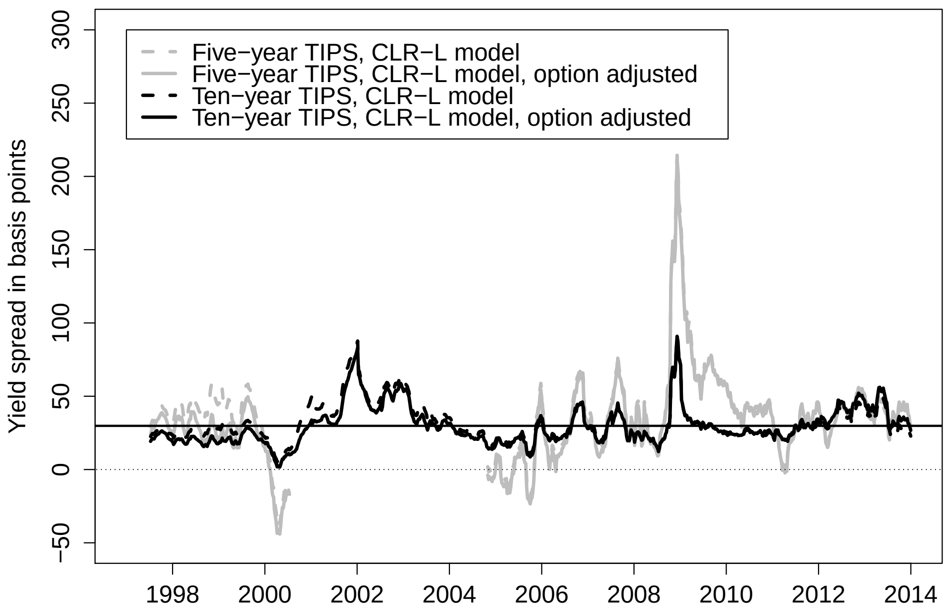


Figure 8: **Estimated On-The-Run TIPS Liquidity Premiums.**
Illustration of the estimated liquidity premiums for the most recently issued (on-the-run) five- and ten-year TIPS on each observation date implied by the CLR-L model with and without deflation option adjustment. TIPS liquidity premiums are measured as the estimated yield difference between the fitted yield-to-maturity of the individual TIPS and the corresponding frictionless yield-to-maturity with the liquidity risk factor turned off. The average ten-year on-the-run TIPS liquidity premium according to the option-adjusted CLR-L model is shown with a solid black horizontal line. The data cover the period from July 11, 1997, to December 27, 2013. Note that between July 14, 2000, and October 29, 2004, there are no five-year TIPS outstanding with more than two years to maturity.

their measure of priced frictions averaged 12 to 14 basis points lower than expected, and they conclude that this suggests that QE can improve market liquidity for the purchased securities.

As explained in CG, the Fed purchased about $26 billion of TIPS during the operation of QE2 from November 3, 2010, until June 29, 2011. However, CG did not estimate TIPS liquidity premiums directly unlike what we do in this paper. Thus, equipped with our estimated average TIPS liquidity premium series shown in Figure 7, we can study how this measure of priced frictions in the TIPS market was impacted by the Fed's TIPS purchases. Following CG we use three variables to control explicitly for sources that reflect bond market liquidity more broadly.

The first variable we consider is the VIX options-implied volatility index. It represents near-term uncertainty about the general stock market as reflected in options on the Standard & Poor's 500 stock price index and is widely used as a gauge of investor fear and risk aversion. The motivation for including this variable is that elevated economic uncertainty would imply increased uncertainty about the future resale price of any security and therefore could cause liquidity premiums that represent investors' guard against such uncertainty to go up.

The second variable included is a market illiquidity measure introduced in the recent paper by HPW.[18] They demonstrate that deviations in bond prices in the Treasury securities market from a fitted yield curve represent a measure of noise and illiquidity caused by limited availability of arbitrage capital. Their analysis suggests that this measure is a priced risk factor across several financial markets, which they interpret to imply that it represents an economy-wide illiquidity measure that should affect all financial markets. If so, this should include the markets for TIPS.

The third variable used is the yield difference between seasoned (off-the-run) Treasury securities and the most recently issued (on-the-run) Treasury security, both with ten years to maturity.[19] For each maturity segment in the Treasury yield curve, the on-the-run security is typically the most traded security and therefore penalized the least in terms of liquidity premiums. For our analysis, the important thing to note is that if there is a wide yield spread between liquid on-the-run and comparable seasoned Treasuries, we would expect to also see large liquidity premiums in TIPS yields relative to those in the Treasury bond market, that is, a widening of our liquidity premium measure.

Similar to CG, we perform a counterfactual analysis. However, as noted by CG, residuals from regressions in levels are serially correlated. In our case, a simple Durbin-Watson test for the regression with all three explanatory variables included gives a value of 0.14, which indicates highly significant positive serial correlation.

To overcome that problem, we follow CG and include the lagged value of our TIPS liquidity premium measure in the regressions, that is, we use an AR(1) specification. Thus, we run regressions of the following type:

$$LP_t(\tau) = \beta_0 + \rho LP_{t-1}(\tau) + \beta^T X_t + \varepsilon_t, \tag{10}$$

where $LP_t(\tau)$ is the estimated average TIPS liquidity premium from the CLR-L model with adjustment for the deflation option values shown in Figure 7, while X_t represents the exogenous explanatory variables. To replicate the analysis in CG, we estimate the regressions on the sample from January 14, 2005, to November 2, 2010, which delivers the estimated coefficients $\widehat{\beta}_0$, $\widehat{\rho}$, and $\widehat{\beta}$ reported in Table 8 that describe the historical statistical relationship before the introduction of the QE2 program.

Based on the historical dynamic relationship implied by the estimated coefficients in equation (10) and reported in Table 8, we analyze whether the shocks to the liquidity premium measure during QE2 were statistically significantly more negative than in the pre-QE2 pe-

[18] The data are publicly available at Jun Pan's website: https://sites.google.com/site/junpan2/publications.

[19] We do not construct off-the-run spreads for the TIPS market since Christensen et al. (2012) show that such spreads have been significantly biased in the years following the peak of the financial crisis due to the value of the deflation protection option embedded in the TIPS contract.

Explanatory variables	Dependent variable: Avg. estimated TIPS liquidity premium							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Constant	0.92	-4.87**	0.35	-0.55	-6.06**	-8.47**	-0.53	-6.07**
	(1.33)	(-4.73)	(0.48)	(-0.66)	(-5.56)	(-7.54)	(-0.61)	(-5.45)
AR(1) coefficient	0.98**	0.89**	0.94**	0.93**	0.93**	0.88**	0.93**	0.93**
	(91.68)	(56.22)	(45.12)	(48.05)	(48.13)	(58.62)	(44.02)	(47.25)
VIX		0.45**			0.61**	0.67**		0.61**
		(7.20)			(7.47)	(9.79)		(7.18)
HPW measure			0.59*		-0.94**		0.05	-0.94*
			(2.34)		(-3.01)		(0.12)	(-2.45)
Off-the-run spread				0.22**		-0.97**	0.21	0.00
				(2.99)		(-6.31)	(1.85)	(0.03)
Adjusted R^2	0.97	0.97	0.97	0.97	0.97	0.97	0.97	0.97

Table 8: **Regression Results for Pre-QE2 Period with AR(1) Specification.**
The table reports the results of regressions with the average estimated TIPS liquidity premium as the dependent variable and an AR(1) term and three measures of market functioning as explanatory variables. T-statistics are reported in parentheses. Asterisks * and ** indicate significance at the 5 percent and 1 percent levels, respectively. The data are weekly covering the period from January 14, 2005, to October 29, 2010, a total of 303 observations.

