Forecasting Value-at-Risk under Temporal and Portfolio Aggregation

Erik Kole*[1,4], Thijs Markwat[2], Anne Opschoor[3,4], and Dick van Dijk[1,4,5]

[1] *Econometric Institute, Erasmus University Rotterdam*
[2] *Robeco Asset Management*
[3] *VU University Amsterdam*
[4] *Tinbergen Institute*
[5] *Erasmus Research Institute of Management*

December 2015

Abstract

We examine the impact of temporal and portfolio aggregation on the quality of Value-at-Risk (VaR) forecasts over a horizon of ten trading days for a well-diversified portfolio of stocks, bonds and alternative investments. The VaR forecasts are constructed based on daily, weekly or biweekly returns of all constituent assets separately, gathered into portfolios based on asset class, or into a single portfolio. We compare the impact of aggregation to that of choosing a model for the conditional volatilities and correlations, the distribution for the innovations and the method of forecast construction. We find that the degree of temporal aggregation is most important. Daily returns form the best basis for VaR forecasts. Modelling the portfolio at the asset or asset class level works better than complete portfolio aggregation, but differences are smaller. The differences from the model, distribution and forecast choices are also smaller compared to temporal aggregation.

Key words: forecast evaluation, aggregation, Value-at-Risk, model comparison

JEL classification: C22, C32, C52, C53, G17

*Corresponding Author: Erasmus University Rotterdam, P.O. Box 1738, NL-3000 DR Rotterdam, The Netherlands. E-mail addresses are `kole@ese.eur.nl`, `t.markwat@robeco.nl`, `a.opschoor@vu.nl`, and `djvandijk@ese.eur.nl`.

1 Introduction

Value-at-Risk (VaR) for a horizon of ten trading days has become the standard downside risk measure for investment portfolios since the Basel Committee on Banking Supervision (1996), and is widely used in the financial sector (see Berkowitz and O'Brien, 2002). Because price movements can be observed at a higher frequency and for all assets in the portfolio, a risk manager has to choose the degree of aggregation of the observations in the process of constructing the VaR forecast. The typical approach in practice first gathers the assets into asset classes like stocks, bonds, and alternative assets. Next, a model for the daily returns is used to construct a one-step VaR forecast. This forecast is then scaled to the required horizon by the square-root-of-time rule. This scaling of the daily VaR forecast is explicitly advised in the Basel Committee on Banking Supervision (1996) (see also Diebold et al., 1997; Daníelsson and Zigrand, 2006). However, many alternatives with a lower or higher degree of temporal and portfolio aggregation are available.

We investigate the importance of the aggregation choice for the quality of ten-day VaR forecasts. Such a VaR forecast is a quantile of the cumulative return distribution over the coming ten days. We focus on the 1% and 5% quantiles, so VaR forecasts with 99% and 95% confidence levels. To construct this distribution, a risk manager can use different degrees of both temporal and portfolio aggregation. We compare their importance to other choices that he has to make. In particular, we consider different models for the conditional volatilities and correlations of the asset returns, different distributions for the innovations in these models, and different methods to construct cumulative multi-step forecasts (see also McAleer, 2009).

The degree of aggregation determines how detailed the serial and cross-sectional dependence of the returns is modelled. A model for all assets at the highest observed frequency describes very precisely how shocks to one particular asset affect its own future return distributions at different horizons, and those of other assets. However, such an extensive model can be tedious to use in practice. Aggregation of assets into portfolios reduces the dimension of the model, whereas temporal aggregation reduces the number of forecasts that have to be accummulated. It will also reduce the complexity of the model, if serial dependence only has short-term consequences.

Consequently, this choice involves a trade-off between precision and efficiency on the one hand and complexity and the risk of misspecification on the other hand.[1] A low degree of

[1] See the surveys by Bhansali (1999) and Chevillon (2007) for a general discussion of iterated versus direct multi-step forecasts.

aggregation particularly improves precision when different aspects of the return distributions vary. For instance, when the effect of shocks on the volatility is more persistent in one (group of) asset(s) than in another, temporally and cross-sectionally aggregated models will lead to imprecise forecasts. A lower degree of temporal aggregation also increases the efficiency, since more observations are available (keeping the estimation window fixed). The flip side of this choice is the risk of misspecification. More complex models are generally less robust to misspecification. The effects of errors in the specification can be amplified by the forecasting horizon. Small errors in a single-period forecast can build up to a large error in a multi-step forecast.

We study the impact of the degree of aggregation on ten-day VaR forecasts for a well-diversified portfolio of eight indexes, related to equities, bonds, and alternative assets. We vary the degree of temporal aggregation between modelling daily returns, weekly returns and biweekly returns, so from no to full temporal aggregation. In the cross-sectional dimension we analyze aggregation into a single portfolio, aggregation into the three main asset classes (stocks, bonds and alternatives) and no aggregation, so modelling the returns on all eight portfolio constituents. We examine all possible combinations of these different levels of temporal and portfolio aggregation. We do not examine the use of intraday data, because they are generally used in models for daily data (see, for example, Giot and Laurent, 2004; Clements et al., 2008; Brownlees and Gallo, 2010). Instead, we focus on models that use one specific frequency. We leave a comparison of models that use mixed data frequencies with or without intraday data (see also Ghysels et al., 2009) for future research.

We compare the impact of aggregation to that of three other necessary and important choices: the model for the conditional volatilities and correlations, the distribution for the innovations and the method for forecast construction. Regarding the first choice, we examine five well-established, relatively simple models from the GARCH family. We combine the univariate GARCH models of Bollerslev (1986) and Glosten et al. (1993) (GJR) with the CCC model of Bollerslev (1990) or the DCC model of Engle (2002) for the correlations. Hansen and Lunde (2005) and Laurent et al. (2012) show that these models perform just as well as more extensive alternatives (see, for example, Kuester et al., 2006; Brownlees and Gallo, 2010). The RiskMetrics approach from JP Morgan and Reuters (1996), which is popular in practice completes the set of models. These five models allow us to determine the importance of asymmetry in the marginal models on the hand, and of dynamics in the correlations on the other. The RiskMetrics approach offers a one-size-fits-all alternative.

As distributions for the innovations in these models we use a copula approach based on the normal, the Student's t and the empirical distribution. While the failure of the normal distribution for risk measurement has been widely documented since Mandelbrot (1963) and Fama (1965), it remains popular in practice. The other two are promising alternatives.

The method for constructing the forecasts is a relevant issue when returns are modelled at a higher frequency than the forecast horizon. In that case, cumulative multi-step forecasts can be constructed by an iterative procedure or by scaling one-step forecasts.[2] In the iterative procedure, h-step forecasts are constructed by iterating forward the one-period-ahead density for h periods.[3] This procedure captures the path dependence that the models imply, but can be time-consuming. A quicker though maybe less accurate alternative is the scaling of a one-period risk forecast. Scaling by the square-root-of-time is the industry standard, even though it can lead to overestimation of the ten-day VaR (see Diebold et al., 1997) as well as underestimation (see Daníelsson and Zigrand, 2006; Wang et al., 2011).

The combination of these five choice aspects leads to 195 methods to forecast the VaR. We use the tests proposed by Christoffersen (1998) and Engle and Manganelli (2004) to check the accuracy of each method in isolation. We check whether a particular method outperforms another by comparing the values of the asymmetric tick-loss function as in Giacomini and Komunjer (2005) based on the test of Diebold and Mariano (1995) in the framework of Giacomini and White (2006), and construct Model Confidence Sets as proposed by Hansen et al. (2011) to assess the importance of a particular choice. These horse races are based on more than five thousand ten-day VaR forecasts for the period 1994–2014. We use the industry standard of a confidence level of 99%, and use 95% as a robustness check.

Our results show that the degree of temporal aggregation is most important, whereas the degree of portfolio aggregation is less consequential. Working with daily returns leads to 99%- and 95%-VaR forecasts that have correct coverage, and perform significantly better than forecasts based on weekly or biweekly returns. While we also find evidence supporting more detailed multivariate models, in particular for 95%-VaR forecasts, differences here are smaller and in many cases not significant. Our results point at models based on the three main asset classes for 99%-VaR forecasts, and models on the asset level for the 95% case.

When forecasts are constructed by iterating based on daily data, the GJR model of

[2]In specific cases, closed-form expressions for the multi-step distributions can be used. See for example Drost and Nijman (1993) for the GARCH(1,1) model, Hafner (2008) for multivariate GARCH models, and Sbrana and Silvestrini (2013) for the RiskMetrics method.

[3]See Marcellino et al. (2006) for iterated forecasts in an autoregressive framework and Ghysels et al. (2009) for iterated forecasts in a GARCH framework.

Glosten et al. (1993) gives the best forecasts, whereas the distribution choice is of less consequence. When forecasts are constructed from weekly or biweekly data, the distribution choice is important for 99%-VaR forecasts, but not for 95% forecasts. In the 99% case, the empirical (Student's t distribution performs best for weekly (biweekly) data. So, at the (bi)weekly frequency the innovations capture the fat tails and asymmetry, whereas the models incorporate these features at the daily frequency. The method of forecast construction is not that important. Our evidence tends to favor iterating over scaling to form forecasts, but differences vary widely based on the other settings. When combined with a normal distribution, scaling leads to worse performance than iterating. The same holds for combinations with a GJR model. Modelling time-variation in correlation hardly has any consequences at all.

We contribute to several strands of the literature. Closest related to our research are the papers that compare different methods for forecasting VaR. We show the importance of the degree of temporal aggregation in the realistic risk management setting of a ten-day VaR forecast, whereas the literature so far only evaluates one-day VaR forecasts based on either daily data or intraday data.[4] Our results for portfolio aggregation complement McAleer and Da Veiga (2008) and Santos et al. (2013). McAleer and Da Veiga (2008) conclude that VaR forecasts for a small equity-only portfolio are best when full aggregation is applied. We also find that aggregation of the equity part into a single portfolio works well, but that further aggregation of the different asset classes does not improve performance. In line with our findings, Santos et al. (2013) report that multivariate models perform better than univariate models, but they do not consider intermediate degrees of aggregation. The importance of the distribution choice is in line with Giot and Laurent (2004); Kuester et al. (2006); Bao et al. (2006); Clements et al. (2008); McAleer and Da Veiga (2008).

Second, we add to the literature that examines multi-period forecasts of variances and covariances. Contrary to the papers that study VaR forecasts, the papers in this area do investigate temporal aggregation, but they generally evaluate the forecasts based on a symmetric loss function.[5] Using an asymmetric loss function, we add a new perspective.[6] Also when tail loss is important, iterated forecasts are preferred, followed by scaled and

[4] Giacomini and Komunjer (2005); Bao et al. (2006); Kuester et al. (2006); McAleer and Da Veiga (2008); Santos et al. (2013) use daily data, whereas Giot and Laurent (2004); Brownlees and Gallo (2010); Clements et al. (2008) also use intraday data.

[5] See the surveys by Andersen et al. (2006); Patton and Sheppard (2009).

[6] See Elliott and Timmermann (2008) for a general discussion about the loss function in relation to forecasting.

then direct forecasts as in Ghysels et al. (2009). In line with Hansen and Lunde (2005); Brownlees et al. (2011); Laurent et al. (2012) we find that leverage effects in GARCH models are important. Contrary to Laurent et al. (2012), we do not find a preference for DCC over CCC. When tail loss is evaluated, the distribution constitutes an important choice opposite to Brownlees et al. (2011) who evaluate forecasts based on a symmetric loss function.

Third, our research is related to the papers in the large field of economic forecasting that compare iterated with direct forecasts, and forecasts of aggregates with aggregating forecasts. Theoretically, iterated forecasts are more efficient if the model is correctly specified, whereas direct forecasts are more robust to misspecification (see Chevillon, 2007). Empirically, Marcellino et al. (2006) report that iterated forecasts outperform direct ones in an analysis of 170 macro time series, because the models for the higher observation frequency can include more lags. We do not include more lags in models for daily data, but in our case the more precise identification and propagation of shocks in the daily models can explain their outperformance. The same explanation applies to the preference of multivariate models over univariate models that we find. In a macro economic setting, Marcellino et al. (2003) conclude that the aggregation of forecasts works better than forecasting aggregates. In a follow-up of Marcellino et al. (2006), Pesaran et al. (2011) show that multivariate models can also improve the forecasts for single non-aggregated variables.

From a practical perspective, our results show that it is best to work with daily data, and to model on the level of either the assets or the asset classes. A GJR-GARCH model with iterated forecasts further improves performance. For 99%-VaR forecasts in particular, it is best to use the empirical distribution function. However, our results also indicate that the RiskMetrics approach based on scaling forecasts constructed from daily data in combination with the empirical distribution is not so bad after all.

The article proceeds as follows. In Section 2 we discuss the different choice aspects in more detail, and present the design of our research. In Section 3 we present the data. We present the results in Section 4 and conclude in Section 5. Appendices provide additional information on the methods and results.

2 Methodology

We take the perspective of a risk manager who is concerned with the risk of a portfolio of n assets over the coming h time periods, during which the portfolio composition does not change. She observes the asset prices at discrete points in time $t = 1, 2, \ldots$. We can approximate the log portfolio return from t to $t+h$ by

$$r^{\mathrm{p}}_{t,h} = \sum_{\tau=1}^{h} \boldsymbol{w}'\boldsymbol{r}_{t+\tau}, \tag{1}$$

where $\boldsymbol{w}$ and $\boldsymbol{r}_t$ are $n \times 1$ vectors that contain the portfolio weights and the one-period log returns. We concentrate on a fully invested long-only portfolio, so we assume $w_i \geq 0$, $i = 1, 2, \ldots, n$ and $\sum_{i=1}^{n} w_i = 1$, though these assumptions are not crucial for our analysis. The approximation holds reasonably well when the variance of the portfolio return is not too large, which holds for our case of a ten-day horizon and long-only portfolios.

The risk of the portfolio is measured in terms of Value-at-Risk. The h-period VaR with confidence level α of the portfolio return $r^{\mathrm{p}}_{t,h}$ is defined as

$$\mathrm{VaR}_\alpha(r^{\mathrm{p}}_{t,h}) = -\sup\{l : \Pr[r^{\mathrm{p}}_{t,h} \leq l | \mathcal{F}_t] \leq 1-\alpha\}, \tag{2}$$

where $\mathcal{F}_t$ denotes the information set at time t. In line with the Basel accords with which financial institutions must comply, we use $\alpha = 99\%$ and $h = 10$ days. Additionally, we consider $\alpha = 95\%$.

Value-at-Risk is negative of the $(1-\alpha)$-quantile of the conditional distribution of $r^{\mathrm{p}}_{t,h}$. To forecast this distribution, the risk manager has to make a number of choices. We investigate the consequences of five different choice aspects. The first two choices concern the degree of temporal and portfolio aggregation. Together, they establish the basic return variable $\boldsymbol{r}^{\mathrm{b}}_{t,k}$ that will be modelled. Third, she has to decide on the time series model to describe the return dynamics. Fourth, she has to choose a distribution for (the innovation in) $\boldsymbol{r}^{\mathrm{b}}_{t,k}$. Fifth, she has to determine how to construct the distribution of $r^{\mathrm{p}}_{t,h}$ from the model for $\boldsymbol{r}^{\mathrm{b}}_{t,k}$. We discuss these five aspects below.

2.1 Choosing the degree of aggregation

The first choice determines the degree of aggregation of the nh random variables that constitute $r^{\mathrm{p}}_{t,h}$. Aggregation can be done both temporally and cross-sectionally in this setting. Temporal aggregation means that k-period returns form the basis of the model instead of single-period returns. Cross-sectional aggregation means a reduction of the portfolio dimension by grouping assets together in m basic portfolios. So instead of the single-period

n-dimensional returns $\boldsymbol{r}_t$, the k-period returns $\boldsymbol{r}^{\mathrm{b}}_{t,k}$ with dimension $m \leq n$ are the basis of the model. Assuming k is a divisor of h, we replace Equation (1) by

$$r^{\mathrm{p}}_{t,h} = \sum_{\tau=1}^{h/k} \boldsymbol{w}^{\mathrm{b}\prime} \boldsymbol{r}^{\mathrm{b}}_{t+(\tau-1)k,k}, \tag{3}$$

where $\boldsymbol{w}_{\mathrm{b}}$ contains the weights on the m basic portfolios, and $\boldsymbol{r}^{\mathrm{b}}_{t,k} = \sum_{\tau=1}^{k} \boldsymbol{r}^{\mathrm{b}}_{t+\tau}$. To construct the basic portfolios we define an $m \times n$ selection matrix $\boldsymbol{S}$, whose element $s_{ij} = 1$ if asset j is part of portfolio i and zero otherwise. Row i indicates which assets constitute portfolio i. Since each asset should be part of exactly one portfolio, all columns of $\boldsymbol{S}$ should sum to one. The weights $\boldsymbol{w}^{\mathrm{b}}$ satisfy $\boldsymbol{w}^{\mathrm{b}} = \boldsymbol{S}\boldsymbol{w}$. The returns on the basic portfolios can be constructed from the asset returns by a weight matrix $\widetilde{\boldsymbol{W}}$

$$\boldsymbol{r}^{\mathrm{b}}_t = \widetilde{\boldsymbol{W}} \boldsymbol{r}_t \quad \text{with } \tilde{w}_{ij} = s_{ij} w_j / \sum_{k=1}^{n} s_{ik} w_k.$$

Row i contains the weights of the assets in basic portfolio i, scaled so they sum to one.

When $k = 1$ and $m = n$ the risk manager aggregates in neither dimension, and models the asset returns in the most detailed way. When $k = h$ she fully aggregates over time, and constructs a one-step ahead forecast of the return distribution. When $m = 1$, the cross-section is fully aggregated, and she models the portfolio return as a univariate random variable. Other choices of k and m reduce the scale of the model, but the distribution of $r^{\mathrm{p}}_{t,h}$ remains the result of a sum of multivariate return distributions.

For temporal aggregation we investigate the cases of no ($k = 1$), weekly ($k = 5$) and full (biweekly) aggregation ($k = h = 10$). For cross-sectional aggregation we analyze the case of no ($m = n$), full ($m = 1$), and aggregation by asset class.

2.2 Choosing the time-series model

The financial econometrics literature has established that the conditional distribution of asset returns is not constant over time. For the time-varying mean of returns (V)ARMA models have been proposed. The family of GARCH models have become the standard for modeling the volatility of returns. Because the number of models has grown very large, we restrict our attention to a couple of relatively simple models that have have performed well.[7]

[7] See Bauwens et al. (2006) and Silvennoinen and Teräsvirta (2009) for an overview of (multivariate) GARCH models. Hansen and Lunde (2005); Laurent et al. (2012) show that relatively simple models are

We specify all models in a multivariate setting, and only discuss the univariate version if necessary.

