Volatility-of-Volatility Risk in Asset Pricing

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Abstract: Exploring the equilibrium model of Bollerslev et al. (2009), this paper investigates the asset pricing implication of the market volatility-of-volatility (VOV) and extends their study in three aspects. First, we empirically construct the VOV and show that it predicts future stock market returns. Second, this paper shows that the covariance between market return and market variance, which is so-called the market leverage effect or the volatility risk for the market, is time-varying and provides predictability for future market returns through the channel of VOV. Third, we develop an asset pricing model that incorporates the pricing effect of VOV risk as well as the volatility risk of Ang et al. (2006) on individual stocks. We find that, in the presence of VOV beta, VIX beta becomes insignificant, consistent with the notion that market turmoil has become more frequent and the market crash risk is better characterized by VOV rather than by VIX. VOV beta also subsumes the pricing power of high-moment and jump betas documented in the literature. Furthermore, crash-prone stocks (whose returns co-move more negatively with the VOV) tend to offer higher returns, about 10 percent higher per annum than those offered by defensive stocks. The higher risk of crash-prone stocks is evident prior to and during market turmoil as investors collectively sell crash-prone stocks and buy defensive stocks. In sum, our study suggests that the VOV is an important source of risk that affects asset prices both in aggregation and in the cross-section.

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Comments welcome

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Exploring the equilibrium model of Bollerslev et al. (2009), this paper investigates the asset pricing implication of the market volatility-of-volatility (VOV) and extends their study in three aspects. First, we empirically construct the VOV and show that it predicts future stock market returns. Second, this paper shows that the covariance between market return and market variance, which is so-called the market leverage effect or the volatility risk for the market, is time-varying and provides predictability for future market returns through the channel of VOV. Third, we develop an asset pricing model that incorporates the pricing effect of VOV risk as well as the volatility risk of Ang et al. (2006) on individual stocks. We find that, in the presence of VOV beta, VIX beta becomes insignificant, consistent with the notion that market turmoil has become more frequent and the market crash risk is better characterized by VOV rather than by VIX. VOV beta also subsumes the pricing power of high-moment and jump betas documented in the literature. Furthermore, crash-prone stocks (whose returns co-move more negatively with the VOV) tend to offer higher returns, about 10 percent higher per annum than those offered by defensive stocks. The higher risk of crash-prone stocks is evident prior to and during market turmoil as investors collectively sell crash-prone stocks and buy defensive stocks. In sum, our study suggests that the VOV is an important source of risk that affects asset prices both in aggregation and in the cross-section.

JEL classification: G12, G13, E44

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1. Introduction

Turmoil in financial markets has become more frequent, raising the need to better understand how investor behavior and the pricing of individual firms are affected by financial crises. While previous studies have shown that market volatility is a pricing factor for individual stocks (e.g. Coval and Shumway, 2001; Ang, Hodrick, Xing, and Zhang, 2006; and Adrian and Rosenberg, 2008), we develop an asset pricing model and hypothesize: (i) Uncertainty about market volatility, which tends to rise sharply during market turmoil,[1] makes volatility beta (the covariance with market volatility) unreliable and weakens its pricing power; and (ii) for hedging against a market crash, investors prefer stocks whose returns co-move more positively with volatility of market volatility.

Our hypothesis is motivated by Bollerslev, Tauchen, and Zhou (2009) and Drechsler and Yaron (2011), who show that, to capture uncertainty about market volatility, it is useful to include an additional volatility-of-volatility (VOV) process in stock market volatility. More importantly, they demonstrate that the VOV contains information for explaining variance risk premium and for predicting stock market returns.

We extend the VOV literature to develop an asset pricing model,[2] which begins with a macroeconomic model that incorporates the seminal long-run risks (LRR) model of Bansal and Yaron (2004) and the variance-of-variance model of Bollerslev, Tauchen, and Zhou (2009). We

[1] As Mr. Olivier Blanchard, IMF's chief economist, points out in The Economist in January 2009, "Crises feed uncertainty. And uncertainty affects behaviour, which feeds the crisis."

[2] The market VOV has important implications for asset pricing because it contains information about future market returns and future market volatility, which shape the investment opportunity set and affects investors' portfolio decisions. Thus, according to the Intertemporal CAPM (ICAPM; Merton, 1973), market VOV should be a pricing factor in the cross-section of stock returns.

solve the macro-finance model explicitly and derive the equilibrium aggregate prices. Then, we use the properties of the aggregate asset prices to characterize the macroeconomic risks, transforming the underlying macro-based model to a market-based model.[3]

The market-based model developed in this paper has several advantages. First, financial data provide useful information because asset prices tell us how market participants value risks. Moreover, financial data convey information to the public in a timely fashion. Hence, the empirical design of our model is compatible with a large literature of multi-factor models explaining cross-sectional monthly stock returns (see, for instance, Fama and French, 1993; Ang, Hodrick, Xing, and Zhang, 2006; Maio and Santa-Clara, 2012; among others).

In our model, the expected stock return of a security i is determined by three sources of risks. These risks are associated with: (i) the return sensitivity to market return,$\mathbb{C}\mathrm{ov}_t[r_{i,t+1}, r_{m,t+1}]$; (ii) the return sensitivity to market variance, $\mathbb{C}\mathrm{ov}_t[r_{i,t+1}, V_{m,t+1}]$, where $V_{m,t} = \mathbb{V}\mathrm{ar}_t[r_{m,t+1}]$; and, (iii) the return sensitivity to variance of market variance, $\mathbb{C}\mathrm{ov}_t[r_{i,t+1}, Q_{m,t+1}]$, where $Q_{m,t} = \mathbb{V}\mathrm{ar}_t[V_{m,t+1}]$. The first term measures the market risk of classical capital asset pricing model (CAPM; Sharpe, 1964; Lintner, 1965). The second term corresponds to the market volatility risk of Ang, Hodrick, Xing, and Zhang (2006). The last term, which is the main focus of our paper, measures the market volatility-of-volatility risk. This model allows us to test our hypothesis posited above.

Specifically, we first investigate how market volatility-of-volatility risk affects cross-sectional stock returns on NYSE, AMEX, and NASDSQ listed stocks over the period from 1996 to 2010. To implement our model, we develop a measure of market volatility-of-volatility

[3] Our model provides a unified framework to help understand the empirical findings that (i) aggregate volatility risk is priced in cross-sectional stock returns (e.g. Ang, Hodrick, Xing, and Zhang, 2006), (ii) variance beta is priced in cross-sectional variance risk premiums (e.g. Carr and Wu, 2009), and (iii) individual variance risk premiums can predict the cross-sectional stock returns (e.g. Bali and Hovakimian, 2009; Han and Zhou, 2011).

using high frequency S&P 500 index option data.[4] We convert the tick-by-tick option data to equally spaced five-minute observations and then use the model-free methodology[5] to estimate the market variance implied by index option prices for each five-minute interval. Thus, for each day, we estimate the market volatility-of-volatility by calculating the realized bipower variance from a series of five-minute model-free implied market variance within the day. The bipower variation, introduced by Bardorff-Nielsen and Shephard (2004), delivers a consistent estimator solely for the continuous component of the volatility-of-volatility whereas the jump component is isolated.[6] In other words, our empirical results are robust to the potential jump risk embedded in volatility (see, for example, Pan, 2002; Eraker, 2008; Drechsler and Yaron 2011; among others).

Consistent with the model, by sorting stocks into quintile portfolios based on return sensitivities to market volatility-of-volatility, we find that stocks in the highest quintile have lower stock returns than stocks in the lowest quintile by 0.88 percent per month. Moreover, we also find evidence consistent with Ang, Hodrick, Xing, and Zhang (2006)'s findings that there is a significant difference of -0.87 percent per month between the stock returns with high volatility risk and the stocks with low volatility risk. Controlling for volatility risk, we still find that the market volatility-of-volatility carries a statistically significant return differentials of -0.97 percent per month. On the other hand, controlling for market volatility-of-volatility risk, we find the

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[4] We use the volatility index, VIX index, from the Chicago Board of Options Exchange (CBOE) as the proxy for the aggregate volatility risk, which has been shown to be a significant priced factor in the cross-sectional stock returns (e.g. Ang, Hodrick, Xing, and Zhang, 2006).

[5] It has been shown that the expectation of market variance can be inferred in a 'model-free' fashion from a collection of option prices without the use of a specific pricing model (see, for example, Carr and Madan 1998; Britten-Jones and Neuberger 2000; Bakshi, Kapadia, and Madan, 2003; Jiang and Tian 2005). The option implied information is forward-looking and the estimate can be obtained using daily or intraday option data.

[6] Measures of realized jump based on the difference between realized variation and bipower variation have been proposed by Barndorff-Nielsen and Shephard (2004), Huang and Tauchen (2005), and Andersen, Bollerslev, and Diebold (2007).

return difference between high volatility risk stocks and low volatility risk stocks is still large in magnitude, at -0.68 percent per month. However, running multivariate regressions, we find that market volatility-of-volatility carry a statistically significant negative price of risk and largely subsumes the valuation effect of volatility risk. Thus, our findings suggest that market volatility-of-volatility is an independently priced risk factor in the cross-sectional stock returns.

To further explore the mechanism that volatility-of-volatility risk affects asset prices, we investigate whether the volatility-of-volatility risk contributes to the asymmetric correlations between returns and market volatility-of-volatility. We refer to the volatility-of-volatility feedback effect as the mechanism that if volatility-of-volatility is priced, an anticipated increase in volatility-of-volatility raises the required rate of return, implying an immediate stock price decline and higher future returns.[7] Consistent with the channel of volatility-of-volatility feedback effect, we find that stocks that co-move more negatively with market volatility-of-volatility have lower returns before the portfolio formation and earn higher post-formation returns than stocks that co-move more positively. More importantly, we find that the return differentials (e.g. the returns of negative exposure stocks minus the returns of positive exposure stocks) before the portfolio formation are negatively correlated with market volatility-of-volatility measured at the portfolio formation date while the correlations between market volatility-of-volatility and the post-formation return differentials are positive. Thus, we can consider the market volatility-of-volatility as a state variable that drives the time-varying risk premium and generates the feedback effect.

Next, we investigate how volatility-of-volatility risk affects cross-sectional variance risk premiums. The variance risk premium is defined as the difference between risk-neutral variance

[7] Our definition of volatility-of-volatility feedback effect follows the definition of volatility feedback effect in the literature (see, e.g. French, Schwert, and Stambaugh 1987; Campbell and Hentschel 1992; Bekaert and Wu 2000; Wu 2001; Bollerslev, Sizova, and Tauchen, 2012; among others).

and realized variance. Define $V_{i,t}$ as the conditional variance of stock i at time t, $V_{i,t} = \mathbb{V}\text{ar}_t[r_{i,t+1}]$. In our model, the variance risk premium of stock i, $VRP_{i,t} \equiv \mathbb{E}_t^{\mathbb{Q}}[V_{i,t+1}] - \mathbb{E}_t[V_{i,t+1}]$, is determined by two sources of risks: (i) the variance sensitivity to market variance, $\mathbb{C}\text{ov}_t[V_{i,t+1}, V_{m,t+1}]$; and, (ii) the variance sensitivity to variance of market variance, $\mathbb{C}\text{ov}_t[V_{i,t+1}, Q_{m,t+1}]$. The first term corresponds to the variance beta of Carr and Wu (2009). The second term measures the risk that individual stock volatility co-moves with the market volatility of volatility. As shown by Carr and Wu (2009), variance risk premium corresponds to a trading strategy that shorts a swap on the realized variance; in particular, $\mathbb{E}_t^{\mathbb{Q}}[V_{i,t+1}]$ is the price for the contract and $\mathbb{E}_t[V_{i,t+1}]$ is the expected payoff. Selling a volatility asset with high volatility sensitivity to market volatility-of-volatility requires high insurance payment since the asset can help hedge the market volatility-of-volatility during market downturns.

Consistent with our model, by sorting stocks into quintile portfolios based on variance sensitivities to market volatility-of-volatility, we find that stock with high sensitivities have higher one-month variance risk premium than stocks with low sensitivities by 67.7 (in percentages squared) per month. The magnitude of the cross-sectional difference in variance risk premium is large compared to the market variance risk premium, which is 17.3 (in percentages squared) per month during our sample period. We study how volatility-of-volatility affects the variance risk premium by running the cross-sectional regressions on the 25 testing portfolios formed on the variance sensitivities to market volatility-of-volatility. We find that the risk price of variance beta with respect to variance of market variance is significantly positive. These findings suggest that market volatility-of-volatility is a priced factor in the cross-sectional variance risk premium.

Our study is related to several strands of the literature. For example, Drechsler and Yaron (2011) show that jump shocks, in a more elaborate LRR model, capture the size and predictive power of the variance premium. Drechsler (2013) show that model uncertainty has a large impact on variance risk premium, helping explain its power to predict stock returns. Nevertheless, none of prior studies provides evidence that volatility-of-volatility is a priced risk factor important for cross-sectional asset pricing.

Our paper is also related to the pricing model with higher moments of the market return as risk factors, as proposed by Chang, Christoffersen, and Jacobs (2013). They find that market skewness is a priced risk factor in the cross section of stock returns. Both our paper and their work extend the investigation of Ang, Hodrick, Xing, and Zhang (2006) and extract implied moments from index option prices. However, our results are robust to the inclusion of market skewness factor while the market skewness risk premium is much weaker in our sample period when we control for our market volatility-of-volatility risk.

In addition, Han and Zhou (2012) examine how firm-level variance risk premiums affect the stock returns in the cross-section, but they do not develop any theory to explain the dependencies. In contrast, our study investigates specifically the pricing of variance of market variance in the joint of cross-sectional stock returns and variance risk premium.

Finally, independent to our study, Baltussen, Van Bekkum, and Van Der Grient (2013) develop a measure of ambiguity, based on firm-level historical volatility of individual option-implied volatility (vol-of-vol). They find that vol-of-vol affects expected stock returns but their results cannot confirm that vol-of-vol is a priced risk factor. Our investigation differs from theirs in two aspects. First, our measure is based on intraday variation of market variance, resulting in a market volatility-of-volatility factor of daily frequency, while their vol-of-vol is based on historical daily information of implied volatility, resulting in a firm-level uncertainty measure of monthly frequency. Second, we find evidence for the rational pricing of market volatility-of-volatility risk, which sharply contrasts their ambiguity interpretation.

The remainder of the paper is organized as follows. The next section describes the economic dynamics and develops our market-based three-factor model for the empirical implementation. Section 3 constructs the measure of market volatility-of-volatility. Section 4 describes the data, presents the summary statistics, and provides evidence for the market return predictability afforded by the market volatility-of-volatility. In section 5, we show empirical evidence on the pricing of variance of market variance risk in cross-sectional stock returns. Section 6 investigates the hedging effect and the flight-to-quality effect for the role of market volatility-of-volatility risk during the period of market crash. Finally, section 7 contains our concluding remarks.

2. A three-factor model

This section describes the economic model. Our model begins with a macroeconomic model that incorporates the seminal long-run risks (LRR) model of Bansal and Yaron (2004) and the variance-of-variance model of Bollerslev, Tauchen, and Zhou (2009). We solve the macro-finance model explicitly and derive the equilibrium aggregate asset prices. Then, we use the properties of aggregate asset prices to characterize the macroeconomic risks and develop a market-based three-factor model for the cross-sectional asset prices.