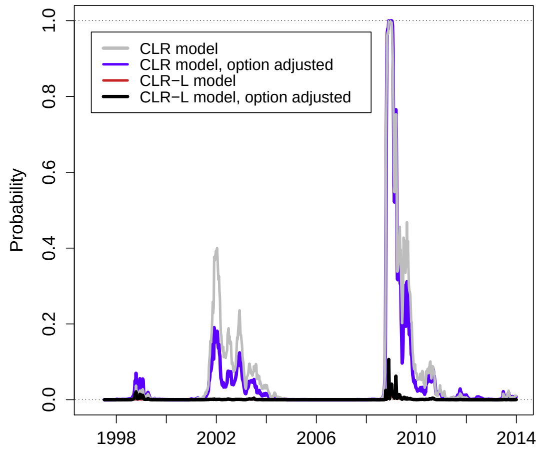
riod. If so, it would suggest that the QE2 TIPS purchases exerted downward pressure on the liquidity premium measure.

Focusing on regression (8) in Table 8, we calculate realized residuals relative to the counterfactual prediction for the period from November 5, 2010, to July 1, 2011, using

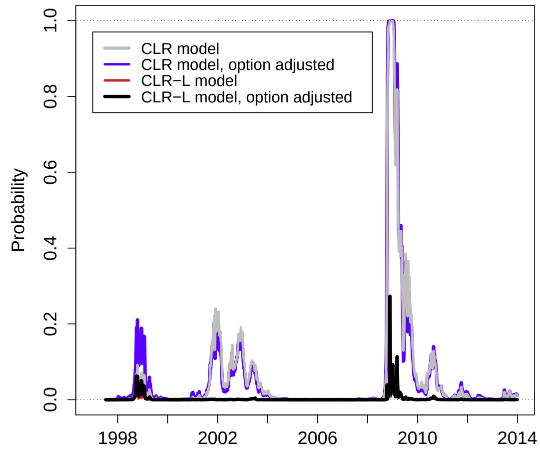
$$\varepsilon_t^R = LP_t(\tau) - \widehat{\beta}_0 - \widehat{\rho} LP_{t-1}(\tau) - \widehat{\beta}^T X_t. \tag{11}$$

Since the residuals from the regressions in Table 8 have fatter tails than the normal distribution (mainly due to the financial crisis), we use a Wilcoxon test of the hypothesis that the mean of the realized residuals in equation (11) is identical to the mean of the residuals in the pre-QE2 regression with the alternative being a lower mean of the realized residuals in light of our previous results. The Wilcoxon test is -1.69 with a p-value of 0.0160. Thus, similar to the results reported by CG, the test suggests that the shocks to the estimated average TIPS liquidity premium experienced during the QE2 program were significantly more negative than what would have been predicted based on the historical dynamic relationships. Therefore, we conclude that the TIPS purchases included in the QE2 program exerted a persistent downward pressure on the priced frictions to trading in the TIPS market.[20] These results suggest that a byproduct of quantitative easing may be improvement in the market liquidity of the assets purchased.

[20] For robustness, we repeated the autoregressive counterfactual analysis using samples starting on January 10, 2003, and January 12, 2007, respectively, and obtained qualitatively similar results, see Appendix C.



(a) Objective deflation probabilities.



(b) Risk-neutral deflation probabilities.

Figure 9: **Estimated One-Year Deflation Probabilities.**
Panel (a) shows the estimated one-year deflation probabilities under the objective P probability measure from the CLR model with and without deflation option adjustment and from the CLR-L model with and without deflation option adjustment. Panel (b) shows the corresponding estimated one-year deflation probabilities under the risk-neutral Q probability measure. The data are weekly covering the period from July 11, 1997, to December 27, 2013.

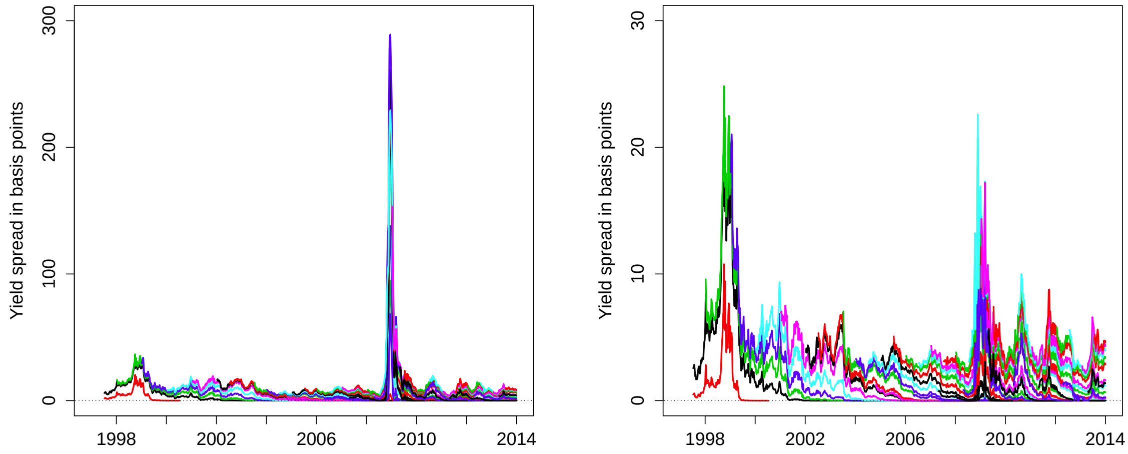
6 Analysis of the Risk of Deflation

In this section, we focus on the lower tail of the model-implied inflation distributions. Specifically, we analyze the effect of accounting for TIPS liquidity risk on the models' assessment of the risk of deflation.

To begin, Figure 9 shows the estimated probabilities of net deflation over the next year according to the four CLR models discussed in the previous section under both the objective P probability measure and the risk-neutral Q probability measure.

We note that such estimates are significantly affected by whether or not the TIPS liquidity premiums are accounted for. When the liquidity factor is omitted, the CLR model suggests that the U.S. economy has experienced three spells of deflation fear, a brief small one in late 1998 (around the time of the Russian sovereign debt crisis), a larger and longer one in 2002-2003, and another, more severe around the peak of the financial crisis in 2008-2009—the two latter spells were also highlighted by Christensen et al. (2012). However, once the TIPS liquidity premium is accounted for, only the first and the last spell show a small uptick in the risk of deflation. If we focus on the risk-neutral distribution that adjusts the objective probabilities with investors' risk premiums, we see a qualitatively similar picture and the differences from accounting for the TIPS liquidity premium are still clearly notable.

Obviously, these differences matter for the model-implied values of the deflation protection options embedded in the TIPS contract as also noted by Christensen et al. (2015). Figures



(a) CLR model, option adjusted.

(b) CLR-L model, option adjusted.

Figure 10: **Estimated Values of TIPS Deflation Protection Options.**
Panel (a) shows the estimated value of the deflation protection option embedded in each TIPS according to the CLR model estimated with adjustment for the deflation option values. Panel (b) shows the corresponding option values according to the CLR-L model estimated with adjustment for the deflation option values. The data are weekly covering the period from July 11, 1997, to December 27, 2013.

10(a) and 10(b) illustrate the deflation protection option values implied by the CLR model and the CLR-L model, respectively, both adjusted for the deflation option values. With the exception of the early years of the TIPS market, the CLR model delivers estimates of the deflation protection option value that are between 5 and 10 times larger than the estimates implied by the CLR-L model. Thus, to assess the severity of tail events such as the risk of deflation, it is crucial to account for the time-varying liquidity premiums in TIPS pricing. Omitting it can lead to severely exaggerated estimates as demonstrated in Figures 9 and 10.

7 Model-Implied Inflation Expectations

In this section, we analyze the properties of the model-implied inflation distributions from the CLR models. First, we evaluate the models' one-year inflation expectations by comparing them to inflation swap rates and survey forecasts. Second, we assess the models' longer-term inflation expectations again by comparing them to survey forecasts. Overall, the purpose is to demonstrate that better market-based measures of inflation expectations can be obtained by incorporating a liquidity risk factor into the CLR model.

