A TEST OF GENERAL ASYMMETRIC DEPENDENCE[†]

LEI JIANG[‡], ESFANDIAR MAASOUMI[§], JIENING PAN[*] AND KE WU[¶]

First draft: March 2015,

This draft: December 9, 2015.

Abstract. We extend the asymmetric correlation test in Hong, Tu, and Zhou (2007) and provide a two-step procedure for testing general asymmetric exceedance dependence between two random variables. The test consists of comparing variables' exceedance correlation and dependence, which is associated with higher order moments. The performance of the test is examined by a Monte Carlo simulation. The example of testing asymmetries in dependence between stock and market returns during a market upturn and downturn is presented. Different from previous findings, our test shows that the asymmetric dependence between individual stock returns and the market return is prevailing.

JEL Classification: C12, C15, C32, G12.

Key words: Asymmetric dependence, Kullback-Leibler entropy, exceedance mutual information.

[†]We want to thank David Jacho-Chavez and Raymond Kan for useful discussions and comments. We would also like to express our appreciation for the comments and suggestions by the seminar participants at Emory University and South University of Science and Technology of China.

[‡]School of Economics and Management, Tsinghua University, China. Email: jianglei@sem.tsinghua.edu.cn.

[§]Department of Economics, Emory University, Atlanta, GA, 30322, USA. Email: esfandiar.maasoumi@emory.edu. Corresponding author, Tel.: +1-404-727-9817; fax: +1-404-727-4639.

[*]School of Finance, Nankai University, China. Email: jiening.pan@nankai.edu.cn.

[¶]Hanqing Advanced Institute of Economics and Finance, Renmin University of China. Email: ke.wu@ruc.edu.cn.

I. Introduction

Empirical studies indicate that individual stock returns and the U.S. market return comove more strongly in market downturns (low returns) than in market upturns (high returns), which indicates the existence of asymmetric dependence (Kroner and Ng, 1998; Longin and Solnik, 2001; Silvapulle and Granger, 2001; and Ang and Chen, 2002). Nevertheless, the dominant paradigm in the literature still maintains joint normal and symmetry assumptions for the return distributions.[1] There remain two open and important questions: how should the asymmetries in dependence be measured and if the observed difference in comovements statistically are significant?

Conditional correlation is one of the most widely-used measures of codependence in the literature. Let R_{1t} and R_{2t} be the returns on two portfolios between period $t-1$ and t; additionally, both of them are assumed to be stationary with $\mathrm{E}(R_i) = \mu_i, \mathrm{Var}(R_i) = \sigma_i^2$, and $i = 1, 2$. The conditional correlation between R_{1t} and R_{2t} when both variables belong to some subset S is defined as

$$\rho_S = corr\left(R_{1t}, R_{2t} | R_{1t}, R_{2t} \in S\right).$$

The conditional correlation is easy to estimate and various tests of determining asymmetric conditional correlation have been proposed and discussed (Ang and Chen, 2002; and Hong, Tu, and Zhou, 2007). Among these tests, Hong, Tu, and Zhou (2007) (hereafter HTZ) first develop a model-free test of asymmetric exceedance correlations.[2] For a vector of exceedance levels $\mathbf{c} = (c_1, c_2, \cdots c_m)'$, HTZ show that under the null hypothesis of symmetric exceedance correlation, the test statistic $J_c = T\left(\hat{\rho}^+ - \hat{\rho}^-\right)' \hat{\Omega}^{-1} \left(\hat{\rho}^+ - \hat{\rho}^-\right)$ asymptotically follows a χ_m^2

[1] In the finance literature, the benchmark scenario of a portfolio choice problem usually assumes that a representative investor is interested in maximizing a portfolio's Sharpe ratio in which only the means and variances of asset returns matter.

[2] For a given non-negative exceedance level c, HTZ test the null hypothesis of $\rho_c^+ = \rho_c^-$, where ρ_c^+ and ρ_c^- are conditional correlations when both of the two variables exceed c standard deviations away from their means; i.e.,

$$\begin{aligned} \rho_c^+ &= corr(R_{1t}, R_{2t} | R_{1t} > \mu_1 + c\sigma_1, R_{2t} > \mu_2 + c\sigma_2), \\ \rho_c^- &= corr(R_{1t}, R_{2t} | R_{1t} < \mu_1 - c\sigma_1, R_{2t} < \mu_2 - c\sigma_2). \end{aligned}$$

distribution. Based on their examination, many commonly used portfolio returns, such as book-to-market portfolios and momentum portfolios, do not exhibit a significant difference in comovement in the market up- and downturns. These results make us doubt the power of the HTZ test.

The HTZ test measures the difference in the conditional correlation, which captures only "linear" dependence. As pointed out by Embrechts, McNeil, and Straumann (2002) and many others, stock returns are not generally elliptically-distributed. The dependence between stock returns cannot be fully captured by a linear correlation. Hence, an insignificant difference in the conditional correlations does not necessarily imply general symmetric codependence. We propose a modified mutual information measure that is constructed from the perspective of the whole return distribution rather than the first two moments. A model-free test is then proposed for testing asymmetric dependence based on this measure. The new method directly tests the equality of the "general" dependence of returns in any domains of the joint distribution. Skaug and Tjøstheim (1993) and Maasoumi and Racine (2008) derive the formal inference theory underlying our general statistic and its bootstrap implementation. More recently, Giannerini, Maasoumi, and Dagum (2015) develop the asymptotic distribution of resampled general information theory test statistics. It includes as a special case the Kullback-Leibler measure of dependence employed in this paper. Our simulation study is innovative as it utilizes copulas to generate controlled levels of "general dependence" to evaluate the power and size of the tests. Furthermore, we find that a bootstrap resampling technique can significantly improve the finite sample performance of the HTZ test, which suggests that the HTZ test after revision can serve as an important ingredient in testing random variables' asymmetric dependence.

Unlike our paper, which tests asymmetries in dependence, Denuit and Scaillet (2004) develop test procedures for positive quadrant dependence via a probability distribution function and copulas. Schmidt and Stadtmüller (2005) use tail copulas to describe the structure of tail dependence and provide a non-parametric estimation process. Tjøstheim and Hufthammer (2013) propose a local Gaussian correlation measure of dependence by approximating a bivariate density locally using densities of Gaussian family. Støve and Tjøstheim (2012) and Støve, Tjøstheim, and Hufthammer (2014) apply the local Gaussian correlation measure to examine various empirical questions in finance. However, their measure is not suitable for testing dependence asymmetries in different domains of the return distribution.

The rest of the paper is organized as follows. Section 2 discusses a potential shortcoming of the conditional correlation measure and introduces the test for general asymmetric dependence based on the mutual information measure. The asymptotic size and finite sample performance of the test statistic are then examined. Section 3 extends the original HTZ test by developing a bootstrap resampling counterpart. The finite sample performance of the test is examined and compared with the results obtained via the asymptotic theory. In section 4, we apply the test to investigate asymmetric dependence in common portfolios sorted by size, book-to-market, and momentum. Section 5 concludes.

II. A Relative Entropy-based Test on Asymmetric Dependence

Symmetry of the comovements between two random variables implies the existence of symmetries of the moments of all orders in sub-domains, assuming they exist. However, a sample moment-based measure of dependence, such as an exceedance correlation, only captures dependence up to a certain order of the moment. As pointed out by Jiang, Wu, and Zhou (2015), using an HTZ type of test, any possible higher order dependence other than the second would be ignored, which implies a potentially large information loss when the underlying distribution is not jointly elliptical.

In this section, we first present a scenario where there only exists a higher order dependence between two random variables and the commonly used exceedance correlation measure fails to describe the dependence between these two random variables. An entropy-based measure of exceedance dependence, which is motivated by the Kullback-Leibler mutual information measure, is then proposed. Based on our measure, we further develop a test for asymmetric dependence. A bootstrap algorithm for obtaining the sampling distribution of the test statistic is also discussed in detail.

II.1. **The Failure of Conditional Correlation.**

Consider a random variable X distributed piece-wisely uniform, following the density function:

$$f_X(x) = \begin{cases} c & X \in (-a, 0), \\ d & X \in [0, b). \end{cases} \tag{1}$$

The random variable Y is a quadratic function of X, plus some white noises:

$$Y = \begin{cases} -e\,(X + a/2)^2 + \varepsilon_1 & X \in (-a, 0), \qquad \varepsilon_1 \sim \text{Normal}(0, \sigma_1^2), \\ f\,(X - b/2)^2 + \varepsilon_2 & X \in [0, b), \qquad \varepsilon_2 \sim \text{Normal}(0, \sigma_2^2), \end{cases} \tag{2}$$

where parameters $a, b, c, d, e,$ and f are positive constants and ε_1 and ε_2 are independent from each other. We are interested in investigating whether the comovements of X and Y are asymmetric at the exceedance level $c = 0$. We choose a set of parameter values, which are in Panel A of Table 1. One can verify that under this set of parameters, the exceedance correlation ρ^+ and ρ^- is simply the correlation calculated by using the observations in the first and the third quadrant, respectively.