2.1. Economic dynamics and equilibrium aggregate asset prices

The underlying setting of our model is a discrete-time endowment economy. The dynamics of consumption growth rate, g_{t+1}, and dividend growth rate, $g_{d,t+1}$, are governed by the following process:

$$
\begin{aligned}
g_{t+1} &= \mu_g + x_t + \sigma_t z_{g,t+1} \\
x_{t+1} &= \rho_x x_t + \varphi_x \sigma_t z_{x,t+1} \\
\sigma_{t+1}^2 &= \mu_\sigma + \rho_\sigma \sigma_t^2 + q_t z_{\sigma,t+1} \\
q_{t+1}^2 &= \mu_q + \rho_q q_t^2 + \varphi_q z_{q,t+1} \\
g_{d,t+1} &= \mu_d + \phi x_t + \varphi_d \sigma_t z_{d,t+1}
\end{aligned}
\tag{1}
$$

where $z_{g,t+1}, z_{x,t+1}, z_{\sigma,t+1}, z_{q,t+1}, z_{d,t+1} \overset{\text{iid}}{\sim} N(0,1)$, x_{t+1} represents the long-run consumption growth, σ_{t+1}^2 is the time-varying economic uncertainty, and q_{t+1}^2 is the economic volatility-of-volatility, which is the conditional variance of the economic uncertainty. The features of the long-run risk and the time-varying economic uncertainty is proposed by Bansal and Yaron (2004), while the additional feature of economic volatility-of-volatility is introduced by Bollerslev, Tauchen, and Zhou (2009).

The representative agent is equipped with recursive preferences of Epstein and Zin (1989). Thus, the logarithm of the Intertemporal Marginal Rate of Substitution (IMRS) is $m_{t+1} = \theta \log(\delta) - \frac{\theta}{\psi} g_{t+1} + (\theta - 1) r_{a,t+1}$, where $r_{a,t+1}$ is the return on consumption claim, and $\theta \equiv (1-\gamma)(1 - 1/\psi)^{-1}$. We assume that $\gamma > 1, \text{and}\, \psi > 1$, and therefore $\theta < 0$. Based on Campbell and Shiller (1988) approximation, $r_{a,t+1} \approx \kappa_0 + \kappa_1 pc_{t+1} - pc_t + g_{t+1}$, where pc_t is the logarithm of price–consumption ratio, which in equilibrium is an affine function of the state variables, $pc_t = A_0 + A_x x_t + A_\sigma \sigma_t^2 + A_q q_t^2$.[8] Substituting the equilibrium consumption return, $r_{a,t+1}$, into the IMRS, the innovation in the pricing kernel is

$$
m_{t+1} - \mathbb{E}_t[m_{t+1}] = -\lambda_g \sigma_t z_{g,t+1} - \lambda_x \sigma_t z_{x,t+1} - \lambda_\sigma q_t z_{\sigma,t+1} - \lambda_q \varphi_q z_{q,t+1}, \tag{2}
$$

where $\lambda_g = \gamma > 0$, $\lambda_x = (1-\theta) A_x \kappa_1 \varphi_x > 0$, $\lambda_\sigma = (1-\theta) A_\sigma \kappa_1 < 0$, $\lambda_q = (1-\theta) A_q \kappa_1 < 0$. The parameters determine the prices for short-run risk (λ_g), long-run risk (λ_x), volatility risk (λ_σ),

[8] The equilibrium solutions are: $A_x = \frac{1-1/\psi}{1-\kappa_1 \rho_x} > 0, A_\sigma = \frac{\theta((1-1/\psi)^2 + A_x^2 \kappa_1^2 \varphi_x^2)}{2(1-\kappa_1 \rho_\sigma)} < 0, \text{and}\ A_q = \frac{\theta A_\sigma^2 \kappa_1^2}{2(1-\kappa_1 \rho_q)} < 0.$

and volatility of volatility risk (λ_q). An analogous expression holds for the stock market return, $r_{m,t+1} = \kappa_{0,m} + \kappa_{1,m} pd_{m,t+1} - pd_{m,t} + g_{d,t+1}$, where $pd_{m,t}$ is the log price–dividend ratio, which in equilibrium is an affine function of the state variables, $pd_{m,t} = A_{0,m} + A_{x,m} x_t + A_{\sigma,m}\sigma_t^2 + A_{q,m} q_t^2$.[9] Since we require that $\theta < 0$, we have $A_{x,m} > 0$, $A_{\sigma,m} < 0$, and $A_{q,m} < 0$.

The innovation in market return can be express as $r_{m,t+1} - \mathbb{E}_t[r_{m,t+1}] = \varphi_d \sigma_t z_{d,t+1} + \beta_{m,x}\sigma_t z_{x,t+1} + \beta_{m,\sigma} q_t z_{\sigma,t+1} + \beta_{m,q}\varphi_q z_{q,t+1}$, wher $\beta_{m,x} = A_{x,m}\kappa_{1,m}\varphi_x > 0, \beta_{m,\sigma} = A_{\sigma,m}\kappa_{1,m} < 0, \beta_{m,q} = A_{q,m}\kappa_{1,m} < 0$. The conditional variance of market return is readily calculated as $V_{m,t} \equiv \mathbb{V}\mathrm{ar}_t[r_{m,t+1}] = (\varphi_d^2 + \beta_{m,x}^2)\sigma_t^2 + \beta_{m,\sigma}^2 q_t^2 + \beta_{m,q}^2 \varphi_q^2$, and the process for innovations in market variance is $V_{m,t+1} - \mathbb{E}_t[V_{m,t+1}] = \beta_{V,\sigma} q_t z_{\sigma,t+1} + \beta_{V,q}\varphi_q z_{q,t+1}$, where $\beta_{V,\sigma} = \varphi_d^2 + \beta_{m,x}^2$, $\beta_{V,q} = \beta_{m,\sigma}^2$. Thus, innovations in market variance are related to both the economic volatility shock and the economic volatility-of-volatility shock. It follows that the market volatility-of-volatility (e.g. the conditional variance of market variance) is $Q_{m,t} \equiv \mathbb{V}\mathrm{ar}_t[V_{m,t+1}] = \beta_{V,\sigma}^2 q_t^2 + \beta_{V,q}^2 \varphi_q^2$, and the process for its innovations is $Q_{m,t+1} - \mathbb{E}_t[Q_{m,t+1}] = \beta_{Q,q}\varphi_q z_{q,t+1}$, where $\beta_{Q,q} = \beta_{V,\sigma}^2$. Note that innovations in market volatility-of-volatility is solely determined by economic variance of variance shock with a scaling factor, $\beta_{Q,q}$. The market volatility-of-volatility-of-volatility (e.g. the conditional variance of variance of market variance), $W_{m,t} \equiv \mathbb{V}\mathrm{ar}_t[Q_{m,t+1}] = \beta_{Q,q}^2 \varphi_q^2$, is constant in our model.

It is straightforward now to derive the equity premium on the market portfolio,

$$\begin{aligned}\mathbb{E}_t[r_{m,t+1}] - r_{f,t} + 0.5\mathbb{V}\mathrm{ar}_t[r_{m,t+1}] &= \mathbb{C}\mathrm{ov}_t[r_{m,t+1}, -m_{t+1}] \\ &= \lambda_x \mathbb{C}\mathrm{ov}_t[r_{m,t+1}, x_{t+1}] + \lambda_\sigma \mathbb{C}\mathrm{ov}_t[r_{m,t+1}, \sigma_{t+1}^2] + \lambda_q \mathbb{C}\mathrm{ov}_t[r_{m,t+1}, q_{t+1}^2]. \quad (3) \\ &= \lambda_x \beta_{m,x}\sigma_t^2 + \lambda_\sigma \beta_{m,\sigma} q_t^2 + \lambda_q \beta_{m,q}\varphi_q^2.\end{aligned}$$

The expected market return consists of three terms. The first two terms are long-run risk

[9] The solutions are: $A_{x,m} = \frac{\phi - 1/\psi}{1-\kappa_{1,m}\rho_x}$, $A_{\sigma,m} = \frac{(1-\theta)A_\sigma(1-\kappa_1\rho_\sigma)+0.5H_{m,\sigma}}{1-\kappa_{1,m}\rho_\sigma}$, and $A_{q,m} = \frac{(1-\theta)A_q(1-\kappa_1\rho_q)+0.5H_{m,q}}{1-\kappa_{1,m}\rho_q}$, where $H_{m,\sigma} = \gamma^2 + \varphi_d^2 + \varphi_x^2(\lambda_x - \beta_{m,x})^2$ and $H_{m,q} = (\lambda_\sigma - \beta_{m,\sigma})^2$.

premium and volatility risk premium, which are the same as in Bansal and Yaron (2004), while the last term represents the volatility-of-volatility risk premium, which corresponds to the work of Bollerslev, Tauchen, and Zhou (2009).[10]

2.2. Volatility-of-volatility, time-varying leverage effect and return predictability

In the literature, the leverage effect (e.g. Black, 1976; Christie, 1982; among others) refers to the negative contemporaneous return-volatility correlation. The model endogenously generates negative contemporaneous correlation between return and volatility-of-volatility as well as a time-varying leverage effect. A straightforward calculation shows that

$$\begin{aligned} \mathbb{C}\mathrm{ov}_t\left[r_{m,t+1}, Q_{m,t+1}\right] &= \beta_{m,q}\beta_{Q,q}\varphi_q^2 < 0, \\ \mathbb{C}\mathrm{ov}_t\left[r_{m,t+1}, V_{m,t+1}\right] &= \beta_{m,\sigma}\beta_{V,\sigma}q_t^2 + \beta_{m,q}\beta_{V,q}\varphi_q^2 < 0, \end{aligned} \tag{4}$$

In the absence of the time-varying economic volatility-of-volatility (e.g. when q_t^2 is constant and φ_q^2=0), the second term of $\mathbb{C}\mathrm{ov}_t\left[r_{m,t+1}, V_{m,t+1}\right]$ is reduced to zero, leading to a smaller value. Thus, the dynamics of economic volatility-of-volatility amplifies the leverage effect.

Moreover, driven by q_t^2, the market leverage effect is time-varying and provides information about future market returns since the time-varying market risk premium is driven by q_t^2 as well. To illustrate, define $CVRV_{m,t} \equiv \mathbb{C}\mathrm{ov}_t\left[r_{m,t+1}, V_{m,t+1}\right]$ as the market leverage effect at time t, and consider the predictive regression of market returns on $CVRV_{m,t}$,

$$r_{m,t+1} - r_{f,t} = \alpha + \beta_{CVRV}^{pred} CVRV_{m,t} + \epsilon_{t+1}. \tag{5}$$

Then, the projection coefficient is

$$\beta_{CVRV}^{pred} = \frac{\mathbb{C}ov\left[\mathbb{E}_t\left[r_{m,t+1}\right] - r_{f,t} + \epsilon_{t+1}, CVRV_{m,t}\right]}{\mathbb{V}ar\left[CVRV_{m,t}\right]} = \frac{\lambda_\sigma}{\beta_{V,\sigma}} < 0. \tag{6}$$

When the time-varying economic volatility-of-volatility is high, there is a high market volatility, a negative contemporaneous market return due to price decline, a negative market leverage effect,

[10] Since we do not assume the square root process for the volatility-of-volatility as Bollerslev, Tauchen, and Zhou (2009) do, the volatility risk in the resulting equity premium does not confound with the volatility-of-volatility risk.

and a high future market return due to risk premium, leading to a negative association between the market leverage effect and the future market return. Similarly, consider the predictive regression of market returns on the market volatility-of-volatility,

$$r_{m,t+1} - r_{f,t} = \alpha + \beta_Q^{pred} Q_{m,t} + \epsilon_{t+1}. \tag{7}$$

Then, the projection coefficient is

$$\beta_Q^{pred} = \frac{\mathbb{C}ov\left[\mathbb{E}_t\left[r_{m,t+1}\right] - r_{f,t} + \epsilon_{t+1}, Q_{m,t}\right]}{\mathbb{V}ar\left[Q_{m,t}\right]} = \frac{\beta_{m,\sigma}\lambda_\sigma}{\beta_{V,\sigma}^2} > 0. \tag{8}$$

When the time-varying economic volatility-of-volatility is high, there is a high market volatility-of-volatility and a high future market return due to risk premium, leading to a positive association between the market volatility-of-volatility and the future market return.

It is worth noting that the market return predictability afforded by the economic volatility-of-volatility through the time-varying market leverage effect or through the time-varying market volatility-of-volatility, which is in line with the mechanism of volatility feedback effect in the literature (see, e.g. Campbell and Hentschel 1992; Bekaert and Wu 2000; Wu 2001; Bollerslev, Sizova, and Tauchen, 2012; among others) through the channel of time-varying risk premium.

Further, the pioneer work of Bollerslev, Tauchen, and Zhou (2009) suggests that the market variance risk premium, which is the proxy for the economic volatility-of-volatility in their equilibrium model, could predict the future market return. However, as indicated in Drechsler and Yaron (2011), the role of the economic volatility-of-volatility in Bollerslev, Tauchen, and Zhou (2009) could be challenged since the jump risk premium could be an alternative state variable that contributes to both the market variance risk premium and the market return predictability. Thus, compared with the indirect measure used in Bollerslev, Tauchen, and Zhou (2009), the continuous component of the market volatility-of-volatility, which is isolated from the jump risk, should shed additional light on the role of the economic volatility-of-volatility.

2.3. A market-based three-factor model for individual stocks

We assume that the innovations in stock return i is $r_{i,t+1} - \mathbb{E}_t[r_{i,t+1}] = \beta_{i,x}\sigma_t z_{x,t+1} + \beta_{i,\sigma} q_t z_{\sigma,t+1} + \beta_{i,q}\varphi_q z_{q,t+1}$. Given the expression for the pricing kernel, the expected stock return can be written as

$$\begin{aligned} &\mathbb{E}_t[r_{i,t+1}] - r_{f,t} + 0.5\mathbb{V}\mathrm{ar}_t[r_{i,t+1}] \\ &\quad = \beta_{i,x}\lambda_x\sigma_t^2 + \beta_{i,\sigma}\lambda_\sigma q_t^2 + \beta_{i,q}\lambda_q\varphi_q^2. \end{aligned} \tag{9}$$

Thus, the expected stock return is determined by three sources of economic risks: economic long-run risk ($\beta_{i,x}$), economic volatility risk ($\beta_{i,\sigma}$), and economic volatility-of- volatility risk ($\beta_{i,q}$).

We now use the properties of the aggregate asset prices to characterize the macroeconomic risks. First of all, in equilibrium, the market volatility-of-volatility risk, which is the return covariance with respect to variance of market variance, is solely determined by the economic volatility-of-volatility risk ($\beta_{i,q}$), i.e. $\mathbb{C}\mathrm{ov}_t[r_{i,t+1}, Q_{m,t+1}] = \beta_{i,q}\beta_{Q,q}\varphi_q^2$. Furthermore, the return sensitivities with respect to market variance and with respect to market return provide additional information for the economic volatility risk and the long-run risk; that is, $\mathbb{C}\mathrm{ov}_t[r_{i,t+1}, V_{m,t+1}] = \beta_{i,\sigma}\beta_{V,\sigma}q_t^2 + \beta_{i,q}\beta_{V,q}\varphi_q^2$, and $\mathbb{C}\mathrm{ov}_t[r_{i,t+1}, r_{m,t+1}] = \beta_{i,x}\beta_{m,x}\sigma_t^2 + \beta_{i,\sigma}\beta_{m,\sigma}q_t^2 + \beta_{i,q}\beta_{m,q}\varphi_q^2$. Substituting out the economic risks gives us the market-based three-factor model:

$$\begin{aligned} &\mathbb{E}_t[r_{i,t+1}] - r_{f,t} + 0.5\mathbb{V}\mathrm{ar}_t[r_{i,t+1}] \\ &\quad = \lambda_m\mathbb{C}\mathrm{ov}_t[r_{i,t+1}, r_{m,t+1}] + \lambda_V\mathbb{C}\mathrm{ov}_t[r_{i,t+1}, V_{m,t+1}] \\ &\qquad + \lambda_Q\mathbb{C}\mathrm{ov}_t[r_{i,t+1}, Q_{m,t+1}], \end{aligned} \tag{10}$$

where

$$\lambda_m = \frac{\lambda_x}{\beta_{m,x}}, \lambda_V = \frac{\lambda_\sigma - \lambda_m\beta_{m,\sigma}}{\beta_{V,\sigma}}, \lambda_Q = \frac{\lambda_q - \lambda_V\beta_{V,q} - \lambda_m\beta_{m,q}}{\beta_{Q,q}}. \tag{11}$$

Thus, the expected stock return is now determined by three sources of risks related to aggregate asset prices. The first term measures the market risk of classical capital asset pricing model (CAPM; Sharpe, 1964; Lintner, 1965). The second term corresponds to the aggregate volatility risk of Ang, Hodrick, Xing, and Zhang (2006). The last term, which is the main focus of this paper, measures the aggregate volatility of volatility risk. The resulting three risk prices in our market-based model, λ_m, λ_V, and λ_Q, are related to the three economic risk prices with a linear transformation.