Figure 11 shows the estimated one-year expected inflation according to the four CLR models with a comparison to the subsequent realizations of year-over-year headline CPI inflation. We note that the standard CLR model produces estimates of one-year expected inflation that

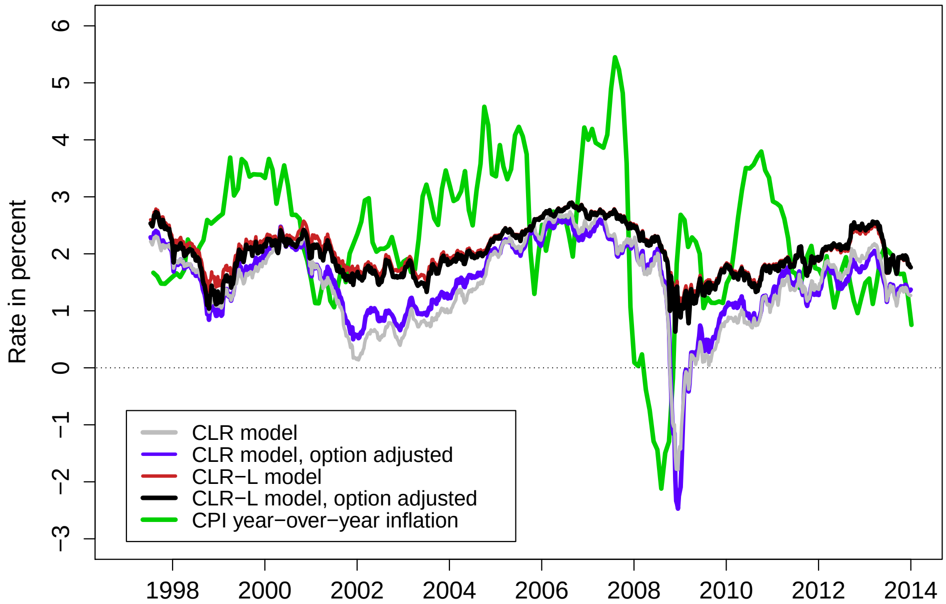


Figure 11: **Estimated One-Year Expected Inflation.**
Illustration of the estimated one-year expected inflation from the CLR model with and without deflation option adjustment and from the CLR-L model with and without deflation option adjustment. The data cover the period from July 11, 1997, to December 27, 2013.

are typically well below the subsequent realizations of year-over-year inflation.

To evaluate the forecast accuracy, Table 9 reports the statistics of the forecast errors from the four CLR models and compare them to the performance of the random walk and the median of forecasts in the Blue Chip Economic Forecast survey. Also, for the most recent period from January 2005 through the end of 2013, the one-year inflation swap rate is included in the forecast evaluation.

Several things are worth highlighting in Table 9. First, the models are able to outperform the random walk at forecasting CPI inflation one year ahead over the full sample; this is the minimum requirement for any model. Second and more importantly, the CLR-L model systematically outperforms the CLR model, and this conclusion is valid whether or not the model estimation accounts for the values of the deflation options embedded in TIPS. Third, for the most recent period since 2005, the CLR-L model even outperforms the Blue Chip Economic Forecasts survey at forecasting CPI inflation one year ahead, while that is not the case in the early 1997-2004 period. Finally, the inflation expectations from the CLR models are competitive relative to inflation swap rates, which underscores the importance of accounting for the inflation risk premium when it comes to using TIPS BEI to forecast future inflation. Overall, these results are very encouraging as they show that incorporating

Model	1997-2004		2005-2013		1997-2013	
	Mean	RMSE	Mean	RMSE	Mean	RMSE
Random Walk	20.85	108.04	-19.16	230.51	-0.97	185.17
Blue Chip	23.37	86.44	-0.54	150.93	10.33	125.79
Inflation swap rate	n.a.	n.a.	44.32	198.07	n.a.	n.a.
CLR model	113.56	147.78	58.25	162.65	83.39	156.07
CLR model, option adjusted	105.34	136.66	65.09	174.21	83.39	158.25
CLR-L model	57.65	110.92	18.10	143.09	36.08	129.47
CLR-L model, option adjusted	47.29	110.07	-3.30	143.63	19.69	129.46

Table 9: **Comparison of CPI Inflation Forecasts.**
The table reports summary statistics for one-year forecast errors of headline CPI inflation. The Blue Chip forecasts are mapped to the end of each month from July 1997 to December 2013, a total of 198 monthly forecasts. The comparable model forecasts are generated on the nearest available business day prior to the end of each month. The subsequent CPI realizations are year-over-year changes starting at the end of the month before the survey month. As a consequence, the random walk forecasts equal the past year-over-year change in the CPI series as of the beginning of the survey month. For the subperiod from January 2005 to December 2013, which represents 108 monthly forecasts, one-year inflation swap rates are also available and their forecast performance is reported, while this is not the case for the period from July 1997 to December 2004 that contains 90 monthly forecasts.

the liquidity factor does improve model performance along this important dimension. Still, we emphasize that this is not conclusive evidence since the model forecasts are full-sample look-back estimates of expected inflation, while both the BC survey and the inflation swap rates reflect real-time assessments of the inflation outlook.

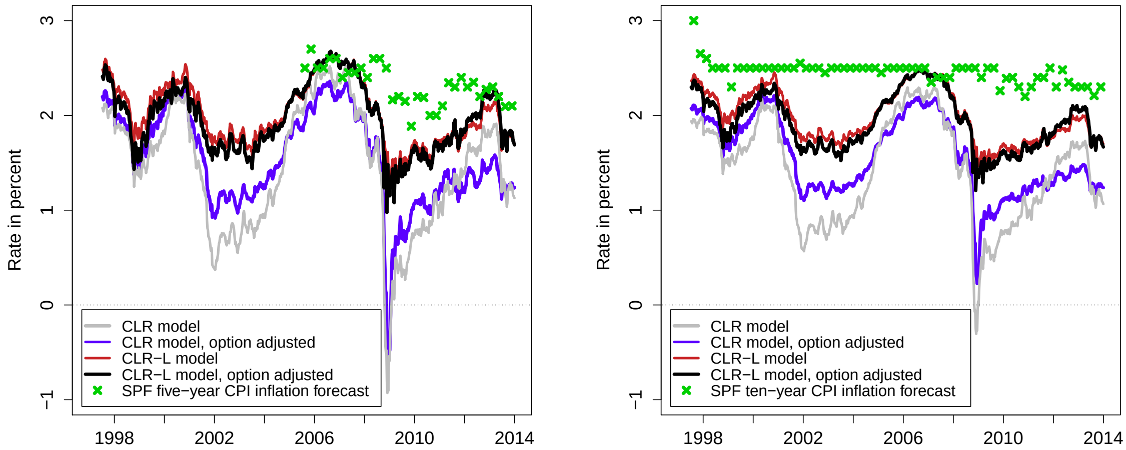
Figure 12 shows the estimated longer-term inflation expectations from the four CLR models with a comparison to the median of the long-term inflation forecasts from the Survey of Professional Forecasters (SPF). Here, we observe results across the four CLR models that are qualitatively similar to the ones shown in Figure 11 in the sense that the standard CLR model generates estimates of longer-term inflation expectations that appear to be too low and very volatile, while the estimates from the CLR-L model are much more stable and of a more reasonable magnitude, although not as high and stable as the SPF forecasts.

In summary, we find that the incorporation of a liquidity factor into the CLR model framework leads to better behaved estimates of financial market participants' inflation expectations in addition to the improvement in model fit documented earlier.

8 Conclusion

In this paper, we extend an established model of nominal Treasury and real TIPS yields with a liquidity risk factor to account for the relative illiquidity of TIPS.

As for the TIPS liquidity premiums, we find that they exhibit large variation across time. At times, they are low and even negative, which seems to coincide with high inflation when



(a) Five-year expected inflation.

(b) Ten-year expected inflation.

Figure 12: **Estimated Five- and Ten-Year Expected Inflation.**
Panel (a) shows the estimated five-year expected inflation from the CLR model with and without deflation option adjustment and from the CLR-L model with and without deflation option adjustment. Panel (b) shows the corresponding estimates of the ten-year expected inflation. The data are weekly covering the period from July 11, 1997, to December 27, 2013.

TIPS are desirable. On the other hand, the TIPS liquidity premiums were large in the early 2000s when the U.S. Treasury committed fully to the TIPS program and increased both the issuance pace and the issuance sizes. Also, the liquidity premiums spiked around the peak of the financial crisis as also emphasized by Campbell et al. (2009) and Fleckenstein et al. (2014). Overall, this suggests that the estimated TIPS liquidity premium series exhibit a time series pattern consistent with the stylized facts documented in the existing literature.