[Insert Figure 1 about here]

Figure 1 shows the scatter plot of one realization of X and Y generated from distribution (1) and (2). It is clear that X and Y co-move with each other in both the first and the third quadrant (for the exceedence level $c = 0$). Moreover, the patterns of comovement are different, which indicates the existence of asymmetries in variables' dependence structures. However, if the exceedance correlation measure is used to describe the two variables' comovements, as in HTZ and many others, we conclude that X and Y co-move symmetrically at

$c = 0$ because ρ_c^+ and ρ_c^- are equal to 0 under this particular set of parameters. The failure of the exceedance correlation measure in this example suggests that in order to accurately test the asymmetric comovement behavior of random variables, we need a new measure of general dependence. This dependence measure should not be moment-based because of the potential information loss associated with the higher order moments and the non-existence of finite moments for certain distributions. Hence, an ideal general dependence measure should be constructed from the distribution perspective, which is able to summarize the dependence structure in any domain of any given distribution, even those with no finite moments.

[Insert Table 1 about here]

II.2. **A Relative Entropy-based Measure of Exceedance Dependence.**

Originating in physics and information theory, entropy has a long history of use as an aggregate measure of information contained in a distribution. During recent years, Kullback-Leibler relative entropy (Kullback and Leibler, 1951) has been employed more frequently in finance and economics research (see, for example, Hansen, 2012; Backus, Chernov, and Zin, 2014). In particular, Kullback-Leibler relative entropy has also been used to construct a widely used mutual information (MI) measure that can measure the mutual stochastic dependence between two random variables.

The MI for random variables R_1 and R_2 is defined as the Kullback-Leibler relative entropy between the joint density $g(R_1, R_2)$ and the product of their marginals $g_1(R_1) \cdot g_2(R_2)$:

$$I(R_1; R_2) \equiv E(\log \frac{g(R_1, R_2)}{g_1(R_1) \cdot g_2(R_2)}) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} g(R_1, R_2) \log \frac{g(R_1, R_2)}{g_1(R_1) \cdot g_2(R_2)} dR_1 dR_2. \tag{3}$$

Essentially, MI measures the expected difference between the log likelihood of $g(R_1, R_2)$ and the product distribution $g_1(R_1) \cdot g_2(R_2)$, which represents independence. To serve as a measure for dependence, the MI measure possesses the following desirable properties. First, MI is theoretically always non-negative, i.e., $I(R_1; R_2) \geq 0$. $I = 0$ if and only if

R_1 and R_2 are independent and this value increases as the dependence between R_1 and R_2 grows.[3] Second, the measure is obtained by comparing the entire distributions $g(R_1, R_2)$ and $g_1(R_1) \cdot g_2(R_2)$. Hence, it captures all higher order dependence between R_1 and R_2 beyond the commonly employed moment-based and/or linear dependence measures. Moreover, it can also be shown that the integrand in the Kullback-Leibler mutual information measure $\log \frac{g(R_1,R_2)}{g_1(R_1)g_2(R_2)}$ is exactly equal to the log copula density (Sklar, 1959).[4] This is a function that is widely used in statistics to capture the general dependence structure between random variables.

Motivated by the fact that $I(R_1, R_2)$ measures the dependence between R_1 and R_2 in the whole sample space $\mathbb{R}^2$, we propose a modified MI measure that is defined on subspaces in $\mathbb{R}^2$ to measure the variables' exceedance dependence. For a given exceedance level c, we define the upper and lower tail exceedance dependence as:

$$\rho_{c,o}^{-} = \int_{-\infty}^{\mu_2 - c\sigma_2} \int_{-\infty}^{\mu_1 - c\sigma_1} g(R_1, R_2) \log \frac{g(R_1, R_2)}{g_1(R_1) \cdot g_2(R_2)} dR_1 dR_2, \tag{4}$$

$$\rho_{c,o}^{+} = \int_{\mu_2 + c\sigma_2}^{+\infty} \int_{\mu_1 + c\sigma_1}^{+\infty} g(R_1, R_2) \log \frac{g(R_1, R_2)}{g_1(R_1) \cdot g_2(R_2)} dR_1 dR_2, \tag{5}$$

where $\rho_{c,o}^{+}$ and $\rho_{c,o}^{-}$ measures the general dependence between R_1 and R_2 in the upper tail [in the subspace $(\mu_1 + c\sigma_1, +\infty) \times (\mu_2 + c\sigma_2, +\infty)$] and lower tail [in the subspace $(-\infty, \mu_1 - c\sigma_1) \times (-\infty, \mu_2 - c\sigma_2)$], respectively.

If the dependence structure of a return distribution is symmetric around a certain exceedance level c, we will have $\rho_{c,o}^{+} = \rho_{c,o}^{-}$. Therefore, testing for asymmetric dependence simply requires testing the following hypothesis:

$$H_0 : \rho_{c,o}^{+} = \rho_{c,o}^{-}. \tag{6}$$

[3] See, for example, Cover and Thomas (2006), page 42, for reference.
[4] The detailed derivation is given in Appendix A.

Although the exceedance MI measures in Eq.(4) and Eq.(5) are not invariant under general linear transformations, the following theorem shows that the invariability holds under simple standardization.

Theorem II.1. *Under the assumptions made in Section II.2, the exceedance MI measure is invariant under simple standardization, i.e., for standardized returns* $X_t = \frac{R_{1t}-\mu_1}{\sigma_1}$ *and* $Y_t = \frac{R_{2t}-\mu_2}{\sigma_2}$, *we have* $\forall c$, $\rho^+_{c,o} = \rho^+_c$ *and* $\rho^-_{c,o} = \rho^-_c$. *Here* ρ^-_c *and* ρ^+_c *are given by:*

$$\rho^-_c = \int_{-\infty}^{-c} \int_{-\infty}^{-c} f(X,Y) \log \frac{f(X,Y)}{f_1(X) f_2(Y)} dX dY,$$

$$\rho^+_c = \int_{c}^{+\infty} \int_{c}^{+\infty} f(X,Y) \log \frac{f(X,Y)}{f_1(X) f_2(Y)} dX dY,$$

where $f(X,Y), f_1(X)$, *and* $f_2(Y)$ *denote the joint and marginal densities for the standardized returns, respectively.*

Proof. See Appendix B. □

Therefore, to compare the exceedance dependence of R_1 and R_2 at any given level c, it is equivalent to test the symmetry of the upper- and lower-tail dependence between standardized returns X and Y at the same exceedance level; i.e.,

$$H_0 : \rho^+_c = \rho^-_c \quad \text{for a given exceedance level } c.$$

Following the literature, we work with the standardized returns X and Y with a zero mean and unit variance in the rest of this paper. The rejection of the null hypothesis will lead to the asymmetric alternative $H_1 : \rho^+_c \neq \rho^-_c$.