The market-based model developed in this paper has several advantages. First, financial data provide useful information because asset prices tell us how market participants value risks. Moreover, financial data convey information to public in a timely fashion. Hence, the empirical design of our model is compatible with a large literature of multi-factor model explaining cross-sectional monthly stock returns (see, for instance, Fama and French, 1993; Ang, Hodrick, Xing, and Zhang, 2006; Maio and Santa-Clara, 2012; among others).

3. Estimation of variance of market variance

In previous section, we propose a market-based three-factor model, which requires the information of market return, market variance, and variance of market variance. To proxy for the first two factors, we use CRSP value-weighted market index and CBOE VIX index, which have been widely used in the literature (see, for example, Ang, Hodrick, Xing, and Zhang, 2006; Chang, Christoffersen, and Jacobs, 2013; Bollerslev, Tauchen, and Zhou, 2009; among others). In this study, we estimate the variance of market variance by calculating the realized bipower variation from a series of five-minute model-free implied variances, using the high-frequency S&P 500 index option data. The details of our empirical settings are described as follows.

First of all, we extract the model-free implied variance, using the spanning methodology proposed by Carr and Madan (2001), Bakshi and Madan (2000), Bakshi, Kapadia, and Madan

(2003), and Jiang and Tian (2005). Bakshi, Kapadia, and Madan (2003) show that the price of a τ-maturity return variance contract, $\bar{V}_t(\tau) \equiv \mathbb{E}_t^{\mathbb{Q}}\left[\mathrm{e}^{-r_{f,t}}\mathrm{Log}\left[\frac{S_{t+\tau}}{S_t}\right]^2\right]$, which is the discounted conditional expectation of the square of market return under the risk-neutral measure, can be spanned by a collection of out-of-money call options and out-of-money put options,

$$\bar{V}_t(\tau) = \int_{S_t}^{\infty} \frac{2(1 - \log[K/S_t])}{K^2} C_t(K;\tau)\,dK + \int_0^{S_t} \frac{2(1 + \log[K/S_t])}{K^2} P_t(K;\tau)\,dK, \tag{12}$$

where $C_t(K;\tau)$ and $P_t(K;\tau)$ are the prices of European calls and puts at time t written on the underlying stock with strike price K and expiration date at $t + \tau$. The conditional variance of market return can be calculated by $IV_t(\tau) = \mathrm{e}^{r_{f,t}}\bar{V}_t(\tau) - \mu_t(\tau)^2$, where $\mu_t(\tau)$ satisfies the risk-neutral valuation relationship, which is related to the first four risk-neutral moments of market returns as described in equation (39) of Bakshi, Kapadia, and Madan (2003).

Second, we use the model-free realized bipower variance, introduced by Bardorff-Nielsen and Shephard (2004), to estimate the variance of market variance. Define the intraday stock return as $r_{t+1,j} \equiv \log[S_{t+j/M}] - \log[S_{t+(j-1)/M}]$, $j = 1, \ldots, M$, where M is the sampling frequency per trading day. Bardorff-Nielsen and Shephard (2004) study two measures of realized variations; the first one is the realized variation, $RV_{t+1} = \sum_{j=1}^{M} r_{t+1,j}^2$, and the second one is the bipower variation, $BV_{t+1} = \frac{\pi}{2}\left(\frac{M}{M-1}\right)\sum_{j=2}^{M}\left|r_{t+1,j}\right|\left|r_{t+1,j-1}\right|$. Andersen, Bollerslev, and Diebold (2002) show that the realized variance converges to the integrated variance plus the jump contributions, i.e. $RV_{t+1} \xrightarrow{M\to\infty} \int_t^{t+1}\sigma^2(s)\,ds + \sum_{j=1}^{N_{t+1}} J_{t+1,j}^2$, where N_{t+1} is the number of return jumps within day t+1 and $J_{t+1,j}^2$ is the jump size. Moreover, Bardorff-Nielsen and Shephard (2004) show that $BV_{t+1} \xrightarrow{M\to\infty} \int_t^{t+1}\sigma^2(s)\,ds$. In other words, bipower variation provides a consistent estimator of the integrated variance solely for the diffusion part.

Our measure for variance of market variance is estimated from a series of five-minute based model-free implied variances. The intraday model-free implied variances are calculated using equation (12), which is denoted as $IV_{t+j/M}(\tau), j = 1, \ldots, M$. Since the process of market variance is a (semi-)martingale, we apply the bipower variation formula on the changes in annualized model-free implied variances and obtain a measure for variance of market variance:

$$VOV_{t+1}(\tau) = \frac{\pi}{2}\left(\frac{M}{M-1}\right)\sum_{j=2}^{M}\left|\Delta v_{t+1,j}(\tau)\right|\left|\Delta v_{t+1,j-1}(\tau)\right| \tag{13}$$

where $\Delta v_{t+1,j}(\tau) \equiv \frac{365}{\tau}\left[IV_{t+j/M}(\tau) - IV_{t+(j-1)/M}(\tau)\right]$. In this way, our empirical results will not be affected by the volatility jumps (or the return jumps embedded in the volatility).

4. Data and descriptive statistics

4.1. Data description

We use the tick-by-tick quoted data on S&P 500 index (SPX) options from CBOE's Market Data Report (MDR) tapes over the time period from January 1996 to December 2010. The underlying SPX prices are also provided in the tapes. We obtain daily data from OptionMetrics for equity options and S&P 500 index options. We use the Zero Curve file, which contains the current zero-coupon interest rate curve, and the Index Dividend file, which contains the current dividend yield, from OptionMetrics to calculate the implied volatility for each tick-by-tick data from CBOE's MDR tapes. Daily and monthly stock return data are from CRSP while intraday transactions data are from TAQ data sets. Financial statement data are from COMPUSTAT. Fama and French (1993) factors and their momentum *UMD* factor are obtained from the online data library of Ken French.[11] *VIX* index is obtained from the website of CBOE.[12] While we use the

[11] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/
[12] http://www.cboe.com/micro/vix/historical.aspx

'new' *VIX* index to calculate the market variance risk premium as proposed by Bollerslev, Tauchen, and Zhou (2009), we also use the 'old' *VIX*, which is based on the S&P 100 options and Black–Scholes implied volatilities, as our volatility factor, following Ang, Hodrick, Xing, and Zhang (2006). We use the index option prices from the Option Price file to replicate the market skewness factor and the market kurtosis factor of Chang, Christoffersen, and Jacobs (2013).

We follow the literature (see, for example, Jiang and Tian 2005; Chang, Christoffersen, and Jacobs, 2013; among others) to filter out index option prices that violate the arbitrage bounds.[13] We also eliminate in-the-money options (e.g. put options with K/S>1.03 and call options with K/S<1.03) because prior study suggests that they are less liquid. We use the daily SPX low and high prices, downloaded from Yahoo Finance,[14] to filter out the MDR data that are outside the [low, high] interval.

For the computation of the market volatility-of-volatility, we first partition the tick-by-tick S&P500 index options data into five-minute intervals. For each maturity within each interval, we linearly interpolate implied volatilities for a fine grid of one thousand moneyness levels (K/S) between 0.01% and 300%[15] and use equations (12) to estimate the model-free implied variance. We then use linearly interpolate maturities to obtain the estimate at a fixed 30-day horizon. For each day, our measure for market volatility-of-volatility (*VOV*) is calculated by using the bipower variation formula of equation (13) with the 81 within-day five-minute annualized 30-day model-free implied variance estimates covering the normal CBOE trading hours from 8:30 a.m. to 3:15 p.m. Central Time. We estimate the daily realized covariance, *CVRV*, between the market

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[13] Moreover, we eliminate all observations for which the ask price is lower than the bid price, the bid price is equal to zero, or the average of the bid and ask price is less than 3/8.

[14] http://finance.yahoo.com/q/hp?s=^GSPC+Historical+Prices

[15] For moneyness levels below or above the available moneyness level in the market, we use the implied volatility of the lowest or highest available strike price.

return and the market variance using the intraday 5-min logarithmic return multiplied by 22 and the 5-min implied variance for S&P 500 index.

The market variance risk premium ($VRP_{m,t}$), following Bollerslev, Tauchen, and Zhou (2009), is defined as the difference between the ex-ante implied variance ($IV_{m,t}$) and the ex-post realized variance ($RV_{m,t}$), i.e. $VRP_{m,t} \equiv IV_{m,t} - RV_{m,t}$. We focus on a fixed maturity of 30 days. Market implied variance ($IV_{m,t}$) is measured by the squared 'new' VIX index divided by 12. Summation of SPX five-minute squared logarithmic returns are used to calculate the market realized variance ($RV_{m,t}$). With eighty five- minute intervals per trading day and the overnight return, we construct the daily market realized variance, using a rolling window of 22 trading days starting from the current day.

To implement our empirical model, we construct innovations in market moments. First, following Ang, Hodrick, Xing, and Zhang (2006), innovations in market volatility (*ΔVIX*) is measured by its first order difference, i.e. $\Delta VIX_{t+1} = VIX_{t+1} - VIX_t$. Chang, Christoffersen, and Jacobs (2013) indicate that taking the first difference is appropriate for *VIX*, whereas an ARMA(1,1) model is need to remove the autocorrelation in the their skewness and kurtosis factors. Following their approach, the innovations in market volatility-of-volatility (*ΔVOV)* is computed as the ARMA(1,1) model residuals of the market volatility-of-volatility. Similar estimation of ARMA(1,1) is performed to obtain the innovations in market leverage, $\Delta CVRV$.

4.2. Descriptive statistics

Figure 1 plots the daily S&P 500 logarithmic return (*rSPX*) and changes in market volatility (*ΔVIX*) over the time period from January 1996 to December 2010. Figure 1 also plots the area of the extreme return shocks, which is defined as the period when *rSPX* is above its time-series 95th percentile value or below its time-series 5th percentile value; and the extreme volatility shocks, which is defined as the period when *ΔVIX* is above its time-series 95th percentile value or below its time-series 5th percentile value. There are clear spikes on the graph—the Asian financial crisis in 1997, the LTCM crisis in 1998, September11, 2001, the WorldCom and Enron bankruptcies in 2001 and 2002, subprime loan crisis in 2007, the recent financial crisis in 2008, and the flash crash in 2010.

Table 1 reports descriptive statistics for the daily factors used in this paper, including the four factors (*MKT*, *SMB*, *HML*, and *UMD*) of Fama and French (1993) and Carhart (1997), the market variance risk premium (*VRP*), the volatility index (*VIX*; Ang, Hodrick, Xing, and Zhang, 2006), our measure of variance of market variance (*VOV*), the market skewness factor (*SKEW*) and market kurtosis factor (*KURT*) of Chang, Christoffersen, and Jacobs (2013), the zero beta straddle factor of Coval and Shumway (2001), and the options return volatility factor (*VOL*) and the options return jump factor (*JUMP*) of Cremers, Halling, and Weinbaum (2015).

In our sample, the mean of 30-day market variance risk premium (*VRP*) is 0.173%, which is slightly smaller than 18.3 in Bollerslev, Tauchen, and Zhou's (2010) sample. The mean of *VOV* is 0.054%, which is much smaller than its standard deviation, 0.563%. The mean of *SKEW* is -1.663 and the mean of *KURT* is 9.313. Thus, the risk-neutral distribution of the market return is asymmetric and has fat tails. The mean of the delta-neutral straddle return (*STR*) is -0.333; the mean of the delta-neutral, gamma-neutral, and vega positive straddle return (*VOL*) is -0.038; the mean of the delta-neutral, vega-neutral, and gamma a positive straddle return (*JUMP*) is -0.210. Thus, our sample estimates are consistent with the findings of Cremers, Halling, and Weinbaum (2015) that the *STR* in Coval and Shumway (2001) is largely affected by the jump risk (*JUMP*).

Panel B reports the Spearman correlations between factors, where the non-return based state variables are measured by their innovations. As expected, *MKT* is negatively correlated with both *ΔVIX* (-0.779) and *ΔVOV* (-0.044), supporting the leverage effect predicted by our model.

Moreover, *VRP* is positively correlated with *ΔVOV* (0.145), consistent with our theory that the variance risk premium and the market volatility-of-volatility are both driven by the economic volatility-of-volatility. *ΔKURT* and *ΔSKEW* are highly correlated with a correlation value of -0.863, which is comparable to -0.83 reported by Chang, Christoffersen, and Jacobs (2013). Further, *STR* has a much higher correlation with *JUMP* (0.924) than that with *VOL* (0.201), which is similar to the findings reported by Cremers, Halling, and Weinbaum (2015). More importantly, *ΔVOV* shows little correlation with *ΔVIX* (0.049), *ΔSKEW* (-0.017), *ΔKURT* (-0.010), *STR* (0.051), *VOL* (0.039), and *JUMP* (0.046), which suggests that *ΔVOV* should be an independent state variable that cannot be explained by these market moments studied in the literature.

4.3. Return predictability

In this section, we check the return predictability afforded by market volatility-of-volatility. The theoretical model suggests that market volatility-of-volatility is positively related to economic volatility-of-volatility. Hence, we should expect that our *VOV* measure can predict future stock returns as market variance risk premium does.

Panel A of Table 2 reports the estimates of the one-period return predictability regression using daily S&P 500 logarithmic returns (*rSPX*) on the lagged realized covariance (multiplied by 22), between the intraday 5-min logarithmic return and the 5-min implied variance for S&P 500 index (*CVRV*), market volatility (*VIX*), variance of market variance (*VOV*), variance risk premium (*VRP*), market skewness (*SKEW*), market kurtosis (*KURT*), and jump risk (*JUMP*). Robust Newey-West (1987) *t*-statistics with sixteen lags that account for autocorrelations are used. Consistent with the theory, in columns from [1] to [4], we find that *CVRV* negatively predicts one-period ahead daily market return. Moreover, in columns from [5] to [8], we find that

VOV positively predicts one-period ahead daily market return in all of the specifications. More importantly, in column [9], we find that *VOV* subsumes the predictive power of *CVRV*, suggesting that *VOV* is the main driving force for the market return predictability provided by the time-varying market leverage effect. In Panel B, we use the monthly S&P 500 logarithmic returns (*rSPX*) as the dependent variable, and the independent variables are sampled at the end of the previous month. We similarly find that *VOV* positively predicts one-period ahead monthly market return in all of the specifications.

Overall, the return predictability supports the volatility-of-volatility feedback effect implied by our model. The evidence for the predictability afforded by the market volatility-of-volatility suggests that economic volatility-of-volatility is an important state variable that affects the aggregate asset prices.