In addition to estimates of the liquidity premium for each individual TIPS, the model delivers estimates of the frictionless nominal and real yields that can be decomposed into investors' frictionless inflation expectations and associated inflation risk premiums. We provide evidence that suggests that such decompositions may lead to superior inflation forecasts than simply using BEI without any adjustments. Furthermore, we find that accounting for TIPS liquidity premiums is crucial for the assessment of tail risks to inflation such as the risk of deflation. Without this adjustment, the severity of such risks is likely to be seriously overstated.

In terms of applications, we note that our model framework can be extended in several ways. First, the model can be modified to allow for stochastic yield volatility as in Christensen et al. (2015) to improve the pricing accuracy of the deflation protection option embedded in TIPS. Second, if respecting the zero lower bound for nominal yields is required, the Gaussian nominal short-rate dynamics can be cast as a shadow-rate AFNS model using the formulas provided in Christensen and Rudebusch (2015). However, this modification would make both model estimation and the calculation of model output more time consuming. Thus, we leave it for future research to explore such extensions even though Appendix D provides a brief summary of preliminary results from using this approach that do look promising.

Finally, we emphasize that our model generalization with a liquidity factor can be combined with any term structure model, in particular we envision this approach applied to sovereign bond markets in Europe such as Denmark and Switzerland where market liquidity is known to be an issue. Again, we leave that for future research.

Appendix A: Analytical Bond Pricing Formula

In this appendix, we provide the analytical formula for the price of individual TIPS within the CLR model with liquidity factor described in Section 3.3. The net present value of one unit of the consumption basket paid at time $t+\tau$ by TIPS i with liquidity sensitivity parameter, β^i, and liquidity decay parameter, $\lambda^{L,i}$, can be calculated by the formula provided in the following proposition.[21]

<u>**Proposition 1:**</u>

The net present value of one unit of the consumption basket paid at time $t+\tau$ by TIPS i with liquidity sensitivity parameter, β^i, and liquidity decay parameter, $\lambda^{L,i}$, is given by

$$\begin{aligned}
P^i(t_0^i, t, T) &= E_t^Q\Big[e^{-\int_t^T (r_s^R + \beta^i(1-e^{-\lambda^{L,i}(s-t_0^i)})X_s^{Liq})ds} e^{(\overline{B}^i)' X_T + \overline{A}^i}\Big] \\
&= \exp\Big(B_1^i(t_0^i,t,T)L_t^N + B_2^i(t_0^i,t,T)S_t + B_3^i(t_0^i,t,T)C_t + B_4^i(t_0^i,t,T)L_t^R + B_5^i(t_0^i,t,T)X_t^{Liq} + A^i(t_0^i,t,T)\Big),
\end{aligned}$$

where

$$\begin{aligned}
B_1^i(t_0^i,t,T) &= \overline{B}_1^i, \\
B_2^i(t_0^i,t,T) &= e^{-\lambda(T-t)}\overline{B}_2^i - \alpha^R \frac{1-e^{-\lambda(T-t)}}{\lambda}, \\
B_3^i(t_0^i,t,T) &= \lambda(T-t)e^{-\lambda(T-t)}\overline{B}_2^i + \overline{B}_3^i e^{-\lambda(T-t)} + \alpha^R\Big[(T-t)e^{-\lambda(T-t)} - \frac{1-e^{-\lambda(T-t)}}{\lambda}\Big], \\
B_4^i(t_0^i,t,T) &= \overline{B}_4^i - (T-t), \\
B_5^i(t_0^i,t,T) &= e^{-\kappa_{Liq}^Q(T-t)}\overline{B}_5^i - \beta^i \frac{1-e^{-\kappa_{Liq}^Q(T-t)}}{\kappa_{Liq}^Q} + \beta^i e^{-\lambda^{L,i}(t-t_0^i)} \frac{1-e^{-(\kappa_{Liq}^Q+\lambda^{L,i})(T-t)}}{\kappa_{Liq}^Q+\lambda^{L,i}}, \\
A^i(t_0^i,t,T) &= \overline{A}^i - \beta^i\theta_{liq}^Q(T-t) + \theta_{liq}^Q\Big[\overline{B}_5^i + \frac{\beta^i}{\kappa_{Liq}^Q} - \beta^i\frac{e^{-\lambda^{L,i}(T-t_0^i)}}{\kappa_{Liq}^Q+\lambda^{L,i}}\Big](1-e^{-\kappa_{Liq}^Q(T-t)}) \\
&\quad +\beta^i \frac{\kappa_{liq}^Q\theta_{liq}^Q}{\kappa_{Liq}^Q+\lambda^{L,i}} \frac{e^{-\lambda^{L,i}(t-t_0^i)} - e^{-\lambda^{L,i}(T-t_0^i)}}{\lambda^{L,i}} + \frac{\sigma_{11}^2}{2}(\overline{B}_1^i)^2(T-t) \\
&\quad +\sigma_{22}^2\Big[\frac{(\alpha^R)^2}{2\lambda^2}(T-t) - \alpha^R\frac{(\alpha^R+\lambda\overline{B}_2^i)}{\lambda^3}[1-e^{-\lambda(T-t)}] + \frac{(\alpha^R+\lambda\overline{B}_2^i)^2}{4\lambda^3}[1-e^{-2\lambda(T-t)}]\Big] \\
&\quad +\sigma_{33}^2\Big[\frac{(\alpha^R)^2}{2\lambda^2}(T-t) + \alpha^R\frac{\alpha^R+\lambda\overline{B}_2^i}{\lambda^2}(T-t)e^{-\lambda(T-t)} - \frac{(\alpha^R+\lambda\overline{B}_2^i)^2}{4\lambda}(T-t)^2e^{-2\lambda(T-t)}\Big] \\
&\quad -\sigma_{33}^2\frac{(\alpha^R+\lambda\overline{B}_2^i)(3\alpha^R+\lambda\overline{B}_2^i+2\lambda\overline{B}_3^i)}{4\lambda^2}(T-t)e^{-2\lambda(T-t)} \\
&\quad +\sigma_{33}^2\frac{(2\alpha^R+\lambda\overline{B}_2^i+\lambda\overline{B}_3^i)^2+(\alpha^R+\lambda\overline{B}_3^i)^2}{8\lambda^3}[1-e^{-2\lambda(T-t)}] - \sigma_{33}^2\alpha^R\frac{2\alpha^R+\lambda\overline{B}_2^i+\lambda\overline{B}_3^i}{\lambda^3}[1-e^{-\lambda(T-t)}] \\
&\quad +\frac{\sigma_{44}^2}{6}[(\overline{B}_4^i)^3 - (\overline{B}_4^i-(T-t))^3] \\
&\quad +\frac{\sigma_{55}^2}{2}\Bigg[\frac{(\beta^i)^2}{(\kappa_{Liq}^Q)^2}(T-t) + \Big[\overline{B}_5^i + \frac{\beta^i}{\kappa_{Liq}^Q} - \beta^i\frac{e^{-\lambda^{L,i}(T-t_0^i)}}{\kappa_{Liq}^Q+\lambda^{L,i}}\Big]^2\frac{1-e^{-2\kappa_{Liq}^Q(T-t)}}{2\kappa_{Liq}^Q} \\
&\qquad +\frac{(\beta^i)^2}{(\kappa_{Liq}^Q+\lambda^{L,i})^2}\frac{e^{-2\lambda^{L,i}(t-t_0^i)}-e^{-2\lambda^{L,i}(T-t_0^i)}}{2\lambda^{L,i}} - 2\frac{\beta^i}{\kappa_{Liq}^Q}\Big[\overline{B}_5^i + \frac{\beta^i}{\kappa_{Liq}^Q} - \beta^i\frac{e^{-\lambda^{L,i}(T-t_0^i)}}{\kappa_{Liq}^Q+\lambda^{L,i}}\Big]\frac{1-e^{-\kappa_{Liq}^Q(T-t)}}{\kappa_{Liq}^Q} \\
&\qquad -2(\beta^i)^2\frac{1}{\kappa_{Liq}^Q}\frac{1}{\kappa_{Liq}^Q+\lambda^{L,i}}\frac{e^{-\lambda^{L,i}(t-t_0^i)}-e^{-\lambda^{L,i}(T-t_0^i)}}{\lambda^{L,i}} \\
&\qquad +2\beta^i\Big[\overline{B}_5^i + \frac{\beta^i}{\kappa_{Liq}^Q} - \beta^i\frac{e^{-\lambda^{L,i}(T-t_0^i)}}{\kappa_{Liq}^Q+\lambda^{L,i}}\Big]e^{-\lambda^{L,i}(t-t_0^i)}\frac{e^{-\lambda^{L,i}(T-t)}-e^{-\kappa_{Liq}^Q(T-t)}}{(\kappa_{Liq}^Q)^2-(\lambda^{L,i})^2}\Bigg].
\end{aligned}$$

[21] The calculations leading to this result are available upon request.