We model the mean of the return distribution by a VAR(p) model

$$\boldsymbol{r}^{\mathrm{b}}_{t,k} = \boldsymbol{\phi} + \sum_{i=1}^{p} \boldsymbol{\Gamma}_i \boldsymbol{r}^{\mathrm{b}}_{t-ik,k} + \boldsymbol{\varepsilon}_{t,k}, \quad \mathrm{E}_t[\boldsymbol{\varepsilon}_{t,k}] = \mathbf{0},\ \mathrm{E}_t[\boldsymbol{\varepsilon}_{t,k}\boldsymbol{\varepsilon}'_{t.k}] = \boldsymbol{\Sigma}_{t,k} \tag{4}$$

where $\boldsymbol{\phi}$ is an m-vector, $\boldsymbol{\Gamma}_i$, $i = 1, \dots p$ are $m \times m$ matrices, and $\boldsymbol{\varepsilon}_{t,k}$ is an m-vector with innovations over the period from t to $t + k$. Conditional on the information available at time t, the innovations have expectation zero and variance $\boldsymbol{\Sigma}_{t,k}$. When no temporal aggregation takes place and returns are daily, we choose $p = 1$. Assets may be traded in different time zones, and a VAR(1) model can accommodate the resulting lagged cross-sectional dependence. In the cases of weekly and fortnightly aggregation, serial dependence in level of returns becomes negligible, so we use a VAR(0) model, that is, we assume a constant mean.

Because (co)variances have a crucial impact on risk measures, we investigate several specifications from the class of multivariate GARCH models. The literature offers a wide variety of models that differ in the effects that shock have on the variance matrix. We are particularly interested in the importance of asymmetric effects of shocks, and the importance of time-varying correlations. Moreover, estimation should remain feasible when no cross-sectional aggregation is applied. Therefore, we focus on GARCH and GJR-GARCH models combined with CCC and DCC, and the RiskMetrics approach. Based on the evidence in Laurent et al. (2012), we do not consider the DCC extensions proposed by Tse and Tsui (2002); Cappiello et al. (2006); Engle and Kelly (2012).

CCC models in the style of Bollerslev (1990) and DCC models in the style of Engle (2002) build the variance matrix $\boldsymbol{\Sigma}_{t,k}$ from m univariate models for the variances $\sigma^2_{i,t,k}$, and a separate model for the correlation matrix $\boldsymbol{R}_{t,k}$,

$$\boldsymbol{\Sigma}_{t,k} = \mathrm{diag}(\boldsymbol{\sigma}_{t,k}) \boldsymbol{R}_{t,k} \, \mathrm{diag}(\boldsymbol{\sigma}_{t,k}), \tag{5}$$

where diag returns a diagonal matrix with the argument vector on the diagonal, and $\boldsymbol{\sigma}_{t,k}$ contains the volatilities $\sigma_{i,t,k}$. We define standardized innovations

$$\eta_{i,t,k} = \varepsilon_{i,t,k} / \sigma_{i,t,k},\ i = 1, \dots, m, \tag{6}$$

not outperformed by more extensive alternatives.

that are still correlated, and uncorrelated innovations $\zeta_{i,t,k}$ that satisfy

$$\boldsymbol{\eta}_{t,k} = \boldsymbol{R}_{t,k}^{1/2}\boldsymbol{\zeta}_{t,k}, \quad \mathrm{E}_{t-k}[\boldsymbol{\zeta}_{t,k}\boldsymbol{\zeta}_{t,k}'] = \boldsymbol{I}, \tag{7}$$

where $\boldsymbol{\eta}_{t,k}$ and $\boldsymbol{\zeta}_{t,k}$ are the m-vectors with elements $\eta_{i,t,k}$ and $\zeta_{i,t,k}$, and $\boldsymbol{R}_{t,k}^{1/2}$ denotes the lower triangular matrix that results from the Choleski decomposition of $\boldsymbol{R}_{t,k}$. We discuss the distributions for the innovations in the next subsection.

The univariate models for $\sigma_{i,t,k}^2$ belong to the GARCH family, in particular the standard GARCH model of Bollerslev (1986), and the extension proposed by Glosten et al. (1993), referred to as GJR-GARCH. The latter can be written as

$$\sigma_{i,t,k}^2 = \omega_i + \big(\alpha_i + \gamma_i I(\varepsilon_{i,t-k,k} < 0)\big)\varepsilon_{i,t-k,k}^2 + \beta_i \sigma_{i,t-k,k}^2, \; i = 1, \ldots, m, \tag{8}$$

where $\omega_i, \alpha_i, \gamma_i, \beta_i > 0$ are parameters, and I is the indicator function that returns 1 if the statement between parentheses is true and zero otherwise. Assuming that the conditional distribution of $\varepsilon_{i,t,k}$ is symmetric, the variance is stationary when $\alpha_i + \frac{1}{2}\gamma_i + \beta_i < 1$. Shocks have an asymmetric effect on the volatility when $\gamma_i \neq 0$. The standard GARCH model results when $\gamma_i = 0$.

The CCC model uses a constant correlation matrix $\boldsymbol{R}_{t,k} = \boldsymbol{R}_k$. The DCC model contains a dynamic specification for the correlation matrix. First, a quasi-correlation matrix $\boldsymbol{Q}_{t,k}$ is constructed from

$$\boldsymbol{Q}_{t,k} = \bar{\boldsymbol{Q}}_k(1 - \alpha_\mathrm{q} - \beta_\mathrm{q}) + \alpha_\mathrm{q}\boldsymbol{\eta}_{t-k,k}\boldsymbol{\eta}_{t-k,k}' + \beta_\mathrm{q}\boldsymbol{Q}_{t-k,k}, \tag{9}$$

where $\alpha_\mathrm{q}, \beta_\mathrm{q} > 0$ are parameters that should satisfy $\alpha_\mathrm{q} + \beta_\mathrm{q} < 1$. $\bar{\boldsymbol{Q}}_k$ is the unconditional correlation matrix of the standardized innovations. While $\boldsymbol{Q}_t$ is positive definite by construction, its diagonal elements are not necessarily equal to one. The correlation matrix $\boldsymbol{R}_{t,k}$ results from scaling $\boldsymbol{Q}_{t,k}$,

$$\boldsymbol{R}_{t,k} = \mathrm{diag}(\boldsymbol{Q}_{t,k})^{-1/2}\boldsymbol{Q}_{t,k}\,\mathrm{diag}(\boldsymbol{Q}_{t,k})^{-1/2}. \tag{10}$$

Finally, we analyze the RisMetrics model, which is widely used in practice. This model directly gives the evolution of the variance matrix,

$$\boldsymbol{\Sigma}_{t,k} = \boldsymbol{\Sigma}_0 + (1-\lambda)\boldsymbol{\varepsilon}_{t-k,k}\boldsymbol{\varepsilon}'_{t-k,k} + \lambda\boldsymbol{\Sigma}_{t-k,k}, \tag{11}$$

where $\boldsymbol{\Sigma}_0$ is a constant matrix and $0 \leq \lambda \leq 1$ is the decay parameter (see JP Morgan and Reuters, 1996 and Mina and Xiao, 2001). Based on Zaffaroni (2008) we include $\boldsymbol{\Sigma}_0$ in the specification. Because the dynamics in this model are determined by a single parameter λ, the variances of all basic returns and the covariances between all pairs of basic returns share the same behavior over time. A comparison with the other models can show how well this one-size-fits-all approach performs.

2.3 Choosing the distribution

The third choice concerns the distribution of the innovations $\boldsymbol{\varepsilon}_{t,k}$. We examine three choices for this distribution, being the normal distribution, the Student's t distribution, and the empirical distribution. We briefly discuss these options, and the implications of their combination with the models of the previous subsection.

The first option uses the multivariate normal distribution, which enjoys wide popularity, despite its exponentially declining tails. Because it is closed under summation, temporal and cross-sectional aggregation of forecasted distributions is straightforward. The combination of the normal distribution with univariate GARCH models and a constant correlation matrix was proposed by Bollerslev (1990). Replacing the constant correlation matrix by the dynamics in Equations (9) and (10) is the DCC model as in Engle (2002). The normal distribution combined with the RiskMetrics model produces the classical RiskMetrics approach (see Mina and Xiao, 2001).

The Student's t-distribution is a popular alternative for the normal distribution, because it has fatter tails.[8] To enhance flexibility, we use a univariate Student's t-distribution for the marginal distribution of each $\varepsilon_{i,t,k}$, with a specific degrees of freedom parameter ν_i, and model the dependence by the Student's t-copula. We transform the univariate pdf to have expectation μ and variance σ^2,

$$\tilde{\psi}(z;\mu,\sigma^2,\nu) = \frac{\Gamma((\nu+1)/2)}{\Gamma(\nu/2)\sqrt{(\nu-2)\pi}\sigma}\left(1+\frac{(z-\mu)^2}{(\nu-2)\sigma^2}\right)^{-\frac{\nu+1}{2}}, \ \nu > 2 \tag{12}$$

[8] Though the literature proposes many alternatives with fatter tails than the normal distribution, we restrict ourselves to the Student's t-distribution because of its good performance reported by Mittnik and Paolella (2000); Giot and Laurent (2004); Bao et al. (2006); Kuester et al. (2006).

where ν is the degrees of freedom, and Γ is the Gamma function. We combine these marginal models with the Student's t-copula,

$$C_{\psi}(v_1, \ldots, v_m; \boldsymbol{\Omega}, \nu) = \Psi_m(\Psi^{-1}(v_1; \nu), \ldots, \Psi^{-1}(v_m; \nu); \boldsymbol{\Omega}, \nu), \ \nu > 2 \tag{13}$$

where $\boldsymbol{\Omega}$ is a correlation matrix, ν the degrees of freedom parameter, Ψ_m the cdf of the (unscaled) multivariate Student's t-distribution, and Ψ^{-1} the inverse of the cdf of the (unscaled) univariate Student's t-distribution. Together, the marginal models and the copula have $m+1$ degrees of freedom parameters, so the degrees of freedom parameters of the marginal models and the copula can all be different. The combination of this copula approach with the DCC specification in Equations (9) and (10) is closely related to one of the models in Jondeau and Rockinger (2006). When the RiskMetrics approach is selected, we use a multivariate Student's t-distribution for $\boldsymbol{\varepsilon}_{t,k}$,

$$\tilde{\psi}_m(\boldsymbol{z}; \boldsymbol{\mu}, \boldsymbol{\Sigma}, \nu) = \frac{\Gamma((\nu+m)/2)}{\Gamma(\nu/2)((\nu-2)\pi)^{m/2}\sqrt{|\boldsymbol{\Sigma}|}} \left(1 + \frac{(\boldsymbol{z}-\boldsymbol{\mu})'\boldsymbol{\Sigma}^{-1}(\boldsymbol{z}-\boldsymbol{\mu})}{(\nu-2)}\right)^{-\frac{\nu+m}{2}}, \quad \nu > 2, \tag{14}$$

where $|\boldsymbol{\Sigma}|$ denotes the determinant of $\boldsymbol{\Sigma}$. This specification is again scaled to ensure $\text{Var}[\boldsymbol{z}] = \boldsymbol{\Sigma}$.

As the third option we propose the empirical distribution. For the CCC and DCC specifications, we use it for the marginal distributions of the standardized innovations $\eta_{i,t,k}$ in Equation (6) in combination with the Gaussian copula. For a set of P realizations in the vector $\boldsymbol{x}$, the empirical cumulative probability function is given by

$$F_P^{\text{emp}}(z; \boldsymbol{x}) = \frac{1}{P+1}\sum_{\tau=1}^{P} I(x_\tau \leq z). \tag{15}$$

The Gaussian copula is given by

$$C_{\phi}(v_1, \ldots, v_m; \boldsymbol{\Omega}) = \Phi_m(\Phi^{-1}(v_1), \ldots, \Phi^{-1}(v_m); \boldsymbol{\Omega}), \tag{16}$$

where $\forall i, v_i \in [0,1]$, $\boldsymbol{\Omega}$ is a correlation matrix, Φ_m the cdf of the m-variate normal distribution, and Φ^{-1} the inverse of the cdf of the univariate normal distribution. This design corresponds with the SCOMDY models in Chen and Fan (2006). In a univariate setting this approach reduces to Filtered Historical Simulation (see also Christoffersen, 2009). When the RiskMetrics-specification is used, the empirical distribution applies to $\zeta_{i,t,k}$ as defined in Equation (7). We do not assume that $\zeta_{i,t,k}$ and $\zeta_{j,t,k}$ are independent for $i \neq j$, and use the multivariate empirical distribution

$$F^{\text{emp}}_{P,m}(\boldsymbol{z};\boldsymbol{X}) = \frac{1}{P+1}\sum_{\tau=1}^{P}\prod_{i=1}^{m} I(x_{i,\tau} \leq z_i), \tag{17}$$

where $\boldsymbol{z}$ is an m-vector and $\boldsymbol{X}$ is a $m \times P$ matrix that contains the set of P observations.

2.4 Choosing the method of constructing forecasts

The final choice relates to the construction of forecasts. The risk manager has to choose how to construct a multi-period forecast, when she has not applied full temporal aggregation to the basic returns. When $h = k$ she constructs a one-step-ahead forecast, and we call this forecast "direct". When $k < h$, she can choose between "iterated" and "scaled" forecasts. Iterated forecasts take into account the path dependence that results from the serial dependence in the return models. The forecasted distribution for $\boldsymbol{r}_{t+jk,k}$, $j \geq 1$ is constructed based on the forecasted distribution for the previous horizon, $\boldsymbol{r}_{t+(j-1)k,k}$. Generally, the convolution of these distributions does not have a closed-form expression, which means that simulations have to be used. Scaling forecasts implies the assumption that the distributions of $\boldsymbol{r}_{t,k}$ and $\boldsymbol{r}_{t,h}$ have the same shape, but the location and scale parameters of the aggregated distribution $\boldsymbol{r}_{t,h}$ are adjusted for the horizon h. When VAR-effects in Equation (4) are absent, the mean and variance are scaled by a factor h/k. Formulated for volatility, this is the well-known "square root of time" rule, that is advocated in the Basel Accords. We discuss the technical aspects of the forecast construction in Appendix B.2, and show in AppendixB.3 how to include the serial dependence implied by the VAR in Equation (4) when scaled forecasts are used.

Iterated forecasts can be more precise but also more time-consuming than scaled forecasts. When portfolios are large, iterated forecasts may become computationally infeasible, which explains the popularity of scaling in practice. However, Diebold et al. (1997); Daníelsson and Zigrand (2006); Wang et al. (2011) shows that scaled forecasts become worse when the horizon increases. For some of the models we consider, for instance the one-dimensional GARCH or RiskMetrics specification, multi-step ahead forecasts can be constructed in closed form (see Drost and Nijman, 1993 for the first and Sbrana and Silvestrini, 2013 for the second), but generally this is not the case. Therefore, scaling remains a relevant forecasting method.

Ghysels et al. (2009) show that MIDAS forecasts dominate other forecasting methods for horizons longer than 10 days (see also Ghysels et al., 2006). Because our forecasting horizon is shorter, we do not investigate the performance of MIDAS techniques.

2.5 Estimation and Empirical Design

We estimate the parameters in all the different models with a multi-step procedure. First, we estimate the VAR-parameters in Equation (4) by ordinary least squares. The VAR residuals are the input for the next estimation step. For the CCC and DCC models we estimate the parameters of the marginal models in Equation (8) (GARCH, or GJR-GARCH) separately for each basic portfolio by (Quasi) Maximum Likelihood (QML). We construct standardized residuals as in Equation (6), and use these as the basis of our Maximum Likelihood estimation of the (unconditional) correlation matrix, and if needed the parameters of the DCC model in Equation (9), and the degrees of freedom in Equation (13). For the RiskMetrics approach, we estimate the parameters in Equation (11) by (Q)ML. We provide a more detailed description in Appendix B.1.

The estimation procedure for the CCC and DCC models is the common approach that is advocated in the literature (see Bollerslev, 1990; Engle, 2002; Chen and Fan, 2006). For copulas this multi-step estimation is referred to as the Inference Functions for Margins method (see Joe, 1997, Ch. 10), or Multi-Stage Maximum Likelihood (see Patton, 2013). Originally, the decay factor λ of the RiskMetrics models is determined by minimizing the root mean squared error (see JP Morgan and Reuters, 1996), but Zaffaroni (2008) shows that this approach is unreliable, and advocates QML instead.

Each forecast is based on a moving window of 1,000 days (so approximately four years of data). When we apply temporal aggregation ($k > 1$), we aggregate the daily observations to 200 weekly ($k = 5$) or 100 biweekly ($k = 100$) observations. We reestimate all model parameters, when 50 days have passed, and assume that the parameters do not change at intermediate points in time. When simulations are needed, we use 10,000 draws.

2.6 Evaluation

At each point in time t, we construct an out-of-sample Value-At-Risk forecast for the portfolio return $r^{\mathrm{p}}_{t,h}$ over the next h days for all combinations of choices. We refer to each choice combination as a method (cf. Giacomini and White, 2006), and denote the forecast by method j as $q_t^j \equiv -\,\mathrm{VaR}_\alpha(r_{t,h}^{\mathrm{p}})$. We evaluate the quality of each forecasting method itself, and compare them with each other. To establish the quality we use two tests, being an unconditional coverage test as in Christoffersen (1998); Kupiec (1995) and the dynamic quantile test in the style of Engle and Manganelli (2004).[9] We rank the methods based on a loss function, and test whether loss differentials are significant in the style of Diebold and Mariano (1995); Giacomini and White (2006). We also use the loss function to determine the Model Confidence Set of Hansen et al. (2011) for different sets of methods.

The unconditional coverage (UC) test of Christoffersen (1998); Kupiec (1995) tests whether the actual fraction of VaR-violations is equal to the predicted proportion of $1-\alpha$. To implement it, we define the VaR violation

$$x_{t+h}^j \equiv I(r_{t,h}^{\mathrm{p}} \leq q_t^j), \tag{18}$$

where I denotes again the indicator function. If VaR method j works correctly, $\mathrm{E}[x_{t+h}^j] = 1-\alpha$. We test this hypothesis against the alternative $\mathrm{E}[x_{t+h}^j] > 1-\alpha$, as regulators care mostly about too many violations.

The dynamic quantile test (DQ) test of Engle and Manganelli (2004) tests whether the VaR-violations are predictable by past information. If VaR method j works correctly, the violation x_{t+h}^j should be unrelated to the forecasted Value-at-Risk q_t^j. Because of the overlap in the forecasting horizon, violations x_{t+h}^j and x_{t+h+1}^j are related by construction. Therefore, we only include q_t^j in the test. We implement the test by a logistic regression as in Berkowitz et al. (2011),

$$\Pr[x_{t+h}^j = 1|\mathcal{F}_t] = \Lambda(c_{0j} + c_{1j} q_t^j), \tag{19}$$

where Λ is the logistic function. We test the hypothesis $c_{1j} = 0$.

To compare two methods with each other, we apply the test of Diebold and Mariano (1995) (DM-test) in the framework of Giacomini and White (2006). We need their framework, because some of the methods that we compare are nested (for example, the GARCH models are nested in the GJR models, and the CCC models in the DCC models). Additionally, it allows for parameter estimation uncertainty. Our use of a rolling window to estimate the model parameters fits in this framework. We also apply the procedure for constructing

[9] The recent quantile-regression based tests as proposed by Gaglianone et al. (2011) has not been extended to the situation of overlapping observations.

a Model Confidence Set as in Hansen et al. (2011, Sec. 3.1.2).