II.3. **The Non-parametric Estimator.**

We now consider how to estimate the exceedance MI measure given the data. Similar to the

MI in Eq.(3), ρ_c^- and ρ_c^+ can also be interpreted as expectations?

$$\begin{aligned}\rho_c^- &= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(X,Y)\log\tfrac{f(X,Y)}{f_1(X)f_2(Y)}\cdot\mathbf{1}(X<-c,Y<-c)dXdY \\ &= E(\log\tfrac{f(X,Y)}{f_1(X)f_2(Y)}\cdot\mathbf{1}(X<-c,Y<-c)),\end{aligned}$$

$$\begin{aligned}\rho_c^+ &= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(X,Y)\log\tfrac{f(X,Y)}{f_1(X)f_2(Y)}\cdot\mathbf{1}(X>c,Y>c)dXdY \\ &= E(\log\tfrac{f(X,Y)}{f_1(X)f_2(Y)}\cdot\mathbf{1}(X>c,Y>c)),\end{aligned}$$

where $\mathbf{1}(\cdot)$ denotes the indicator function. ρ_c^- and ρ_c^+ can be estimated by their sample analogues. For a random sample of returns that consists of T observations $\{X_t,Y_t\}_{t=1}^T$, we can express the sample exceedance dependence as:

$$\hat{\rho}_c^- = \frac{1}{T}\sum_{t=1}^{T}\log\frac{\hat{f}(X_t,Y_t)}{\hat{f}_1(X_t)\hat{f}_2(Y_t)}\mathbf{1}(X_t<-c,Y_t<-c), \tag{7}$$

$$\hat{\rho}_c^+ = \frac{1}{T}\sum_{t=1}^{T}\log\frac{\hat{f}(X_t,Y_t)}{\hat{f}_1(X_t)\hat{f}_2(Y_t)}\mathbf{1}(X_t>c,Y_t>c), \tag{8}$$

where the probability density functions $\hat{f}(X_t,Y_t), \hat{f}_1(X_t)$, and $\hat{f}_2(Y_t)$ are estimated by robust non-parametric kernel estimators as proposed in Rosenblatt (1956) and Parzen (1962). Kernel estimation provides consistent estimators for the joint density of a set of random variables. Given a series of m-dimensional random vectors Z that consists of T observations $z_1, z_2, \ldots, z_T$, the Parzen-Rosenblatt kernel density estimator of $f(z)$ is:

$$\hat{f}(z) = \frac{1}{Th_1h_2\cdots h_m}\cdot\sum_{t=1}^{T}K\left(\frac{z_t-z}{h}\right), \tag{9}$$

where $K\left(\frac{z_t-z}{h}\right)\equiv\prod_{i=1}^{m}k\left(\frac{z_{i,t}-z_i}{h_i}\right)$. $k(\cdot)$ is a symmetric non-negative bounded function and h_i is the bandwidth (or smooth parameter). Various studies (see, for example, Epanechnikov, 1969) suggest that different kernel functions have very little impact on estimations; hence, we use the popular Gaussian kernel $k(z)=\frac{1}{\sqrt{2\pi}}e^{-z^2/2}$. On selecting the bandwidth, we choose to use the likelihood cross-validation method. Specifically, it solves the following maximum likelihood problem:

$$\max_{h_1,h_2,\cdots,h_m} \mathcal{L} = \sum_{t=1}^{T} \ln\left[\hat{f}_{-t}(z)\right], \tag{10}$$

where

$$\hat{f}_{-t}(z) = \frac{1}{Th_1h_2\cdots h_m} \cdot \sum_{s\neq t}^{T} K\left(\frac{z_t - z}{h}\right), \tag{11}$$

which is equal to $\hat{f}(z)$ without the t-th realization. For stationary and weakly dependent data such as stock returns, the estimated density in Eq.(9) converges to the actual density at a fairly fast speed (see, e.g., Li and Racine, 2006, for technical details).

II.4. **Test Statistic and Its Sampling Distribution.**

Let $\hat{\theta} = \hat{\rho}_c^+ - \hat{\rho}_c^-$. Symmetries in exceedance dependence can be tested using an intuitive t-type test statistic:

$$\hat{t} = \frac{\hat{\theta}}{\hat{\sigma}_\theta}. \tag{12}$$

Although the asymptotic theory for the MI measure under the null hypothesis of independence has been developed in previous research [see, for example, Robinson (1991) and Hong and White (2005)], the asymptotic distribution for the exceedance MI measures $\hat{\rho}_c^+$ and $\hat{\rho}_c^-$ are unknown when allowing for general dependence. Moreover, various studies, including Rilstone (1991) and Robinson (1991), report that inferences based on the asymptotic distribution are not reliable in finite samples. Part of the reason is that the test statistic does not depend on bandwidth asymptotically, as the optimal bandwidth $\hat{h}$ vanishes when the number of observations $T \to \infty$. In finite samples, however, the test statistic is highly sensitive to $\hat{h}$, which varies across different approaches for bandwidth selection. Following the suggestion of Racine (1997), Hong and White (2005), and many others, we construct the sampling distribution for $\hat{t}$ using the pivotal bootstrap resampling approach.[5]

We follow Künsch (1989) to take into account the dependent structure in weakly-dependent time series data using a bootstrap procedure with overlapping blocks. Stationarity is ensured by letting the length of each block be randomly sampled from the geometric distribution

[5]General properties of the bootstrap resampling approach can be found in Efron (1982). Horowitz (2001) provides excellent reviews of the literature.

(Politis and Romano, 1994), whose mean is determined by the algorithm proposed in Politis and White (2004) and Patton, Politis, and White (2009). We use their method because it is designed to minimize the mean squared error of the estimated long-run variance of the time series. The details of the bootstrap resampling procedure are described below.

We compute $\hat{\theta}_0$ using Eq.(7) and Eq.(8). The standard error $\hat{\sigma}_\theta$, which is crucial for achieving asymptotic refinement, is obtained by the nested resampling method (Hinkley and Shi, 1989; Efron and Tibshirani, 1993). In this paper, we use the stationary geometric bootstrap to create a sequence of B_1 nested samples from the original data sample. For each of the B_1 nested samples, we calculate its sample estimates for $\hat{\theta}$ and form a sequence $\{\hat{\theta}_0^{(i)}\}_{i=1}^{B_1}$. The standard error for the original data sample is simply the sample standard deviation of these B_1 nested samples:

$$\hat{\sigma}_{\theta_0} = \frac{1}{B_1 - 1} \sum_{i=1}^{B_1} \left(\hat{\theta}_0^{(i)} - \overline{\hat{\theta}_0^{(i)}} \right)^2 .$$

Given both $\hat{\theta}_0$ and $\hat{\sigma}_{\theta_0}$, $\hat{t}_0$ can be directly computed by Eq.(12).

For the sampling distribution of the t-statistic, we first generate B bootstrap samples from the original data set using the stationary block bootstrap. For the jth bootstrap sample, we calculate $\hat{\theta}_j$ following Eq.(7) and Eq.(8). $\hat{\sigma}_{\theta_j}$ is also constructed using the nested block bootstrap resampling with the same mean in the underlying geometric distribution. We create B_1 nested bootstrap samples by resampling from the given bootstrap sample and calculating $\hat{\theta}_j^{(i)}$ for each nested sample. $\hat{\sigma}_{\theta_j}$ is then simply computed by the sample standard deviation of $\{\hat{\theta}_j^{(i)}\}_{i=1}^{B_1}$. Following Horowitz (2001), the t-statistics from the bootstrap samples are adjusted for the sampling bias:

$$\hat{t}_j = \frac{\hat{\theta}_j - \hat{\theta}_0}{\hat{\sigma}_{\theta_j}}.$$

We then estimate the empirical distribution F for $\{\hat{t}_j\}_{j=1}^{B}$ and report the percentile of $\hat{t}_0$ under F. For a given level of significance α, the null hypothesis of symmetric dependence will be rejected if $\hat{t}_0$ is located in the upper $1 - \alpha/2$ or lower $\alpha/2$ percentile.

II.5. **Simulation Results.**

A valid test should possess the following asymptotic properties: first, as the sample size increases, the probability of falsely rejecting a true null hypothesis should converge to the nominal size; second, the power of the test increases monotonically with the sample size and converges to 1 when the sample size approaches infinity. We examine both the asymptotic size and finite sample performance (size and power) of the asymmetric dependence test using the simulation and present the results in this subsection.

To examine the size and power of the asymmetric dependence test in finite samples, data with different asymmetry levels in exceedance dependence are needed. A natural choice for our simulation is to generate samples using parametric copulas with different comovement behaviors at certain exceedance levels.

To study the asymptotic size of our test, a copula with symmetric exceedance dependence is needed; i.e., $\rho_c^- = \rho_c^+$. In this paper, we use Student's t copula:

$$C_d(u, v; \rho) = t_{d,\rho}(t_d^{-1}(u), t_d^{-1}(v)),$$

where $t_{d,\rho}(\cdot)$ is the cdf of the bivariate Student's t distribution with d degree of freedom and $\rho \in (-1, 1)$ is the correlation coefficient between the marginal distributions. Compared with the Gaussian copula, the t copula has fatter tails and thus is an ideal candidate for examining the empirical size of our test in large samples.