Appendix B: Factor Structure of TIPS Yield Data

In this appendix, we analyze the factor structure of TIPS yields.

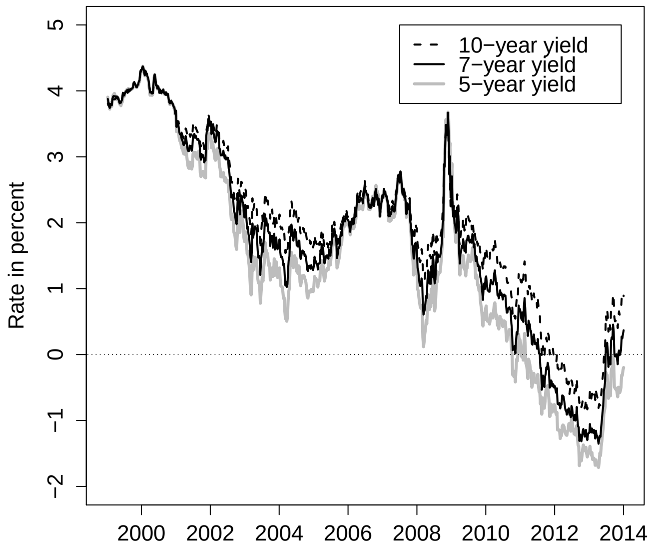


Figure 13: **Real TIPS Yields.**
Illustration of the weekly real TIPS zero-coupon bond yields covering the period from January 8, 1999, to December 27, 2013. The yields shown have maturities in five years, seven years, and ten years, respectively.

To do so, we use a sample of smoothed zero-coupon TIPS yields constructed by the method described in Gürkaynak, Sack, and Wright (2010)[22] and briefly detailed here. For each business day a zero-coupon yield curve of the Svensson (1995)-type

$$y_t(\tau) = \beta_0 + \frac{1 - e^{-\lambda_1 \tau}}{\lambda_1 \tau}\beta_1 + \Big[\frac{1 - e^{-\lambda_1 \tau}}{\lambda_1 \tau} - e^{-\lambda_1 \tau}\Big]\beta_2 + \Big[\frac{1 - e^{-\lambda_2 \tau}}{\lambda_2 \tau} - e^{-\lambda_2 \tau}\Big]\beta_3$$

is fitted to price a large pool of underlying TIPS. Thus, for each business day, we have the fitted values of the four factors $(\beta_0(t), \beta_1(t), \beta_2(t), \beta_3(t))$ and the two parameters $(\lambda_1(t), \lambda_2(t))$. From this data set zero-coupon yields for any relevant maturity can be calculated as long as the maturity is within the range of maturities used in the fitting process. As demonstrated by Gürkaynak, Sack, and Wright (2010), this model fits the underlying pool of bonds extremely well. By implication, the zero-coupon yields derived from this approach constitute a very good approximation to the true underlying TIPS zero-coupon yield curve (our results confirms this). We construct real TIPS zero-coupon bond yields with the following six maturities: 5-year, 6-year, 7-year, 8-year, 9-year, and 10-year. We use weekly data and limit our sample to the period from January 8, 1999, to December 27, 2013.[23] The summary statistics are provided in Table 10, while Figure 13 illustrates the constructed time

[22]The Board of Governors in Washington DC frequently updates the factors and parameters of this method, see the related website http://www.federalreserve.gov/pubs/feds/2006/index.html

[23]Note that the GSW TIPS yield database only is available starting in early January 1999. The low number of TIPS before then prevents the construction of the yield curve for that early period of the TIPS market.

Maturity in months	Mean in %	St. dev. in %	Skewness	Kurtosis
60	1.49	1.59	-0.11	2.22
72	1.62	1.52	-0.17	2.32
84	1.74	1.46	-0.22	2.40
96	1.84	1.40	-0.25	2.47
108	1.92	1.34	-0.27	2.54
120	2.00	1.29	-0.28	2.59

Table 10: **Summary Statistics for the Real TIPS Yields.**
Summary statistics for the sample of weekly real TIPS zero-coupon bond yields covering the period from January 8, 1999, to December 27, 2013, a total of 782 observations.

Maturity	Loading on		
in months	First P.C.	Second P.C.	Third P.C.
60	0.45	0.68	-0.50
72	0.43	0.28	0.34
84	0.42	-0.01	0.49
96	0.40	-0.22	0.29
108	0.38	-0.39	-0.10
120	0.37	-0.51	-0.55
% explained	99.66	0.34	0.00

Table 11: **Eigenvectors of the First Three Principal Components in Real TIPS Yields.**
The loadings of yields of various maturities on the first three principal components are shown. The final row shows the proportion of all bond yield variability accounted for by each principal component. The data consist of weekly real TIPS zero-coupon bond yields from January 8, 1999, to December 27, 2013.

series of the 5-year, 7-year, and 10-year TIPS zero-coupon yields.

Researchers have typically found that three factors are sufficient to model the time-variation in the cross section of nominal Treasury bond yields (e.g., Litterman and Scheinkman, 1991). Indeed, for our weekly real TIPS yield data, 99.99% of the total variation is accounted for by three factors. Table 11 reports the eigenvectors that correspond to the first three principal components of our data. The first principal component accounts for 99.7% of the variation in the real TIPS yields, and its loading across maturities is uniformly negative. Thus, like a level factor, a shock to this component changes all yields in the same direction irrespective of maturity. The second principal component accounts for 0.3% of the variation in these data and has sizable negative loadings for the shorter maturities and sizable positive loadings for the long maturities. Thus, like a slope factor, a shock to this component steepens or flattens the yield curve. Finally, the third component, which accounts for only 0.0% of the variation, has a hump shaped factor loading as a function of maturity, which is naturally interpreted as a curvature factor. This motivates the unique factor structure in the CLR model of nominal and real yields that preserves the AFNS model structure with its level, slope, and curvature structure for the nominal Treasury yields, while accommodating the level, slope, and curvature structure in the TIPS yield data documented above in a very parsimonious way.