For both tests, we use the asymmetric tick loss function as in Giacomini and Komunjer (2005)

$$L_\alpha(e^j_{t+h}) = (1 - \alpha - I(e^j_{t+h} < 0))e^j_{t+h}, \tag{20}$$

where $e^j_{t+h} \equiv r^{\mathrm{p}}_{t,h} - q^j_t$. Because of the asymmetry of the loss function, realizations below q^j_t, that is, VaR violations, lead to a larger loss. Next, we define the loss differential between methods j and j' as

$$d^{j,j'}_{\alpha,t} \equiv L_\alpha(e^j_t) - L_\alpha(e^{j'}_t). \tag{21}$$

We test $\mathrm{E}[d^{j,j'}_{\alpha,t}] = 0$ by the statistic proposed by Diebold and Mariano (1995). When the null-hypothesis is rejected and $\mathrm{E}[d^{j,j'}_{\alpha,t}] < 0$, method j outperforms method j'.

Because we make ten-day VaR forecasts for every day in our sample, all tests in this section suffer from overlapping data. Therefore, we use the procedure of Newey and West (1987) with a Bartlett kernel and 14 leads and lags in the calculation of variances and standard errors. We use a significance level of 5% in our tests, unless stated otherwise.

3 Data

We base our empirical analysis on a well-diversified investment portfolio. This portfolio invests 50% in bonds, 30% in equities, 10% in real estate and the remaining 10% in commodities. The investments in bonds are split over US government bonds (30%) and US corporate bonds (20%). The equity part can be further divided based on regions: Europe (12%), United States (10.5%), Pacific (3.75%) and Emerging Markets (3.75%). Appendix A provides details about our used indexes. Our sample covers the period January 1, 1990–February 28, 2014. We delete holidays, giving a sample size of 6021 daily observations.[10]

The summary statistics for daily returns in Table 1 indicate substantial differences between bonds and the other assets. Bonds show a considerably lower return than the other asset classes. The lowest average return comes from Pacific equities (-0.50% per year), the highest from US equities. Bond volatilities range from 4.48% to 5.40% per year, whereas

[10] Because we analyse an international portfolio, we define a holiday as day when the values of two or more indexes do not change. Because of the dominance of the US in the portfolio, most holidays are US public or government holidays.

the volatilities of the other classes range from 16.57% to 21.71%. Almost all series exhibit negative skewness (between -0.18 and -0.61), except the Pacific equity series. Excess kurtosis is present in each return series (ranging from 2.48 to 10.18). Consequently, the Jarque-Bera test rejects the hypothesis of normality for all series. The significance of Ljung-Box statistics indicates significant autocorrelation for all series. The autocorrelation is largely driven by the first lag and is positive for all series, except for US equity and for commodity returns.

[Table 1 about here.]

Because we are interested in the effects of temporal and portfolio aggregation on predictive densities in general and risk in particular, we calculate summary statistics for the portfolio as a whole, and for weekly and biweekly returns. Of course, the average portfolio return corresponds directly with a weighted average of the returns of each asset class. The volatilities of the portfolio returns shows the effects of diversification, as it is much lower than the non-bond classes. However, diversification breaks down when extreme returns occur. The portfolio returns are stronger left skewed than almost all constituent series, and the kurtosis coefficient is close to the maximum among them. Clustering over time is also stronger for the portfolio, with a high first order autocorrelation coefficient and a high Ljung-Box statistic.

Using weekly or biweekly returns instead of daily returns has a small effect on the volatility of the series. If returns were i.i.d., the scaled volatilities would be independent of the interval over which the returns are observed. When returns exhibit positive (negative) autocorrelation, the volatility of aggregated returns will be higher (lower). For Emerging Market equities, real estate, and the portfolio as a whole, we see an increase in volatility, which implies positive autocorrelation. For the other classes, volatilities go down, implying negative autocorrelation.

The coefficients for skewness and kurtosis show large deviations from the values that would result if daily returns were i.i.d. Lau and Wingender (1989) derive that when k i.i.d. returns are aggregated, the skewness coefficient is scaled by $1/\sqrt{k}$ and the coefficient of excess kurtosis by $1/k$. Instead of a decrease for skewness, the left skewness of all series increases for biweekly and weekly returns compared to daily returns. The kurtosis coefficients for weekly returns decrease, but much less than for the i.i.d. case. These results indicate that extreme returns, and in particular negative extreme returns tend to cluster. Also, the hypothesis of a normal distribution is rejected for (bi)weekly data.

The statistics for the portfolio as a whole show that extreme and negative returns tend to cluster over time and in the cross section. The volatility estimates do not vary much with the frequency of observation, and are considerably lower than the average volatilities of the asset classes. When returns are aggregated, the degree of left skewness goes up instead of down, and the coefficient of excess kurtosis does not decrease either. Hence, while scaling daily or weekly volatilities to biweekly volatilities may work, scaling the density seems to be incorrect, in particular when risk measures are needed. The results for the portfolio returns show that cross-sectional aggregation does not mitigate this effect.

We show the correlations between the different asset classes at different frequencies in Table 2. We see a clear block structure in the correlation matrix. Corporate bonds and government bonds are highly correlated with each other, but correlations with the other assets is slightly negative. The equity returns are also highly correlated with each other and with real estate. Commodities is a category on its own. Aggregating the assets into the larger categories of bonds, equities and alternatives corresponds with this block structure.

[Table 2 about here.]

Correlations that are calculated with weekly or a biweekly returns generally show the same picture as the correlations for daily data, though some equity correlations show a substantial increase. This applies in particular to all correlations with Pacific Equities, reflecting that daily returns are not synchronized.

We conclude from this initial analysis that asset returns show several forms of temporal dependence. Linear dependence over time, as measured by autocorrelation, is small but significant. Scaling volatility is therefore not correct, but deviations are generally not very large. The results for skewness and kurtosis on the other hand are more troublesome. Instead of going down, both measures increase, both for temporal and for portfolio aggregation. These increases indicate that extreme, and in particular extreme negative returns tend to cluster. As a consequence, risks are larger for a (bi)weekly than for a daily horizon. Simply scaling daily risk measures might therefore not be appropriate.

4 Results

Our analysis boils down to a large-scale horse race. We analyze the effect of three temporal aggregation options (daily, weekly and biweekly), three portfolio aggregation options (no aggregation, aggregation into the three main asset classes, and aggregation into a single portfolio), five model options (CCC-GARCH, DCC-GARCH, CCC-GJR, DCC-GJR and

RiskMetrics), three distribution options (normal, empirical, and Student's t), and two forecasting options (iterated and scaling). The forecasting options are available when daily or weekly returns are used. For biweekly returns only a one-step-ahead forecast is needed. In total this leads to 195 different methods. We evaluate the quality of the resulting VaR forecasts in different ways, where we focus on the impact of the different choices. We do not aim at finding the specific combination of options that beats all others. We examine the VaR forecasts with a 99% confidence level in detail, and briefly consider the forecasts with the 95% confidence level.

4.1 VaR forecasts with a 99% confidence level

4.1.1 Absolute forecast quality

We first investigate how the five different choices impact the coverage of the VaR forecasts in Tables 3a to 3e, and test equality to the theoretical 1% level against the one-sided alternative of more violations. Too many violations can lead to penalties for a financial institution, whereas too few violations may be undesirable, but are less likely to cause problems. To show how the different choices influence the results, we report the total number of rejections for a particular significance level in Table 3f. The interpretation of this summary panel deserves some care, because the tests are not independent.

[Table 3 about here.]

[Table 3 (continued) about here.]

The different panels in Table 3 show that temporal aggregation leads to more violations. For daily returns combined with iterated forecasts (see Table 3a), one method out of 39 leads to less violations than the theoretical 1%, and 12 lead to violations that significantly exceed 1% with p-values below 0.05. The violation frequencies vary between 0.78 and 1.87%. For biweekly aggregation (Table 3e), 30 methods lead to significantly more violations than 1%, though some methods still end up close to it, as the frequencies range from 1.10 to 2.75%. The results for weekly returns lie in between. Together, the results indicate that less temporal aggregation, so a more detailed observation of the time-series dynamics, leads to better coverage ratios.

The relation of portfolio aggregation with the coverage generally shows a U-shape, though with some exceptions. A split into three basic assets, so modelling the main asset classes, leads to the lowest violation frequency with a minimum of 1.00% and a maximum of 2.31%. Forty out of 75 fractions deviate significantly from 1%. Aggregating the assets into a single portfolio generally leads to larger frequencies, ranging from 0.78% to 2.75%, with 31 out of 45 being significant. Modelling all asset also leads to frequencies (range 1.15–2.33%), with 45 out of 75 being significant.

The model choice shows some interesting results. First, the one-size-fits-all approach of RiskMetrics works quite well, both univariately and multivariately, with 9 out of 15 and 13 out of 30 rejections at the 0.05 level, and with coverage ratios ranging from 0.78 to 2.29%. Second, replacing GARCH by GJR, so including an asymmetric effect of shocks, improves the coverage ratios, and also leads to 9 rejections in the univariate setting. Exchanging the CCC and DCC models does not influence the coverage ratios much. The combination DCC-GJR gives results that are at par with RiskMetrics, with only 15 out of 30 fractions significant at the 0.05 level, and coverage ratios in the range 1.00–2.31%. However, the differences in coverage are relatively small. In case of the CCC-GARCH model they range from 1.12 to 2.55%. Though the performance of the RiskMetrics and DCC-GJR models is similar, the RiskMetrics model works particularly well when combined with scaling, whereas the DCC-GJR model works better in combination with iterated forecasts.

The choice for the distribution impacts coverage substantially. The empirical distribution comes first, the Student's t-distribution second, and the normal distribution last. When daily returns are combined with iterated forecasts (see Table 3a), the differences between the distributions are not that large, but in Tables 3b to 3e the differences are considerable. Table 3f confirms this result. When the empirical distribution is used, only 9 out of 65 fractions are significant at the 5% level, compared to no less than 50 for the Student's t and even 57 for the normal distribution. Moreover, when the violation frequencies exceed 1%, those of the empirical distribution are the lowest.

The choice between scaled or iterated forecasts particularly influences the results when daily data is used. Scaling a forecast from one to ten steps ahead generally leads to worse coverage ratios than constructing iterated forecasts. This effect holds particularly when the dynamics are specified in more detail, and are largest for the DCC-GJR model. When RiskMetrics is used constructing scaled or iterated forecasts does not matter much. Differences are also smaller for weekly data. Because the variance process in the RiskMetrics method is not mean-reverting, this result is not surprising.

Next, we apply the dynamic quantile (DQ) test based on Equation (19) to investigate whether the forecast quality varies with the magnitude of the forecast. Table 4 shows that only 19 of the 195 estimated coefficients are significant at the 0.10 level, so the null-hypothesis of no relation between a VaR forecast and a violation is generally not rejected. Nevertheless, the estimates and in particular their signs give useful insights into the forecast quality. A positive sign means that risk is overestimated in times of distress, because a larger than average VaR forecast (a value for q_t^j below average) decreases the probability of a violation. Risk is then underestimated for quiet times. For negative signs, the interpretation is reversed. We report the total number of positive and negative signs in Table 4f.

[Table 4 about here.]

[Table 4 (continued) about here.]

Temporal aggregation has a clear effect on the forecast quality. The use of daily returns leads to a slight underestimation of risk in times of distress for 34 methods, but to overestimation for only five. These results indicate that models based on daily returns generally revert a bit too fast to the steady-state volatility. A closer inspection of Table 4a shows that the five positive coefficients are all for forecasts based on RiskMetrics, where mean-reversion is absent. The negative coefficients are mostly insignificant, but a richer GARCH specification may improve the forecasts. For weekly returns, the number of positive and negative coefficients are about equal. For biweekly returns all coefficients are positive, and 13 out of 39 are significant. Here, mean reversion is too slow, and the evidence is somewhat stronger.

Full portfolio aggregation leads more often to positive than to negative coefficients (27 versus 18), but this pattern reverses for less aggregation. When all assets are considered, 50 methods lead to negative coefficients and 25 to positive ones. When three basic assets are used, positive and negative coefficients are more balanced. Also in the cross-sectional dimension, methods with a low degree of aggregation lead more often to an underestimation of VaR in times of distress.

The results for the different models show again the effect of mean reversion. In the univariate case, we find more evidence for overestimation of VaR in times of distress, compared to the multivariate cases. A more precise identification of the nature of the shocks may help, but it is not clear which specification is best, as the differences between the four multivariate GARCH models are quite small. The asymmetry captured by the GJR model has small effects on iterated forecasts (daily or weekly) but leads to underestimation of risk in times of distress when forecasts are scaled. The difference between CCC and DCC models is generally larger when eight assets are considered than three. In the CCC models correlation is constant, so it leads more often to underestimation than the DCC models. The RiskMetrics model, which does not imply mean reversion, leads to overestimation.

Overall, we conclude that the results of the DQ-test are mostly related to the mean reversion implied by the models. When mean reversion is relatively strong, the DQ-test indicates a slight underestimation of risk in times of distress, though mostly insignificant. This effect is clearest when daily data and/or CCC models are chosen. When mean reversion is weaker and shocks become more persistent, the DQ-tests signal overestimation in times of distress. Significance is a bit stronger, though the majority of methods show insignificant coefficients. The differences between the distributions are small. All show about as many positive as negative coefficients. The same conclusion applies to iteration versus scaling.

Summarizing, we find that temporal aggregation has a large impact on the forecast quality. Using weekly or biweekly data leads to too frequent violations, though more often during quiet times. Portfolio aggregation has less impact. Modelling on the level of asset classes gives the best results for coverage and time-variation. The model choice is also of less importance. Differences between the combinations of CCC/DCC and GARCH/GJR methods are small, but still indicate a preference for the least restricted DCC-GJR method. Differences with RiskMetrics are somewhat larger. The choice for the distribution is again more important. In particular distributions with fat tails lead to better coverage ratios. The forecasting method is important when daily data are used, with iterated forecasts outperforming scaled forecasts.

We conclude that more detailed information on shocks as provided by daily data on the asset or asset class level, and a more detailed specification of their propagation in the form of a richer model and iterated forecasts, improve forecast quality. Increased aggregation means that sometimes shocks are missed, whereas the one-size-fits-all approach of the RiskMetrics methods, and the scaling of forecasts lead to a too strong extrapolation of the effects of shocks. On the other hand, we find that detailed information, models and forecasting increase the risk of forecast errors. So far, the benefits of an increased level of detail outweigh these errors.

4.1.2 Relative forecast quality

We now turn to a statistical comparison of the different methods. For each method, we calculate the average asymmetric tick loss as in Equation (20), reported in Table 5. We then test whether the loss of two methods is equal using the DM-test. To limit the number of comparisons, we restrict the tests to methods that differ in only one aspect. We report a summary of the results in Table 6, and refer to appendix C.1 for the full set of results.

[Table 5 about here.]

[Table 6 about here.]

Comparing the losses in Table 5 for different degrees of temporal aggregation clearly indicate a preference for detailed observations. A higher degree of temporal aggregation leads to a higher average loss. For a considerable number of tests, the difference is significant (see Table 6a). The loss differential for daily compared to weekly observations is significantly negative at the 10% level for 13 our of 39 tests, of which five are significant at the 5% level. It does not matter whether forecasts are iterated or scaled. Results are stronger when comparing daily to biweekly observations: 26 (15) out of 39 loss differentials are significant at the 10% (5%) level when forecasts are iterated, and 22 (15) when scaled. Using weekly observations also beats using biweekly observations with 35 (22) significant test outcomes at the 10% (5%) level for iterated forecasts and 29 (17) for scaled forecasts.

We also see a preference for a low degree of portfolio aggregation, though less convincing than for temporal aggregation. Modelling at the asset level generally gives the lowest losses. For 51 (56) out of 75 times, modelling the assets separately produces a lower loss than aggregation into asset classes (a single portfolio). However, only few differentials are significant: modelling eight assets leads to six (13) loss differentials that are significantly lower at then 10% level than modelling three asset classes (the portfolio as a whole), and one (four) that are significantly higher. Modelling at the asset class level leads to lower losses than modelling at the portfolio level in 47 cases (of which five are significant), and to higher loses in 28 cases (with six being significant). These results are consistent with the preference for multivariate models reported by Santos et al. (2013).

When it comes to model choice, some striking results stand out. First, the average losses for the GJR-models are lower than for the GARCH models. From the in total 145 comparisons, this applies to 140 tests, of which 30 and 16 are significant at the 10 and

5% level. So, the asymmetric effect in the GJR model improves the quality of the VaR forecasts. The DCC models perform only slightly better than the CCC models. The average loss differential favors the DCC models 36 times (with one being significant) compared to 24 times the CCC model (with 2 being significant). Finally, the CCC/DCC-GARCH/GJR models lead to lower losses than the RiskMetrics models for 138 out of 150 tests. Table 5 shows that RiskMetrics only produces lower losses when compared with a univariate GARCH or GJR-model (10 and 2 out of 15 tests). So in particular in a multivariate setting, the one-size-fits-all approach of RiskMetrics is too inflexible.

The results for the choice of distribution confirm the well-known failure of the normal distribution. The empirical and the Student's t-distributions lead to average losses that are lower than the normal distribution in 52 and 60 out of 65 tests. However, only a limited number of tests provide significant evidence against the normal distribution, indicating that the loss differentials are typically close to zero. The preference for the empirical or the Student's t-distribution is less clear-cut, as 40 tests show a preference for the empirical and 25 a preference for the Student's t-distribution, though most outcomes are not significant. A more detailed inspection of the tests does not show a particular pattern related to the temporal or portfolio aggregation, or the chosen model.

The tests for the forecasting method show that for daily observations iterated forecasts lead to lower losses than scaled forecasts for 27 out of 39 tests, with 6 (2) being significant at the 10% (5%) level. For weekly data, tests are undecided, as iterated forecasts lead to lower losses in 19 tests (with 6 being significant), and scaled forecasts in 20 tests (with none being significant). Of course, scaling a forecasts to ten steps ahead is likely to differ more from an iterated forecast than scaling to two steps.

Our results so far are based on pairwise comparisons. Though the set of test outcomes in Table 6 gives an indication of the importance of the different choices, the tests are not independent. To properly account for the dependence, we construct a Model Confidence Set as proposed by Hansen et al. (2011), based again on the asymmetric tick loss function in Equation (20). We start with all methods and subsequently remove the method whose average loss is significantly larger than for all other methods. We use a critical value of 10%, so all methods with an MCS p-value below 10% are removed. We present the results in Table 7.

[Table 7 about here.]

The results confirm the importance of the observation frequency. None of the methods based on daily observations are removed, neither when iterated nor when scaled forecasts are constructed. A lower frequency leads to more removals: 11 for weekly observations combined with iterated forecasts, 14 for the combination with scaled forecasts, and 25 for biweekly observations. So, less temporal aggregation clearly leads to better results. Moreover, when daily observations are used, the loss differentials that result from the other choices do not lead to significant differences. Also when we start the MCS procedure with 78 daily methods, no methods are removed from this set.

Using observations at the asset level, or to a lesser extent at the asset class level, can compensate for the low detail in the temporal dimension. For the portfolio level, 23 out of the 27 weekly/biweekly methods are removed from the MCS. For the asset class level we find that 18 out of 45 methods are removed, whereas for the asset level this declines further to nine. Still, the degree of portfolio aggregation is less consequential than the degree of temporal aggregation.