To study the power of our test, we use a distribution with different levels of asymmetric dependence. Following HTZ and many others, random samples are generated using the mixture copula. Different levels of asymmetric dependence are achieved by mixing the Gaussian copula, which has symmetric exceedance dependence around its mean, with the Clayton copula, which exhibits stronger left-tail dependence. The mixture Gaussian-Clayton copula has the following specification:

$$C_{mix}(u, v; \rho, \tau, \kappa) = \kappa C_{nor}(u, v; \rho) + (1 - \kappa) C_{clay}(u, v; \tau), \quad \kappa \in [0, 1]. \tag{13}$$

The parameter ρ in the Gaussian copula is the correlation coefficient and the parameter τ governs the dependence between the marginal distributions in the Clayton copula. A higher τ indicates stronger left-tail dependence. The parameter κ represents the weight we place on the Gaussian copula. Different levels of asymmetric dependence can be achieved by adjusting κ. When $\kappa = 1$, Eq.(13) reduces to the Gaussian copula with symmetric tail dependence. Asymmetries in tail dependence gradually increase as κ decreases. When $\kappa = 0$, Eq.(13) reduces to the Clayton copula, which displays the strongest asymmetric tail dependence. In this paper, we consider $\kappa = 0, 0.25, 0.375, 0.5,$ and 1 in the simulation; which represents five levels of dependence asymmetries, from the highest to the lowest.

The parametric copula only determines the dependence structure between two random variables. In order to obtain the joint density, the marginal density of each random variable is needed. Since our ultimate goal is to investigate whether asymmetric dependence exists between a stock portfolio and market returns, marginal distributions that mimic portfolio return distributions are used in our simulation. As in HTZ and Jiang, Wu, and Zhou (2015), the value-weighted size 5 portfolio is selected as the benchmark. Following the finance literature, we model the marginal distributions of a stock return with a GARCH (1,1) specification with no ARMA components.

In our simulations, we first fit the copula-GARCH model to the portfolio and market returns and obtain Maximum Likelihood Estimates (MLE) of parameters in the copula and GARCH specification. The true data-generating process (DGP) is assumed to follow the copula-GARCH model with all parameters set at the MLE. One thousand simulated random samples are generated under the same DGP for each sample size T, on which the properties of the test are examined.

II.5.1. *Asymptotic Size.*

The asymptotic size of our test is examined on simulated random samples of size 1,000 and 1,500, respectively. Table 2 shows the probabilities of rejecting the null hypothesis of symmetric dependence at the exceedance level $c = 0$ under the nominal sizes of 10%, 5%, and 1%. The rejecting probabilities are computed as the portion of rejection decisions made in 1,000 simulated random samples. For each random sample, the inference is based on 199 stationary bootstraps. Since the random samples are generated using the t-copula, the null hypothesis of symmetric exceedance dependence is correct under the true DGP. The rejecting probabilities are thus empirical sizes for each corresponding nominal level. Simulation results are reported in Table 2. When the sample size T=1,000, the empirical sizes read 10.3%, 5.2%, and 1.4% for the 10%, 5%, and 1% nominal levels, respectively. This suggests that the sizes of our test are quite accurate for a fairly large sample size. If we further increase the sample size to 1,500, we obtain almost identical empirical and nominal sizes, which are 10%, 5%, and 1.1% for 10%, 5%, and 1%, respectively. The simulation results in this section clearly show that the empirical sizes of our test converge monotonically to the sizes at all nominal levels as the sample size increases. In other words, our test exhibits accurate size asymptotically.

[Insert Table 2 about here]

II.5.2. *Finite Sample Performance.*

The finite sample performance of our test is evaluated with the simulated copula-GARCH samples whose dependence structure follows the mixture copula as shown in Eq.(13). Three sample sizes ($T = 240, 420$, and 600) are considered, which corresponds to 20, 35, and 50 years of monthly data, respectively. In our simulation, the empirical size and power are computed as the relative frequency of rejecting the null hypothesis of a symmetric exceedance dependence at the exceedance level $c = 0$ in 1,000 simulated samples. Statistical inferences are made based on 199 stationary block bootstraps. To mitigate the concern that different bandwidth selection methods may have an impact on the final result, we also examine the robustness of our test results with respect to different bandwidth selection approaches.

Table 3 reports the empirical size and power of our asymmetric dependence test at the nominal levels of 10%, 5%, and 1%. For any given nominal size when asymmetric dependence exists in the true DGP, which corresponds to $\kappa = 0, 0.25, 0.375$, and 0.5 in the simulation, the empirical power of our test increases monotonically with the sample size. When $\kappa = 0, 0.25$, and 0.375 in our simulation, which corresponds to a stronger asymmetric exceedance dependence, our test possesses excellent power even for the smallest sample size of $T = 240$, where we are able to detect the asymmetries in exceedance dependence 83.1% of the time at the conventional 5% level of significance. As the sample size increases to 600, we are even able to reject almost all simulated samples at the 1% level. When $\kappa = 0.5$, the test's power decreases slightly, especially for the smallest sample; however, the power recovers as the sample size increases. When $T = 600$, the power of our test reaches about 70% at the 5% level, which is more than twice as large as the power in $T = 240$. The size of the test also looks promising as the falsely rejecting probabilities are very close to the corresponding nominal sizes for all three sample sizes. Combining findings in this section with the results of the asymptotic size, we are able to claim that our bootstrap-based asymmetric dependence test is both asymptotically accurate and consistent.

[Insert Table 3 about here]

Although the asymmetric dependence test performs well in finite samples, the robustness of all results requires further investigation. As we discussed in section II.4, the main reason for not using the asymptotic distribution is that the value of our kernel-based test statistic relies on the choice of bandwidth parameters in finite samples. Hence, it is worthwhile to investigate the impact of different bandwidth selection methods on the test.

Aside from the likelihood cross validation approach adopted in the previous simulation, the optimal bandwidth in the kernel estimation can also be obtained using the least square cross validation (LSCV) and plug-in method. We apply both approaches in the kernel estimation and keep the rest settings the same. Table 4 provides the empirical size and power of the asymmetric dependence test under different bandwidth selection approaches. The results of the least square cross validation and the plug-in approach are presented in Panel A and Panel B, respectively. Comparing the results of both approaches to those obtained by the likelihood cross validation, we find similar patterns across all three tables: the power of the asymmetric dependence test increases with the sample size no matter which bandwidth selection method is used. Moreover, the rejection probabilities are generally of the same magnitude for all three bandwidth selection methods. For example when $\kappa = 0.5$, the empirical power of 600 samples at the 5% nominal level is 69.4%, 63.8%, and 72.5% for the three bandwidth selection approaches. The empirical size under different bandwidth selection approaches also shows a promising pattern. The plug-in method uses the bandwidth calculated under the normal density assumption; therefore, it provides the most accurate empirical size in finite samples. The empirical size from the LSCV approach is relatively less accurate in the sample, but we can still see that the empirical size converges to the nominal level as the sample size increases.

In a nutshell, our simulation results indicate that changing bandwidth selection methods has almost no impact on the testing results.

[Insert Table 4 about here]

III. A Resampling Extension of Hong, Tu, and Zhou (2007)

Our asymmetric dependence test possesses great power in small samples when the asymmetry in DGP is relatively higher; i.e., for $\kappa \leq 37.5\%$ in the mixture copula process (13). When the asymmetry in DGP is relatively lower; i.e., for $\kappa = 50\%$, we observe a large decrease in the empirical power, especially when the sample size is small. The non-parametric estimator in Eq.(7) and Eq.(8) converges at a slower rate than $\sqrt{T}$, which is the convergence rate of a linear estimator. Hence, any failure in rejecting the null hypothesis of symmetric dependence may be simply due to the slower convergence rate of the non-parametric estimator.

On the other hand, the null hypothesis of symmetric dependence would be rejected if any moment exhibits asymmetries. Moreover, when the degree of asymmetry decreases, the linear correlation can better serve as a dependence measure. Hence, in this section, we develop a bootstrap version of the HTZ test that works well in finite samples. We conduct both the original and bootstrap HTZ test on simulated samples. The finite sample performance is subsequently compared and discussed.

III.1. **An Improved HTZ Test in Finite Samples: A Bootstrap Approach.**

The theoretical attractiveness of the original HTZ test does not guarantee its usefulness in practice. As we will shortly see, the performance of the original HTZ test is very poor in small samples, which is mainly due to the fact that the sampling distribution of the original HTZ test statistic is obtained via the asymptotic theory.

We approximate the sampling distribution of the HTZ test statistic using a suitable bootstrap resampling approach. The HTZ test statistic is pivotal; hence, the bootstrap sampling distribution is able to achieve asymptotic refinement once we have imposed the null hypothesis by subtracting the finite-sample bias of the HTZ test statistic. As in the asymmetric dependence test, we use the stationary block bootstrap approach to take into account the dependence structure embedded in the data.