Appendix C: Robustness of CG Regressions

Explanatory variables	Dependent var.: Avg. est. TIPS liquidity premium 2003-2011							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Constant	0.66	-4.51**	0.15	-0.80	-5.07**	-8.12**	-0.64	-4.89**
	(1.22)	(-5.40)	(0.26)	(-1.13)	(-5.79)	(-8.68)	(-0.87)	(-5.42)
AR(1) coefficient	0.98**	0.90**	0.94**	0.94**	0.92**	0.89**	0.93**	0.92**
	(108.28)	(66.02)	(56.58)	(58.36)	(57.20)	(68.13)	(53.72)	(55.81)
VIX		0.41**			0.49**	0.63**		0.51**
		(7.73)			(7.46)	(10.73)		(7.29)
HPW measure			0.57**		-0.49*		0.24	-0.37
			(2.77)		(-2.04)		(0.84)	(-1.31)
Off-the-run spread				0.19**		-0.94**	0.14	-0.07
				(3.13)		(-7.17)	(1.68)	(-0.85)
Adjusted R^2	0.97	0.97	0.97	0.97	0.97	0.97	0.97	0.97

Explanatory variables	Dependent var.: Avg. est. TIPS liquidity premium 2007-2011							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Constant	1.33	-7.85**	0.42	-0.80	-8.95**	-11.91**	-0.72	-9.00**
	(1.18)	(-4.66)	(0.35)	(-0.57)	(-5.17)	(-6.96)	(-0.50)	(-5.05)
AR(1) coefficient	0.98**	0.87**	0.93**	0.93**	0.91**	0.86**	0.92**	0.91**
	(67.55)	(43.21)	(33.11)	(35.92)	(35.81)	(45.86)	(32.12)	(35.02)
VIX		0.58**			0.72**	0.82**		0.72**
		(6.84)			(6.94)	(9.22)		(6.73)
HPW measure			0.69**		-0.89*		0.17	-0.93
			(2.01)		(-2.32)		(0.35)	(-1.97)
Off-the-run spread				0.25*		-1.09**	0.21	0.02
				(2.49)		(-5.75)	(1.49)	(0.14)
Adjusted R^2	0.96	0.97	0.96	0.96	0.97	0.97	0.96	0.97

Table 12: **Regression Results for Pre-QE2 Period with AR(1) Specification.**
The top panel reports the results of regressions with the average estimated TIPS liquidity premium as the dependent variable and an AR(1) term and three measures of market functioning as explanatory variables using a weekly sample from January 10, 2003, to October 29, 2010, a total of 408 observations. The bottom panel reports the results of similar regressions using a weekly sample from January 12, 2007, to October 29, 2010, a total of 199 observations. T-statistics are reported in parentheses. Asterisks * and ** indicate significance at the 5 percent and 1 percent levels, respectively.

The top panel is based on a sample from January 10, 2003, to October 29, 2010, while the bottom panel is based on a shorter sample from January 12, 2007, to October 29, 2010, a total of 408 and 199 observations, respectively.

For the 2003-2011 sample the Wilcoxon test described in the main text is -1.77 with a p-value of 0.0062, while the Wilcoxon test for the 2007-2011 sample is -0.60 with a p-value of 0.2605. Thus, the counterfactual analysis produces a statistically significant difference in the residuals during the QE2 period when a long sample of data is used.

Appendix D: Results for the Shadow-Rate B-CLR-L Model

In this appendix, we estimate a version of the CLR-L model where the nominal short rate is interpreted as a shadow short rate, s_t, that may be negative, while the short rate used for discounting nominal cash flows is its truncated version, $r_t^N = \max\{0, s_t\}$. Following Kim and Singleton (2012), we refer to this model as the B-CLR-L model. Up front we note that since computation of deflation option values is complex and time consuming within the B-CLR-L model, we only compare the CLR-L and B-CLR-L models without adjustment for the deflation option values.

K^P	$K^P_{\cdot,1}$	$K^P_{\cdot,2}$	$K^P_{\cdot,3}$	$K^P_{\cdot,4}$	$K^P_{\cdot,5}$	θ^P		Σ
$K^P_{1,\cdot}$	0.2016	0	0	0	0	0.0574	σ_{11}	0.0060
	(0.1123)					(0.0089)		(0.0001)
$K^P_{2,\cdot}$	0	0.0538	0	0	0	-0.0247	σ_{22}	0.0109
		(0.0824)				(0.0402)		(0.0003)
$K^P_{3,\cdot}$	0	0	0.3253	0	0	-0.0359	σ_{33}	0.0258
			(0.1381)			(0.0198)		(0.0005)
$K^P_{4,\cdot}$	0	0	0	0.3161	0	0.0323	σ_{44}	0.0067
				(0.1580)		(0.0082)		(0.0001)
$K^P_{5,\cdot}$	0	0	0	0	0.5715	0.0092	σ_{55}	0.0126
					(0.1699)	(0.0090)		(0.0006)

Table 13: **Estimated Dynamic Parameters for the B-CLR-L Model.**
The top panel shows the estimated parameters of the K^P matrix, θ^P vector, and diagonal Σ matrix for the shadow-rate B-CLR-L model. The estimated value of λ is 0.4965 (0.0019), while $\alpha^R = 0.7189$ (0.0046), $\kappa^Q_{liq} = 1.0674$ (0.0439), and $\theta^Q_{liq} = 0.0012$ (0.0000). The numbers in parentheses are the estimated parameter standard deviations.

The estimated dynamic parameters reported in Table 13 are very similar to those reported in the main text for the CLR-L model.

Regarding the liquidity sensitivity parameters reported in Table 14, we note that the differences between the CLR-L model and the B-CLR-L model are, in general, smaller before the financial crisis when the zero lower bound did not matter much.

The four estimated paths for (L_t^N, S_t, C_t, L_t^R) are mostly indistinguishable from each other across the two models. This is shown in Figure 14. The main difference is that the curvature factor is more negative in the shadow-rate B-CLR-L model since 2011 when short- and medium-term Treasury yields started to be severely constrained by zero lower bound.

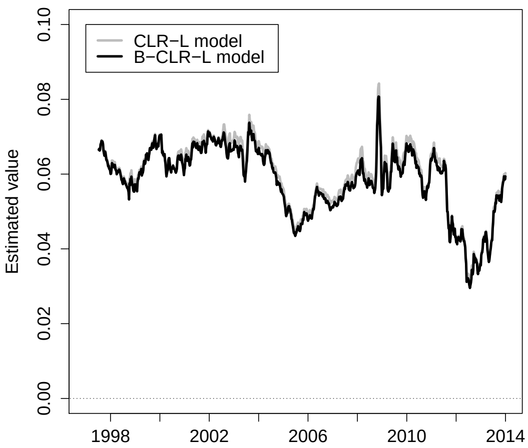
For the same reasons, the estimated path for the TIPS liquidity factor across the two models shown in Figure 15 are very similar throughout the entire sample period.

As shown in Figure 16, the B-CLR-L model also provides improvements in the estimated TIPS liquidity premium in addition to the improvement in the overall model fit already discussed above. The average estimated TIPS liquidity premium is reduced from 42.32 basis points to 38.18 basis points, or about 10 percent.

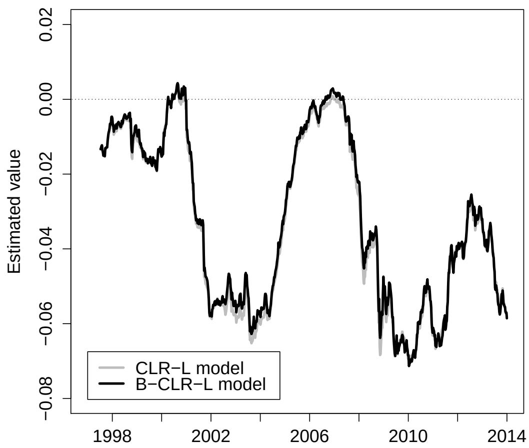
Overall, we conclude that there are benefits from accounting for the zero lower bound of nominal yields by using a shadow-rate modeling approach to those yields, which is consistent with the findings of Christensen and Rudebusch (2015, 2016).