5. Pricing volatility-of-volatility risk in the cross-sectional stock returns

This section examines how market volatility-of-volatility risk affects cross-sectional average returns. Based on our market-based three-factor model with their empirical proxies, at the end of each month, we estimate the regression for each stock *i* using daily returns:

$$r_{i,t+1} - r_{f,t+1} = \alpha_i + \beta_{i,MKT} MKT_{t+1} + \beta_{i,VIX} \Delta VIX_{t+1} + \beta_{i,VOV} \Delta VOV_{t+1} + \varepsilon_{i,t+1}. \tag{14}$$

We construct a set of testing assets that are sufficiently disperse in exposure to aggregate volatility-of-volatility innovations by sorting firms on $\beta_{i,VOV}$ loadings over the past month using the regression (14) with daily data. Our empirical model is an extension of Ang, Hodrick, Xing, and Zhang (2006). Following their work, we run the regression for all common stocks on NYSE, AMEX, and NASDAQ with more than 17 daily observations. After the portfolio formation, we calculate the value-weighted daily and monthly stock returns for each portfolio. If market volatility-of-volatility is a priced risk factor, we should expect to see a monotonic decreasing pattern in the portfolio returns.

5.1. Portfolios sorted on market volatility-of-volatility risk

Table 3 provides the performance of portfolios sorted on $\beta_{i,VIX}$ and $\beta_{i,VOV}$. We sort stocks into quintile portfolios based on $\beta_{i,VIX}$, from the lowest (quintile 1) to the highest (quintile 5), and independently sort stocks into quintile portfolios based on $\beta_{i,VOV}$. In Panel A, each of the quintile portfolios sorted on $\beta_{i,VOV}$ is then averaged over the five portfolios intersected with the quintile portfolios sorted on $\beta_{i,VIX}$, resulting in $\beta_{i,VOV}$ quintile portfolios controlling for $\beta_{i,VIX}$. In Panel B, similar approach yields $\beta_{i,VIX}$ quintile portfolios controlling for $\beta_{i,VOV}$.

Consistent with the model, controlling for volatility risk loadings ($\beta_{i,VIX}$), we find that stocks with positive return sensitivities to market volatility-of-volatility (quintile 5) have lower stock returns than stocks with negative return sensitivities (quintile 1) by 0.97 percent per month with t- statistic of -2.42. Controlling for Fama-French (1993) and Carhart (1997) four factor model, the "5-1" long-short portfolio still gives a significant alpha of -1.05 percent per month with a *t*-statistic of -2.93. On the other hand, controlling for market volatility-of-volatility risk loadings ($\beta_{i,VoV}$), we find that the return difference between stocks with high volatility risk and stocks with low volatility risk is also significantly negative, at -0.68 percent per month with *t*-value of -2.43. Thus, these findings suggest that the market volatility-of-volatility risk is a pricing factor independent with the aggregate volatility factor.

5.2. Controlling for firm characteristics, high moment risks, and jump risk

To check whether our results are robust to firm characteristics and other competing systematic risks, Table 4 shows performance of portfolios sorted on $\beta_{i,VoV}$, controlling for market capitalization (*Size*), book-to-market ratio (*B/M*), past 11-month returns (*RET_2_12*), past one-month return (*RET_1*) and Amihud's illiquidity (*ILLIQ*), respectively. Regarding to other competing systematic risks, we consider the Chang, Christoffersen, and Jacobs's (2013) market skewness risk ($\beta_{i,SKEW}$) and market kurtosis risk ($\beta_{i,KURT}$), Coval and Shumway's (2001) volatility risk with respect to the zero beta straddle factor ($\beta_{i,VOL}$), Cremers, Halling, and Weinbaum's (2015) volatility risk ($\beta_{i,VOL}$) and jump risk ($\beta_{i,JUMP}$), and Kelly and Jiang's (2014) return tail risk ($\beta_{i,TAIL}$). We sort stocks into quintile portfolios based on $\beta_{i,VOV}$, from the lowest (quintile 1) to the highest (quintile 5), and independently sort stocks into quintile portfolios based on each control variable.

Panel A presents the results for $\beta_{i,VOV}$ quintile portfolios controlling for the quintile portfolios sorted on each control variable. The Fama-French (1993) and Carhart (1997) four factor alpha of the "5-1" long–short portfolio remains significant controlling for firm characteristics, i.e. at -0.52 percent with a *t*-statistic of -2.29 controlling for *Size*, at -0.89 percent with a *t*-statistic of -2.66 controlling for *B/M*, at -0.74 percent with a *t*-statistic of -2.36 controlling for *RET_2_12*, at -0.76 percent with a *t*-statistic of -2.25 controlling for *RET_1*, and at -0.57 percent with a *t*-statistic of -2.32 controlling for *ILLIQ*. Hence, the low returns to high $\beta_{i,VOV}$ stocks are not completely driven by the existing well-known firm characteristics.

Panel A further shows that the four factor alpha of the "5-1" long–short portfolio sorted by $\beta_{i,VOV}$ remains significant controlling for other competing systematic risks,, i.e. at -0.82 percent with a *t*-statistic of -2.37 controlling for $\beta_{i,SKEW}$, at -0.74 percent with a *t*-statistic of -1.96 controlling for $\beta_{i,KURT}$, at -0.75 percent with a *t*-statistic of -1.99 controlling for $\beta_{i,STR}$, at -0.93 percent with a *t*-statistic of -2.54 controlling for $\beta_{i,VOL}$, at -0.89 percent with a *t*-statistic of -2.35 controlling for $\beta_{i,JUMP}$, and at -0.74 percent with a *t*-statistic of -2.14 controlling for $\beta_{i,TAIL}$. Hence, the low returns to high $\beta_{i,VOV}$ stocks are less likely to be explained by the existing high-moment risks, jump risk, or tail risk in the literature.

Table 4 also presents the results for the quintile portfolios sorted on each of the competing systematic risks without and with controlling for $\beta_{i,VOV}$ in Panel B and in Panel C, respectively. In our sample period, as shown in Panel B, only $\beta_{i,SKEW}$ and $\beta_{i,JUMP}$ are significantly priced, with their four factor "5-1" alphas at -0.65 (t = -2.11) and at -0.84 (t = -2.37), respectively. Controlling for $\beta_{i,VOV}$, as shown in Panel C, the four factor "5-1" alpha for $\beta_{i,JUMP}$ remains significantly negative at -0.66 (t = -2.09), but the four factor "5-1" alpha for $\beta_{i,SKEW}$ becomes significant at -0.33 (t = -1.16). Thus, the market skewness risk is less likely to explain the market volatility-of-volatility risk, whereas substantial part of the skewness return differential can be explained by the market volatility-of-volatility risk.

In summary, the return differential associated with $\beta_{i,VOV}$ cannot be fully explained by either firm characteristics or other competing systematic risks. On the other hand, while $\beta_{i,SKEW}$ and $\beta_{i,JUMP}$ are the only two other competing systematic risks that carry significant unconditional return differentials in our sample period, $\beta_{i,VOV}$ can largely subsume that pricing effect of $\beta_{i,SKEW}$.

5.3. Price of market volatility-of-volatility risk

We apply the two-pass regressions of Fama-MacBeth (1973) to estimate the price of market volatility-of-volatility risk. Our main set of test assets are the 25 portfolios formed on intersection of $\beta_{i,VIX}$ quintile portfolios and $\beta_{i,VOV}$ quintile portfolios. For each portfolio, we estimate the time-series regression of equation (14) using the post-formation daily value-weighted portfolio returns to obtain the post-formation factor loadings. We then conduct the cross-sectional regression:

$$\mathbb{E}[r_p] - r_f = \lambda_{MKT}\beta_{p,MKT} + \lambda_{VIX}\beta_{p,VIX} + \lambda_{VOV}\beta_{p,VOV}. \quad (15)$$

The dependent variable is the monthly value-weighted portfolio return and the independent variables are the post-ranking return betas estimated from equation (14) using full-sample daily portfolio returns. Robust Newey and West (1987) t-statistics with six lags that account for autocorrelations are used. The cross-sectional regression gives the estimates of risk prices, i.e. λ_{MKT}, λ_{VIX}, and λ_{VOV}.

Table 5 presents the Fama–MacBeth (1973) factor premiums for the volatility-of-volatility factor (*ΔVOV*), with controlling for the market factor (*MKT*), the volatility factor (*ΔVIX*), Fama-French and Carhart factors (*SMB*, *HML*, and *UMD*), Chang, Christoffersen, and Jacobs's (2013) market skewness factor (*ΔSKEW*) and market kurtosis factor (*ΔKURT*), and Cremers, Halling, and Weinbaum's (2015) jump factor (*JUMP*). In column [2], controlling for the market risk and the market volatility risk we find that λ_{VOV} is negative (-3.69) with a significant *t*-statistic of -2.36, which accounts for -3.66×0.21= -0.70 percent per month of the "5-1" return in Table 3. Further controlling for Fama-French and Carhart factors, as reported in column [3], λ_{VoV} is still significantly negative (-3.87) with a *t*-statistic of -2.69. Controlling for all of the other factors, as shown in column [4], λ_{VOV} remains significantly negative (-3.60) with a *t*-statistic of -2.62, which accounts for -3.60×0.19= -0.68 percent, but none of other factors are significantly priced in this test portfolios. In contrast, λ_{VIX} is only significant in column [1] with a t-statistic of -3.26 when other risk factors are not included. Thus, our empirical findings suggest that market volatility-of-volatility indeed is an independently priced risk factor relative to aggregate volatility factor.

To pairwise compare the pricing effect of the market volatility-of-volatility with that of any other competing risk factor, we consider additional test portfolios that potentially have substantial disperse exposures in the both factors. Since $\beta_{i,SKEW}$ and $\beta_{i,JUMP}$ are the only two other competing systematic risks that carry significant unconditional return differentials in our sample period as shown in Table 4, we further compare the pricing effect of the market volatility-of-volatility with the market skewness factor and with the jump risk factor. In column [5], when we use the 25 portfolios independently sorted on $\beta_{i,VOV}$ and $\beta_{i,SKEW}$, we find that λ_{VOV} is significantly negative (-1.88) with a *t*-statistic of -1.75, but λ_{SKEW} is insignificant with a *t*-statistic of 1.09. Further, in column [6], when we use the 25 portfolios independently sorted on $\beta_{i,VOV}$ and $\beta_{i,JUMP}$, we similarly find that λ_{VOV} remains significantly negative (-4.55) with a *t*-statistic of -3.22, but λ_{JUMP} is insignificant with a *t*-statistic of 0.16. Therefore, the market volatility-of-volatility appears to be an important pricing factor that can largely subsume the pricing power of the market skewness factor and the jump risk factor.

5.4. Firm-level Fama-MacBeth regressions

In this section, we examine whether the pricing of market volatility-of-volatility risk is robust to the firm-level analysis. We employ individual stocks as the set of test assets to avoid potentially spurious results that could arise when the test portfolios are constructed toward a specific model (Lewellen, Nagel, and Shanken, 2010). Furthermore, a stock-level analysis could increase the power of the test by controlling for several individual characteristics at the same time. We test our market-based three factor model at firm-level with the following cross-sectional regression:

$$\begin{aligned} r_{i,t+1} - r_{f,t+1} &= c_0 + \lambda_{MKT}\beta_{i,MKT,t} + \lambda_{VIX}\beta_{i,VIX,t} + \lambda_{VOV}\beta_{i,VOV,t} \\ &+ c_{FIRM}\, FirmCharac_{i,t} + c_{VOL}\, VolatilityCharac_{i,t} + \varepsilon_{i,t+1}, \end{aligned} \tag{16}$$

where the dependent variable is the monthly individual stock returns; $\beta_{i,MKT,t}$, $\beta_{i,VIX,t}$, and $\beta_{i,VoV,t}$ are post-ranking betas estimated from the 25 portfolios formed on intersection of $\beta_{i,VIX}$ quintile portfolios and $\beta_{i,VoV}$ quintile portfolios. Following the methodology of Fama and French (1992), we assign each of the 25 portfolio-level post-ranking beta estimates to the individual stocks within the portfolio at that time. Thus, individual stock betas vary over time because the portfolio compositions change each month. Robust Newey and West (1987) *t*-statistics with six lags that account for autocorrelations are used.

$FirmCharac_{i,t}$ denotes a set of firm characteristic variables that consist of *Size*, *B/M*, *RET_1*, *RET_2_12*, and *ILLIQ*. We also check whether our results are robust to existing well-known volatility spreads that affect cross-sectional stock returns. We construct the implied-realized volatility spread (*IVOL-TVOL*), which is, as described in Bali and Hovakimian (2009), defined as the average of implied volatilities by at-the-money call and put minus the total volatility calculated using daily returns in the previous month; the call-put implied volatility spread (*CIVOL-PIVOL*), which is, as described in Bali and Hovakimian (2009) and Yan (2011), defined as the at-the-money call implied volatility minus the at-the-money put implied volatility. Since we extract the volatility data from OptionMetrics Volatility Surface file as Yan (2011) do, we choose the 30-day maturity put and call options with deltas equal to -0.5 and 0.5, respectively. Thus, $VolatilityCharac_{i,t}$ denotes a set of volatility characteristic variables that include *IVOL-TVOL*, and *CIVOL-PIVOL*.

Table 6 reports the results from the firm-level Fama-MacBeth regressions. In column [2], we find that λ_{VoV} is negative (-3.06) with a significant *t*-statistic of -4.09. Controlling for the market volatility risk, as reported in column [3], λ_{VoV} is still significantly negative (-3.06) with a *t*-statistic of -4.12, which accounts for -3.06×0.19= -0.58 percent per month of the “5-1” return in Table 3. Controlling for all of the other variables, as shown in column [5], λ_{VoV} remains significantly negative (-2.85) with a *t*-statistic of -2.55, which accounts for -2.85×0.19= -0.54 percent. Thus, the firm-level evidence confirms our results that the market volatility-of-volatility is a priced risk factor in the cross-sectional stock returns.

5.5. Market conditions and volatility-of-volatility risk premium

Figure 2 plots the volatility-of-volatility risk premium and the volatility risk premium over the time period January 1996 through December 2010, where the volatility-of-volatility risk premium (*rVOV*) is measured by the "5-1" portfolio returns of the $\beta_{i,VOV}$ portfolios, with controlling for the $\beta_{i,VIX}$ portfolios; the volatility risk premium (*rVIX*) is measured by the "5-1" portfolio returns of the $\beta_{i,VIX}$ portfolios, with controlling for the $\beta_{i,VOV}$ portfolios. As can be seen in Figure 2, the negative returns for *rVOV* mainly come from the market crash periods in 2000 and in 2008 and the cumulative returns for *rVOV* are relatively flat during the other time. In contrast, the negative returns for *rVIX* are relatively persistent across time except for the market crash periods in 2000 and in 2008 where *rVIX* carries positive returns.

We further examine how the volatility-of-volatility risk premium and the volatility risk premium are related to market conditions. The monthly S&P 500 logarithmic returns (*rSPX*) are used for the market dummies and the monthly changes in *VIX* (*ΔVIX*) are used for the volatility dummies. The dummy of market crash, *rSPX_DXTR*, takes the value of one when *rSPX* is below its time-series 5^{th} percentile value; the dummy of down market, *rSPX_DOWN*, takes the value of one when *rSPX* is below its time-series mean but is still above its time-series 5^{th} percentile value; the dummy of the upper extreme volatility shock, *ΔVIX_UXTR*, takes the value of one when *ΔVIX* is above its time-series 95^{th} percentile value; the dummy of the upper volatility shock, *ΔVIX_UP*, takes the value of one when *ΔVIX* is above its time-series mean but is still below its time-series 95^{th} percentile value; the dummy of the extreme volatility shock, *ΔVIX_XTR*, takes the value of one when *ΔVIX* is above its time-series 95^{th} percentile value or below its time-series 5^{th} percentile value.