The results for the model choice are in line with the outcomes in Table 6. From the 50 removed methods, twenty methods use RiskMetrics. Twelve of the removed methods use the CCC-GARCH model, indicating that this approach can be improved by the asymmetric effect in the GJR-specification, and the dynamic dependence of the DCC models. The results from the MCS construction indicate that correlation dynamics are somewhat more important than the asymmetry of the GJR-specification, contrary to our previous analysis. Overall, richer specifications help capturing relevant aspects of the data.

The differences between the distributions are again small. While more methods with the empirical distribution have been removed (23, compared to 12 for the normal and 15 for the Student's t-distribution), these additional removals are concentrated in the biweekly frequency. Hundred observations are apparently too few to approximate the true distribution by the empirical distribution. At the weekly frequency, 200 observations are available, and the number of removals is about equal for each distribution.

4.2 95% VaR forecasts

We repeat our analyses for VaR forecasts with a confidence level of 95%. We focus here on the main results, and provide the full set of results in Appendix C.2. In the discussion below, we always use the test outcomes at the 5% significance level, except for the results of the Model Confidence Set, which uses again the 10% significance level.

In terms of absolute forecasting quality, the different methods show better coverage ratios than for the 99% VaR forecasts. Based on their 95% VaR-forecasts, only 32 methods (out of 195) are rejected, compared to 116 based on their 99% VaR forecasts. This drop in the number of rejections is concentrated in the methods that use the normal distribution, as they go down from 57 to only 7. For the Student's t-distribution, the number of rejections decreases from 50 to 22, indicating that the normal distribution now leads to better coverage. When we use a significance level of 10%, the empirical distribution outperforms both the normal and Student's t-distribution with 8 compared to 34 and 43 rejections.

The rejections based on the 95% VaR forecasts are more concentrated in the methods that use biweekly observations (18 out of 32), which further strengthens the preference for less time aggregation. The options for the portfolio aggregation and for the model are more ore less equally affected, and show similar drops in the number of rejections.

The DQ-tests do again not find much evidence of time variation of the VaR exceedances. Based on the 95% VaR forecasts, 22 (0) coefficients are significantly positive (negative), compared to 14 (5) based on the 99% VaR forecasts. However, we find more positive coefficients for the 95% case (145 compared to 88). This finding indicates more overestimation of risk in times of distress. The differences with the 99% case may point at changes in the shape of the distributions, but we do not investigate this issue in more detail.

The pairwise comparison of the different methods based on their average tick loss leads to more insights, because the average tick losses are estimated with more precision. The DM-tests find significantly lower losses for methods that use daily data. For example, methods based on daily data with iterated forecasts lead to an average loss that lies significantly below methods based on weekly data with iterated forecasts 13 times, compared to 5 times based on 99% Var forecasts, and below methods based on biweekly data 29 times, compared to 15.

The DM-tests also indicate a stronger preference for less portfolio aggregation. Modelling on the asset level leads more often to lower losses, and to more loss differentials that are significant. The average loss resulting from modelling on the asset level is now 5 (15) times significantly lower than modelling on the asset class (portfolio) level, compared to 0 (11) for the 99% Var forecasts.

The loss differentials between methods that differ in the distribution that they use become smaller and significant for less instances. Based on the 99% VaR forecasts, the normal distribution leads to higher losses than the Student's t-distribution 60 times out of 65. For the 95% VaR forecasts this number drops to 35. The choice for the distribution becomes less relevant.

The results of comparing the model choices and the forecast aggregation do not change much. The models with the asymmetric GJR-specification perform better than symmetric GARCH, the difference between models with constant or dynamic correlations are small, and the RiskMetrics model leads to higher losses than the other models. Scaled forecasts lead more often to lower losses than iterated foreacsts, but the differentials are mostly insignificant.

The Model Confidence Set becomes smaller when based on 95% VaR forecasts, because 93 out of 195 methods have been removed, compared to only 50 based on the 99% forecasts. These removals are concentrated in the methods that use a lower data frequency. Of the 39 methods with biweekly data (weekly data, scaled / iterated forecasts), 38 (31 / 21) are now removed, compared to 25 (14 / 11) based on the 99% VaR forecasts. From the 39 methods with daily data and iterated (scaled) forecasts, only 2 (1) methods are removed. The Model Confidence Set does not show a clear preference for the other choices.

We conclude that the degree of the time aggregation is the most important choice, both for 99% and 95% VaR forecasts. Using daily data leads to the best coverage and the lowest losses. The degree of portfolio aggregation is less consequential. Modelling the portfolio as a whole leads to the worst results, whereas the differences between modelling at the asset or asset class level are smaller. The importance of the model choice depends on the choices for aggregation. When daily observations for the assets or asset classes are used, models that precisely specify the effects of shocks are clearly preferable. When the observations are more aggregated, the model choice matters less. The choice for the distribution is important for 99% VaR forecasts, with the empirical and Student's t-distribution outperforming the normal distribution. For the 95% VaR forecasts, the distribution matters less. For daily data, iterated forecasts perform better than scaled forecasts, but for weekly data, differences are small.

5 Conclusion

In this paper we investigate the importance of the degree of aggregation for forecasting 99% and 95% ten-day ahead Value-at-Risk (VaR). We analyze the effect of aggregating daily returns to weekly and biweekly returns, and aggregation of assets into the main asset classes and into a single portfolio. We compare the impotance of aggregation to those of choosing a time-series model (asymmetric or symmetric GARCH, constant or time-varying correlation, or the RiskMetrics approach), choosing a distribution (the Gaussian, Student's t, or the empirical distribution), and choosing a method for forecast construction (iterated or scaled).

Our main finding is that the degree of temporal aggregation is the most important choice. Modelling daily returns leads to better VaR forecasts than modelling weekly or biweekly returns. This result shows that the dynamics in the distribution of returns is best captured at the daily level. A higher degree of aggregation leads to a loss of important details. The chosen model and distribution also matter for the forecasts. When daily returns are chosen, the model is the second-most important choice. Because the dynamics are present in great detail, the model should accurately capture them. Because weekly or biweekly returns obscure the return dynamics, the chosen distribution becomes more important than the model.

We find that multivariate models outperform univariate models, confirming Santos et al. (2013). However, the improvement in forecasting quality that results from mutlivariate models is not that large, and often not significant. The impact of portfolio aggregation is less than the degree of temporal aggregation, the model and distribution choice. Iterating forecasts are better than scaled and direct forecasts. This holds in particular for the 99% VaR estimates. Scaled forecasts generally perform better than direct forecasts. This last finding are in line with Ghysels et al. (2009).

Based on our results, we advise to construct models on the asset class level based on daily data. The model would rather contain asymmetric effects on the marginal level, but correlation dynamics are not consequential. The distribution should allow for fat tails, in particular when forecasting VaR at the 99% level. It is also best to iterate forecasts. However, applying the RiskMetrics approach with scaling does make forecasts that much worse.

References

Andersen, T. G., Bollerslev, T., Christoffersen, P. F., and Diebold, F. X. (2006). Volatility and correlation forecasting. In Elliot, G., Granger, C. W., and Timmermann, A., editors, *Handbook of Economic Forecasting*, volume 1, chapter 15, pages 777–878. Elsevier North Holland, Amsterdam, Netherlands.

Bao, Y., Lee, T.-H., and Saltoglu, B. (2006). Evaluating predictive performance of value-at-risk models in emerging markets: A reality check. *Journal of Forecasting*, 25(2):101–128.

Basel Committee on Banking Supervision (1996). Amendment to the capital accord to incorporate market risks. Technical Report Committee Report 24, Basel Committee on Banking Supervision, Basel, Switzerland.

Bauwens, L., Laurent, S., and Rombouts, J. (2006). Multivariate GARCH models: a survey. *Journal of Applied Econometrics*, 21:79–109.

Berkowitz, J., Christoffersen, P., and Pelletier, D. (2011). Evaluating value-at-risk models with desk-level data. *Management Science*, 57(12):2213–2227.

Berkowitz, J. and O'Brien, J. (2002). How accurate are value-at-risk models at commercial banks? *Journal of Finance*, 57(3):1093–1111.

Bhansali, R. (1999). Parameter estimation and model selection for multistep prediction of a time series. In Ghosh, S., editor, *Asymptotics, Nonparametrics, and Time Series*, Statistics: textbooks and monographs, chapter 6, pages 201–225. Marcel Dekker, New York.

Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics*, 31:307–327.

Bollerslev, T. (1990). Modelling the coherence in short-run nominal exchange rates: A multivariate generalized arch model. *Review of Economics and Statistics*, 72(3):498–505.

Brownlees, C. T., Engle, R. F., and Kelly, B. T. (2011). A practical guide to volatility forecasting through calm and storm. *Journal of Risk*, 14(2):1–20.

Brownlees, C. T. and Gallo, G. M. (2010). Comparison of volatility measures: a risk management perspective. *Journal of Financial Econometrics*, 8(1):29–56.

Campbell, J. Y. and Viceira, L. M. (2005). Long-horizon mean-variance analysis: User guide. *Financial Analysts Journal*, 61(1):Supplement, 29 pages.

Cappiello, L., Engle, R. F., and Sheppard, K. (2006). Asymmetric dynamics in the correlations of global equity and bond returns. *Journal of Financial Econometrics*, 4(4):537–572.

Chen, X. and Fan, Y. (2006). Estimation and model selection of semiparametric copula-based multivariate dynamic models under copula misspecification. *Journal of Econometrics*, 135:125–154.

Chevillon, G. (2007). Direct multi-step estimation and forecasting. *Journal of Economic Surveys*, 21(4):746–785.

Christoffersen, P. (2009). Value-at-risk models. In Andersen, T. G., Davis, R. A., Kreiss, J.-P., and Mikosch, T., editors, *Handbook of Financial Time Series*, pages 753–766. Springer.

Christoffersen, P. F. (1998). Evaluating interval forecasts. *International Economic Review*, 39(4):841–862.

Clements, M. P., Galvão, A. B., and Kim, J. H. (2008). Quantile forecasts of daily exchange rate returns from forecasts of realized volatility. *Journal of Empirical Finance*, 15(4):729–750.

Daníelsson, J. and Zigrand, J. (2006). On time-scaling of risk and the square-root-of-time rule. *Journal of Banking & Finance*, 30(10):2701–2713.

Diebold, F., Hickman, A., Inoue, A., and Schuermann, T. (1997). Converting 1-day volatility to h-day volatility: Scaling by $\sqrt{n}$ is worse than you think. *Wharton Working Paper*, 97-34.

Diebold, F. X. and Mariano, R. S. (1995). Comparing predictive accuracy. *Journal of Business & Economic Statistics*, 13(3):134–144.

Drost, F. and Nijman, T. (1993). Temporal aggregation of GARCH processes. *Econometrica*, 61:909–927.

Elliott, G. and Timmermann, A. (2008). Economic forecasting. *Journal of Economic Literature*, 46(1):3–56.

Engle, R. and Kelly, B. (2012). Dynamic equicorrelation. *Journal of Business & Economic Statistics*, 30(2):212–228.

Engle, R. F. (2002). Dynamic conditional correlation - a simple class of multivariate generalized autoregressive conditional heteroskedasticity models. *Journal of Business & Economics Statistics*, 20(3):339–350.

Engle, R. F. and Manganelli, S. (2004). CAViaR: Conditional autoregressive value at risk by regression quantiles. *Journal of Business & Economic Statistics*, 22(4):367–381.

Fama, E. F. (1965). The behavior of stock market prices. *Journal of Business*, 38(1):34–105.

Gaglianone, W. P., Lima, L. R., Linton, O., and Smith, D. R. (2011). Evaluating Value-at-Risk models via quantile regression. *Journal of Business & Economic Statistics*, 29(1):150–160.

Ghysels, E., Rubia, A., and Valkanov, R. (2009). Multi-period forecasts of volatility: Direct, iterated, and mixed-data approaches. Working paper, Rady School of Management, University of California San Diego.

Ghysels, E., Santa-Clara, P., and Valkanov, R. (2006). Predicting volatility: Getting the most out of return data sampled at different frequencies. *Journal of Econometrics*, 131:59–95.

Giacomini, R. and Komunjer, I. (2005). Evaluation and combination of conditional quantile forecasts. *Journal of Business & Economic Statistics*, 23(4):416–431.

Giacomini, R. and White, H. (2006). Tests of conditional predictive ability. *Econometrica*, 74(6):1545–1578.

Giot, P. and Laurent, S. (2004). Modelling daily value-at-risk using realized volatility and ARCH type models. *Journal of Empirical Finance*, 11(3):379–398.

Glosten, L. R., Jagannathan, R., and Runkle, D. E. (1993). On the relation between the expected value and the volatility of the nominal excess return on stocks. *Journal of Finance*, 48(5):1779–1801.

Hafner, C. M. (2008). Temporal aggregation of multivariate garch processes. *Journal of Econometrics*, 142(1):467–483.

Hansen, P. R. and Lunde, A. (2005). A forecast comparison of volatility models: does anything beat a GARCH (1, 1)? *Journal of Applied Econometrics*, 20(7):873–889.

Hansen, P. R., Lunde, A., and Nason, J. M. (2011). The model confidence set. *Econometrica*, 79(2):453–497.

Joe, H. (1997). *Multivariate Models and Dependence Concepts*, volume 73 of *Monographs on Statistics and Applied Probability*. Chapman & Hall, London, UK.

Jondeau, E. and Rockinger, M. (2006). The copula-GARCH model of conditional dependencies: An international stock market application. *Journal of International Money and Finance*, 25:827–853.

JP Morgan and Reuters (1996). Riskmetrics–technical document. Technical report, Morgan Guaranty Trust Company and Reuters Ltd.

Kuester, K., Mittnik, S., and Paolella, M. S. (2006). Value-at-risk prediction: A comparison of alternative strategies. *Journal of Financial Econometrics*, 4(1):53–89.

Kupiec, P. (1995). Techniques for verifying the accuracy of risk measurement models. *The Journal of Derivatives*, 3:73–84.

Lau, H.-S. and Wingender, J. R. (1989). The analytics of the intervaling effect on skewness and kurtosis of stock returns. *The Financial Review*, 24(2):215–233.

Laurent, S., Rombouts, J. V., and Violante, F. (2012). On the forecasting accuracy of multivariate garch models. *Journal of Applied Econometrics*, 27(6):934–955.

Mandelbrot, B. (1963). The variation of certain speculative prices. *Journal of Business*, 36(4):394–419.

Marcellino, M., Stock, J. H., and Watson, M. W. (2003). Macroeconomic Forecasting in the Euro Area. *European Economic Review*, 47:1–18.

Marcellino, M., Stock, J. H., and Watson, M. W. (2006). A comparison of direct and iterated multistep ar methods for forecasting macroeconomic time series. *Journal of Econometrics*, 135:499–526.

McAleer, M. (2009). The ten commandments for optimizing value-at-risk and daily capital charges. *Journal of Economic Surveys*, 23(5):831–849.

McAleer, M. and Da Veiga, B. (2008). Single-index and portfolio models for forecasting value-at-risk thresholds. *Journal of Forecasting*, 27(3):217.

Mina, J. and Xiao, J. Y. (2001). Return to RiskMetrics: The evolution of a standard. Technical report, RiskMetrics, New York, NY, USA.

Mittnik, S. and Paolella, M. S. (2000). Conditional density and value-at-risk prediction of Asian currency exchange rates. *Journal of Forecasting*, 19(4):313–333.

Newey, W. K. and West, K. D. (1987). A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix. *Econometrica*, 55(3):703–708.

Patton, A. (2013). Copula methods for forecasting multivariate time series. In Elliott, G. and Timmermann, A., editors, *Handbook of Economic Forecasting*, volume 2, chapter 16, pages 899–960. Elsevier North Holland, Amsterdam, Netherlands.

Patton, A. J. and Sheppard, K. (2009). Evaluating volatility and correlation forecasts. In Andersen, T., Davis, R., Kreiss, J.-P., and Mikosch, T., editors, *Handbook of Financial Time Series*, pages 801–838. Springer-Verlag, Berlin, Heidelberg.

Pesaran, M. H., Pick, A., and Timmermann, A. (2011). Variable selection, estimation and inference for multi-period forecasting problems. *Journal of Econometrics*, 164(1):173–187.

Santos, A. A. P., Nogales, F. J., and Ruiz, E. (2013). Comparing univariate and multivariate models to forecast portfolio value-at-risk. *Journal of Financial Econometrics*, 11(2):400–441.

Sbrana, G. and Silvestrini, A. (2013). Aggregation of exponential smoothing processes with an application to portfolio risk evaluation. *Journal of Banking & Finance*, 37(5):1437–1450.

Silvennoinen, A. and Teräsvirta, T. (2009). Multivariate GARCH models. In Andersen, T. G., Davis, R. A., Kreiss, J.-P., and Mikosch, T., editors, *Handbook of Financial Time Series*, pages 201–229. Springer-Verlag, Berlin, Heidelberg.

Tse, Y. K. and Tsui, A. K. C. (2002). A multivariate generalized autoregressive conditional heteroscedasticity model with time-varying correlations. *Journal of Business & Economic Statistics*, 20(3):351–362.

Wang, J.-N., Yeh, J.-H., and Cheng, N. Y.-P. (2011). How accurate is the square-root-of-time rule in scaling tail risk: A global study. *Journal of Banking & Finance*, 35(5):1158–1169.

Zaffaroni, P. (2008). Large-scale volatility models: theoretical properties of professionals' practice. *Journal of Time Series Analysis*, 29(3):581–599.