For a given exceedance level c, consider $\theta = \rho_c^+ - \rho_c^-$. $\theta_0 = 0$ under the null hypothesis of the symmetric exceedance correlation, which implies no adjustment is needed when calculating the test statistic. The test statistic of the original sample is $J_{c,0} = T\left(\hat{\theta}_0 - \theta_0\right)' \hat{\Omega}_0^{-1} \left(\hat{\theta}_0 - \theta_0\right)$, where $\hat{\theta}_0$ is simply the difference between the sample right and left exceedance correlation and the covariance matrix $\hat{\Omega}_0$ is estimated by the nested resampling method (Hinkley and Shi, 1989; Efron and Tibshirani, 1993). More specifically, we use the stationary geometric bootstrap to create a sequence of B_1 nested samples from the original data sample. For each of the B_1 nested samples, we calculate the sample estimates for $\hat{\theta}$ and form a sequence $\{\hat{\theta}_0^{(i)}\}_{i=1}^{B_1}$. The covariance matrix Ω_0 for the original data sample is simply estimated by:

$$\hat{\Omega}_0 = \frac{1}{B_1 - 1} \sum_{i=1}^{B_1} \left(\hat{\theta}_0^{(i)} - \overline{\hat{\theta}_0^{(i)}}\right) \left(\hat{\theta}_0^{(i)} - \overline{\hat{\theta}_0^{(i)}}\right)',$$

where $\overline{\hat{\theta}_0^{(i)}} = \frac{1}{B_1}\sum_{i=1}^{B_1}\hat{\theta}_0^{(i)}$. Given both $\hat{\theta}_0$ and $\hat{\Omega}_0$, $\hat{J}_{c,0}$ can be directly computed.

For the sampling distribution of J_c, we first generate B bootstrap samples from the original data set using the stationary block bootstrap. For the ith bootstrap sample, we estimate $\hat{\theta}_i$ using the sample conditional correlation. The estimation of the covariance matrix $\hat{\Omega}_i$ is conducted in the same manner as the original sample. The test statistic of J_c is calculated after adjusting for the sampling bias $\hat{\theta}_0$ via $\hat{J}_{c,i} = T\left(\hat{\theta}_i - \hat{\theta}_0\right)'\hat{\Omega}_i^{-1}\left(\hat{\theta}_i - \hat{\theta}_0\right)$. We collect $\left\{\hat{J}_{c,i}\right\}_{i=1}^{B}$ and form its empirical cumulative distribution function (ECDF) F. We report the percentile of $\hat{J}_{c,0}$ under F for a given level of significance α; the null hypothesis of the symmetric exceedance correlation will be rejected if $\hat{J}_{c,0}$ is located in the upper $1-\alpha$ percentile, as in the case of the χ^2 distribution.

III.2. **Finite Sample Performance of the HTZ Test.**

We conduct tests based on the asymptotic theory and bootstrap resampling with the same simulated samples of sizes $T = 240, 420$, and 600. In our simulation, the empirical size and power are computed as the relative frequency of rejecting the null hypothesis of symmetric correlation at the exceedance level $c = 0$ in 1,000 simulated random samples. For each random sample, the p-value of the asymptotic HTZ test statistic is obtained from its asymptotic distribution, which is χ_1^2, as shown in Hong, Tu, and Zhou (2007). The bootstrap HTZ tests are based on 199 stationary bootstraps.

[Insert Table 5 about here]

In Table5, we present the empirical size and power of the HTZ tests. The performance of the original HTZ test is reported in Panel A of the table. Unsurprisingly, the test performs poorly in finite samples, even for the largest sample size of 600. The empirical size of the test is distorted at all commonly used nominal levels for all three sample sizes. The test based on the asymptotic theory fails to cause any rejection in the finite sample. Moreover, when the level of asymmetry is low; e.g., $\kappa = 50\%$, the power of the asymptotic HTZ test is very low. It almost cannot detect the asymmetry for all sample sizes at the 1%

level; for the conventional 5% level of significance, the asymptotic HTZ test only rejects 1.4% of the samples when $T = 600$. In Panel B of the table, we report the results of the bootstrap HTZ test on the same simulated samples. Empirical size suggests that the problem of size distortion lessens for the bootstrap HTZ test. At the 10% (5%) nominal level, for instance, the empirical sizes are 10.1% (4.0%), 9.5% (4.3%), and 11.2% (5.4%) for samples with different sizes, respectively. At the 1% level, although the empirical size is slightly distorted for $T = 240$ (0.6%) and 420 (0.7%), the concern is mitigated for $T = 600$ (1.2%). Meanwhile, the test power improves dramatically. When the level of asymmetry is high ($\kappa = 0, 0.25$), we are able to make correct decision more than 95% of the time even when the sample size is small ($T = 240$). The rejection power increases monotonically with the sample size; for a fairly large sample ($T = 600$), the rejecting power of the bootstrap HTZ test is still about 82.7% (at the conventional 5% level) for the samples with the most indistinguishable asymmetry ($\kappa = 0.5$).

Comparing the results in Table 5, we show that our bootstrap version HTZ test provides a more accurate empirical size and stronger rejecting power than the asymptotic-based original HTZ test. Our test can serve as a useful tool for researchers, especially in empirical studies where the sample size is not large enough for the asymptotic theory to be valid.

III.3. **A Two-step Procedure for Testing General Asymmetric Dependence.**

In our simulation, the power of the asymmetric dependence test is lower than the bootstrap HTZ test, especially when the level of asymmetry is weak. The exceedance correlation measure is estimated using sample moments, which converge at the usual rate of $\sqrt{T}$. However, the kernel estimation, while consistent, converges at a lower rate. Therefore, although the general test of asymmetric dependence subsumes the correlation-based null, the power difference in the previous section implies that the failure of rejecting the symmetric dependence $\rho_c^+ = \rho_c^-$ may simply be the result of a slower convergence rate of the non-parametric estimator. In order to test the existence of general asymmetric dependence, both tests are needed.

To take advantages of both tests, a two-step contingent procedure can be motivated in practice for testing asymmetries in general exceedance dependence. For a given exceedance level, one may first conduct the asymmetric dependence test. If the null hypothesis of the symmetric exceedance dependence is rejected, one may infer asymmetric inference and stop. If one is interested to learn more about the nature of the dependence, perhaps as a guide to modeling, or if the asymmetric dependence test fails to reject its null, we continue with the bootstrap HTZ test. We retain the null hypothesis of symmetric dependence only if we fail to reject the null hypothesis by the general test.

We now reconsider the example presented in Section II.1 and apply our two-step method to test for asymmetric dependence. As shown in the upper half of Panel B of Table 1, we first apply the asymmetric dependence test on the simulated samples; the empirical power of the test is close to 70.3% at the conventional 5% level, indicating we have a high probability in making a correct decision to reject the null hypothesis of symmetric dependence. We are also interested in knowing the dependence structure by continuing the second bootstrap HTZ test on the same simulated sample. The lower half of Panel B in Table 1 reports the empirical power of the bootstrap HTZ test. The null hypothesis of the symmetric exceedance correlation is correct and hence the bootstrap HTZ test merely reports the empirical size of the test. The testing results show that the empirical size is close to the nominal size for all commonly used nominal levels, which suggests that we are very unlikely to reject the null hypothesis of the symmetric correlation. We can therefore further proceed to propose a model that only contains a higher order dependence to describe the data.

IV. Asymmetric Dependence in Stock Returns

In this section, we apply our two-step general asymmetric dependence test to equity portfolios sorted by size, book-to-market, and momentum in order to investigate whether asymmetric comovement is a common phenomenon in stock returns. It should be noted that previous empirical studies report mixed results.

IV.1. **Data.**

Similar to Ang and Chen (2002) and Hong, Tu, and Zhou (2007), we consider excess returns for value-weighted size and book-to-market decile portfolios and equal-weighted decile momentum portfolios that are formed based on cumulative returns from 12 to 2 months prior to formation. The CRSP (Center for Research in Security Prices) value-weighted index return, which includes all stocks listed in the NYSE/AMEX/NASDAQ, is used as a proxy for the market return. All returns are recorded in excess of the rate on one-month T-bill. The entire data set is available on Kenneth French's website.[6] Monthly observations are from January 1965 to December 2013, which totals 588 observations.