Table 7 shows the regressions of *rVIX* and *rVOV* on these dummy variables for market conditions as well as shocks for the state variables. In column [1] of Panel A, we find that *rVIX* tends to be more negative when the market return is negative but not extreme as *rSPX_DOWN* has a significantly negative coefficient of -0.016 (*t* = -2.12) while *rSPX_DXTR* has an insignificant coefficient of -0.011 (*t* = -0.56). As reported in column [2] and column [3], we also find that the negative returns of *rVIX* are related to the non-extreme increase in volatility as *ΔVIX_UP* has a significantly negative coefficient of -0.013 (*t* = -1.74) in column [2] and -0.013 (*t* = -1.71) in column [3] while *ΔVIX_UXTR* or *ΔVIX_XTR* has an insignificant coefficient. On the other hand, as reported in column [4], we find that *rVOV* tends to be more negative when the market return is extremely negative as *rSPX_DXTR* has a significantly negative coefficient of -0.048 (*t* = -2.33) while *rSPX_DOWN* has an insignificant coefficient of 0.005 (*t* = 0.82). Moreover, as reported in column [5] and column [6], we find that the negative returns of *rVOV* are related to extreme volatility shocks as *ΔVIX_XTR* has a significantly negative coefficient of -0.032 (*t* = -1.97) while *ΔVIX_UP* has an insignificant coefficient.

In summary, we find that the negative returns for the volatility-of-volatility risk premium are associated with the market crash while the negative returns for the volatility risk premium are associated with the non-extreme negative market returns. Thus, the pricing effect of volatility-of-volatility risk is different from that of the volatility risk and the relative pricing effects of these two depend on different states of market conditions.

5.6. Market crash and the price of volatility risk : reconciliation with AHXZ (2006)

Our results are consistent with AHXZ (2006) that the portfolios sorted by $\beta_{i,VIX}$ have significant return differentials in supportive of the negative volatility risk premium. However, we also find evidence that market price of volatility risk, $\lambda_{i,VIX}$, is insignificant once the market volatility-of-volatility is included into the model. Since we find evidence that the market volatility-of-volatility risk premium is more pronounced during the period of market crash and in which time the volatility risk premium tends to be attenuated, we examine whether market price of volatility risk, $\lambda_{i,VIX}$, is priced without presence of market crash.

Table 8 presents the Fama–MacBeth (1973) factor premiums using the daily full-sample post-formation value-weighted returns. The test portfolios are the 25 portfolios independently sorted on $\beta_{i,VOV}$ and $\beta_{i,VIX}$ that are re-estimated for each day using daily returns over the past 22 days. Portfolios are rebalanced daily and are value weighted. Similar to the previous findings when the factor premium is estimated from the full sample period, $\lambda_{i,VIX}$ is insignificant as reported in column [1] and in column [2]. However, when we exclude the sample period of market crash, where the daily the daily S&P 500 return is below its time-series 5^{th} percentile value, we find that $\lambda_{i,VIX}$ becomes significantly negative, at -8.55 with a *t*-statistic of -3.77 even with control for the market volatility-of-volatility factor. Further controlling for Fama-French and Carhart (1997) factors, $\lambda_{i,VIX}$ remains significantly negative, at -11.99 with a *t*-statistic of -3.03. Thus, we find evidence that market price of volatility risk, $\lambda_{i,VIX}$, is priced when we exclude the sample period of market crash.

In summary, these findings suggest that while the volatility-of-volatility premium is more pronounced during the period of market crash, the volatility risk premium remains important during the non-crash period. In other words, consistent with AHXZ (2006) as well as our three factor model, the volatility-of-volatility risk premium should be the complement rather than the substitute for the volatility risk premium.

6. Event-time evolution of portfolios sorted on market volatility-of-volatility risk

If market volatility-of-volatility risk is a priced state variable, then an anticipated increase in market volatility-of-volatility raises the required rate of return, implying an immediate stock price decline and higher future returns. In particular, for any state variable that negatively affects the investment opportunity set, Merton's ICAPM implies stocks with more positive return sensitivities to the state variable should earn lower future stock returns since the these stocks tend to have high returns during the bad times and therefore provide hedge for the low market returns. Thus, we examine the event time portfolio returns during the period of the market crash when the market return is below its 5^{th} percentile value.

To identify the timely volatility-of-volatility shocks, at the end of each day, we sort stocks into quintile portfolios based on $\beta_{i,VIX}$, from the lowest (quintile 1) to the highest (quintile 5), and independently sort stocks into quintile portfolios based on $\beta_{i,VOV}$. For each day, $\beta_{i,VIX}$ and $\beta_{i,VOV}$ are re-estimated using daily stock returns over the past 22 days and portfolios are rebalanced daily. Each of the quintile portfolios sorted on $\beta_{i,VOV}$ is then averaged over the five portfolios intersected with the quintile portfolios sorted on $\beta_{i,VIX}$; similarly, each of the quintile portfolios sorted on $\beta_{i,VIX}$ is then averaged over the five portfolios intersected with the quintile portfolios sorted on $\beta_{i,VOV}$.

6.1. Hedging effect and volatility-of-volatility risk premium

Consistent with the theory of hedging effect for the market crash, as reported in Table 9, we find that the "5-1" returns of $\beta_{i,VOV}$ quintile portfolios are significantly positive in event days from -5 to 0. Thus, these findings suggest that the theory of hedging effect for the market crash could explain the negative volatility-of-volatility risk premium. On the other hand, we find that the "5-1" returns of $\beta_{i,VIX}$ quintile portfolios are significantly negative in event days from -5 to 0. In other words, stocks with more positive return sensitivities to market volatility are more risky during the market crash, and therefore they should earn higher expected returns compensated for that risk. Thus, the theory of hedging effect for the market crash implies that the volatility risk premium should be positive in the presence of market crash, which could substantially offset the negative volatility risk premium well documented in the literature.

In summary, we find evidence in supportive of the theory of hedging effect for the market crash for the negative volatility-of-volatility risk premium. In contrast, we find that the volatility risk premium tends to be positive during the market crash, suggesting that the attenuated volatility risk premium in our more recent sample period may be attributed to the increasingly frequent market turmoil.

6.2. Flight-to-quality effect and volatility-of-volatility risk premium

We examine the evolution of trading activity in event time for portfolios sorted on $\beta_{i,VIX}$ and $\beta_{i,VOV}$. The theory of flight-to-quality for the market crash implies that stocks with more positive return sensitivities to the state variable should have more net buying trading volume during the market crash since the these stocks can provide hedge for the low market returns and are therefore flavored by investors.

Table 10 shows two measures of the event-time daily equal-weighted portfolio abnormal order imbalance ranging from -5 to 5 in days. For each stock in each day, we define the order imbalance in dollar volume (in million), *OIBDLR*, as the difference between buy dollar volume and sell dollar volume in the day and then define the order imbalance in proportion*, %OIBDLR*, as *OIBDLR* divided by the sum of buy dollar volume and sell dollar volume. The abnormal order imbalance in proportion are defined by subtracting their averaged value over the past 5 days from order imbalance and order imbalance in proportion, respectively.

Consistent with the theory of flight-to-quality for the market crash, as reported in Table 10, we find that the "5-1" returns of $\beta_{i,VOV}$ quintile portfolios are significantly positive abnormal order imbalance in proportion in event days from -5 to 0. In contrast, we find that the "5-1" returns of $\beta_{i,VIX}$ quintile portfolios are significantly negative abnormal order imbalance in proportion in event days 0. Thus, these findings suggest that the flight-to-quality for the market crash is more likely to be associated with the volatility-of-volatility risk premium rather than the volatility risk premium.

7. Conclusions

Market volatility-of-volatility appears to be a state variable that is important for asset pricing. We develop a market-based three-factor model, in which market risk, market volatility risk, and market volatility-of-volatility risk determine the cross-sectional asset prices. We find that market volatility-of-volatility risk is priced in the cross-sectional stock returns. Stocks with negative larger return exposure to market volatility-of-volatility have substantially higher future stock returns, even after we account for exposures to the Fama and French four factors, market skewness factor, firm characteristics and volatility characteristics. We also find that market volatility-of-volatility risk is priced in the cross-sectional variance risk premium.

Our measure of market volatility-of-volatility generates leverage effect and feedback effect. Stocks with negative larger return exposure to market volatility-of-volatility have substantially lower contemporaneous stock returns, which suggests that market volatility-of-volatility is priced such that an anticipated increase in market volatility-of-volatility risk raises the required rate of return, leading to an immediate stock price decline and higher future returns. Our evidence on return predictability for the aggregate market portfolio supports feedback effect implied by our model. The predictability evidence afforded by the market volatility-of-volatility also suggests that economic volatility-of-volatility is an important state variable.

Reference

Adrian, T., and Rosenberg, J., 2008, Stock returns and volatility: pricing the short-run and long-run components of market risk, *Journal of Finance* 63, 2997–3030.

Amihud, Y., 2002, Illiquidity and stock returns: Cross-section and time-series effects, *Journal of Financial Markets* 5, 31—56.

Andersen, T. G., T. Bollerslev, and F. X. Diebold. 2007. Roughing It Up: Including Jump Components in the Measurement, Modeling, and Forecasting of Return Volatility. *Review of Economics and Statistics* 89:701–20.

Ang, A. R. J. Hodrick, Y. Xing, and X. Zhang, 2006. The Cross-Section of Volatility and Expected Returns. *Journal of Finance* 61: 25—299.

Bakshi, G., and D. Madan. 2006. A Theory of Volatility Spread. *Management Science* 52: 1945—56.

Bakshi, G., and N. Kapadia, 2003. Delta-hedged gains and the negative market volatility risk premium. *Review of Financial Studies* 2: 527—566.

Bakshi, G., N. Kapadia, and D. Madan, 2003. Stock Return Characteristics, Skew Laws, and the Differential Pricing of Individual Equity Options. *Review of Financial Studies* 16: 101—143.

Bali, T. and A. Hovakimian, 2009, Volatility Spreads and Expected Stock Returns, *Management Science*, 55: 1797—1812.

Baltussen, G., S. Van Bekkum, and B. Van Der Grient, 2013. Unknown Unknowns: Vol-of-Vol and the Cross-Section of Stock Returns. AFA 2013 San Diego Meetings Paper.

Bansal, R., and A. Yaron. 2004. Risks for the Long Run: A Potential Resolution of Asset Pricing Puzzles. *Journal of Finance* 59: 1481—1509.

Bates, D., 2003. Empirical Option Pricing: A Retrospection. *Journal of Econometrics* 116: 387—404.

Barndorff-Nielsen, O., P. R. Hansen, A, Lunde, and N. Shephard. 2009. Realized kernels in practice: trades and quotes. *Econometrics Journal* 12: C1—C32.

Barndorff-Nielsen, O., and N. Shephard. 2004. Power and Bipower Variation with Stochastic Volatility and Jumps. *Journal of Financial Econometrics* 2: 1—37.

Bekaert, G., and G. Wu. 2000. Asymmetric Volatility and Risk in Equity Markets. *Review of Financial Studies* 13: 1–42.

Black, F., 1976. Studies of Stock Price Volatility Changes. *Proceedings of the 1976 Meetings of the American Statistical Association, Business and Economical Statistics Section*, 177–181

Bollerslev, T., N. Sizova, and G. Tauchen, 2012. Volatility in Equilibrium: Asymmetries and Dynamic Dependencies. *Review of Finance* 16: 31—80.

Bollerslev, T., G. Tauchen, and H. Zhou, 2009. Expected Stock Returns and Variance Risk Premia. *Review of Financial Studies* 22: 4463—4492.

Bollerslev T., M. Gibson, and H. Zhou, 2011. Dynamic Estimation of Volatility Risk Premia and Investor Risk Aversion from Option-Implied and Realized Volatilities. *Journal of Econometrics* 160: 235—245.

Bollerslev, T., J. Marrone, L. Xu, and H. Zhou, 2013. Stock Return Predictability and Variance Risk Premia: Statistical Inference and International Evidence. *Journal of Financial and Quantitative Analysis* (forthcoming).

Bollerslev, T., and V. Todorov, 2011. Tails, Fears, and Risk Premia. *Journal of Finance* 66: 2165—2211.

Breeden, D. T., 1979. An Intertemporal Asset Pricing Model with Stochastic Consumption and Investment Opportunities. *Journal of Financial Economics* 7: 265—296.

Britten-Jones, M., and A. Neuberger. 2000. Option Prices, Implied Price Processes, and Stochastic Volatility. *Journal of Finance* 55:839—866.

Campbell, J. Y., and L. Hentschel, 1992, No News is Good News: An Asymmetric Model of Changing Volatility in Stock Returns, *Journal of Financial Economics*, 31, 281–318.

Campbell, J. Y., and R. J. Shiller, 1988. The Dividend-Price Ratio and Expectations of Future Dividends and Discount Factors. *Review of Financial Studies* 1: 195—228.

Carr, P., and D. Madan. 1998. Towards a Theory of Volatility Trading. In R. Jarrow (ed.), Volatility: New Estimation Techniques for Pricing Derivatives, chap. 29, pp. 417—427. London: Risk Books.

Carr, P., and L. Wu. 2009. Variance Risk Premia. *Review of Financial Studies* 22: 1311—1341.

Chang, B. Y., P. Christoffersen, and K. Jacobs, 2013. Market Skewness Risk and the Cross Section of Stock Returns. *Journal of Financial Economics* 107: 46—68.

Christie, A. A., 1982. The Stochastic Behavior of Common Stock Variances—Value, Leverage and Interest Rate Effects. *Journal of Financial Economics* 10: 407–432.

Coval, J., and T. Shumway, 2001, Expected option returns, *Journal of Finance* 56, 983–1009.

Cremers, M., M. Halling, D. Weibaum, 2015, Aggregate Jump and Volatility Risk in the Cross-Section of Stock Returns, Journal of Finance 70: 577–614.

Drechsler, I., and A. Yaron, 2011. What's Vol Got to Do with It. *Review of Financial Studies* 24: 1—45.

Drechsler, I., 2013. Uncertainty, Time-Varying Fear, and Asset Prices. *Journal of Finance*, *forthcoming*.

Duan, J.C., and J.Wei, 2009. Systematic Risk and the Price Structure of Individual Equity Options. *Review of Financial Studies* 22: 1981—2006.

Epstein, L.G., and S. E. Zin, 1989. Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework. *Econometrica* 57: 937—969.

Eraker, B. 2008. Affine General Equilibrium Models. *Management Science* 54:2068–80.

Fama, E.F., French, K., 1992. The cross-section of expected stock returns. *Journal of Finance* 47, 427—465.

Fama, E. F., and K. R. French, 1993, Common risk factors in the returns on stocks and bonds, *Journal of Financial Economics* 33, 3—56.

Fama, E. F., and J. D. MacBeth, 1973, Risk return, and equilibrium: Empirical tests, *Journal of Political Economy* 71, 607–636.

French, K. R., G. W. Schwert, and R. F. Stambaugh. 1987. Expected Stock Returns and Volatility. *Journal of Financial Economics* 19: 3–30.

Han, Bing, and Yi Zhou, 2012, Variance risk premium and cross-section of stock returns, Working paper, University of Texas at Austin.

Huang, X., and G. Tauchen. 2005. The Relative Contribution of Jumps to Total Price Variance. *Journal of Financial Econometrics* 3:456–99.

Jiang, G., and Y. Tian. 2005. Model-Free Implied Volatility and Its Information Content. *Review of Financial Studies* 18: 1305—1342.

Jones, C. S. 2003. The Dynamics of Stochastic Volatility: Evidence from Underlying and Options Markets. *Journal of Econometrics* 116: 181—224.

Kelly, B., H. Jiang. 2014. Tail risk and asset prices. *Review of Financial Studies* 27, 2841–2871.

Linter, J. 1965. The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets. Review of Economics and Statistics 47: 13—37.

Lewellen, J., S. Nagel, and J. Shanken. 2010. A skeptical appraisal of asset pricing tests. *Journal of Financial Economics* 96: 175—194.

Lucas, Robert, 1978, Assets prices in an exchange economy, *Econometrica* 46, 1429–1445.