Table 1: Summary Statistics of Asset and Portfolio Returns

	Govt. bonds	Corp. bonds	EU equities	US equities	Pacific equities	EM equities	real estate	commo-dities	portfolio
Weight (in %)	30	20	12	10.5	3.75	3.75	10	10	100
(a) daily returns									
Mean (% p.a.)	-0.02	0.45	5.24	7.06	-0.50	6.24	3.16	4.00	2.38
Volatiliy (% p.a.)	4.48	5.40	20.18	18.35	21.24	19.15	16.57	21.71	6.87
Skewness	-0.18	-0.27	-0.14	-0.30	-0.01	-0.54	-0.50	-0.61	-0.59
Excess kurtosis	2.19	2.48	9.24	8.43	5.27	10.18	10.13	8.73	9.45
Jarque-Bera	1240	1610	21485	17963	6974	26308	26026	19527	22796
ACF(1)	0.022	0.004	0.007	-0.065	0.015	0.212	0.137	-0.012	0.154
ACF(2)	-0.031	-0.012	-0.039	-0.023	-0.031	0.044	0.035	0.004	0.008
ACF(3)	-0.018	-0.008	-0.022	0.010	-0.016	0.018	0.026	-0.002	0.016
Ljung-Box	46.80	35.30	77.76	87.67	49.94	371.1	176.8	39.64	205.0
(b) weekly returns									
Volatiliy (% p.a.)	4.42	5.33	19.73	17.11	20.98	22.73	18.74	21.55	7.74
Skewness	-0.25	-0.39	-0.73	-0.66	-0.20	-0.80	-1.05	-0.60	-0.98
- implied by daily	-0.08	-0.12	-0.06	-0.13	0.00	-0.24	-0.22	-0.27	-0.26
Excess kurtosis	0.85	2.04	5.01	6.10	2.42	5.59	8.79	3.30	7.56
- implied by daily	0.44	0.50	1.85	1.69	1.05	2.04	2.03	1.75	1.89
Jarque-Bera	49.59	262.2	1729	2103	329.4	1927	4235	661.3	3645
ACF(1)	-0.064	-0.027	-0.079	-0.103	-0.052	0.029	0.010	-0.032	0.008
ACF(2)	0.058	0.090	0.025	0.045	0.030	0.092	0.058	0.043	0.067
Ljung-Box	30.31	40.13	50.31	42.63	31.42	54.99	53.71	37.13	48.96
(c) biweekly returns									
Volatiliy (% p.a.)	4.27	5.26	18.93	16.20	20.42	23.05	18.82	21.20	7.77
Skewness	-0.28	-0.64	-1.06	-0.97	-0.21	-0.97	-1.35	-0.68	-1.45
- implied by weekly	-0.18	-0.27	-0.51	-0.47	-0.14	-0.56	-0.74	-0.42	-0.69
- implied by daily	-0.06	-0.08	-0.04	-0.09	0.00	-0.17	-0.16	-0.19	-0.19
Excess kurtosis	0.66	2.73	5.41	6.47	2.34	4.22	9.28	2.92	10.63
- implied by weekly	0.42	1.02	2.50	3.05	1.21	2.80	4.40	1.65	3.78
- implied by daily	0.22	0.25	0.92	0.84	0.53	1.02	1.01	0.87	0.95
Jarque-Bera	22.87	235.8	1004	1483	188.2	587.5	2635	285.4	3538
ACF(1)	0.05	0.11	0.00	-0.01	0.02	0.13	0.09	0.07	0.09
Ljung-Box	25.74	27.55	29.57	23.66	21.73	42.97	45.06	28.32	36.60

Means and volatilities for daily returns in panel(a) are annualized assuming 250 trading days. ACF(q) denotes the autocorrelation for lag q. Ljung-Box statistics are calculated for 20 lags. Critical values for 95% and 99% confidence levels are 31.4 and 37.6 based on a χ^2_{20} distribution. The statistics in panel (b) are the averages of the statistics for each day of the week. The statistics in panel (c) are the averages of the statistics for each of the ten possible starting days. Implied coefficients of skewness and of excess kurtosis for the aggregation of k returns are calculated by scaling the non-aggregated coefficients by $1/\sqrt{k}$ and $1/k$, based on Lau and Wingender (1989).

Table 2: Correlations of Asset Returns

	Govt. bonds	Corp. bonds	EU equities	US equities	Pacific equities	EM equities	Real estate	Commo-dities
				(a) daily returns				
Govt. bonds	1	0.91	-0.15	-0.16	-0.02	-0.14	-0.13	-0.14
Corp. bonds		1	-0.08	-0.06	0.06	-0.02	-0.04	-0.12
EU equities			1	0.51	0.12	0.43	0.56	0.14
US equities				1	0.38	0.63	0.61	0.25
Pac equities					1	0.53	0.52	0.10
EM equities						1	0.62	0.22
Real estate							1	0.19
Commodities								1
				(b) weekly returns				
Govt. bonds	1	0.85	-0.11	-0.10	-0.06	-0.14	-0.05	-0.15
Corp. bonds		1	0.05	0.06	0.07	0.05	0.12	-0.07
EU equities			1	0.74	0.59	0.73	0.74	0.25
US equities				1	0.46	0.64	0.67	0.18
Pacific equities					1	0.61	0.67	0.19
EM equities						1	0.71	0.25
Real estate							1	0.23
Commodities								1
				(c) biweekly returns				
Govt. bonds	1	0.80	-0.10	-0.10	-0.07	-0.13	-0.02	-0.16
Corp. bonds		1	0.15	0.16	0.13	0.15	0.24	-0.03
EU equities			1	0.77	0.62	0.73	0.75	0.26
US equities				1	0.53	0.67	0.69	0.18
Pacific equities					1	0.63	0.68	0.22
EM equities						1	0.71	0.27
Real estate							1	0.25
Commodities								1

The correlations in panel (b) are the averages of the correlations based on weekly returns for each day of the week. The correlations in panel (c) are the averages of the correlations based on biweekly returns for each of the ten possible starting days.

Table 3: Empirical coverage of the VaR forecasts

(a) daily returns, iterated forecasts

	Normal			Empirical			Student's t		
Asset split	1	3	8	1	3	8	1	3	8
CCC-GARCH	1.69^{b}	1.71^{b}	1.87^{b}	1.57^{c}	1.45^{c}	1.65^{b}	1.61^{b}	1.65^{b}	1.87^{b}
	(0.38)	(0.38)	(0.38)	(0.37)	(0.34)	(0.35)	(0.37)	(0.37)	(0.39)
DCC-GARCH		1.69^{b}	1.87^{b}		1.41	1.55^{c}		1.67^{b}	1.61^{b}
		(0.37)	(0.39)		(0.34)	(0.35)		(0.37)	(0.36)
CCC-GJR	1.08	1.19	1.41	1.06	1.04	1.23	1.18	1.14	1.53^{c}
	(0.29)	(0.31)	(0.33)	(0.29)	(0.28)	(0.31)	(0.31)	(0.29)	(0.34)
DCC-GJR		1.18	1.31		1.00	1.16		1.16	1.37
		(0.30)	(0.32)		(0.28)	(0.29)		(0.30)	(0.32)
RiskMetrics	1.37	1.45	1.85^{b}	1.43	1.37	1.39	0.78	1.37	1.57^{c}
	(0.35)	(0.37)	(0.42)	(0.35)	(0.36)	(0.35)	(0.25)	(0.37)	(0.39)

(b) daily returns, scaled forecasts

	Normal			Empirical			Student's t		
Asset split	1	3	8	1	3	8	1	3	8
CCC-GARCH	1.97^{a}	1.87^{b}	2.07^{a}	1.31	1.37	1.53^{c}	1.69^{b}	1.57^{c}	1.73^{b}
	(0.42)	(0.39)	(0.41)	(0.33)	(0.33)	(0.36)	(0.38)	(0.36)	(0.37)
DCC-GARCH		1.91^{b}	1.97^{a}		1.37	1.45^{c}		1.73^{b}	1.65^{b}
		(0.40)	(0.40)		(0.34)	(0.34)		(0.37)	(0.37)
CCC-GJR	1.93^{a}	1.83^{b}	2.01^{a}	1.51^{c}	1.45^{c}	1.59^{b}	1.63^{b}	1.67^{b}	1.67^{b}
	(0.40)	(0.38)	(0.39)	(0.35)	(0.34)	(0.35)	(0.37)	(0.36)	(0.35)
DCC-GJR		1.87^{b}	1.79^{b}		1.53^{c}	1.43^{c}		1.65^{b}	1.67^{b}
		(0.39)	(0.37)		(0.35)	(0.33)		(0.36)	(0.36)
RiskMetrics	1.75^{b}	1.59^{c}	1.95^{b}	1.18	1.33	1.18	0.84	1.33	1.35
	(0.38)	(0.39)	(0.42)	(0.32)	(0.35)	(0.34)	(0.26)	(0.36)	(0.36)

(c) weekly returns, iterated forecasts

	Normal			Empirical			Student's t		
Asset split	1	3	8	1	3	8	1	3	8
CCC-GARCH	2.29^{a}	1.99^{a}	2.03^{a}	1.77^{b}	1.27	1.33	2.07^{a}	1.87^{b}	1.93^{b}
	(0.46)	(0.42)	(0.43)	(0.42)	(0.36)	(0.35)	(0.45)	(0.41)	(0.42)
DCC-GARCH		1.93^{b}	2.03^{a}		1.23	1.23		1.83^{b}	1.89^{b}
		(0.42)	(0.42)		(0.35)	(0.34)		(0.40)	(0.41)
CCC-GJR	1.85^{b}	1.73^{b}	1.67^{b}	1.45	1.16	1.23	1.71^{b}	1.65^{b}	1.71^{b}
	(0.39)	(0.38)	(0.38)	(0.36)	(0.33)	(0.33)	(0.39)	(0.38)	(0.38)
DCC-GJR		1.71^{b}	1.75^{b}		1.14	1.16		1.55^{c}	1.65^{b}
		(0.38)	(0.39)		(0.32)	(0.34)		(0.36)	(0.37)
RiskMetrics	2.05^{b}	1.89^{b}	2.03^{b}	1.73^{b}	1.37	1.29	1.83^{b}	1.77^{b}	1.69^{c}
	(0.45)	(0.43)	(0.47)	(0.42)	(0.38)	(0.37)	(0.41)	(0.43)	(0.43)

This table shows the empirical coverage and results of the unconditional coverage tests of Christoffersen (1998). Models are estimated with a moving window of 1,000 daily returns, 200 weekly returns or 100 biweekly returns. For every day in the sample, we construct a violation indicator that equals one when the ten-day realized portfolio loss exceeds the forecasted VaR_α with $\alpha = 99\%$, and zero otherwise, as in Equation (18). We report the coverage (in %), and test whether it is equal to $1 - \alpha$ against the alternative of strictly more violations. We report Newey and West (1987) standard errors calculated with 14 leads and lags in parentheses. Superscripts a, b, c denote rejection of the null-hypothesis with a significance level of 1%, 5% and 10%. The results are based on 5,021 forecasts. We report the number of significant violations for the different choices in panel (f) on the next page. The abbreviations uv. and mv. in panel (f) stand for univariate and multivariate.

Table 3: Results for the unconditional coverage test – *continued*

(d) weekly returns, scaled forecasts

	Normal			Empirical			Student's t		
Asset split	1	3	8	1	3	8	1	3	8
CCC-GARCH	2.41^a	2.15^a	2.21^a	1.75^b	1.12	1.25	2.13^a	1.91^b	1.91^b
	(0.47)	(0.43)	(0.44)	(0.41)	(0.34)	(0.35)	(0.46)	(0.41)	(0.41)
DCC-GARCH		2.19^a	2.19^a		1.18	1.18		1.91^b	1.95^b
		(0.44)	(0.44)		(0.35)	(0.34)		(0.41)	(0.42)
CCC-GJR	2.31^a	2.23^a	2.13^a	1.59^c	1.23	1.27	1.91^b	1.91^b	1.81^b
	(0.43)	(0.44)	(0.43)	(0.37)	(0.34)	(0.35)	(0.40)	(0.40)	(0.39)
DCC-GJR		2.29^a	2.09^a		1.27	1.25		1.83^b	1.85^b
		(0.44)	(0.42)		(0.35)	(0.34)		(0.39)	(0.39)
RiskMetrics	2.15^a	1.91^b	2.03^b	1.67^c	1.33	1.27	1.77^b	1.69^b	1.65^c
	(0.46)	(0.43)	(0.47)	(0.41)	(0.36)	(0.36)	(0.40)	(0.41)	(0.42)

(e) biweekly returns, direct forecasts

	Normal			Empirical			Student's t		
Asset split	1	3	8	1	3	8	1	3	8
CCC-GARCH	2.55^a	2.21^a	2.33^a	1.83^b	1.12	1.29	2.25^a	2.13^a	2.07^a
	(0.48)	(0.44)	(0.46)	(0.42)	(0.32)	(0.35)	(0.47)	(0.45)	(0.44)
DCC-GARCH		2.27^a	2.33^a		1.18	1.35		2.19^a	2.01^b
		(0.45)	(0.46)		(0.32)	(0.35)		(0.44)	(0.44)
CCC-GJR	2.75^a	2.31^a	2.17^a	2.07^a	1.14	1.29	2.33^a	2.07^a	1.89^b
	(0.48)	(0.44)	(0.44)	(0.43)	(0.32)	(0.34)	(0.45)	(0.44)	(0.41)
DCC-GJR		2.31^a	2.15^a		1.10	1.35		2.09^a	1.85^b
		(0.45)	(0.44)		(0.32)	(0.35)		(0.43)	(0.41)
RiskMetrics	2.29^a	1.93^b	2.29^a	2.09^a	1.65^c	1.99^b	2.07^a	1.83^b	2.13^b
	(0.48)	(0.45)	(0.49)	(0.44)	(0.42)	(0.46)	(0.45)	(0.44)	(0.50)

(f) Number of significant violations

	significance level			
Temporal aggregation	1%	5%	10%	tests
Daily, iterated	0	12	17	39
Daily, scaled	5	22	30	39
Weekly, iterated	5	26	28	39
Weekly, scaled	12	26	29	39
Biweekly, direct	22	30	31	39

	significance level			
Model	1%	5%	10%	tests
GARCH, uv.	7	13	14	15
GJR, uv.	5	9	11	15
RiskMetrics, uv.	4	9	10	15
CCC-GARCH	9	20	23	30
DCC-GARCH	7	20	22	30
CCC-GJR	6	17	19	30
DCC-GJR	5	15	18	30
RiskMetrics, mv.	1	13	18	30

	significance level			
Portfolio aggregation	1%	5%	10%	tests
Portfolio level	16	31	35	45
Asset class level	13	40	47	75
Asset level	15	45	53	75

	significance level			
Distribution	1%	5%	10%	tests
Normal	32	57	58	65
Empirical	2	9	21	65
Student's t	10	50	56	65

See table note on previous page.

Table 4: Results of the dynamic quantile tests

(a) daily returns, iterated forecasts

Asset split	Normal 1	Normal 3	Normal 8	Empirical 1	Empirical 3	Empirical 8	Student's t 1	Student's t 3	Student's t 8
CCC-	-0.086	-0.067	-0.102	-0.130	-0.095	-0.097	-0.107	-0.061	-0.078
GARCH	(0.093)	(0.109)	(0.114)	(0.087)	(0.108)	(0.124)	(0.085)	(0.115)	(0.124)
DCC-		-0.066	-0.070		-0.102	-0.098		-0.055	-0.035
GARCH		(0.102)	(0.106)		(0.097)	(0.106)		(0.113)	(0.139)
CCC-	-0.076	-0.068	-0.101	-0.134^{b}	-0.062	-0.161^{c}	-0.073	-0.061	-0.088
GJR	(0.081)	(0.092)	(0.100)	(0.066)	(0.101)	(0.085)	(0.079)	(0.092)	(0.105)
DCC-		-0.056	-0.095		-0.082	-0.119		-0.069	-0.055
GJR		(0.091)	(0.096)		(0.092)	(0.088)		(0.087)	(0.107)
RiskMetrics	-0.049	0.037	0.026	-0.089	0.009	0.034	-0.123	0.008	-0.021
	(0.120)	(0.178)	(0.163)	(0.101)	(0.148)	(0.159)	(0.091)	(0.162)	(0.140)

(b) daily returns, scaled forecasts

Asset split	Normal 1	Normal 3	Normal 8	Empirical 1	Empirical 3	Empirical 8	Student's t 1	Student's t 3	Student's t 8
CCC-	-0.095	-0.123	-0.092	-0.081	-0.095	-0.054	-0.100	-0.122	-0.030
GARCH	(0.091)	(0.101)	(0.116)	(0.099)	(0.112)	(0.140)	(0.089)	(0.102)	(0.159)
DCC-		-0.107	-0.052		-0.079	-0.059		-0.114	-0.036
GARCH		(0.093)	(0.116)		(0.110)	(0.120)		(0.084)	(0.135)
CCC-	-0.061	-0.132	-0.111	-0.089	-0.153^{c}	-0.085	-0.068	-0.116	-0.074
GJR	(0.083)	(0.089)	(0.087)	(0.079)	(0.080)	(0.106)	(0.085)	(0.098)	(0.109)
DCC-		-0.124	-0.071		-0.131^{c}	-0.064		-0.127^{c}	-0.077
GJR		(0.081)	(0.106)		(0.073)	(0.103)		(0.077)	(0.100)
RiskMetrics	-0.075	0.032	0.044	-0.067	0.025	0.019	-0.134	0.005	-0.021
	(0.097)	(0.187)	(0.172)	(0.109)	(0.123)	(0.134)	(0.087)	(0.160)	(0.141)

(c) weekly returns, iterated forecasts

Asset split	Normal 1	Normal 3	Normal 8	Empirical 1	Empirical 3	Empirical 8	Student's t 1	Student's t 3	Student's t 8
CCC-	0.078	0.030	-0.030	0.036	-0.051	-0.061	0.046	0.053	-0.017
GARCH	(0.135)	(0.118)	(0.103)	(0.101)	(0.083)	(0.088)	(0.133)	(0.127)	(0.104)
DCC-		0.035	-0.011		-0.055	-0.052		0.057	-0.045
GARCH		(0.122)	(0.108)		(0.083)	(0.093)		(0.134)	(0.089)
CCC-	0.062	0.002	-0.061	0.028	-0.008	-0.039	0.058	0.002	-0.079
GJR	(0.112)	(0.085)	(0.091)	(0.103)	(0.094)	(0.084)	(0.140)	(0.096)	(0.076)
DCC-		0.033	-0.055		-0.037	-0.095		0.023	-0.060
GJR		(0.100)	(0.089)		(0.071)	(0.072)		(0.116)	(0.081)
RiskMetrics	0.172	0.176	0.139	0.195	0.192	0.235^{c}	0.190	0.203	0.120
	(0.164)	(0.183)	(0.149)	(0.172)	(0.160)	(0.131)	(0.191)	(0.198)	(0.144)

This table reports the results of the dynamic quantile test of Engle and Manganelli (2004). We conduct a logistic regression of the VaR-violations on a constant and the forecasted VaR_α (in %) for a confidence level of $\alpha = 0.99$, as in Equation (19). We report the estimated coefficient on the forecasted VaR_α, with standard errors based on Newey and West (1987) with 14 leads and lags in parentheses. Superscripts a, b, c denote significance of the estimates with a significance level of 1%, 5% and 10%. We report the number of negative, significantly negative, positive and significantly positive coefficients in panel (f) on the next page, where we use a significance level of 10%.