IV.2. **Empirical Results.**

We apply the two-step general asymmetric dependence test to the equity portfolios. The exceedance level is set at 0 for both tests. P-values of the test are calculated based on 399 stationary bootstrap resamplings. Table 6 provides the sample estimates of the ρ_c^+, ρ_c^-, $\rho_c^- - \rho_c^+$, and the p-values of the two-step general asymmetric dependence test on all three sets of portfolios. We are able to reject the null hypothesis of symmetric dependence for the 1st to the 5th smallest size portfolios (Panel A) at the 5% level. In the second step of the bootstrap HTZ test, we further reject the null of the asymmetric correlation for size 6-8 portfolios. Our finding supports the necessity of including the bootstrap HTZ test in the second step if we fail to reject the null of symmetric dependence in the first step. This finding is consistent with previous literature. Since our proxy for the market portfolio is a value-weighted index, larger firms co-move with the market symmetrically. Therefore, the asymmetries in general dependence vanish as the firm size increases. For value-weighted book-to-market portfolios (Panel B), the general asymmetric dependence test rejects symmetry for the tenth BE/ME portfolios. The symmetric dependence hypothesis can be rejected for the fifth, eighth, and ninth BE/ME portfolios if we apply the bootstrap HTZ test in the second step. Our findings are consistent with Ang and Chen (2002) and Jondeau

[6]We are grateful to Kenneth French for making the data available at
http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

(2015). Value stocks exhibit more asymmetric comovement with the market. However, Hong, Tu, and Zhou (2007) report that they fail to detect any asymmetry for all book-to-market portfolios using the asymptotic HTZ test, which indicates the impreciseness of the asymptotic theory-based test in finite samples. In Panel C, we present the results for equal-weighted momentum portfolios. Both the bootstrap HTZ test and the asymmetric dependence test show the overwhelming favor of the alternative hypothesis of asymmetry. This means that a statistically significant difference exists in the equal-weighted momentum portfolios at the 1% level under our general asymmetric dependence test.

[Insert Table 6 about here]

V. Conclusion

Whether or not individual stock returns significantly co-move with the market return more during market downturns than during upturns, and whether the phenomenon is prevailing enough to be considered in asset management is of great current interest. Ang and Chen (2002) give positive answers to both questions, but their test concerns joint normality rather than asymmetry. Hong, Tu, and Zhou (2007) give an opposite answer based on a model-free test; however, their result lacks sufficient power in finite samples because it relies on first order asymptotic theory.

In this paper, we propose a model-free test of general asymmetric dependence between stock returns and the market return. Our general test may be used alone or in a two-step procedure for testing asymmetries in the exceedance correlation and exceedance mutual information, respectively. We find greater test power in finite samples than is shown in the previous model-free test by Hong, Tu, and Zhou (2007). The power comes from finite sample refinement using bootstrap resampling. Furthermore, our general method is able to test higher order dependence asymmetries, which is crucial in view of the fact that stock returns are not elliptically distributed (Embrechts, McNeil, and Straumann, 2002).

Using the new test of general asymmetric dependence, we test common U.S. portfolios sorted by size, book-to-market ratios and momentum. We find that most of the time, although asymmetries cannot be detected using the asymptotic theory-based HTZ test, those portfolios are in fact asymmetric in dependence. Portfolio managers should pay more attention to risk-hedging down markets due to this asymmetry.

Appendix A. Connections between Mutual Information and Copulas

A copula density is defined as the second-order partial derivatives of a copula function with respect to the uniform marginal distribution functions. Consider two random variables X and Y and a copula function $C(U,V)$, where $U = F_x(X)$ and $V = F_y(Y)$ are uniform marginal distributions. Mathematically, the copula density can be written as:

$$c(u,v) \equiv \frac{\partial}{\partial F_y}\left(\frac{\partial C}{\partial F_x}\right) = \frac{\partial^2 C(u,v)}{\partial F_y \partial F_x}. \tag{14}$$

Note that if we transform the original variables via their CDF functions, then the joint density function of the transformed variables U and V are actually the copula density $c(u,v)$. We know that any joint distribution can be written in terms of copulas; i.e.,

$$C(u,v) = F(x,y). \tag{15}$$

Taking second-order partial derivatives with respect to x and y, respectively, on both sides yields:

$$\frac{\partial}{\partial y}\left(\frac{\partial C(u,v)}{\partial x}\right) = f(x,y), \tag{16}$$

$$\frac{\partial^2 C(u,v)}{\partial F_y \partial F_x}\frac{\partial F_x}{\partial x}\frac{\partial F_y}{\partial y} = f(x,y), \tag{17}$$

$$c(u,v) f(x) f(y) = f(x,y). \tag{18}$$

The last equation follows from the definition of copula density given in Eq.(14). Dividing the product of the marginal density functions on both sides of Eq.(18), we have:

$$c(u,v) = \frac{f(x,y)}{f(x)f(y)}. \tag{19}$$

Note that the RHS of Eq.(19) is exactly the integrand in the Kullback-Leibler mutual information measure (3). The above equation tells us that the MI measures are equal to the copula densities. As we know that copula density captures the general dependence structure between random variables, the MI-based test we propose can also capture the dependence structure.

Appendix B. Proofs of Theorem II.1

Proof. We want to show that the exceedance dependence measure is invariant under simple standardization. Without loss of generality, we will prove the equality for upper tail dependence measures $\rho^+_{c,o}$ and ρ^+_c in detail. The same proof also works for lower tail dependence measures $\rho^-_{c,o}$ and ρ^-_c.

Under simple standardization

$$X = \frac{R_1 - \mu_1}{\sigma_1} \text{ and } Y = \frac{R_2 - \mu_2}{\sigma_2}. \tag{20}$$

The following equalities hold for the marginal densities g_1, g_2 and f_1, f_2:

$$\begin{aligned} g_1(R_1) &= \frac{1}{\sigma_1} f_1\left(\frac{R_1 - \mu_1}{\sigma_1}\right) = \frac{1}{\sigma_1} f_1(X), \\ g_2(R_2) &= \frac{1}{\sigma_2} f_2\left(\frac{R_2 - \mu_2}{\sigma_2}\right) = \frac{1}{\sigma_2} f_2(Y). \end{aligned}$$

For the joint density g and f, we have

$$g(R_1, R_2) = f\left(\frac{R_1 - \mu_1}{\sigma_1}, \frac{R_2 - \mu_2}{\sigma_2}\right) \cdot |J|,$$

where J is the Jacobian of the transformation, which is defined as

$$J = \begin{bmatrix} \frac{\partial X}{\partial R_1} & \frac{\partial X}{\partial R_2} \\ \frac{\partial Y}{\partial R_1} & \frac{\partial Y}{\partial R_2} \end{bmatrix}.$$

Particularly under the simple standardization (20),

$$\det(J) = \begin{vmatrix} \frac{1}{\sigma_1} & 0 \\ 0 & \frac{1}{\sigma_2} \end{vmatrix} = \frac{1}{\sigma_1 \sigma_2}.$$

Hence,

$$\begin{aligned} \rho_{c,o}^{+} &= \int_{\mu_2 + c\sigma_2}^{+\infty} \int_{\mu_1 + c\sigma_1}^{+\infty} g(R_1, R_2) \log \frac{g(R_1, R_2)}{g_1(R_1) g_2(R_2)} dR_1 dR_2 \\ &= \int_{c}^{+\infty} \int_{c}^{+\infty} |J| \cdot f(X, Y) \log \frac{|J| \cdot f(X, Y)}{\frac{1}{\sigma_1 \sigma_2} \cdot f_1(X) f_2(Y)} \cdot \frac{1}{|J|} dX dY \\ &= \rho_c^{+}. \end{aligned}$$

Similarly, we also have:

$$\rho_{c,o}^{-} = \rho_c^{-}.$$

□

References

Ang, A., and J. Chen. 2002. Asymmetric correlations of equity portfolios. *Journal of Financial Economics* 63:443–94.

Backus, D., M. Chernov, and S. Zin. 2014. Sources of entropy in representative agent models. *The Journal of Finance* 69:51–99.

Cover, T. M., and J. A. Thomas. 2006. *Elements of information theory.* 2 ed. Wiley-Interscience.

Denuit, M., and O. Scaillet. 2004. Nonparametric tests for positive quadrant dependence. *Journal of Financial Econometrics* 2:422–50.

Efron, B. 1982. *The jackknife, the bootstrap, and other resampling plans*, vol. 38 of *CBMS-NSF Regional Conference Series in Applied Mathematics.* SIAM.

Efron, B., and R. J. Tibshirani. 1993. *An introduction to the bootstrap.* New York: Chapman and Hall.

Embrechts, P., A. McNeil, and D. Straumann. 2002. Correlation and dependence in risk management: properties and pitfalls. *Risk management: value at risk and beyond* 176–223.