Maio, P., and P. Santa-Clara, 2012. Multifactor Models and their Consistency with the ICAPM. *Journal of Financial Economics* 106: 586—613.

Merton, R. C., 1973. An Intertemporal Capital Asset Pricing Model. *Econometrica* 41: 867—887.

Pan, J., 2002. The Jump-Risk Premia Implicit in Options: Evidence from an Integrated Time-Series Study. *Journal of Financial Economics* 63: 3—50.

Rubinstein, M., 1976. The Valuation of Uncertain Income Streams and the Pricing of Options. *Bell Journal of Economics* 7: 407—425.

Sadka, R., 2006. Momentum and post-earnings announcement drift anomalies: the role of liquidity risk. *Journal of Financial Economics* 80: 309—349.

Sharpe, W. F., 1964. Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk. *Journal of Finance* 19: 425—442.

Todorov, V., 2010. Variance Risk Premium Dynamics: The Role of Jumps. *Review of Financial Studies* 23: 345—383.

Wu, G., 2001. The Determinants of Asymmetric Volatility. *Review of Financial Studies* 14, 837–859.

Yan, S., 2011. Jump risk, stock returns, and slope of implied volatility smile. *Journal of Financial Economics* 99: 216—233.

Table 1 Properties of the daily factors.

We report summary statistics and Spearman correlations for the daily factors, including the four factors (*MKT*, *SMB*, *HML*, and *UMD*) of Fama and French (1993) and Carhart (1997), the market variance risk premium (*VRP*), the *VIX* index, our measure of variance of market variance (*VOV*), the realized covariance (multiplied by 22), between the intraday 5-min logarithmic return and the 5-min implied variance for S&P 500 index (*CVRV*), the market skewness factor (*SKEW*) and market kurtosis factor (*KURT*) of Chang, Christoffersen, and Jacobs (2013), the zero beta straddle factor of Coval and Shumway (2001), and the options return volatility factor (*VOL*) and the options return jump factor (*JUMP*) of Cremers, Halling, and Weinbaum (2015). *ΔVIX* is the first difference of *VIX*; *ΔCVRV, ΔVOV*, *ΔSKEW*, and *ΔKURT* are the residuals from fitting an ARMA(1,1) regression using *VOV*, *SKEW*, and *KURT*, respectively. The sample period is from January 1996 to December 2010.

	Panel A: Summary statistics												
	MKT (%)	*SMB (%)*	*HML (%)*	*UMD (%)*	*VRP (%)*	*VIX (%)*	*CVRV (%)*	*VOV (%)*	*SKEW*	*KURT*	*STR (%)*	*VOL (%)*	*JUMP (%)*
Mean	0.023	0.010	0.016	0.024	0.173	23.098	-0.086	0.054	-1.663	9.313	-0.333	-0.038	-0.210
Median	0.070	0.030	0.020	0.070	0.151	22.150	-0.021	0.003	-1.637	8.672	-1.305	-0.074	-0.752
Std.Dev.	1.300	0.629	0.682	1.035	0.212	9.509	0.441	0.563	0.485	3.466	6.293	1.601	4.575

	Panel B: Spearman correlation												
	MKT	*SMB*	*HML*	*UMD*	*VRP*	*ΔVIX*	*ΔCVRV*	*ΔVOV*	*ΔSKEW*	*ΔKURT*	*STR*	*VOL*	*JUMP*
MKT	1.000												
SMB	0.038	1.000											
HML	-0.278	-0.082	1.000										
UMD	-0.048	0.052	-0.077	1.000									
VRP	-0.223	-0.050	0.007	0.062	1.000								
ΔVIX	-0.779	0.031	0.209	0.021	0.181	1.000							
ΔCVRV	0.244	0.036	-0.083	0.052	-0.203	-0.223	1.000						
ΔVOV	-0.044	-0.004	-0.038	0.036	0.145	0.050	-0.199	1.000					
ΔSKEW	-0.236	-0.021	0.026	0.034	0.075	0.248	-0.033	-0.017	1.000				
ΔKURT	0.311	0.014	-0.057	-0.018	-0.106	-0.307	0.077	-0.011	-0.863	1.000			
STR	-0.245	-0.057	0.038	-0.023	0.094	0.526	-0.153	0.051	0.123	-0.140	1.000		
VOL	-0.259	-0.078	-0.021	0.007	0.118	0.343	-0.107	0.039	0.086	-0.100	0.201	1.000	
JUMP	-0.170	-0.034	0.035	-0.020	0.069	0.410	-0.121	0.046	0.097	-0.111	0.924	-0.110	1.000

Table 2 *CVRV*, *VOV*, and future stock returns

This table reports the estimates of the one-period return predictability regression using daily S&P 500 logarithmic returns (*rSPX*) on the lagged realized covariance (multiplied by 22), between the intraday 5-min logarithmic return and the 5-min implied variance for S&P 500 index (*CVRV*), market volatility (*VIX*), variance of market variance (*VOV*), variance risk premium (*VRP*), market skewness (*SKEW*), market kurtosis (*KURT*), and jump risk (*JUMP*). The dependent variable in Panel A is the daily *rSPX* multiplied by 22. In Panel B, the dependent variable is the monthly *rSPX* and the independent variables are sampled at the end of the month. Robust Newey and West (1987) *t*-statistics are reported in parentheses. The sample period is from January 1996 to December 2010.

	Panel A: Dependent variable= daily rSPX (t)								
	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
Intercept	1.092	1.139	5.806	5.883	-0.766	-1.110	2.522	2.630	0.147
	(0.47)	(0.48)	(1.75)	(1.78)	(-0.42)	(-0.56)	(0.87)	(0.91)	(0.09)
VIX (*t*-1)	-0.052	-0.182	-0.216	-0.220	0.037	-0.058	-0.078	-0.083	-0.009
	(-0.44)	(-1.35)	(-1.53)	(-1.56)	(0.41)	(-0.54)	(-0.69)	(-0.74)	(-0.10)
CVRV (*t*-1)	**-5.839**	**-6.714**	**-7.010**	**-6.996**					-2.305
	(-2.01)	**(-2.90)**	**(-3.14)**	**(-3.13)**					(-0.86)
VOV (*t*-1)					**5.319**	**5.057**	**5.098**	**5.152**	**4.460**
					(2.29)	**(2.26)**	**(2.30)**	**(2.33)**	**(1.73)**
VRP (*t*-1)		**16.685**	**16.696**	**15.541**		**14.756**	**14.696**	**13.423**	
		(3.97)	**(4.05)**	**(3.67)**		**(3.97)**	**(3.98)**	**(3.56)**	
SKEW (*t*-1)			2.270	2.232			2.117	2.073	
			(1.52)	(1.49)			(1.39)	(1.36)	
KURT (*t*-1)			-0.012	0.004			0.042	0.059	
			(-0.06)	(0.02)			(0.19)	(0.26)	
JUMP (*t*-1)			0.107	0.067			0.132	0.088	
			(0.93)	(0.60)			(1.11)	(0.77)	
rSPX (*t*-1)				**-0.036**				**-0.039**	
				(-1.83)				**(-2.08)**	
Adj. R^2	0.006	0.020	0.021	0.022	0.011	0.021	0.022	0.023	0.011

	Panel B: Dependent variable= monthly rSPX (t)									
	[1]		[2]		[3]		[4]		[5]	
Intercept	0.961	(0.86)	0.511	(0.73)	1.647	(1.16)	1.662	(1.20)	1.243	(0.89)
VIX (t-1)	-0.028	(-0.49)	-0.042	(-1.35)	-0.045	(-1.39)	-0.046	(-1.41)	-0.020	(-0.55)
VOV (t-1)	**1.768**	**(2.91)**	**1.294**	**(2.44)**	**1.415**	**(2.65)**	**1.383**	**(2.34)**	**1.476**	**(2.30)**
VRP (t-1)			**4.420**	**(5.33)**	**4.398**	**(4.79)**	**4.375**	**(4.61)**	**3.999**	**(3.70)**
SKEW (t-1)					0.851	(0.57)	0.809	(0.55)	0.467	(0.31)
KURT (t-1)					0.037	(0.21)	0.033	(0.19)	-0.046	(-0.24)
JUMP (t-1)							0.007	(0.10)	0.013	(0.19)
rSPX (t-1)									0.118	(1.53)
Adj. R^2	0.006		0.052		0.044		0.038		0.043	

Table 3 Two-way sorted portfolios on $\boldsymbol{\beta_{i,VIX}}$ and $\boldsymbol{\beta_{i,VOV}}$

At the end of each month, we run the following regression for each stock using daily returns:

$$r_{i,t+1} - r_{f,t+1} = \alpha_i + \beta_{i,MKT}MKT_{t+1} + \beta_{i,VIX}\Delta VIX_{t+1} + \beta_{i,VOV}\Delta VOV_{t+1} + \varepsilon_{i,t+1}.$$

We sort stocks into quintile portfolios based on $\beta_{i,VIX}$, from the lowest (quintile 1) to the highest (quintile 5), and independently sort stocks into quintile portfolios based on $\beta_{i,VOV}$. Portfolios are rebalanced monthly and are value weighted. In Panel A, each of the quintile portfolios sorted on $\beta_{i,VOV}$ is then averaged over the five portfolios intersected with the quintile portfolios sorted on $\beta_{i,VIX}$, resulting in $\beta_{i,VOV}$ quintile portfolios controlling for $\beta_{i,VIX}$. In Panel B, similar approach yields $\beta_{i,VIX}$ quintile portfolios controlling for $\beta_{i,VOV}$. The column "5-1" refers to the hedge portfolio that longs portfolio 5 and shorts portfolio 1. Using the post-formation daily portfolio returns, we estimate the same time-series regression as above to obtain the post-formation factor loadings $\beta_{p,VIX}$ and $\beta_{p,VOV}$ for each portfolio. Using the post-formation monthly portfolio returns, we compute the excess returns and the risk-adjusted returns of each portfolio with respect to Fama-French and Carhart four factors (*MKT*, *SMB*, *HML*, and *UMD*). Robust Newey–West (1987) *t*-statistics are in parentheses. The sample period is from January 1996 to December 2010.

	Portfolios ranking						
	1	*2*	*3*	*4*	*5*	*5-1*	*t-stat*
	Panel A: Performance of $\beta_{i,VoV}$ sorted portfolio, controlling for $\beta_{i,VIX}$						
Excess return	0.90	0.65	0.35	0.28	-0.08	**-0.97**	**(-2.42)**
α-CAPM	0.33	0.19	-0.06	-0.17	-0.67	**-1.00**	**(-2.37)**
α-FF3	0.23	0.15	-0.10	-0.20	-0.69	**-0.92**	**(-2.51)**
α-FFC4	0.39	0.18	-0.09	-0.20	-0.66	**-1.05**	**(-2.93)**
$\beta_{p,VOV}$	-0.03	-0.01	0.02	0.02	0.16	**0.19**	**(2.33)**
Pre-formation characteristics							
Size($b)	1.14	2.71	3.15	3.01	1.45	**0.30**	**(5.43)**
B/M	1.12	0.90	0.86	0.83	1.05	-0.08	(-1.20)
RET_2_12	12.76	14.80	14.98	14.35	12.55	-0.21	(-0.14)
RET_1	2.27	1.10	0.62	0.29	1.23	**-1.03**	**(-4.38)**
ILLIQ(10^6)	8.06	3.35	2.96	3.41	7.99	-0.07	(-0.41)
	Panel B: Performance of $\beta_{i,VIX}$ sorted portfolio, controlling for $\beta_{i,VoV}$						
Excess return	0.66	0.58	0.48	0.40	-0.02	**-0.68**	**(-2.43)**
α-CAPM	0.14	0.15	0.06	-0.09	-0.65	**-0.79**	**(-3.15)**
α-FF3	0.15	0.15	0.02	-0.17	-0.75	**-0.89**	**(-3.52)**
α-FFC4	0.25	0.20	0.03	-0.17	-0.70	**-0.95**	**(-3.41)**
$\beta_{p,VIX}$	0.03	-0.01	0.00	0.03	0.13	**0.10**	**(3.37)**
Pre-formation characteristics							
Size($b)	1.40	3.16	3.30	2.52	1.08	**-0.32**	**(-4.22)**
B/M	1.07	0.91	0.82	0.84	1.13	0.06	(1.05)
RET_2_12	11.31	14.95	15.33	14.95	12.90	1.58	(0.75)
RET_1	1.28	0.71	0.76	0.86	1.90	**0.62**	**(2.35)**
ILLIQ(10^6)	8.61	3.05	2.69	2.95	8.48	-0.13	(-0.71)

Table 4 Two-way sorted portfolios on $\beta_{i,VoV}$ and control variables

This table shows performance of portfolios sorted on $\beta_{i,VOV}$, with controlling market capitalization (*Size*), book-to-market ratio (*B/M*), past 11-month returns (*RET_2_12*), past one-month return (*RET_1*), and Amihud's illiquidity (*ILLIQ*), Chang, Christoffersen, and Jacobs's (2013) market skewness risk ($\beta_{i,SKEW}$) and market kurtosis risk ($\beta_{i,KURT}$), Coval and Shumway's (2001) volatility risk with respect to the zero beta straddle factor ($\beta_{i,STR}$), Cremers, Halling, and Weinbaum's (2015) volatility risk ($\beta_{i,VOL}$) and jump risk ($\beta_{i,JUMP}$), and Kelly and Jiang's (2014) return tail risk ($\beta_{i,TAIL}$), respectively. We sort stocks into quintile portfolios based on $\beta_{i,VOV}$, from the lowest (quintile 1) to the highest (quintile 5), and independently sort stocks into quintile portfolios based on each control variable. Portfolios are rebalanced monthly and are value weighted. Panel A presents the results for $\beta_{i,VOV}$ quintile portfolios controlling for the quintile portfolios sorted on each control variable. The results for the quintile portfolios sorted on each of the competing systematic risks without and with controlling for $\beta_{i,VOV}$ quintile portfolios are presented in Panel B and in Panel C, respectively. The column "5-1" refers to the hedge portfolio that longs portfolio 5 and shorts portfolio 1. Using the post-formation monthly portfolio returns, we compute the risk-adjusted returns of each portfolio with respect to Fama-French and Carhart four factors (*MKT*, *SMB*, *HML*, and *UMD*). Robust Newey–West (1987) *t*-statistics are in parentheses. The sample period is from January 1996 to December 2010.

	Portfolios ranking						
Control variables	*1*	*2*	*3*	*4*	*5*	*5-1*	*t-stat*
	Panel A: Performance of $\beta_{i,VoV}$ sorted portfolio, controlling for each control variable						
Size	0.94	0.89	0.79	0.74	0.41	**-0.52**	**(-2.29)**
B/M	1.20	0.68	0.44	0.52	0.31	**-0.89**	**(-2.66)**
RET_2_12	0.73	0.54	0.45	0.31	-0.02	**-0.74**	**(-2.36)**
RET_1	0.77	0.66	0.37	0.46	0.01	**-0.76**	**(-2.25)**
ILLIQ	0.87	0.84	0.76	0.67	0.30	**-0.57**	**(-2.32)**
$\beta_{i,SKEW}$	0.88	0.67	0.38	0.43	0.05	**-0.82**	**(-2.37)**
$\beta_{i,KURT}$	0.77	0.58	0.47	0.35	0.03	**-0.74**	**(-1.96)**
$\beta_{i,STR}$	0.90	0.74	0.38	0.43	0.15	**-0.75**	**(-1.99)**
$\beta_{i,VOL}$	0.91	0.70	0.41	0.35	-0.01	**-0.93**	**(-2.54)**
$\beta_{i,JUMP}$	0.93	0.74	0.37	0.46	0.04	**-0.89**	**(-2.35)**
$\beta_{i,TAIL}$	0.81	0.71	0.47	0.39	0.08	**-0.74**	**(-2.14)**

Table 4 (continued.)