Table 4: Results of the dynamic quantile test − *continued*

(d) weekly returns, scaled forecasts

Asset split	Normal 1	Normal 3	Normal 8	Empirical 1	Empirical 3	Empirical 8	Student's t 1	Student's t 3	Student's t 8
CCC-	0.059	−0.009	−0.033	0.136	−0.031	−0.013	0.024	−0.020	−0.045
GARCH	(0.136)	(0.110)	(0.105)	(0.142)	(0.096)	(0.112)	(0.120)	(0.103)	(0.093)
DCC-		−0.029	−0.026		−0.026	−0.047		0.012	−0.022
GARCH		(0.105)	(0.106)		(0.090)	(0.091)		(0.106)	(0.094)
CCC-	0.026	−0.079	−0.076	0.060	−0.088	−0.060	0.038	−0.066	−0.090
GJR	(0.104)	(0.081)	(0.090)	(0.140)	(0.071)	(0.088)	(0.118)	(0.082)	(0.081)
DCC-		−0.073	−0.088		−0.081	−0.055		−0.040	−0.045
GJR		(0.079)	(0.084)		(0.067)	(0.078)		(0.082)	(0.091)
RiskMetrics	0.152	0.132	0.135	0.187	0.114	0.261	0.127	0.072	0.103
	(0.171)	(0.122)	(0.152)	(0.163)	(0.079)	(0.165)	(0.179)	(0.097)	(0.139)

(e) biweekly returns, direct forecasts

Asset split	Normal 1	Normal 3	Normal 8	Empirical 1	Empirical 3	Empirical 8	Student's t 1	Student's t 3	Student's t 8
CCC-	0.223	0.180	0.138	0.393^b	0.212^c	0.191^c	0.324	0.161	0.208
GARCH	(0.191)	(0.162)	(0.150)	(0.165)	(0.121)	(0.113)	(0.232)	(0.166)	(0.162)
DCC-		0.190	0.137		0.267^c	0.189^c		0.172	0.195
GARCH		(0.176)	(0.151)		(0.141)	(0.113)		(0.176)	(0.160)
CCC-	0.172	0.190	0.086	0.377^b	0.233	0.112	0.227	0.223	0.086
GJR	(0.159)	(0.174)	(0.143)	(0.188)	(0.144)	(0.122)	(0.208)	(0.191)	(0.145)
DCC-		0.202	0.094		0.208	0.105		0.189	0.078
GJR		(0.192)	(0.153)		(0.144)	(0.116)		(0.181)	(0.146)
RiskMetrics	0.298	0.327	0.667^a	0.465^a	0.720^a	0.533^a	0.382^b	0.353^c	0.361^c
	(0.222)	(0.203)	(0.254)	(0.165)	(0.204)	(0.186)	(0.190)	(0.197)	(0.207)

(f) Number of negative and positive coefficients

Temporal aggregation	neg.	sign. neg.	pos.	sign. pos.
Daily, iterated	34	2	5	0
Daily, scaled	34	3	5	0
Weekly, iterated	16	0	23	1
Weekly, scaled	23	0	16	0
Biweekly, direct	0	0	39	13

Model	neg.	sign. neg.	pos.	sign. pos.
GARCH, uv.	6	0	9	1
GJR, uv.	6	1	9	1
RiskMetrics, uv.	6	0	9	2
CCC-GARCH	22	0	8	2
DCC-GARCH	21	0	9	2
CCC-GJR	22	2	8	0
DCC-GJR	22	2	8	0
RiskMetrics, mv.	2	0	28	6

Portfolio aggregation	neg.	sign. neg.	pos.	sign. pos.
Portfolio level	18	1	27	4
Asset class level	39	3	36	4
Asset level	50	1	25	6

Distribution	neg.	sign. neg.	pos.	sign. pos.
Normal	34	0	31	1
Empirical	38	4	27	10
Student's t	35	1	30	3

See table note on previous page.

Table 5: Average asymmetric tick loss

(a) daily returns, iterated forecasts

	Normal			Empirical			Student's t		
Asset split	1	3	8	1	3	8	1	3	8
CCC-GARCH	5.88	5.47	5.60	5.82	5.44	5.50	5.85	5.45	5.55
DCC-GARCH		5.52	5.51		5.45	5.39		5.53	5.47
CCC-GJR	5.50	5.28	5.27	5.48	5.31	5.23	5.49	5.29	5.23
DCC-GJR		5.35	5.20		5.39	5.15		5.25	5.17
RiskMetrics	5.58	6.04	6.10	5.68	5.98	5.86	5.67	6.01	6.13

(b) daily returns, scaled forecasts

	Normal			Empirical			Student's t		
Asset split	1	3	8	1	3	8	1	3	8
CCC-GARCH	5.92	5.59	5.65	5.66	5.42	5.41	5.78	5.47	5.46
DCC-GARCH		5.65	5.57		5.50	5.34		5.55	5.46
CCC-GJR	5.74	5.59	5.54	5.59	5.49	5.30	5.66	5.49	5.36
DCC-GJR		5.63	5.43		5.52	5.23		5.43	5.28
RiskMetrics	5.74	6.06	6.17	5.60	5.95	6.12	5.64	6.00	6.02

(c) weekly returns, iterated forecasts

	Normal			Empirical			Student's t		
Asset split	1	3	8	1	3	8	1	3	8
CCC-GARCH	7.11	6.68	6.71	6.90	6.33	6.35	6.86	6.55	6.68
DCC-GARCH		6.68	6.69		6.35	6.32		6.57	6.66
CCC-GJR	6.63	6.50	6.41	6.39	6.23	6.16	6.44	6.23	6.30
DCC-GJR		6.51	6.39		6.27	6.12		6.30	6.22
RiskMetrics	7.09	7.06	7.33	6.95	7.03	7.46	6.69	7.05	7.14

(d) weekly returns, scaled forecasts

	Normal			Empirical			Student's t		
Asset split	1	3	8	1	3	8	1	3	8
CCC-GARCH	7.28	6.79	6.79	6.75	6.33	6.28	6.84	6.60	6.61
DCC-GARCH		6.79	6.76		6.38	6.33		6.55	6.63
CCC-GJR	6.75	6.68	6.55	6.27	6.18	6.08	6.37	6.36	6.28
DCC-GJR		6.70	6.51		6.26	6.08		6.32	6.29
RiskMetrics	7.15	7.06	7.36	6.90	6.88	7.40	6.66	7.04	7.08

(e) biweekly returns, direct forecasts

	Normal			Empirical			Student's t		
Asset split	1	3	8	1	3	8	1	3	8
CCC-GARCH	7.42	7.16	7.18	7.55	7.49	7.09	7.22	7.07	7.05
DCC-GARCH		7.13	7.18		7.44	7.10		7.04	7.04
CCC-GJR	7.17	7.00	6.95	7.15	7.18	6.86	6.88	6.89	6.80
DCC-GJR		6.97	6.93		7.19	6.84		6.85	6.77
RiskMetrics	7.59	7.59	8.45	7.80	8.31	8.82	7.49	7.67	8.41

This table shows the average value of the asymmetric tick loss function of Giacomini and Komunjer (2005) as in Equation (20) for $\alpha = 0.99$. All values have been multiplied by 100.

Table 6: Results of the Diebold-Mariano tests

Method A	Method B	# of tests	neg.	sign. neg. at 1%	5%	10%	pos.	sign. pos. at 1%	5%	10%
(a) Temporal Aggregation										
Daily, iterated	Weekly, iterated	39	39	0	5	13	0	0	0	0
Daily, iterated	Biweekly, direct	39	39	2	15	26	0	0	0	0
Daily, scaled	Weekly, scaled	39	39	0	5	13	0	0	0	0
Daily, scaled	Biweekly, direct	39	39	1	15	22	0	0	0	0
Weekly, iterated	Biweekly, direct	39	39	9	22	35	0	0	0	0
Weekly, scaled	Biweekly, direct	39	39	8	17	29	0	0	0	0
(b) Portfolio Aggregation										
Asset level	Asset class level	75	51	0	0	6	24	0	0	1
Asset level	Portfolio level	75	56	4	11	13	19	0	3	4
Asset class level	Portfolio level	75	47	2	4	5	28	3	4	6
(c) Model choice										
CCC-GARCH	CCC-GJR	45	2	0	0	0	43	2	8	12
CCC-GARCH	DCC-GJR	30	2	0	0	0	28	0	2	6
DCC-GARCH	DCC-GJR	30	1	0	0	0	29	0	3	5
DCC-GARCH	CCC-GJR	30	0	0	0	0	30	1	3	7
CCC-GARCH	DCC-GARCH	30	13	0	0	0	17	0	0	1
CCC-GJR	DCC-GJR	30	11	0	2	2	19	0	0	0
CCC-GARCH	RiskMetrics	45	35	3	11	18	10	0	0	0
DCC-GARCH	RiskMetrics	30	30	3	9	16	0	0	0	0
CCC-GJR	RiskMetrics	45	43	5	11	22	2	0	0	0
DCC-GJR	RiskMetrics	30	30	3	9	14	0	0	0	0
(d) Distribution choice										
Normal	Empirical	65	13	1	1	1	52	0	0	1
Normal	Student's t	65	5	0	0	0	60	0	3	17
Empirical	Student's t	65	40	0	0	0	25	1	3	4
(e) Forecast Aggregation										
Daily, iterated	Daily, scaled	39	27	0	2	6	12	0	0	0
Weekly, iterated	Weekly, scaled	39	19	0	1	6	20	0	1	1

This table shows the results of the test that the expected value of asymmetric tick loss function of Giacomini and Komunjer (2005) as in Equation (20) for methods A and B are equal. We calculate the loss differential by substracting the loss of method B from the loss of method A. We report the number of negative and positive average loss differentials, and the number of times these differentials are significant for different significance levels, based on the statistic proposed by Diebold and Mariano (1995), evaluated in the setting of Giacomini and White (2006). Standard errors of the average loss differential are based on Newey and West (1987) with 14 leads and lags. We compare pairs of methods that differ in only one of the five choice aspects, so the other four choices are the same.

Table 7: Models removed from the Model Confidence Set

(a) weekly returns, iterated forecasts

	Normal			Empirical			Student's t		
Asset split	1	3	8	1	3	8	1	3	8
CCC-GARCH	0.085			0.083			0.091		
DCC-GARCH									
CCC-GJR							0.096		
DCC-GJR									
RiskMetrics	0.089	0.099		0.088	0.08	0.072	0.071	0.097	

(b) weekly returns, scaled forecasts

	Normal			Empirical			Student's t		
Asset split	1	3	8	1	3	8	1	3	8
CCC-GARCH	0.083			0.075			0.089		
DCC-GARCH					0.093				
CCC-GJR	0.089	0.093							
DCC-GJR		0.089			0.093				
RiskMetrics	0.089			0.08	0.075	0.062	0.078	0.094	

(c) biweekly returns, direct forecasts

	Normal			Empirical			Student's t		
Asset split	1	3	8	1	3	8	1	3	8
CCC-GARCH	0.082			0.061	0.059	0.081	0.083	0.093	
DCC-GARCH					0.056	0.079		0.093	
CCC-GJR	0.084			0.069	0.062	0.086	0.085	0.097	
DCC-GJR					0.060	0.085		0.093	
RiskMetrics	0.082		0.089	0.062	0.078	0.075	0.067		0.089

(d) Number of removed and maintained methods

Temporal aggr.	Out	In	Total	Model	Out	In	Total
Daily iterated	0	39	39	CCC-GARCH	12	33	45
Daily scaled	0	39	39	DCC-GARCH	4	26	30
Weekly iterated	11	28	39	CCC-GJR	9	36	45
Weekly scaled	14	25	39	DCC-GJR	5	25	30
Biweekly direct	25	14	39	RiskMetrics	20	25	45

Portfolio aggr.	Out	In	Total	Distribution	Out	In	Total
Portfolio level	23	22	45	Normal	12	53	65
Asset class level	18	57	75	Empirical	23	42	65
Asset level	9	66	75	Student's t	15	50	65

This table presents the MCS p-value with which a model has been removed from the Model Confidence Set. We follow the procedure of Hansen et al. (2011, Sec. 3.1.2), with the tick loss function in (20) and a significance level of 10%. The procedure starts with the complete set of 195 methods. Panels for daily observations combined with iterated or scaled forecasts are absent, because no methods with these combinations have been removed. Panel (d) presents the number of methods with a particular choice that have been removed from ("Out") and are maintained in ("In") the Model Confidence Set.

A Portfolio construction

This section contains detailed information on the construction of the typical pension fund portfolio. For the returns on the US bond portfolios we use the Barclays US Aggregate Bond Index and the Barclays Capital US Corporate Bond Index. The Aggregate Bond Index is a predominate index benchmark for US bond investors, and acts as a benchmark index for many US index funds. It comprises four major subindexes: US Government Index, US Credit Index, US Mortgage Backed Securities Index and US Asset Backed Securities Index. The Corporate Bond Index covers investment-grade bonds that are denominated in US Dollar.

We use the MSCI Europe Index to cover the European stock market. This index captures large and mid cap representation across the following 15 Developed Markets (DM) countries in Europe: Austria, Belgium, Denmark, Finland, France, Germany, Ireland, Italy, the Netherlands, Norway, Portugal, Spain, Sweden, Switzerland and the UK. The US equity part is based MSCI USA Index, a free float adjusted market capitalization index that is designed to measure large and mid cap US equity market performance. The equity part for the Pacific area represents the large and mid cap across Australia, Hong Kong, Japan, New Zealand and Singapore. Likewise, the MSCI EM Index summarizes the equity returns from 23 Emerging Markets.

The return on the real estate investment is constructed from the EPRA/NAREIT Developed Europe Index, which is a subset of the FTSE EPRA/NAREIT Developed Index that incorporates Real Estate Investment Trusts (REITs) and Real Estate Holding & Development companies. The index represent general trends in eligible real estate equities worldwide.

The investment in commodities is approximated by the the S&P GSCI Index (formerly the Goldman Sachs Commodity Index). This Index serves as a benchmark for investment in the commodity markets and as a measure of commodity performance over time. Currently, it comprises 24 commodities from all commodity sectors - energy products, industrial metals, agricultural products, livestock products and precious metals with a high exposure to energy products.

For all series, total returns are based on the closing prices of each day. All return series are obtained from Thomson Reuters Datastream.

B Methodological details

B.1 Esimation

In this subsection, we discuss the parameter estimation for all combinations of univariate and multivariate models on the one hand and distributions on the other hand.

Univariate case

We first estimate the parameters of the univariate version of Equation (4) by ordinary least squares (OLS), and construct a series of residuals $\hat{\varepsilon}_{t,k}$. Based on this series, we estimate the parameters of the GJR or GARCH model in Equation (8) or of the RiskMetrics model in Equation (11) as the second pass. When these models are combined with the normal or Student's t-distribution for the innovations we use Maximum Likelihood (ML) estimation. When the Student's t-distribution is used, we base the likelihood function on the scaled pdf in Equation (12). When these models are combined with the empirical distribution, we use Quasi Maximum Likelihood (QML) estimation.

Multivariate RiskMetrics

We estimate the parameters of the VAR-model of Equation (4) by OLS, and construct a series of residuals $\hat{\boldsymbol{\varepsilon}}_{t,k}$. In the second pass, we estimate the parameters in Equation (11) based on this series. When RiskMetrics is combined with the normal or Student's t-distribution, we apply ML estimation. In the case of the Student's t-distribution, the likelihood function is scaled, as in Equation (14). When the empirical distribution is chosen, we use QML estimation.

CCC model

We estimate the parameters of the VAR-model of Equation (4) by OLS, and construct a series of residuals $\hat{\boldsymbol{\varepsilon}}_{t,k}$. In the second pass, we estimate the parameters of the marginal models (GJR or GARCH) for each series $\hat{\varepsilon}_{i,t,k}$, $i = 1, 2, \ldots, m$ in the same way as in the univariate case, and construct series of standardized residuals $\hat{\eta}_{i,t,k}$. The third pass depends on the distribution. When the normal distribution is used, we estimate the correlation matrix by the sample correlation of $\hat{\boldsymbol{\eta}}_{t,k}$ (cf. Bollerslev, 1990). When the Student's t or empirical distribution is used, we first apply the probability integral transform (PIT) to the residuals of each marginal model based on Equation (12) or Equation (15). Next, we use ML based on the transformed series to estimate the parameters of the Student's t in Equation (13) (cf. Patton, 2013, Sec. 3.1) or the parameters of the Gaussian copula in Equation (16) (cf. Patton, 2013, Sec. 3.2; Chen and Fan, 2006).

DCC model

In this case, we follow the same first two passes as for the CCC model. We then construct the target correlation matrix $\bar{\boldsymbol{Q}}_k$ in Equation (9) as the sample correlation of $\hat{\boldsymbol{\eta}}_{t,k}$. The final pass depends again on the distribution. When the normal distribution is used, we estimate the DCC parameters in (9) by ML estimation based on the normal distribution function (cf. Engle, 2002). For the other two distribution, we apply the PIT, and then use ML based on the transformed series to estimate the DCC parameters (and the degrees of freedom in the case of the Student's t-copula).

B.2 Forecasting methods

We build the forecasts of the portfolio return distribution from forecasts for the expected returns, and forecasts for the unexpected part, $u^{\mathrm{p}}_{t,h} \equiv r^{\mathrm{p}}_{t,h} - \mathrm{E}_t[r^{\mathrm{p}}_{t,h}]$. The Value-at-Risk for the portfolio return follows as

$$\mathrm{VaR}_\alpha(r^{\mathrm{p}}_{t,h}) = -\,\mathrm{E}_t[r^{\mathrm{p}}_{t,h}] + \mathrm{VaR}_\alpha(u^{\mathrm{p}}_{t,h}). \tag{B.1}$$

The expected portfolio return obeys

$$\mathrm{E}_t[r^{\mathrm{p}}_{t,h}] = \boldsymbol{w}^{\mathrm{b}\prime}\boldsymbol{\mu}^{\mathrm{b}}_{t,h/k,k}, \tag{B.2}$$

where $\boldsymbol{\mu}^{\mathrm{b}}_{t,\kappa,k}$ is the conditional κ-steps ahead cumulative expected return with $\kappa = h/k$ that follows from Equation (4). We shows in Appendix B.3 that

$$\begin{aligned}\boldsymbol{\mu}^{\mathrm{b}}_{t,k,\kappa} \equiv & \mathrm{E}_t\left[\sum_{j=0}^{\kappa-1} \boldsymbol{r}^{\mathrm{b}}_{t+jk,k}\right] \\ =&(\boldsymbol{I}_m - \boldsymbol{\Gamma})^{-2}\left(\kappa\boldsymbol{I}_m - (\kappa+1)\boldsymbol{\Gamma} + \boldsymbol{\Gamma}^{\kappa+1}\right)\boldsymbol{\phi} + \\ & (\boldsymbol{I}_m - \boldsymbol{\Gamma})^{-1}(\boldsymbol{\Gamma} - \boldsymbol{\Gamma}^{\kappa+1})\boldsymbol{r}^{\mathrm{b}}_{t-k,k}.\end{aligned} \tag{B.3}$$

The forecasts for the unexpected part are derived in closed form or based on simulations. We discuss the different cases in the next subsections.

B.2.1 One-step-ahead forecasts

One-step ahead forecasts result when full temporal aggregation is applied to the modelled returns. They are also used when forecasts are scaled. For some cases, VaR can be calculated analytically, for others simulations are needed. We provide an overview below, and discuss the construction for a one-step-ahead forecast k

	Normal	Student's t		Empirical	
		$m=1$	$m>1$	$m=1$	$m>1$
CCC or DCC	Closed form	Closed form	Simulation C	Closed form	Simulation C
RiskMetrics	Closed form	Closed form	Closed form	Closed form	Simulation E

In the closed-form cases, Value-at-Risk is calculated as

$$\mathrm{VaR}_\alpha(u^{\mathrm{p}}_{t,k}) = -\sqrt{\boldsymbol{w}^{\mathrm{b}\prime}\boldsymbol{\Sigma}_{t,k}\boldsymbol{w}^{\mathrm{b}}}F^{-1}(\alpha;\boldsymbol{\theta}). \tag{B.4}$$

Here $\boldsymbol{\Sigma}_{t,k}$ denotes the forecasted variance, which depends on the specification, CCC, DCC or RiskMetrics. F^{-1} denotes the inverse of a standardized cumulative distribution function, and $\boldsymbol{\theta}$ is a vector containing all parameters needed to calculate this inverse. When a normal distribution is assumed, this vector is empty. When the Student's t-distribution is used, the cdf should be based on Equation (12), and $\boldsymbol{\theta}$ contains the degree of freedom parameter. When the empirical distribution is used, $\boldsymbol{\theta}$ contains a set of realizations (cf. Equation (15)). We use here that the normal and Student's t-distribution are closed under summation.