Epanechnikov, V. A. 1969. Nonparametric estimates of a multivariate probability density. *Theory of Probability and its Applications* 14:153–8.

Giannerini, S., E. Maasoumi, and E. B. Dagum. 2015. Entropy testing for nonlinear serial dependence in time series. *Biometrika* 102:677–93.

Hansen, L. P. 2012. Dynamic valuation decomposition within stochastic economies. *Econometrica* 80:911–67.

Hinkley, D. V., and S. Shi. 1989. Importance sampling and the nested bootstrap. *Biometrika* 76:435–46.

Hong, Y., J. Tu, and G. Zhou. 2007. Asymmetries in stock returns: Statistical tests and economic evaluation. *Review of Financial Studies* 20:1547–81.

Hong, Y., and H. White. 2005. Asymptotic distribution theory for nonparametric entropy measures of serial dependence. *Econometrica* 73:837–901.

Horowitz, J. L. 2001. The bootstrap. *Handbook of Econometrics* 5:3159–228.

Jiang, L., K. Wu, and G. Zhou. 2015. Asymmetry in stock returns: An entropy measure. *Working Paper, Emory University* .

Jondeau, E. 2015. Asymmetry in tail dependence of equity portfolios. *Computational Statistics & Data Analysis* forthcoming.

Kroner, K. F., and V. K. Ng. 1998. Modeling asymmetric comovements of asset returns. *Review of Financial Studies* 11:817–44.

Kullback, S., and R. Leibler. 1951. On information and sufficiency. *The Annals of Mathematical Statistics* 79–86.

Künsch, H. R. 1989. The jackknife and the bootstrap for general stationary observations. *The Annals of Statistics* 17:1217–41.

Li, Q., and J. S. Racine. 2006. *Nonparametric econometrics: Theory and practice*. Princeton University Press.

Longin, F., and B. Solnik. 2001. Extreme correlation of international equity markets. *Journal of Finance* 56:649–76.

Maasoumi, E., and J. S. Racine. 2008. A robust entropy-based test of asymmetry for discrete and continuous processes. *Econometric Reviews* 28:246–61.

Parzen, E. 1962. On estimation of a probability density function and mode. *The Annals of Mathematical Statistics* 33:1065–76.

Patton, A., D. N. Politis, and H. White. 2009. Correction to "automatic block-length selection for the dependent bootstrap". *Econometric Reviews* 28:372–5.

Politis, D. N., and J. P. Romano. 1994. The stationary bootstrap. *Journal of the American Statistical Association* 89:1303–13.

Politis, D. N., and H. White. 2004. Automatic block-length selection for the dependent bootstrap. *Econometric Reviews* 23:53–70.

Racine, J. S. 1997. Consistent significance testing for nonparametric regression. *Journal of Business and Economic Statistics* 15:369–79.

Rilstone, P. 1991. Nonparametric hypothesis testing with parametric rates of convergence. *International Economic Review* 32:209–27.

Robinson, P. M. 1991. Consistent nonparametric entropy-based testing. *Review of Economic Studies* 58:437–53.

Rosenblatt, M. 1956. Remarks on some nonparametric estimates of a density function. *The Annals of Mathematical Statistics* 27:832–7.

Schmidt, R., and U. Stadtmüller. 2005. Non-parametric estimation of tail dependence. *Scandinavian Journal of Statistics* 33:307–35.

Silvapulle, P., and C. W. Granger. 2001. Large returns, conditional correlation and portfolio diversification: a value-at-risk approach. *Quantitative Finance* 1:542–51.

Skaug, H. J., and D. Tjøstheim. 1993. A nonparametric test of serial independence based on the empirical distribution function. *Biometrika* 80:591–602.

Sklar, M. 1959. *Fonctions de répartition à n dimensions et leurs marges.* Université Paris 8.

Støve, B., and D. Tjøstheim. 2012. Measuring asymmetries in financial returns: An empirical investigation using local gaussian correlation. *Working Paper, University of Bergen* .

Støve, B., D. Tjøstheim, and K. O. Hufthammer. 2014. Using local gaussian correlation in a nonlinear re-examination of financial contagion. *Journal of Empirical Finance* 25:62–82.

Tjøstheim, D., and K. O. Hufthammer. 2013. Local gaussian correlation: A new measure of dependence. *Journal of Econometrics* 172:33–48.

Table 1. Failure of the Exceedance Correlation

Panel A. Value of parameters in Distribution (1) and (2)		Panel B. Test on Asymmetries in Distribution (1) and (2)		
a	2.582		Nominal size	Power
b	1.162		10%	0.802
c	0.120	Asy. Dep.	5%	0.703
d	0.594		1%	0.429
e	2.000			
f	4.444		10%	0.092
σ_1	0.250	HTZ	5%	0.041
σ_2	0.250		1%	0.010

Note: Panel A of this table reports the parameter values we choose in distribution (1) and (2). Under this set of parameters, the random variables X and Y have zero means and unit standard deviations. Panel B presents the probabilities of rejecting the null hypothesis using the asymmetric dependence test and bootstrap HTZ test under different nominal sizes, respectively. The rejection probabilities are defined as the relative frequencies of rejection made in 1,000 simulated random samples. For each random sample, the inferences are made based on 199 stationary bootstrap resamplings and the exceedance level $c = 0$ in all scenarios.

Table 2. Asymptotic Size of the Asymmetric Dependence Test

Sample size (T)		1000			1500	
Nominal size	10%	5%	1%	10%	5%	1%
Empirical size	0.103	0.052	0.014	0.100	0.050	0.011

Note: The table reports the probabilities of rejecting the null hypothesis of symmetric exceedance dependence under different nominal sizes, which are estimated based on the statistical inferences made in 1,000 simulated random samples. The random samples are generated by a t copula, which exhibits symmetric tail dependence. For each random sample, the inferences are made based on 199 stationary bootstrap resamplings and the exceedance level $c = 0$ in all scenarios.

Table 3. Size and Power of the Asymmetric Dependence Test in Finite Samples

		Weight on Normal Copula (κ %)				
Sample size (T)	Nominal size	100% (Size)	50%	37.50%	25%	0%
240	10%	0.122	0.455	0.897	0.992	0.999
	5%	0.062	0.319	0.831	0.977	0.999
	1%	0.016	0.121	0.562	0.918	0.978
420	10%	0.110	0.668	0.986	1.000	1.000
	5%	0.056	0.523	0.974	1.000	1.000
	1%	0.011	0.261	0.862	0.997	0.997
600	10%	0.119	0.808	0.999	1.000	1.000
	5%	0.061	0.694	0.997	1.000	1.000
	1%	0.014	0.380	0.961	0.999	1.000

Note: The table reports the probabilities of rejecting the null hypothesis of the symmetric exceedance dependence under different nominal sizes, which are estimated based on the statistical inferences made in 1,000 simulated random samples. All random samples are generated by the mixture copula in Eq.(13), whose degree of asymmetry in the exceedance dependence is governed by the parameter κ. When $\kappa = 1$, Eq.(13) reduces to a Gaussian copula with symmetric tail dependence. In all other cases, Eq.(13) produces distributions with asymmetric tail dependence. For each random sample, the inferences are made based on 199 stationary bootstrap resamplings and the exceedance level $c = 0$ in all scenarios.

Table 4. Size and Power of the Asymmetric Dependence Test Using Different Bandwidth Selection Methods

Panel A. Asymmetric Dependence Test with the LSCV Bandwidth Selection Approach

Sample size (T)	Nominal size	Weight on Normal Copula (κ %)				
		100% (Size)	50%	37.5%	25%	0%
240	10%	0.142	0.434	0.880	0.990	0.998
	5%	0.087	0.323	0.803	0.974	0.995
	1%	0.021	0.121	0.526	0.887	0.934
420	10%	0.135	0.638	0.979	1.000	1.000
	5%	0.066	0.486	0.964	1.000	1.000
	1%	0.015	0.232	0.832	0.992	0.994
600	10%	0.127	0.769	0.998	1.000	1.000
	5%	0.068	0.638	0.996	1.000	1.000
	1%	0.020	0.339	0.950	1.000	1.000

Panel B. Asymmetric Dependence Test with the Plug-in Bandwidth Selection Approach

Sample size (T)	Nominal size	Weight on Normal Copula (κ %)				
		100% (Size)	50%	37.5%	25%	0%
240	10%	0.106	0.458	0.847	0.965	0.999
	5%	0.048	0.320	0.751	0.923	0.996
	1%	0.012	0.119	0.451	0.761	0.961
420	10%	0.090	0.686	0.978	1.000	1.000
	5%	0.046	0.551	0.959	0.999	1.000
	1%	0.009	0.269	0.804	0.981	0.998
600	10%	0.093	0.820	0.997	1.000	1.000
	5%	0.053	0.725	0.994	1.000	1.000
	1%	0.011	0.438	0.941	0.999	1.000

Note: The table reports the probabilities of rejecting the null hypothesis of the symmetric exceedance dependence under different nominal sizes, which are estimated based on the statistical inferences made in 1,000 simulated random samples. All random samples are generated by the mixture copula in eq.(13), whose degree of asymmetry in the exceedance dependence is governed by the parameter κ. When $\kappa = 1$, Eq.(13) reduces to a Gaussian copula with symmetric tail dependence. In all other cases, Eq.(13) produces distributions with asymmetric tail dependence. For each random sample, the optimal bandwidth is obtained using the least square cross validation and plug-in method, respectively. The inferences are made based on 199 stationary bootstrap resamplings and the exceedance level $c = 0$ in all scenarios.