TIPS security	CLR-L model				B-CLR-L model			
	β^i	Std	$\lambda^{L,i}$	Std	β^i	Std	$\lambda^{L,i}$	Std
(1) 3.375% 1/15/2007 TIPS	1	n.a.	0.7047	0.3563	1	n.a.	9.9990	0.1953
(2) 3.625% 7/15/2002 TIPS*	0.8260	0.1333	8.9414	2.1275	0.9909	0.0727	9.9732	0.1933
(3) 3.625% 1/15/2008 TIPS	2.1317	0.4677	0.1320	0.0493	2.9551	0.1906	0.0915	0.0084
(4) 3.875% 1/15/2009 TIPS	3.0805	0.8843	0.0988	0.0402	5.5806	0.1896	0.0536	0.0025
(5) 4.25% 1/15/2010 TIPS	2.0739	0.1818	0.2360	0.0415	2.9151	0.1375	0.1620	0.0124
(6) 3.5% 1/15/2011 TIPS	2.3928	0.1943	0.2143	0.0320	3.0939	0.1129	0.1835	0.0148
(7) 3.375% 1/15/2012 TIPS	2.4185	0.1833	0.2384	0.0364	3.1411	0.1073	0.2192	0.0202
(8) 3% 7/15/2012 TIPS	2.3956	0.1686	0.2604	0.0412	3.0614	0.1057	0.2573	0.0285
(9) 1.875% 7/15/2013 TIPS	3.0073	0.3761	0.1781	0.0434	3.3641	0.1439	0.2447	0.0298
(10) 2% 1/15/2014 TIPS	5.3622	1.2267	0.0838	0.0264	4.3294	0.1810	0.1752	0.0163
(11) 2% 7/15/2014 TIPS	2.5352	0.1962	0.3410	0.0657	3.0783	0.0827	0.4269	0.0523
(12) 0.875% 4/15/2010 TIPS*	1.9994	0.0802	9.9988	2.2979	2.4305	0.0476	6.7981	0.1985
(13) 1.625% 1/15/2015 TIPS	3.4597	0.3973	0.1871	0.0371	3.9026	0.1571	0.2508	0.0255
(14) 1.875% 7/15/2015 TIPS	2.1376	0.1231	0.9495	0.3939	2.7339	0.0505	0.9966	0.1702
(15) 2% 1/15/2016 TIPS	2.5252	0.1953	0.3848	0.0677	3.3522	0.1188	0.3612	0.0384
(16) 2.375% 4/15/2011 TIPS*	1.9328	0.0798	5.2441	2.0697	2.3612	0.0288	5.1394	0.2048
(17) 2.5% 7/15/2016 TIPS	1.8922	0.1077	6.2509	2.1911	2.4192	0.0507	9.9040	0.1875
(18) 2.375% 1/15/2017 TIPS	1.9108	0.1052	9.9794	2.0559	2.4390	0.0435	9.6821	0.1892
(19) 2% 4/15/2012 TIPS*	1.8565	0.0826	9.9975	1.9748	2.3282	0.0318	9.9956	0.2109
(20) 2.625% 7/15/2017 TIPS	1.5773	0.0929	9.9943	2.3952	1.9619	0.0544	9.9975	0.2206
(21) 1.625% 1/15/2018 TIPS	1.9053	0.1815	0.4481	0.1237	2.2358	0.1091	0.5509	0.0976
(22) 0.625% 4/15/2013 TIPS*	5.3129	1.1659	0.1526	0.0450	3.9105	0.1718	0.3385	0.0263
(23) 1.375% 7/15/2018 TIPS	1.2974	0.1566	0.8968	0.3108	1.4006	0.0752	1.5200	0.1996
(24) 2.125% 1/15/2019 TIPS	28.0660	4.1001	0.0100	0.0018	7.4718	0.3449	0.0480	0.0041
(25) 1.25% 4/15/2014 TIPS*	38.6112	4.1291	0.0230	0.0030	6.8318	0.3620	0.2245	0.0190
(26) 1.875% 7/15/2019 TIPS	1.4761	0.3116	0.4666	0.3420	1.2743	0.0945	1.5946	0.3604
(27) 1.375% 1/15/2020 TIPS	29.9390	4.7859	0.0100	0.0018	29.6283	0.3914	0.0100	0.0009
(28) 0.5% 4/15/2015 TIPS*	23.8180	4.8797	0.0372	0.0084	5.8353	0.4020	0.2974	0.0348
(29) 1.25% 7/15/2020 TIPS	1.8997	1.0819	0.3470	0.5347	1.4301	0.2521	0.5288	0.3542
(30) 1.125% 1/15/2021 TIPS	3.0499	1.6842	0.2461	0.3036	21.8655	0.4200	0.0167	0.0021
(31) 0.125% 4/15/2016 TIPS*	9.8655	6.2960	0.0990	0.0757	7.4286	0.5241	0.2207	0.0243
(32) 0.625% 7/15/2021 TIPS	2.1138	0.7052	0.6286	0.9480	1.6662	0.4191	0.4946	0.4278
(33) 0.125% 1/15/2022 TIPS	3.1227	1.6588	0.4219	0.6766	3.7027	0.5698	0.1772	0.0639
(34) 0.125% 4/15/2017 TIPS*	17.4890	8.2002	0.0530	0.0272	9.1689	0.6160	0.1662	0.0188
(35) 0.125% 7/15/2022 TIPS	2.2077	0.2444	5.4838	8.7416	1.5656	0.2316	3.2359	0.6449
(36) 0.125% 1/15/2023 TIPS	2.7209	0.2336	9.9980	9.7693	2.0871	0.2028	9.9976	0.6948
(37) 0.125% 4/15/2018 TIPS*	24.6981	10.9722	0.0416	0.0199	2.9358	0.1990	2.8988	0.7812
(38) 0.375% 7/15/2023 TIPS	1.8409	0.3227	9.9575	16.7198	1.1464	0.4334	9.9908	1.0889

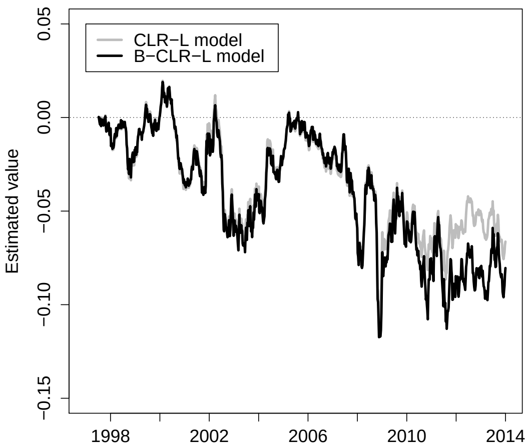
Table 14: **Estimated Liquidity Sensitivity Parameters.**
The estimated β^i sensitivity and $\lambda^{L,i}$ decay parameters for each TIPS from the CLR-L model with and without deflation option adjustment are shown. Also reported are the estimated parameter standard deviations. Asterisk * indicates five-year TIPS. The sample used in each model estimation is weekly covering the period from July 11, 1997, to December 27, 2013. All numbers are measured in basis points.



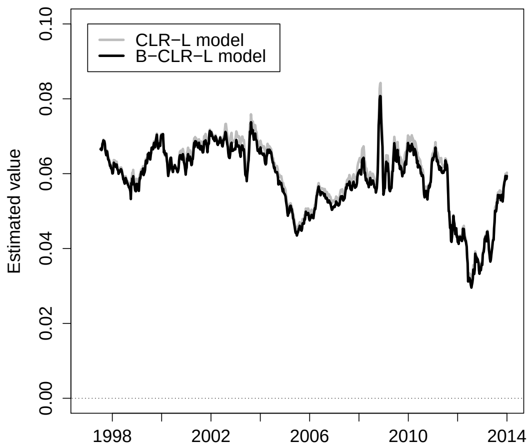
(a) L_t^N.



(b) S_t.



(c) C_t.



(d) L_t^R.

Figure 14: **Estimated State Variables.**
Illustration of the estimated state variables that affect the frictionless nominal and real yields from the CLR-L model and the B-CLR-L model. The data are weekly covering the period from July 11, 1997, to December 27, 2013.

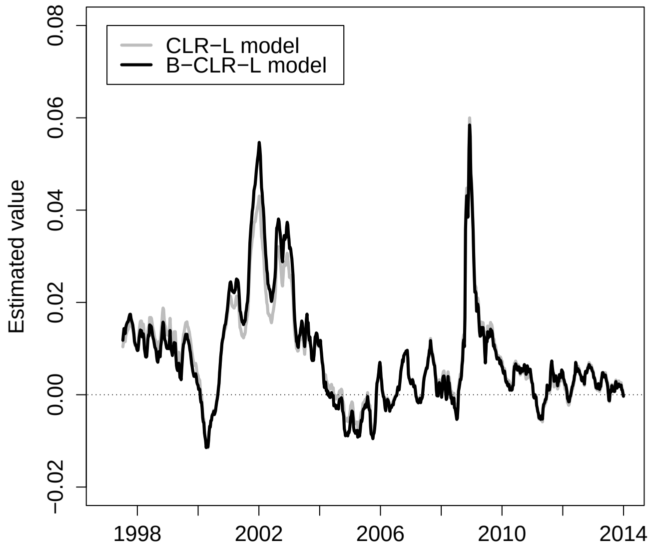


Figure 15: **Estimated Liquidity Factor.**
Illustration of the estimated liquidity factor, X_t^{liq}, from the CLR-L model and the B-CLR-L model. The data are weekly covering the period from July 11, 1997, to December 27, 2013.