When the CCC or DCC approaches are combined with the Student's t- or empirical distribution, copula-based simulations are needed to calculate $\mathrm{VaR}_\alpha(u_{t,k})$. Each simulation s consist of four steps.

C1. Generate a one-step-ahead forecast for the correlation matrix $\boldsymbol{R}_{t,k}$ and the volatilities $\sigma_{i,t,k}$.

C2. Draw a random m-vector $\tilde{\boldsymbol{v}}_s$ from the copula (Gaussian or Student's t-), using the forecasted correlation matrix $\boldsymbol{R}_{t,k}$.

C3. Transform each draw $\tilde{v}_{s,i}$ by applying the standardized inverse cdf of the respective marginal distribution, $\tilde{\eta}_{s,i} = F_i^{-1}(\tilde{v}_{s,i};\boldsymbol{\theta}_i)$.

C4. Multiply $\tilde{\eta}_{s,i}$ by the forecasted volatility, $\tilde{\varepsilon}_{s,i} = \sigma_{i,t,k}\tilde{\eta}_{s,i}$.

C5. Pre-multiply the vector $\tilde{\boldsymbol{\varepsilon}}_s$ by the weights to produce a random draw from the distribution of $u^{\mathrm{p}}_{t,h}$, $\tilde{u}^{\mathrm{p}}_s = \boldsymbol{w}^{\mathrm{b}\prime}\tilde{\boldsymbol{\varepsilon}}_s$.

Repeating these steps S times approximates the distribution of $u^{\mathrm{p}}_{t,k}$. The α-quantile of the simulated distribution is a forecast for $\mathrm{VaR}_\alpha(u^{\mathrm{p}}_{t,k})$.

When the empirical distribution is combined with RiskMetrics, a different simulation procedure is needed.

E1. Generate a one-step-ahead forecast for the variance matrix $\boldsymbol{\Sigma}_{t,k}$.

E2. Take a random draw $\tilde{\boldsymbol{\zeta}}_s$ from the empirical distribution that corresponds with $\boldsymbol{\zeta}_{t,k}$.

E3. Multiply $\tilde{\boldsymbol{\zeta}}_s$ with the Choleski decomposition of the forecasted variance matrix, $\tilde{\boldsymbol{\varepsilon}}_s = \boldsymbol{\Sigma}^{1/2}_{t,k}\tilde{\boldsymbol{\zeta}}_s$.

E4. As in step C5 of the copula-based simulation.

This procedure is again repeated S times to approximate the distribution of $u^{\mathrm{p}}_{t,k}$, from which the α-quantile is taken.

B.2.2 Iterated multi-step-ahead forecasts

Iterated forecasts take the complete serial dependence that is implied by the different models into account. Because of this serial dependence, closed-form expression for VaR are not available and simulations are needed. For the CCC or DCC models, a simulation s consists of the following steps

I1. Set $j = 1$. Generate a one-step-ahead forecast for the correlation matrix $\boldsymbol{R}_{t,k,s} = \boldsymbol{R}_{t,k}$ and the volatilities $\sigma_{i,t,k,s} = \sigma_{i,t,k}$.

I2. Draw a random m-vector $\tilde{\boldsymbol{v}}_{s,j}$ from the copula (Gaussian or Student's t-), using the forecasted correlation matrix $\boldsymbol{R}_{t+(j-1)k,k,s}$.

I3. Transform each draw $\tilde{v}_{s,j,i}$ by applying the standardized inverse cdf of the respective marginal distribution, $\tilde{\eta}_{s,j,i} = F_i^{-1}(\tilde{v}_{s,j,i}; \boldsymbol{\theta}_i)$.

I4. Multiply $\tilde{\eta}_{s,j,i}$ by the forecasted volatility, $\tilde{\varepsilon}_{s,j,i} = \sigma_{i,t+(j-1)k,k,s}\tilde{\eta}_{s,i}$.

I5. Construct one-step-ahead forecasts for the correlation matrix $\boldsymbol{R}_{t+jk,k,s}$ based on $\boldsymbol{R}_{t+(j-1)k,k,s}$ and $\tilde{\boldsymbol{\eta}}_{s,j}$, and the volatilities $\sigma_{i,t+jk,k,s}$ based on $\sigma_{i,t+(j-1)k,k,s}$ and $\tilde{\varepsilon}_{s,j,i}$. If $j < \kappa$, increase j by one, and go to step I2 Otherwise continue.

I6. Construct the draw from the distribution $u^{\mathrm{p}}_{t,h}$ as

$$\tilde{u}^{\mathrm{p}}_s = \boldsymbol{w}^{\mathrm{b}\prime} \sum_{j=1}^{\kappa} \tilde{\boldsymbol{\varepsilon}}_{s,j}.$$

Repeating these steps S times approximates the distribution of $u^{\text{p}}_{t,h}$. The α-quantile of the simulated distribution is a forecast for $\text{VaR}_\alpha(u^{\text{p}}_{t,h})$.

For the RiskMetrics approach, the procedure for simulation s goes as follows.

R1. Set $j = 1$. Generate a one-step-ahead forecast for the variance matrix $\boldsymbol{\Sigma}_{t,k,s} = \boldsymbol{\Sigma}_{t,k}$.

R2. Draw a random m-vector $\tilde{\boldsymbol{\zeta}}_{s,j}$ from the standardized distribution for $\boldsymbol{\zeta}_{t,k}$.

R3. Multiply $\tilde{\boldsymbol{\zeta}}_{s,j}$ with the Choleski decomposition of the forecasted variance matrix, $\tilde{\boldsymbol{\varepsilon}}_{s,j} = \boldsymbol{\Sigma}^{1/2}_{t+(j-1)k,k,s}\tilde{\boldsymbol{\zeta}}_{s,j}$.

R4. Construct a one-step-ahead forecast for the variance matrix $\boldsymbol{\Sigma}_{t+jk,k,s}$ based on $\boldsymbol{\Sigma}_{t+(j-1)k,k,s}$ and $\tilde{\boldsymbol{\varepsilon}}_{s,j}$. If $j < \kappa$, increase j by one, and go to step R2. Otherwise continue with step I6.

Repeating this procedure S times approximates the distribution $u^{\text{p}}_{t,h}$, from which the α-quantile is taken.

B.2.3 Scaled multi-step-ahead forecasts

Contrary to iterated forecasts, scaled forecasts ignore the serial dependence between the innovations in the return process. Scaled forecasts are based on the assumption that the distribution of $u^{\text{p}}_{t,h}$ is the same as the distribution of $u^{\text{p}}_{t,k}$ with a scale parameter that is adjusted for the horizon h. A scaled forecast is hence a one-step-ahead forecasts combined with an adjustment of the scale. When the one-step-ahead forecast can be constructed in closed form as in Equation (B.4), the scaled forecast takes the form

$$\text{VaR}_\alpha(u^{\text{p}}_{t,h}) = -\sqrt{(\boldsymbol{w}^{\text{b}} \otimes \boldsymbol{w}^{\text{b}})'\boldsymbol{S}_{h/k}(\boldsymbol{\Gamma})\,\text{vec}(\boldsymbol{\Sigma}_{t,k})}F^{-1}(\alpha;\boldsymbol{\theta}), \tag{B.5}$$

where $\boldsymbol{S}_{h/k}(\boldsymbol{\Gamma})$ is the $m^2 \times m^2$ scaling factor, and $\boldsymbol{\Sigma}_{t,k}$ is the one-step-ahead forecast of the variance matrix. We derive this scaling factor in Appendix B.3.

When simulations are used to construct a one-step-ahead forecast, scaling takes place in the initialization steps C1 and E1. In both cases, a scaled forecast of the variance matrix is constructed as

$$\boldsymbol{\Sigma}_{t,h} = \text{unvec}\big(\boldsymbol{S}_{h/k}(\boldsymbol{\Gamma})\,\text{vec}(\boldsymbol{\Sigma}_{t,k})\big). \tag{B.6}$$

This scaled variance matrix can be directly used in the RiskMetrics simulation procedure. For the copula simulation procedure, the (scaled) variance matrix is split in a (scaled) correlation matrix and scaled volatility forecasts.

When VAR-effects are absent, so $\boldsymbol{\Gamma} = \mathbf{O}$, the scaling factor simplifies to $\boldsymbol{S}_{h/k}(\mathbf{O}) = h/k\boldsymbol{I}_{m^2}$, which implies a multiplication of the variance matrix by h/k. In that case, the VaR-forecast itself can be scaled,

$$\mathrm{VaR}_\alpha(u^{\mathrm{p}}_{t,h}) = \sqrt{h/k}\,\mathrm{VaR}_\alpha(u^{\mathrm{p}}_{t,k}),$$

which is the familiar "square root of time"-rule.

B.3 VAR-forecasts and variance scaling

For the construction of the Value-at-Risk forecasts, analytical results for forecasts by VAR models are relevant. When the variance of the innovations in a VAR-model are constant, multi-step-ahead forecasts for the (cumulative) process can be constructed in closed form. These forecasts indicate how the variance should be scaled, moving from one- to n-step-ahead forecasts. We derive the scaling factor in this appendix (see also Campbell and Viceira, 2005).

We use slightly different notation in this section for clarity. We consider an m-vector $\boldsymbol{y}_t$ that is observed at each period t, and follows a VAR of order 1,

$$\boldsymbol{y}_{t+1} = \boldsymbol{\phi} + \boldsymbol{\Gamma}\boldsymbol{y}_t + \boldsymbol{\varepsilon}_{t+1}, \quad \mathrm{E}_t[\boldsymbol{\varepsilon}_{t+1}] = \mathbf{0}_m \ \mathrm{E}_t[\boldsymbol{\varepsilon}_{t+1}\boldsymbol{\varepsilon}'_{t+1}] = \boldsymbol{\Omega}, \tag{B.7}$$

where $\boldsymbol{\phi}$ is an m-vector, $\boldsymbol{\Gamma}$ is a $m \times m$ matrix, and $\boldsymbol{\varepsilon}_{t+1}$ is an m-vector with innovations that are independent over time. We assume that the eigenvalues of $\boldsymbol{\Gamma}$ are inside the unit circle, and that $\boldsymbol{\Omega}$ is invertible.

The forward solution $\boldsymbol{y}_{t+n}$ follows as

$$\begin{aligned}\boldsymbol{y}_{t+n} &= \sum_{i=0}^{n-1} \boldsymbol{\Gamma}^i(\boldsymbol{\phi} + \boldsymbol{\varepsilon}_{t+n-i}) + \boldsymbol{\Gamma}^\kappa \boldsymbol{y}_t \\ &= (\boldsymbol{I}_m - \boldsymbol{\Gamma})^{-1}(\boldsymbol{I}_m - \boldsymbol{\Gamma}^n)\boldsymbol{\phi} + \boldsymbol{\Gamma}^n\boldsymbol{y}_t + \sum_{i=0}^{n-1}\boldsymbol{\Gamma}^i\boldsymbol{\varepsilon}_{t+n-i},\end{aligned} \tag{B.8}$$

where $\boldsymbol{I}_m$ is an identity matrix of size m, and the inverse of $(\boldsymbol{I}_m - \boldsymbol{\Gamma})$ exists because the $\boldsymbol{\Gamma}$ is convergent. The conditional expectation of $\boldsymbol{y}_{t+n}$ is thus given by

$$\mathrm{E}[\boldsymbol{y}_{t+n}|\boldsymbol{y}_t] = (\boldsymbol{I}_m - \boldsymbol{\Gamma})^{-1}(\boldsymbol{I} - \boldsymbol{\Gamma}^n)\boldsymbol{\phi} + \boldsymbol{\Gamma}^n\boldsymbol{y}_t. \tag{B.9}$$

Next, we define the cumulative process

$$\boldsymbol{z}_{t,n} \equiv \sum_{i=1}^{n} \boldsymbol{y}_{t+i}. \tag{B.10}$$

Its expectation conditional on a value $\boldsymbol{y}_t$ follows as

$$\begin{aligned}
\mathrm{E}[\boldsymbol{z}_{t,n}|\boldsymbol{y}_t] =& \sum_{i=1}^{n} \mathrm{E}[\boldsymbol{y}_{t+i}|\boldsymbol{y}_t] = \sum_{i=1}^{n} \left((\boldsymbol{I}_m - \boldsymbol{\Gamma})^{-1}(\boldsymbol{I} - \boldsymbol{\Gamma}^i)\boldsymbol{\phi} + \boldsymbol{\Gamma}^i \boldsymbol{y}_t\right) \\
=& (\boldsymbol{I}_m - \boldsymbol{\Gamma})^{-1} \left(n\boldsymbol{I}_m - \boldsymbol{\Gamma}\sum_{i=0}^{n-1} \boldsymbol{\Gamma}^i\right) \boldsymbol{\phi} + \boldsymbol{\Gamma}\sum_{i=0}^{n-1} \boldsymbol{\Gamma}^i \boldsymbol{y}_t \\
=& (\boldsymbol{I}_m - \boldsymbol{\Gamma})^{-2} \left(n(\boldsymbol{I}_m - \boldsymbol{\Gamma}) - \boldsymbol{\Gamma}(\boldsymbol{I}_m - \boldsymbol{\Gamma}^n)\right) \boldsymbol{\phi} + \boldsymbol{\Gamma}(\boldsymbol{I}_m - \boldsymbol{\Gamma})^{-1}(\boldsymbol{I}_m - \boldsymbol{\Gamma}^n)\boldsymbol{y}_t \\
=& (\boldsymbol{I}_m - \boldsymbol{\Gamma})^{-2} \left(n\boldsymbol{I}_m - (n+1)\boldsymbol{\Gamma} + \boldsymbol{\Gamma}^{n+1}\right) \boldsymbol{\phi} + \\
& (\boldsymbol{I}_m - \boldsymbol{\Gamma})^{-1}(\boldsymbol{\Gamma} - \boldsymbol{\Gamma}^{n+1})\boldsymbol{y}_t.
\end{aligned} \tag{B.11}$$

The deviation of $\boldsymbol{z}_{t,n}$ from its conditional average can be written as

$$\begin{aligned}
\boldsymbol{z}_{t,n} - \mathrm{E}[\boldsymbol{z}_{t,n}|\boldsymbol{y}_t] &= \sum_{i=1}^{n}\sum_{j=0}^{i-1} \boldsymbol{\Gamma}^j \boldsymbol{\varepsilon}_{t+i-j} = \sum_{i=1}^{n}\sum_{j=0}^{n-i} \boldsymbol{\Gamma}^j \boldsymbol{\varepsilon}_{t+i} \\
&= \sum_{i=1}^{n} (\boldsymbol{I}_m - \boldsymbol{\Gamma})^{-1}(\boldsymbol{I}_m - \boldsymbol{\Gamma}^{n-i+1})\boldsymbol{\varepsilon}_{t+i}.
\end{aligned} \tag{B.12}$$

Because the innovation vectors are i.i.d., the variance follows as

$$\mathrm{Var}[\boldsymbol{z}_{t,n}] = \sum_{i=1}^{n} (\boldsymbol{I}_m - \boldsymbol{\Gamma})^{-1}(\boldsymbol{I}_m - \boldsymbol{\Gamma}^i)\boldsymbol{\Omega} \left((\boldsymbol{I}_m - \boldsymbol{\Gamma})^{-1}(\boldsymbol{I}_m - \boldsymbol{\Gamma}^i)\right)',$$

For comfortable matrices $\boldsymbol{A}$, $\boldsymbol{B}$, and $\boldsymbol{C}$, $\mathrm{vec}(\boldsymbol{ABC}) = (\boldsymbol{C} \otimes \boldsymbol{A})\,\mathrm{vec}(\boldsymbol{B})$, which applied to the previous equation yields

$$\begin{aligned}
\mathrm{vec}(\mathrm{Var}[\boldsymbol{z}_{t,n}]) =& \sum_{i=1}^{n} \left(\left((\boldsymbol{I}_m - \boldsymbol{\Gamma})^{-1}(\boldsymbol{I}_m - \boldsymbol{\Gamma}^i)\right) \otimes \left((\boldsymbol{I}_m - \boldsymbol{\Gamma})^{-1}(\boldsymbol{I}_m - \boldsymbol{\Gamma}^i)\right)\right) \mathrm{vec}(\boldsymbol{\Omega}) \\
=& \left((\boldsymbol{I}_m - \boldsymbol{\Gamma})^{-1} \otimes (\boldsymbol{I}_m - \boldsymbol{\Gamma})^{-1}\right) \cdot \\
& \sum_{i=1}^{n} \left((\boldsymbol{I}_m - \boldsymbol{\Gamma}^i) \otimes (\boldsymbol{I}_m - \boldsymbol{\Gamma}^i)\right) \mathrm{vec}(\boldsymbol{\Omega})
\end{aligned} \tag{B.13}$$

The summation in this expression can be reduced as follows[11]

$$
\begin{aligned}
&\sum_{i=1}^{n} \big((\boldsymbol{I}_m - \boldsymbol{\Gamma}^i) \otimes (\boldsymbol{I}_m - \boldsymbol{\Gamma}^i)\big) \\
&= n\boldsymbol{I}_{m^2} - \boldsymbol{I}_m \otimes \sum_{i=1}^{n} \boldsymbol{\Gamma}^i - \sum_{i=1}^{n} \boldsymbol{\Gamma}^i \otimes \boldsymbol{I}_m + \sum_{i=1}^{n} \boldsymbol{\Gamma}^i \otimes \boldsymbol{\Gamma}^i \\
&= n\boldsymbol{I}_{k^2} - \boldsymbol{I}_m \otimes (\boldsymbol{I}_m - \boldsymbol{\Gamma})^{-1}(\boldsymbol{\Gamma} - \boldsymbol{\Gamma}^{n+1}) - (\boldsymbol{I}_m - \boldsymbol{\Gamma})^{-1}(\boldsymbol{\Gamma} - \boldsymbol{\Gamma}^{n+1}) \otimes \boldsymbol{I}_m + \\
&\quad (\boldsymbol{I}_{k^2} - \boldsymbol{\Gamma} \otimes \boldsymbol{\Gamma})^{-1}\big((\boldsymbol{\Gamma} \otimes \boldsymbol{\Gamma}) - (\boldsymbol{\Gamma} \otimes \boldsymbol{\Gamma})^{n+1}\big). \qquad \text{(B.14)}
\end{aligned}
$$

The conditional variance of the weighted sum $\boldsymbol{w}'\boldsymbol{z}_{t,n}$ can be calculated as

$$
\operatorname{Var}[\boldsymbol{w}'\boldsymbol{z}_{t,n}|\boldsymbol{y}_t] = \boldsymbol{w}' \operatorname{Var}[\boldsymbol{z}_{t,n}]\boldsymbol{w} = (\boldsymbol{w}\otimes\boldsymbol{w})' \operatorname{vec}(\operatorname{Var}[\boldsymbol{z}_{t,n}]) = (\boldsymbol{w}\otimes\boldsymbol{w})'\boldsymbol{S}_n(\boldsymbol{\Gamma}) \operatorname{vec}(\boldsymbol{\Omega}), \quad \text{(B.15)}
$$

where $\boldsymbol{S}_n$ is the $m^2 \times m^2$ scaling matrix for horizon n depending on $\boldsymbol{\Gamma}$,

$$
\boldsymbol{S}_n(\boldsymbol{\Gamma}) = ((\boldsymbol{I}_m - \boldsymbol{\Gamma}) \otimes (\boldsymbol{I}_m - \boldsymbol{\Gamma}))^{-1} \sum_{i=1}^{n} \big((\boldsymbol{I}_m - \boldsymbol{\Gamma}^i) \otimes (\boldsymbol{I}_m - \boldsymbol{\Gamma}^i)\big), \qquad \text{(B.16)}
$$

as follows from Equation (B.13). Equation (B.14) can be substituted for the summation. Because the one-period variance $\boldsymbol{\Omega}$ and the weights $\boldsymbol{w}$ are on opposite sides of the scaling matrix $\boldsymbol{S}_n(\boldsymbol{\Phi})$, we cannot scale the variance of the weighted sum $\boldsymbol{w}'\boldsymbol{\Omega}\boldsymbol{w}$.