Control variables	*Portfolios ranking* 1	2	3	4	5	*5-1*	*t-stat*
	Panel B: Performance of each control variable sorted portfolio						
$\beta_{i,SKEW}$	0.88	0.43	0.40	0.37	0.23	**-0.65**	**(-2.11)**
$\beta_{i,KURT}$	0.17	0.38	0.48	0.45	0.57	0.40	(1.38)
$\beta_{i,STR}$	0.75	0.71	0.59	0.13	0.21	-0.55	(-1.49)
$\beta_{i,VOL}$	0.33	0.44	0.56	0.44	0.51	0.18	(0.60)
$\beta_{i,JUMP}$	0.87	0.71	0.42	0.33	0.02	**-0.84**	**(-2.37)**
$\beta_{i,TAIL}$	0.42	0.36	0.63	0.50	0.23	-0.19	(-0.61)
	Panel C: Performance of each control variable sorted portfolio, controlling for $\beta_{i,VoV}$						
$\beta_{i,SKEW}$	0.66	0.52	0.35	0.55	0.33	-0.33	(-1.16)
$\beta_{i,KURT}$	0.32	0.35	0.45	0.46	0.63	0.31	(1.39)
$\beta_{i,STR}$	0.79	0.67	0.59	0.21	0.34	-0.44	(-1.31)
$\beta_{i,VOL}$	0.37	0.41	0.56	0.44	0.59	0.22	(0.91)
$\beta_{i,JUMP}$	0.82	0.70	0.42	0.44	0.16	**-0.66**	**(-2.09)**
$\beta_{i,TAIL}$	0.50	0.45	0.61	0.58	0.33	-0.17	(-0.58)

Table 5 The price of market volatility-of-volatility risk

This table presents the Fama–MacBeth (1973) factor premiums for the volatility-of-volatility factor (*ΔVOV*), with controlling for the market factor (*MKT*), the volatility factor (*ΔVIX*), Fama-French and Carhart factors (*SMB*, *HML*, and *UMD*), Chang, Christoffersen, and Jacobs's (2013) market skewness factor (*ΔSKEW*) and market kurtosis factor (*ΔKURT*), and Cremers, Halling, and Weinbaum's (2015) jump factor (*JUMP*). We estimate the first stage return betas using the daily full-sample post-formation value-weighted returns. Then, we regress the cross-sectional monthly portfolio returns on daily return betas from the first stage, using Fama–MacBeth (1973) cross-sectional regression. Three sets of test portfolios are considered. From column [1] to column [4], the test portfolios are the 25 portfolios independently sorted on $\beta_{i,VOV}$ and $\beta_{i,VIX}$. Column [5] uses the 25 portfolios independently sorted on $\beta_{i,VOV}$ and $\beta_{i,SKEW}$. Column [6] uses the 25 portfolios independently sorted on $\beta_{i,VOV}$ and $\beta_{i,JUMP}$. Portfolios are rebalanced monthly and are value weighted. Robust Newey–West (1987) *t*-statistics that account for autocorrelations are in parentheses. The sample period is from January 1996 to December 2010.

	Fama-MacBeth cross-sectional regressions											
Test portfolios	$\beta_{i,VOV} \times \beta_{i,VIX}$ [1]		$\beta_{i,VOV} \times \beta_{i,VIX}$ [2]		$\beta_{i,VOV} \times \beta_{i,VIX}$ [3]		$\beta_{i,VOV} \times \beta_{i,VIX}$ [4]		$\beta_{i,VOV} \times \beta_{i,SKEW}$ [5]		$\beta_{i,VOV} \times \beta_{i,JUMP}$ [6]	
MKT	0.53	(1.32)	0.56	(1.39)	0.54	(1.35)	0.52	(1.30)	0.66	(1.64)	0.47	(1.09)
ΔVIX	**-5.28**	**(-3.26)**	-3.09	(-1.58)	-3.78	(-0.91)	-2.81	(-0.67)	7.76	(1.31)	-2.78	(-0.71)
ΔVOV			**-3.69**	**(-2.36)**	**-3.87**	**(-2.69)**	**-3.60**	**(-2.62)**	**-1.88**	**(-1.75)**	**-4.55**	**(-3.22)**
SMB					-0.95	(-1.28)	-1.10	(-1.54)	-0.51	(-0.60)	0.54	(0.46)
HML					-0.29	(-0.44)	-0.29	(-0.42)	-1.15	(-1.58)	0.34	(0.25)
UMD					-1.78	(-1.61)	-1.85	(-1.64)	2.54	(1.55)	0.84	(0.58)
ΔSKEW							0.01	(0.01)	2.51	(1.09)	-0.69	(-0.50)
ΔKURT							-16.43	(-1.18)	-0.40	(-0.03)	-9.64	(-1.03)
JUMP							16.29	(0.82)	5.00	(0.21)	3.11	(0.16)
Adj. R^2	0.23		0.52		0.56		0.55		0.46		0.50	

Table 6 Firm-level Fama-MacBeth regressions

This table reports the results for the firm-level Fama-MacBeth regressions. We run the following cross-sectional regression:

$$r_{i,t+1} - r_{f,t+1} = c_0 + \lambda_{MKT}\beta_{i,MKT,t} + \lambda_{VIX}\beta_{i,VIX,t} + \lambda_{VOV}\beta_{i,VOV,t} + c_{FIRM}FirmCharac_{i,t} + c_{VOL}VolatilityCharac_{i,t} + \varepsilon_{i,t+1},$$

where the dependent variable is the monthly individual stock returns; $\beta_{i,MKT,t}$, $\beta_{i,VIX,t}$, and $\beta_{i,VOV,t}$ are post-ranking betas estimated from the 25 portfolios formed on intersection of $\beta_{i,VIX}$ quintile portfolios and $\beta_{i,VoV}$ quintile portfolios; $FirmCharac_{i,t}$ consists of *Size*, *B/M*, *RET_2_12*, *RET_1*, and *ILLIQ*; $VolatilityCharac_{i,t}$ includes the implied-realized volatility spread (*IVOL-TVOL*) and the call-put implied volatility spread (*CIVOL-PIVOL*). Following the methodology of Fama and French (1992), we assign each of the 25 portfolio-level post-ranking beta estimates to the individual stocks within the portfolio at that time. Robust Newey and West (1987) *t*-statistics that account for autocorrelations are reported in parentheses. The sample period is from January 1996 to December 2010.

	Fama-MacBeth regressions: individual stocks									
	[1]		[2]		[3]		[4]		[5]	
Intercept	-1.83	(-1.34)	-2.72	(-3.57)	-2.28	(-1.74)	1.52	(1.15)	1.49	(1.12)
Log(Size($b))	0.00	(-0.05)	0.00	(-0.03)	0.00	(-0.08)	-0.04	(-0.46)	-0.03	(-0.43)
Log(B/M)	**0.30**	**(1.96)**	**0.30**	**(1.97)**	**0.30**	**(1.96)**	0.16	(1.06)	0.16	(1.05)
RET_2_12	0.16	(0.40)	0.17	(0.42)	0.16	(0.42)	0.30	(0.68)	0.27	(0.59)
RET_1	**-3.82**	**(-5.64)**	**-3.78**	**(-5.62)**	**-3.79**	**(-5.61)**	**-1.95**	**(-2.45)**	**-1.66**	**(-2.13)**
ILLIQ(10^6)	**0.02**	**(3.78)**	**0.02**	**(3.79)**	**0.02**	**(3.79)**	-0.03	(-0.21)	-0.02	(-0.16)
$\beta_{i,MKT}$	2.23	(1.57)	**3.14**	**(3.89)**	**2.71**	**(1.94)**	-0.83	(-0.59)	-0.77	(-0.55)
$\beta_{i,VIX}$	1.84	(0.46)			1.77	(0.47)	**8.29**	**(1.89)**	**8.25**	**(1.88)**
$\beta_{i,VOV}$			**-3.06**	**(-4.09)**	**-3.06**	**(-4.12)**	**-2.95**	**(-2.61)**	**-2.85**	**(-2.55)**
IVOL-TVOL									**0.58**	**(2.08)**
CIVOL-PIVOL									**5.08**	**(8.22)**
Adj. R^2	0.05		0.05		0.05		0.08		0.08	
No. obs	824,426		824,426		824,426		310,221		310,221	

Table 7 Market conditions and volatility-of-volatility risk premium

This table shows the regressions of the volatility-of-volatility risk premium and the volatility risk premium on variables related to market conditions. We sort stocks into quintile portfolios based on $\beta_{i,VIX}$, from the lowest (quintile 1) to the highest (quintile 5), and independently sort stocks into quintile portfolios based on $\beta_{i,VOV}$. All portfolios are rebalanced monthly and are value weighted. The volatility-of-volatility risk premium ($rVOV$) is measured by the "5-1" portfolio returns of the $\beta_{i,VOV}$ portfolios, with controlling for the $\beta_{i,VIX}$ portfolios; the volatility risk premium (*rVIX*) is measured by the "5-1" portfolio returns of the $\beta_{i,VIX}$ portfolios, with controlling for the $\beta_{i,VOV}$ portfolios. The monthly S&P 500 logarithmic returns (*rSPX*) are used for the market dummies and the monthly changes in VIX (ΔVIX) are used for the volatility dummies. The dummy of market crash, $rSPX\_DXTR$, takes the value of one when $rSPX$ is below its time-series 5th percentile value; the dummy of down market, $rSPX\_DOWN$, takes the value of one when $rSPX$ is below its time-series mean but is still above its time-series 5th percentile value; the dummy of the upper extreme volatility shock, *ΔVIX_UXTR*, takes the value of one when ΔVIX is above its time-series 95th percentile value; the dummy of the upper volatility shock, ΔVIX_UP, takes the value of one when ΔVIX is above its time-series mean but is still below its time-series 95th percentile value; the dummy of the extreme volatility shock, ΔVIX_XTR, takes the value of one when ΔVIX is above its time-series 95th percentile value or below its time-series 5th percentile value. Panel A presents the results for $rVIX$, Panel B present the results for $rVOV$, and Panel C shows the Spearmen correlations among these variables. Robust Newey–West (1987) t-statistics are in parentheses. The sample period is from January 1996 to December 2010.

	Panel A: Time series regressions					
	[1]	[2]	[3]	[4]	[5]	[6]
	rVIX	*rVIX*	*rVIX*	*rVOV*	*rVOV*	*rVOV*
Intercept	-0.051	0.019	0.021	-0.937	-0.831	-0.508
	(-0.14)	(0.05)	(0.06)	(-2.33)	(-1.82)	(-1.27)
rSPX_DOWN	**-0.016**			0.005		
	(-2.12)			(0.82)		
rSPX_DXTR	-0.011			**-0.048**		
	(-0.56)			**(-2.33)**		
ΔVIX_UP		**-0.013**	**-0.013**		0.001	-0.003
		(-1.74)	**(-1.71)**		(0.11)	(-0.38)
ΔVIX_UXTR		-0.030			-0.034	
		(-1.55)			(-1.56)	
ΔVIX_XTR			-0.014			**-0.032**
			(-1.10)			**(-1.97)**
Adj. R^2	0.014	0.020	0.008	0.049	0.017	0.035

	Panel B: Spearman correlation					
	rSPX_DOWN	*rSPX_DXTR*	*ΔVIX_UP*	*ΔVIX_UXTR*	*ΔVIX_XTR*	*rVIX*
rSPX_DXTR	-0.18	1.00				
ΔVIX_UP	0.35	0.01	1.00			
ΔVIX_UXTR	0.04	0.53	-0.20	1.00		
ΔVIX_XTR	-0.09	0.32	-0.30	0.65	1.00	
rVIX	-0.20	-0.02	-0.07	-0.09	-0.06	1.00
rVOV	0.11	-0.19	0.01	-0.12	-0.17	-0.11

Table 8 Market crash and the price of volatility risk

This table presents the Fama–MacBeth (1973) factor premiums for the volatility-of-volatility factor (*ΔVOV*), with controlling for the market factor (*MKT*), the volatility factor (*ΔVIX*), Fama-French and Carhart factors (*SMB*, *HML*, and *UMD*). The test portfolios are the 25 portfolios independently sorted on $\beta_{i,VOV}$ and $\beta_{i,VIX}$ and the portfolios are rebalanced monthly and value weighted. We estimate the first stage return betas using the daily full-sample post-formation value-weighted returns. Then, we regress the cross-sectional daily portfolio returns on daily return betas from the first stage, using Fama–MacBeth (1973) cross-sectional regression. This table presents results for the full sample period in column [1] and column [2] and results for the period when the daily S&P 500 return is above its time-series 5th percentile value in column [3] and column [4]. The dependent variables are multiplied by 22. Robust Newey–West (1987) *t*-statistics that account for autocorrelations are in parentheses. The sample period is from January 1996 to December 2010.

	Fama-MacBeth cross-sectional regressions							
	Full sample period				Excluding market crash			
	[1]		[2]		[3]		[4]	
MKT	**0.69**	**(1.71)**	0.65	(1.62)	**4.07**	**(7.66)**	**3.95**	**(7.90)**
ΔVIX	-2.91	(-1.33)	-4.05	(-1.00)	**-8.55**	**(-3.77)**	**-11.99**	**(-3.03)**
ΔVOV	**-3.54**	**(-2.61)**	**-3.77**	**(-3.18)**	**-2.92**	**(-2.07)**	**-2.91**	**(-2.37)**
SMB			-0.85	(-1.07)			-0.27	(-0.35)
HML			-0.23	(-0.30)			-0.48	(-0.56)
UMD			-1.97	(-1.60)			-1.19	(-0.94)
Adj. R^2	0.47		0.54		0.72		0.73	

Table 9 Returns in event time for portfolios sorted on $\beta_{i,VIX}$ and $\beta_{i,VoV}$

At the end of each day, we sort stocks into quintile portfolios based on $\beta_{i,VIX}$, from the lowest (quintile 1) to the highest (quintile 5), and independently sort stocks into quintile portfolios based on $\beta_{i,VOV}$. For each day, $\beta_{i,VIX}$ and $\beta_{i,VOV}$ are re-estimated using daily stock returns over the past 22 days. Portfolios are rebalanced daily and are value weighted. Each of the quintile portfolios sorted on $\beta_{i,VOV}$ is then averaged over the five portfolios intersected with the quintile portfolios sorted on $\beta_{i,VIX}$; similarly, each of the quintile portfolios sorted on $\beta_{i,VIX}$ is then averaged over the five portfolios intersected with the quintile portfolios sorted on $\beta_{i,VOV}$.The column "5-1" refers to the hedge portfolio that longs portfolio 5 and shorts portfolio 1. This table shows the event-time daily value-weighted portfolio returns ranging from -5 to 5 in days. The event time 0 is defined by the period when the daily S&P 500 return is below its time-series 5th percentile value. Robust Newey–West (1987) *t*-statistics that account for autocorrelations are in parentheses. The sample period is from January 1996 to December 2010.