Maturity in months	CLR-L model		B-CLR-L model	
	Mean	RMSE	Mean	RMSE
3	-1.15	7.40	-0.83	6.83
6	-0.85	2.70	-0.76	2.81
12	1.19	7.03	0.86	6.64
24	2.78	6.15	2.47	5.26
36	1.35	3.62	1.33	3.16
48	-0.62	2.98	-0.60	3.05
60	-1.95	3.68	-2.03	3.68
72	-2.32	3.82	-2.51	3.68
84	-1.79	3.17	-2.01	2.93
96	-0.59	2.37	-0.75	2.09
108	1.03	3.07	1.03	2.94
120	2.86	5.31	3.08	5.27
All yields	0.00	4.59	-0.06	4.30

Table 15: **Summary Statistics of Fitted Errors of Nominal Yields.**
The mean fitted errors and the root mean squared fitted errors (RMSE) of nominal U.S. Treasury yields from the CLR-L model and the B-CLR-L model estimations are shown. The full sample used in each model estimation is weekly covering the period from July 11, 1997, to December 27, 2013. All numbers are measured in basis points.

TIPS security	CLR-L model		B-CLR-L model	
	Mean	RMSE	Mean	RMSE
(1) 3.375% 1/15/2007 TIPS	2.53	4.93	2.71	5.28
(2) 3.625% 7/15/2002 TIPS*	3.25	4.01	3.26	3.93
(3) 3.625% 1/15/2008 TIPS	2.14	4.48	2.36	4.62
(4) 3.875% 1/15/2009 TIPS	1.29	2.67	1.42	2.61
(5) 4.25% 1/15/2010 TIPS	0.77	2.94	0.90	2.92
(6) 3.5% 1/15/2011 TIPS	-0.09	4.33	0.35	3.91
(7) 3.375% 1/15/2012 TIPS	0.12	5.16	0.29	5.13
(8) 3% 7/15/2012 TIPS	-0.18	4.98	-0.03	4.84
(9) 1.875% 7/15/2013 TIPS	-0.76	6.63	-0.52	6.46
(10) 2% 1/15/2014 TIPS	0.53	3.79	0.92	3.78
(11) 2% 7/15/2014 TIPS	-0.14	4.43	0.18	4.64
(12) 0.875% 4/15/2010 TIPS*	1.86	4.33	2.87	4.76
(13) 1.625% 1/15/2015 TIPS	0.77	4.36	1.21	4.28
(14) 1.875% 7/15/2015 TIPS	0.07	4.38	0.52	4.38
(15) 2% 1/15/2016 TIPS	1.06	4.67	1.40	4.79
(16) 2.375% 4/15/2011 TIPS*	4.67	12.20	5.01	12.59
(17) 2.5% 7/15/2016 TIPS	-0.50	5.44	0.08	4.93
(18) 2.375% 1/15/2017 TIPS	1.92	4.39	2.38	4.70
(19) 2% 4/15/2012 TIPS*	5.62	11.25	5.75	11.18
(20) 2.625% 7/15/2017 TIPS	0.54	3.70	0.97	3.52
(21) 1.625% 1/15/2018 TIPS	0.45	3.73	0.85	3.72
(22) 0.625% 4/15/2013 TIPS*	0.32	11.55	0.69	11.04
(23) 1.375% 7/15/2018 TIPS	0.27	4.49	0.77	4.48
(24) 2.125% 1/15/2019 TIPS	-0.08	3.14	0.58	3.18
(25) 1.25% 4/15/2014 TIPS*	0.10	4.19	0.70	3.85
(26) 1.875% 7/15/2019 TIPS	0.00	2.32	0.68	2.30
(27) 1.375% 1/15/2020 TIPS	-0.62	3.71	0.25	2.58
(28) 0.5% 4/15/2015 TIPS*	0.38	3.31	1.13	3.72
(29) 1.25% 7/15/2020 TIPS	-0.32	2.67	0.60	1.90
(30) 1.125% 1/15/2021 TIPS	-0.56	3.85	0.22	2.53
(31) 0.125% 4/15/2016 TIPS*	-0.22	3.67	0.73	3.79
(32) 0.625% 7/15/2021 TIPS	-0.17	2.72	0.36	2.18
(33) 0.125% 1/15/2022 TIPS	0.11	2.47	0.61	1.74
(34) 0.125% 4/15/2017 TIPS*	-0.01	2.58	0.58	2.88
(35) 0.125% 7/15/2022 TIPS	0.15	3.84	0.69	2.57
(36) 0.125% 1/15/2023 TIPS	-0.03	5.93	0.81	3.63
(37) 0.125% 4/15/2018 TIPS*	-0.68	3.47	0.43	3.30
(38) 0.375% 7/15/2023 TIPS	0.44	2.77	1.26	2.44
All TIPS yields	0.65	4.89	1.06	4.81
Max log L	118,915.3		119,815.1	

Table 16: **Summary Statistics of Fitted Errors of TIPS Yields.**
The mean fitted errors and the root mean squared fitted errors (RMSE) of the yield-to-maturity for individual TIPS according to the CLR-L model and the B-CLR-L model. Asterisk * indicates five-year TIPS. The sample used in each model estimation is weekly covering the period from July 11, 1997, to December 27, 2013. All numbers are measured in basis points.

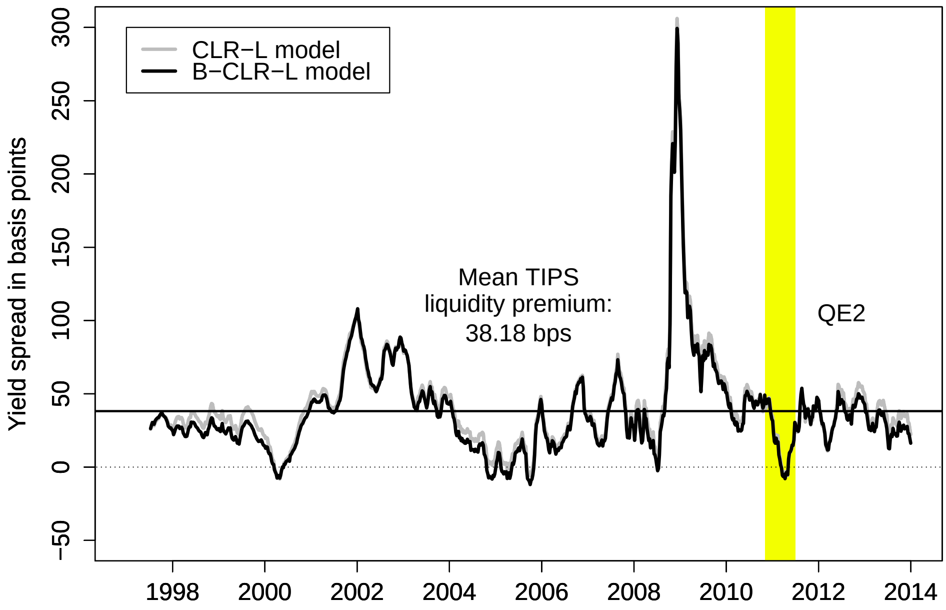


Figure 16: **Average Estimated TIPS Liquidity Premium.**
Illustration of the average estimated TIPS liquidity premium for each observation date implied by the CLR-L model and the B-CLR-L model. TIPS liquidity premiums are measured as the estimated yield difference between the fitted yield-to-maturity of individual TIPS and the corresponding frictionless yield-to-maturity with the liquidity risk factor turned off. The average TIPS liquidity premium according to the B-CLR-L model is shown with a solid black horizontal line. The data cover the period from July 11, 1997, to December 27, 2013.

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