Table 5. Size and Power Comparison: Asymptotic and Bootstrap HTZ Test

HTZ Asymmetric Correlation Test with c=0

		Panel A. Asymptotic Theory					Panel B. Stationary Bootstrap				
Sample	Nominal	Weight on Normal Copula (κ %)					Weight on Normal Copula (κ %)				
size (T)	size	100% (Size)	50%	37.5%	25%	0%	100% (Size)	50%	37.5%	25%	0%
	10%	0.000	0.013	0.204	0.691	0.933	0.101	0.582	0.923	0.989	0.998
240	5%	0.000	0.004	0.083	0.487	0.849	0.040	0.409	0.831	0.960	0.994
	1%	0.000	0.000	0.008	0.166	0.576	0.006	0.130	0.509	0.771	0.931
	10%	0.000	0.031	0.455	0.943	0.992	0.095	0.801	0.994	1.000	0.999
420	5%	0.000	0.008	0.240	0.827	0.971	0.043	0.687	0.981	0.999	0.998
	1%	0.000	0.001	0.031	0.428	0.852	0.007	0.355	0.807	0.980	0.996
	10%	0.000	0.068	0.731	0.998	0.998	0.112	0.910	0.999	1.000	1.000
600	5%	0.000	0.014	0.426	0.969	0.993	0.054	0.827	0.995	1.000	1.000
	1%	0.000	0.000	0.084	0.740	0.960	0.012	0.541	0.942	0.995	0.999

Note: The table reports the probabilities of rejecting the null hypothesis of the symmetric exceedance correlation under different nominal sizes, which are estimated based on the statistical inferences made in 1,000 simulated random samples. All random samples are generated by the mixture copula in Eq.(13), whose degree of asymmetries in exceedance dependence is governed by the parameter κ. When $\kappa = 1$, Eq.(13) reduces to a Gaussian copula with a symmetric tail dependence. In all other cases, Eq.(13) produces distributions with asymmetric tail dependence. For each random sample, the sampling distribution is obtained using both asymptotic theory and 199 stationary bootstrap resamplings. The exceedance level $c = 0$ in all scenarios.

Table 6. Asymmetric Dependence in Returns of Commonly Used Portfolios

Panel A. Value-weighted Size Portfolios

Portfolio	ρ_c^-	ρ_c^+	$\Delta\rho_c$	Asym. Dep.	Boot. HTZ
Size 1	0.317	0.221	0.096	0.000	0.000
Size 2	0.395	0.304	0.091	0.000	0.000
Size 3	0.433	0.363	0.069	0.000	0.000
Size 4	0.437	0.387	0.051	0.005	0.000
Size 5	0.479	0.435	0.044	0.008	0.000
Size 6	0.534	0.485	0.048	0.053	0.010
Size 7	0.584	0.560	0.024	0.514	0.038
Size 8	0.644	0.610	0.034	0.193	0.048
Size 9	0.684	0.715	-0.032	0.286	0.356
Size 10	0.681	0.721	-0.040	0.218	0.150

Panel B. Value-weighted Book-to-Market Portfolios

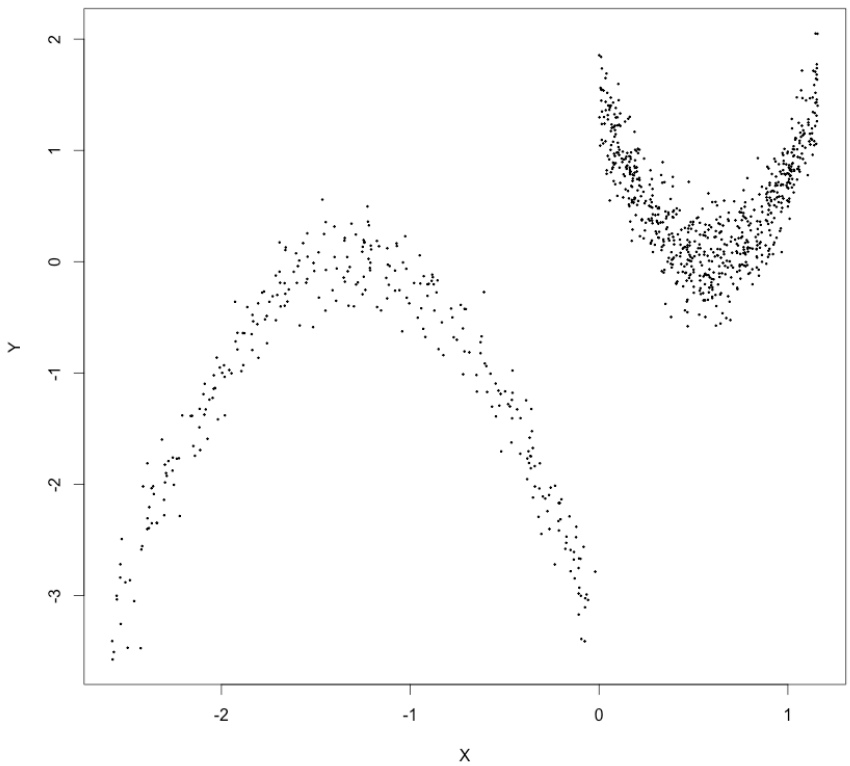
Portfolio	ρ_c^-	ρ_c^+	$\Delta\rho_c$	Asym. Dep.	Boot. HTZ
BE/ME 1	0.458	0.470	-0.012	0.679	0.378
BE/ME 2	0.571	0.577	-0.006	0.799	0.065
BE/ME 3	0.560	0.543	0.017	0.584	0.058
BE/ME 4	0.521	0.515	0.007	0.807	0.125
BE/ME 5	0.452	0.448	0.004	0.920	0.028
BE/ME 6	0.478	0.469	0.009	0.784	0.226
BE/ME 7	0.424	0.421	0.002	0.947	0.158
BE/ME 8	0.410	0.372	0.037	0.203	0.023
BE/ME 9	0.412	0.359	0.053	0.060	0.030
BE/ME 10	0.324	0.267	0.057	0.005	0.033

Panel C. Equal-weighted Momentum Portfolios

Portfolio	ρ_c^-	ρ_c^+	$\Delta\rho_c$	Asym. Dep.	Boot. HTZ
L	0.277	0.190	0.087	0.000	0.000
2	0.345	0.268	0.078	0.000	0.000
3	0.371	0.303	0.068	0.003	0.000
4	0.399	0.332	0.067	0.015	0.000
5	0.390	0.331	0.059	0.015	0.000
6	0.410	0.353	0.057	0.023	0.000
7	0.420	0.353	0.067	0.003	0.000
8	0.409	0.333	0.075	0.003	0.000
9	0.408	0.302	0.106	0.000	0.000
W	0.350	0.270	0.080	0.000	0.000

Note: The table reports the p-values of the asymmetric dependence and bootstrap HTZ test on commonly used portfolios sorted by size, book-to-market, and momentum at the exceedance level 0. The exceedance dependence measures ρ_c^+ and ρ_c^- and the difference $\Delta\rho_c = \rho_c^- - \rho_c^+$ are reported as well. The sample period is from January 1965 to December 2013. P-values of both tests are computed based on 399 stationary bootstrap resamplings.

FIGURE 1. Variables with Symmetric Exceedance Correlation but Asymmetric Comovements



Note: The figure shows the scatter plot for the random variables X and Y, which are drawn independently from the distribution (1) and (2). The parameter values are chosen such that the exceedance correlation between X and Y is symmetric at the level $c = 0$. However, this figure clearly suggests that X and Y have different comovement patterns.