	$\beta_{i,VIX}$-sorted portfolios					*$\beta_{i,VOV}$-sorted portfolios*				
Event time	*1*	*3*	*5*	*5-1*	*t-stat*	*1*	*3*	*5*	*5-1*	*t-stat*
	Excess returns averaged over the market crash period									
-5	0.52	0.15	-0.29	**-0.81**	**(-3.12)**	-0.52	0.10	0.77	**1.28**	**(4.99)**
-4	0.52	0.13	-0.36	**-0.89**	**(-3.31)**	-0.54	0.09	0.75	**1.28**	**(5.34)**
-3	0.62	0.10	-0.38	**-1.00**	**(-3.46)**	-0.51	0.09	0.75	**1.26**	**(5.13)**
-2	0.62	0.13	-0.36	**-0.98**	**(-3.53)**	-0.51	0.10	0.83	**1.34**	**(5.43)**
-1	0.65	0.13	-0.38	**-1.02**	**(-3.78)**	-0.66	0.10	0.93	**1.59**	**(5.39)**
+0	0.69	0.12	-0.40	**-1.09**	**(-3.87)**	-0.68	0.13	0.95	**1.63**	**(5.08)**
+1	0.24	0.20	0.19	-0.05	(-0.38)	0.27	0.17	0.14	-0.13	(-0.89)
+2	0.22	0.21	0.23	0.01	(0.09)	0.26	0.18	0.21	-0.05	(-0.43)
+3	0.24	0.21	0.20	-0.05	(-0.45)	0.27	0.21	0.25	-0.03	(-0.27)
+4	0.25	0.19	0.17	-0.08	(-0.71)	0.24	0.21	0.24	0.00	(-0.02)
+5	0.27	0.19	0.15	-0.12	(-1.08)	0.23	0.21	0.27	0.04	(0.39)

Table 10 Trading activity in event time for portfolios sorted on $\boldsymbol{\beta_{i,VIX}}$ and $\boldsymbol{\beta_{i,VoV}}$

At the end of each day, we sort stocks into quintile portfolios based on $\beta_{i,VIX}$, from the lowest (quintile 1) to the highest (quintile 5), and independently sort stocks into quintile portfolios based on $\beta_{i,VOV}$. For each day, $\beta_{i,VIX}$ and $\beta_{i,VOV}$ are re-estimated using daily stock returns over the past 22 days. Portfolios are rebalanced daily and are value weighted. Each of the quintile portfolios sorted on $\beta_{i,VOV}$ is then averaged over the five portfolios intersected with the quintile portfolios sorted on $\beta_{i,VIX}$; similarly, each of the quintile portfolios sorted on $\beta_{i,VIX}$ is then averaged over the five portfolios intersected with the quintile portfolios sorted on $\beta_{i,VOV}$.The column "5-1" refers to the hedge portfolio that longs portfolio 5 and shorts portfolio 1. This table shows two measures of the event-time daily equal-weighted portfolio abnormal order imbalance ranging from -5 to 5 in days. For each stock in each day, we define the order imbalance in dollar volume (in million), *OIBDLR*, as the difference between buy dollar volume and sell dollar volume in the day and then define the order imbalance in proportion*, %OIBDLR*, as *OIBDLR* divided by the sum of buy dollar volume and sell dollar volume. The abnormal order imbalance in proportion are defined by subtracting their averaged value over the past 5 days from order imbalance in proportion. The event time 0 is defined by the period when the daily S&P 500 return is below its time-series 5th percentile value. Robust Newey–West (1987) *t*-statistics that account for autocorrelations are in parentheses. The sample period is from January 1996 to December 2010.

	$\beta_{i,VIX}$*-sorted portfolios*					$\beta_{i,VOV}$*-sorted portfolios*				
Event time	*1*	*3*	*5*	*5-1*	*t-stat*	*1*	*3*	*5*	*5-1*	*t-stat*
	Abnormal %OIBDLR averaged over the market crash period									
-5	1.18	0.33	-1.02	**-2.20**	**(-2.51)**	-1.49	0.44	1.61	**3.11**	**(4.76)**
-4	1.23	0.40	-1.22	**-2.45**	**(-2.71)**	-1.52	0.23	1.59	**3.11**	**(4.75)**
-3	1.26	0.23	-1.22	**-2.48**	**(-2.69)**	-1.55	0.21	1.61	**3.16**	**(4.57)**
-2	1.30	0.29	-1.25	**-2.55**	**(-2.82)**	-1.40	0.39	1.66	**3.06**	**(4.38)**
-1	1.42	0.41	-1.31	**-2.73**	**(-2.93)**	-1.47	0.45	1.84	**3.31**	**(4.77)**
+0	1.48	0.52	-1.37	**-2.85**	**(-3.07)**	-1.82	0.37	2.29	**4.11**	**(5.15)**
+1	-0.15	0.53	-0.02	0.13	(0.41)	0.08	0.58	-0.18	-0.25	(-0.59)
+2	-0.25	0.75	-0.28	-0.03	(-0.11)	0.03	0.56	-0.16	-0.19	(-0.50)
+3	-0.23	0.52	-0.21	0.02	(0.06)	-0.21	0.51	-0.01	0.20	(0.59)
+4	-0.04	0.68	-0.26	-0.22	(-0.65)	-0.15	0.39	-0.02	0.13	(0.47)
+5	-0.09	0.62	-0.16	-0.07	(-0.19)	-0.18	0.37	0.13	0.31	(1.17)

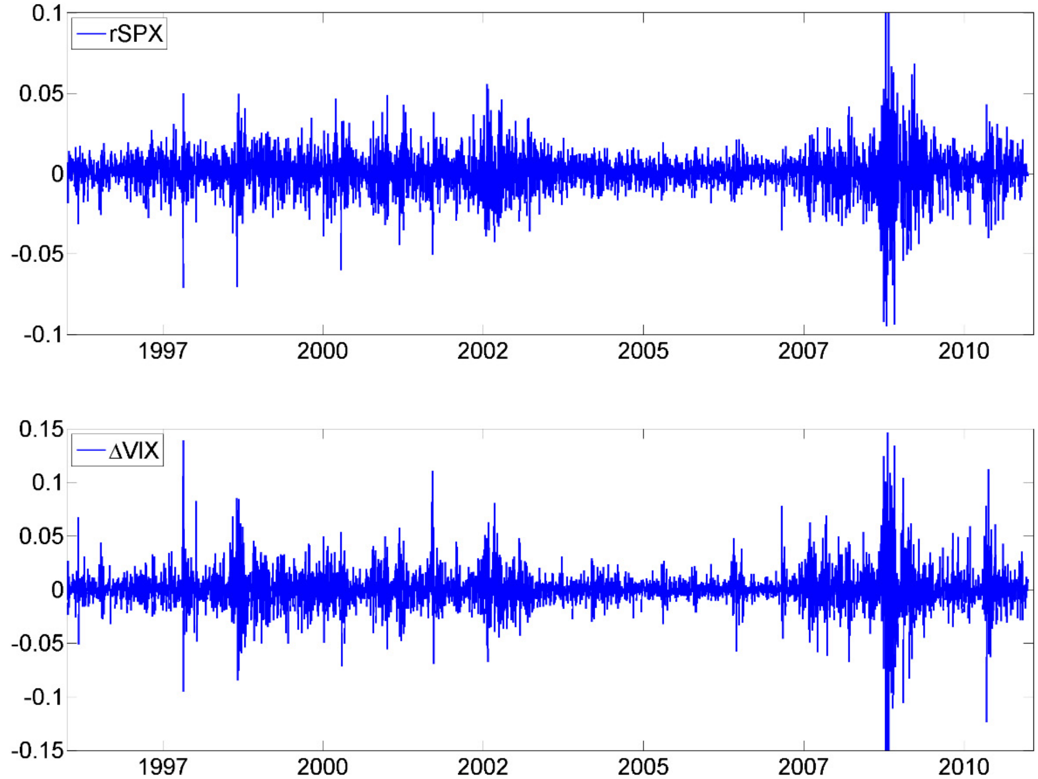

Figure 1. Daily market return (*rSPX*) and market volatility (*VIX*). We plot daily market return (*rSPX*) and changes in market volatility (*ΔVIX*) over the time period January 1996 through December 2010. The *rSPX* extreme is the dummy variable that takes the value of one when *rSPX* is above its time-series 95th percentile value or below its time-series 5th percentile value; the *ΔVIX* extreme is the dummy variable that takes the value of one when *ΔVIX* is above its time-series 95th percentile value or below its time-series 5th percentile value

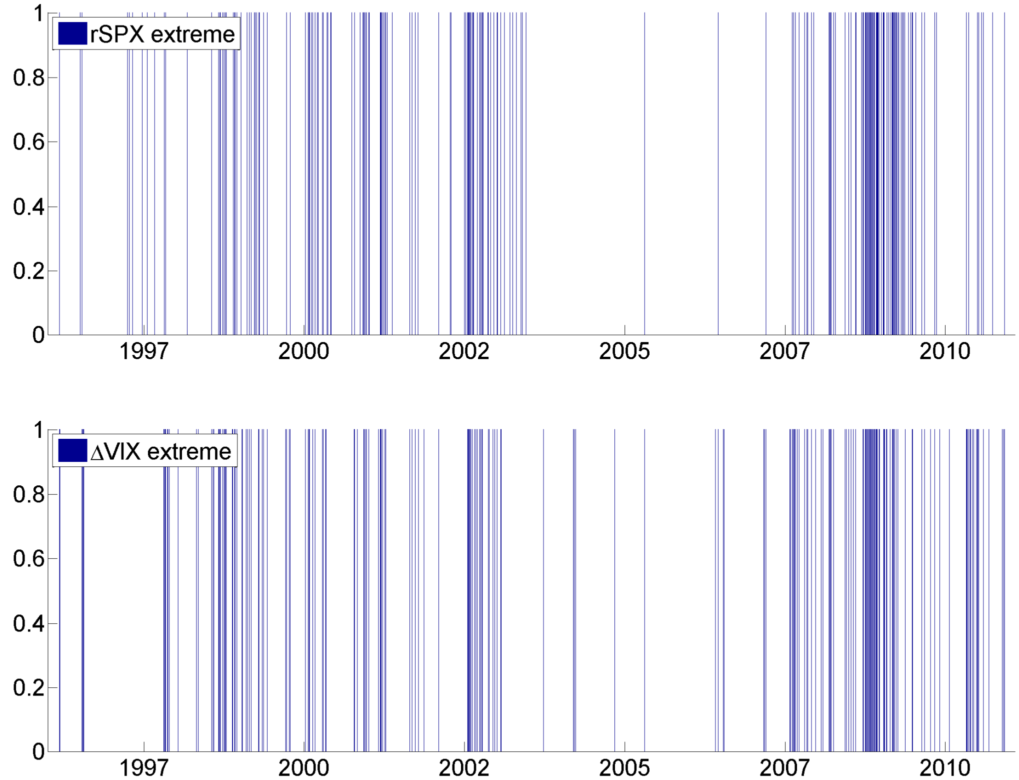

Figure 1. (Continued.)

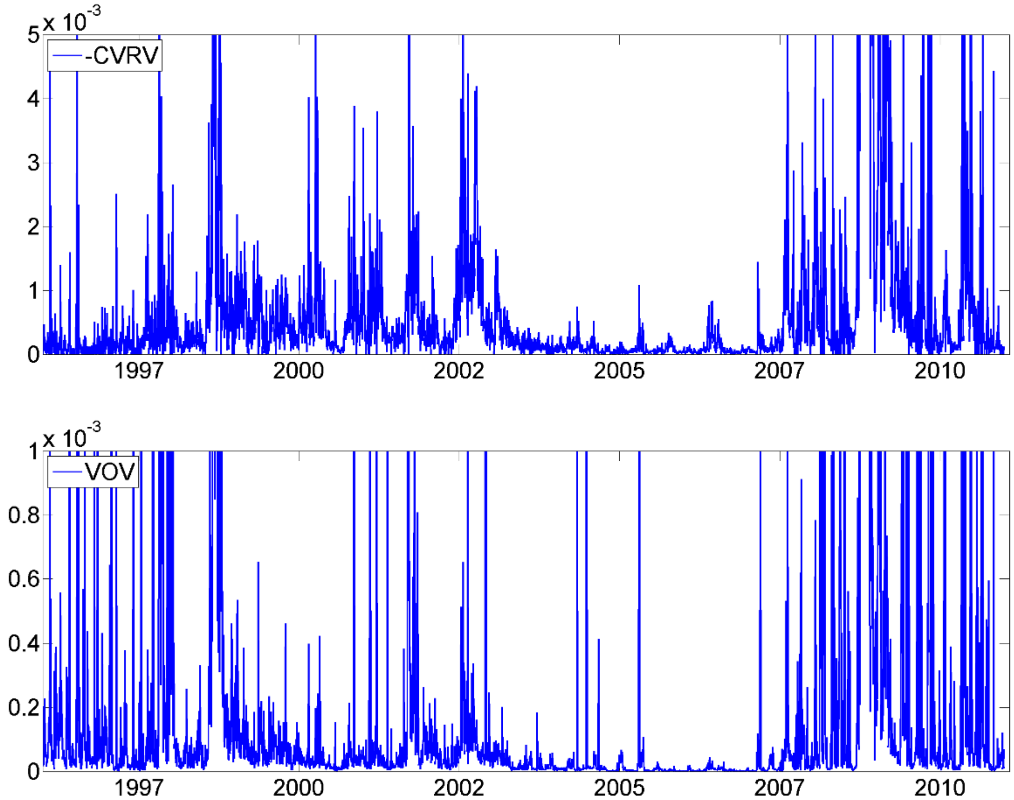

Figure 2 Daily market covariance between return and variance (*CVRV*), and market volatility-of-volatility (*VOV*). We plot daily market covariance between return and variance (*CVRV*) and market volatility-of-volatility (*VOV*) over the time period January 1996 through December 2010. We partition the tick-by-tick S&P500 index options data into five-minute intervals, and then we estimate the model-free implied variance for each interval. For each day, we use the bipower variation formula on the five-minute based annualized 30-day model-free implied variance estimates within the day, resulting in our daily measure of market volatility-of-volatility (*VOV*). We estimate the covariance between the market return and the market variance (*CVRV*) using the realized covariation formula on the five-minute based S&P500 index returns and the five-minute based annualized 30-day model-free implied variance within the day.

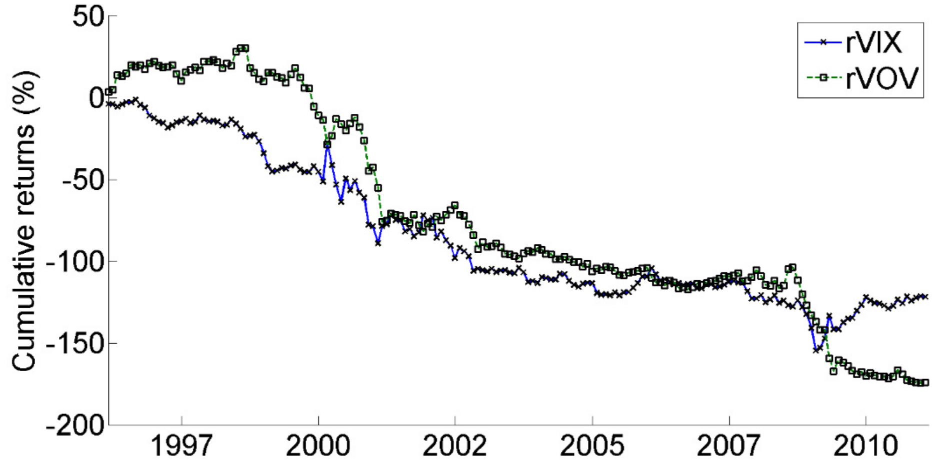

Figure 2. The volatility-of-volatility risk premium (*rVOV*) and the volatility risk premium (*rVIX*). We plot the volatility-of-volatility risk premium and the volatility risk premium over the time period January 1996 through December 2010. We sort stocks into quintile portfolios based on $\beta_{i,VIX}$, from the lowest (quintile 1) to the highest (quintile 5), and independently sort stocks into quintile portfolios based on $\beta_{i,VOV}$. Portfolios are rebalanced monthly and are value weighted. The volatility-of-volatility risk premium (*rVOV*) is measured by the "5-1" portfolio returns of the $\beta_{i,VOV}$ portfolios, with controlling for the $\beta_{i,VIX}$ portfolios; the volatility risk premium (*rVIX*) is measured by the "5-1" portfolio returns of the $\beta_{i,VIX}$ portfolios, with controlling for the $\beta_{i,VOV}$ portfolios.