This scaling factor is an extension of the traditional square-root-of-time rule for scaling the volatility. When VAR-effects are absent, i.e., $\boldsymbol{\Gamma} = \mathbf{O}$, $\boldsymbol{S}_n(\mathbf{O}) = n\boldsymbol{I}_{m^2}$, and scaling is simply by n, $\operatorname{Var}[\boldsymbol{w}'\boldsymbol{z}_{t,n}|\boldsymbol{y}_t] = n\boldsymbol{w}'\boldsymbol{\Omega}\boldsymbol{w} = n \operatorname{Var}[\boldsymbol{w}'\boldsymbol{z}_{t,1}|\boldsymbol{y}_t]$. When $m = 1$, the VAR model reduces to an AR(1)-model with autoregressive coefficient ρ. In that case, the scaling factor reduces to (cf. Wang et al., 2011)

$$
\boldsymbol{S}_n(\rho) = \frac{1}{(1-\rho)^2}\left(n - 2\rho\frac{1-\rho^n}{1-\rho} + \rho^2\frac{1-\rho^{2n}}{1-\rho^2}\right). \qquad \text{(B.17)}
$$

C Additional results

C.1 Results of Diebold-Mariano tests of 99% VaR forecasts

[Table C.1 about here.]

[Table C.1 (continued) about here.]

[11] See http://www.ee.ic.ac.uk/hp/staff/dmb/matrix/intro.html for an overview of matrix operations.

[Table C.2 about here.]

[Table C.2 (continued) about here.]

[Table C.3 about here.]

[Table C.3 (continued) about here.]

[Table C.4 about here.]

[Table C.4 (continued) about here.]

[Table C.5 about here.]

C.2 Results 95% VaR forecasts

[Table C.6 about here.]

[Table C.6 (continued) about here.]

[Table C.7 about here.]

[Table C.7 (continued) about here.]

[Table C.8 about here.]

[Table C.9 about here.]

[Table C.10 about here.]

[Table C.10 (continued) about here.]

[Table C.11 about here.]

[Table C.11 (continued) about here.]

[Table C.12 about here.]

[Table C.12 (continued) about here.]

[Table C.13 about here.]

[Table C.13 (continued) about here.]

[Table C.14 about here.]

[Table C.15 about here.]

[Table C.15 (continued) about here.]

Table C.1: Results of the DM-tests for temporal aggregation

(a) daily vs. weekly returns, iterated forecasts

	Normal			Empirical			Student's t		
Asset split	1	3	8	1	3	8	1	3	8
CCC-GARCH	-1.229^{b}	-1.208	-1.106	-1.083^{b}	-0.888	-0.845	-1.009^{b}	-1.099	-1.125
	(0.587)	(0.825)	(0.763)	(0.508)	(0.648)	(0.668)	(0.500)	(0.802)	(0.775)
DCC-GARCH		-1.164	-1.182		-0.894	-0.937		-1.039	-1.185
		(0.771)	(0.817)		(0.596)	(0.697)		(0.719)	(0.810)
CCC-GJR	-1.121^{c}	-1.216	-1.140	-0.912^{c}	-0.924	-0.931	-0.945^{c}	-0.945	-1.069
	(0.658)	(0.812)	(0.726)	(0.509)	(0.621)	(0.595)	(0.501)	(0.695)	(0.741)
DCC-GJR		-1.154	-1.187		-0.876	-0.965		-1.047	-1.057
		(0.753)	(0.771)		(0.561)	(0.630)		(0.713)	(0.752)
RiskMetrics	-1.510^{c}	-1.019^{c}	-1.238	-1.276^{c}	-1.057^{b}	-1.599^{b}	-1.028^{c}	-1.041^{c}	-1.011
	(0.851)	(0.610)	(0.903)	(0.718)	(0.513)	(0.798)	(0.566)	(0.625)	(0.656)

(b) daily vs. weekly returns, scaled forecasts

	Normal			Empirical			Student's t		
Asset split	1	3	8	1	3	8	1	3	8
CCC-GARCH	-1.360^{b}	-1.192	-1.141	-1.090^{b}	-0.911	-0.873	-1.055^{c}	-1.134	-1.147
	(0.655)	(0.857)	(0.817)	(0.494)	(0.647)	(0.703)	(0.549)	(0.805)	(0.832)
DCC-GARCH		-1.142	-1.189		-0.874	-0.992		-1.001	-1.171
		(0.784)	(0.842)		(0.546)	(0.768)		(0.715)	(0.874)
CCC-GJR	-1.011^{c}	-1.090	-1.018	-0.683^{c}	-0.693	-0.778	-0.708^{c}	-0.864	-0.914
	(0.561)	(0.715)	(0.715)	(0.363)	(0.439)	(0.583)	(0.398)	(0.627)	(0.690)
DCC-GJR		-1.065	-1.083		-0.740^{c}	-0.849		-0.891	-1.007
		(0.651)	(0.732)		(0.433)	(0.612)		(0.600)	(0.725)
RiskMetrics	-1.414^{c}	-0.992	-1.187	-1.304^{c}	-0.937^{b}	-1.277^{b}	-1.024^{b}	-1.038^{c}	-1.053
	(0.813)	(0.618)	(0.885)	(0.686)	(0.458)	(0.600)	(0.522)	(0.628)	(0.682)

(c) daily vs. biweekly returns, iterated forecasts

	Normal			Empirical			Student's t		
Asset split	1	3	8	1	3	8	1	3	8
CCC-GARCH	-1.534^{b}	-1.689	-1.576^{c}	-1.734^{a}	-2.049^{b}	-1.588^{c}	-1.376^{b}	-1.623	-1.493
	(0.675)	(1.048)	(0.951)	(0.569)	(0.864)	(0.832)	(0.612)	(1.008)	(0.951)
DCC-GARCH		-1.612^{c}	-1.672		-1.990^{b}	-1.713^{c}		-1.513^{c}	-1.567
		(0.970)	(1.019)		(0.783)	(0.889)		(0.911)	(1.006)
CCC-GJR	-1.667^{b}	-1.712	-1.686^{c}	-1.671^{b}	-1.874^{b}	-1.632^{c}	-1.391^{b}	-1.605	-1.576
	(0.836)	(1.184)	(1.003)	(0.711)	(0.940)	(0.871)	(0.702)	(1.030)	(1.015)
DCC-GJR		-1.611	-1.735		-1.796^{b}	-1.692^{c}		-1.605	-1.602
		(1.067)	(1.060)		(0.834)	(0.933)		(1.006)	(1.020)
RiskMetrics	-2.010^{b}	-1.551	-2.352^{c}	-2.122^{a}	-2.338^{b}	-2.955^{b}	-1.819^{b}	-1.659^{c}	-2.278^{c}
	(0.994)	(1.010)	(1.369)	(0.819)	(1.019)	(1.343)	(0.893)	(0.994)	(1.212)

This table shows the results of the test that the expected value of the asymmetric tick loss function of Giacomini and Komunjer (2005) as in Equation (20) with $\alpha = 0.99$ for two methods are equal. The two methods differ in their degree of temporal aggregation as stated in the headings of the different panels. We report the loss differential (multiplied by 100), calculated by substracting the loss of second method from the loss of the first one. We report standard errors of the average loss differential in parentheses, based on Newey and West (1987) with 14 leads and lags. We test that the average loss differential is zero based on the statistic proposed by Diebold and Mariano (1995), evaluated in the setting of Giacomini and White (2006). Superscripts ^{a},^{b} ,^{c} denote the rejection with a significance level of 1%, 5% and 10%.

Table C.1: Results of the DM-tests for temporal aggregation – *continued*

(d) daily vs. biweekly returns, scaled forecasts

Asset split	Normal			Empirical			Student's t		
	1	3	8	1	3	8	1	3	8
CCC-	-1.497^{b}	-1.569	-1.532	-1.895^{a}	-2.070^{b}	-1.682^{c}	-1.441^{b}	-1.604	-1.586
GARCH	(0.704)	(1.054)	(0.989)	(0.690)	(0.899)	(0.934)	(0.674)	(1.025)	(1.053)
DCC-		-1.475	-1.612		-1.940^{b}	-1.756^{c}		-1.486	-1.583
GARCH		(0.953)	(1.036)		(0.775)	(0.980)		(0.916)	(1.066)
CCC-	-1.430^{b}	-1.408	-1.418	-1.557^{b}	-1.699^{b}	-1.562^{c}	-1.217^{b}	-1.399	-1.442
GJR	(0.722)	(1.022)	(0.948)	(0.659)	(0.788)	(0.881)	(0.614)	(0.944)	(0.981)
DCC-		-1.335	-1.508		-1.668^{b}	-1.614^{c}		-1.424	-1.486
GJR		(0.920)	(0.988)		(0.729)	(0.914)		(0.895)	(0.971)
RiskMetrics	-1.857^{b}	-1.531	-2.277^{c}	-2.205^{b}	-2.368^{b}	-2.694^{b}	-1.848^{b}	-1.668^{c}	-2.384^{c}
	(0.925)	(1.000)	(1.349)	(0.878)	(1.072)	(1.239)	(0.856)	(1.008)	(1.279)

(e) weekly vs. biweekly returns, iterated vs direct forecasts

Asset split	Normal			Empirical			Student's t		
	1	3	8	1	3	8	1	3	8
CCC-	-0.305^{c}	-0.482^{c}	-0.470^{b}	-0.651^{a}	-1.161^{a}	-0.742^{a}	-0.367^{b}	-0.524^{b}	-0.369^{c}
GARCH	(0.170)	(0.272)	(0.222)	(0.154)	(0.247)	(0.183)	(0.187)	(0.257)	(0.199)
DCC-		-0.447^{c}	-0.490^{b}		-1.096^{a}	-0.776^{a}		-0.473^{c}	-0.382^{c}
GARCH		(0.257)	(0.237)		(0.222)	(0.209)		(0.241)	(0.218)
CCC-	-0.545^{b}	-0.496	-0.545^{c}	-0.760^{a}	-0.950^{a}	-0.701^{b}	-0.446^{c}	-0.660^{c}	-0.507^{c}
GJR	(0.247)	(0.417)	(0.307)	(0.232)	(0.342)	(0.292)	(0.246)	(0.360)	(0.294)
DCC-		-0.458	-0.548^{c}		-0.920^{a}	-0.727^{b}		-0.557^{c}	-0.545^{c}
GJR		(0.368)	(0.315)		(0.307)	(0.315)		(0.317)	(0.287)
RiskMetrics	-0.500^{b}	-0.532	-1.114^{b}	-0.845^{a}	-1.281^{b}	-1.356^{b}	-0.792^{b}	-0.618	-1.267^{b}
	(0.247)	(0.441)	(0.532)	(0.214)	(0.551)	(0.583)	(0.369)	(0.413)	(0.595)

(f) weekly vs. biweekly returns, scaled vs direct forecasts

Asset split	Normal			Empirical			Student's t		
	1	3	8	1	3	8	1	3	8
CCC-	-0.138	-0.377	-0.391^{c}	-0.805^{a}	-1.160^{a}	-0.810^{a}	-0.386^{c}	-0.470^{c}	-0.438^{c}
GARCH	(0.137)	(0.254)	(0.207)	(0.243)	(0.281)	(0.249)	(0.200)	(0.268)	(0.240)
DCC-		-0.333	-0.423^{c}		-1.067^{a}	-0.764^{a}		-0.485^{c}	-0.412^{c}
GARCH		(0.236)	(0.226)		(0.261)	(0.229)		(0.252)	(0.219)
CCC-	-0.419^{c}	-0.318	-0.400	-0.874^{b}	-1.007^{a}	-0.784^{b}	-0.508^{c}	-0.535	-0.528^{c}
GJR	(0.218)	(0.377)	(0.263)	(0.343)	(0.376)	(0.314)	(0.272)	(0.346)	(0.312)
DCC-		-0.270	-0.425		-0.928^{a}	-0.765^{b}		-0.533^{c}	-0.479^{c}
GJR		(0.349)	(0.285)		(0.330)	(0.315)		(0.321)	(0.268)
RiskMetrics	-0.443^{b}	-0.538	-1.090^{b}	-0.900^{a}	-1.431^{b}	-1.417^{b}	-0.824^{b}	-0.631	-1.331^{b}
	(0.223)	(0.431)	(0.530)	(0.267)	(0.659)	(0.678)	(0.395)	(0.427)	(0.638)

See table note on previous page. This table shows the results of the test that the expected value of the asymmetric tick loss function of Giacomini and Komunjer (2005) as in Equation (20) with $\alpha = 0.99$ for two methods are equal. The two methods differ in their degree of

Table C.2: Results of the DM-tests for portfolio aggregation

(a) daily returns, iterated forecasts

Test	Normal			Empirical			Student's t		
	1 vs. 3	1 vs. 8	3 vs. 8	1 vs. 3	1 vs. 8	3 vs. 8	1 vs. 3	1 vs. 8	3 vs. 8
CCC-GARCH	0.411	0.281	−0.130	0.377	0.316	−0.061	0.397	0.294	−0.103
	(0.356)	(0.259)	(0.175)	(0.359)	(0.283)	(0.158)	(0.391)	(0.284)	(0.187)
DCC-GARCH	0.370	0.378	0.009	0.363	0.432	0.068	0.320	0.372	0.052
	(0.308)	(0.319)	(0.114)	(0.318)	(0.336)	(0.110)	(0.319)	(0.331)	(0.115)
CCC-GJR	0.221	0.237	0.016	0.164	0.245	0.080	0.205	0.263	0.058
	(0.305)	(0.187)	(0.231)	(0.297)	(0.204)	(0.202)	(0.283)	(0.255)	(0.153)
DCC-GJR	0.150	0.306	0.156	0.084	0.325	0.241^c	0.241	0.325	0.084
	(0.215)	(0.229)	(0.129)	(0.215)	(0.267)	(0.128)	(0.292)	(0.282)	(0.143)
RiskMetrics	-0.460^c	-0.515^b	−0.055	−0.298	-0.183^c	0.114	−0.348	−0.463	−0.115
	(0.278)	(0.258)	(0.157)	(0.198)	(0.106)	(0.230)	(0.388)	(0.482)	(0.144)

(b) daily returns, scaled forecasts

Test	Normal			Empirical			Student's t		
	1 vs. 3	1 vs. 8	3 vs. 8	1 vs. 3	1 vs. 8	3 vs. 8	1 vs. 3	1 vs. 8	3 vs. 8
CCC-GARCH	0.327	0.274	−0.054	0.238	0.250	0.012	0.313	0.321	0.008
	(0.326)	(0.256)	(0.152)	(0.281)	(0.275)	(0.118)	(0.346)	(0.311)	(0.133)
DCC-GARCH	0.270	0.355	0.086	0.153	0.314	0.161	0.228	0.322	0.094
	(0.255)	(0.295)	(0.105)	(0.200)	(0.315)	(0.149)	(0.262)	(0.327)	(0.125)
CCC-GJR	0.153	0.205	0.052	0.104	0.289	0.185	0.174	0.304	0.130
	(0.242)	(0.217)	(0.148)	(0.190)	(0.254)	(0.130)	(0.280)	(0.288)	(0.127)
DCC-GJR	0.110	0.316	0.206	0.070	0.361	0.291^c	0.234	0.384	0.150
	(0.169)	(0.252)	(0.135)	(0.153)	(0.296)	(0.170)	(0.262)	(0.311)	(0.135)
RiskMetrics	−0.328	-0.437^b	−0.109	-0.351^c	-0.528^b	−0.177	−0.367	−0.386	−0.019
	(0.224)	(0.206)	(0.139)	(0.198)	(0.206)	(0.157)	(0.327)	(0.353)	(0.118)

(c) weekly returns, iterated forecasts

Test	Normal			Empirical			Student's t		
	1 vs. 3	1 vs. 8	3 vs. 8	1 vs. 3	1 vs. 8	3 vs. 8	1 vs. 3	1 vs. 8	3 vs. 8
CCC-GARCH	0.432^b	0.404^a	−0.028	0.572^b	0.554^a	−0.019	0.307^c	0.178	−0.129
	(0.175)	(0.145)	(0.086)	(0.260)	(0.182)	(0.140)	(0.157)	(0.156)	(0.148)
DCC-GARCH	0.434^b	0.425^a	−0.009	0.553^b	0.577^a	0.025	0.290^c	0.196	−0.094
	(0.178)	(0.150)	(0.078)	(0.267)	(0.194)	(0.128)	(0.163)	(0.147)	(0.143)
CCC-GJR	0.126	0.218^b	0.091	0.152	0.225^b	0.073	0.205^c	0.139	−0.067
	(0.196)	(0.111)	(0.141)	(0.199)	(0.106)	(0.143)	(0.122)	(0.144)	(0.144)
DCC-GJR	0.118	0.241^b	0.123	0.120	0.271^b	0.152	0.139	0.213^c	0.075
	(0.183)	(0.116)	(0.122)	(0.196)	(0.134)	(0.121)	(0.114)	(0.124)	(0.111)
RiskMetrics	0.031	−0.242	−0.273	−0.078	-0.507^a	-0.428^a	−0.362	−0.447	−0.085
	(0.120)	(0.302)	(0.247)	(0.109)	(0.165)	(0.144)	(0.445)	(0.550)	(0.170)

This table shows the results of the test that the expected value of the asymmetric tick loss function of Giacomini and Komunjer (2005) as in Equation (20) with $\alpha = 0.99$ for two methods are equal. The two methods differ in their degree of portfolio aggregation. We consider aggregation into a portfolio (labeled "1"), three asset classes (labeled "3"), and no aggregation (labeled "8"). We report the loss differential (multiplied by 100), calculated by substracting the loss of second method from the loss of the first one. We report standard errors of the average loss differential in parentheses, based on Newey and West (1987) with 14 leads and lags. We test that the average loss differential is zero based on the statistic proposed by Diebold and Mariano (1995), evaluated in the setting of Giacomini and White (2006). Superscripts ^{a},^{b},^{c} denote the rejection with a significance level of 1%, 5% and 10%.

Table C.2: Results of the DM-tests for portfolio aggregation – *continued*

(d) weekly returns, scaled forecasts

Test	Normal			Empirical			Student's t		
	1 vs. 3	1 vs. 8	3 vs. 8	1 vs. 3	1 vs. 8	3 vs. 8	1 vs. 3	1 vs. 8	3 vs. 8
CCC-GARCH	0.494^{b}	0.493^{a}	−0.002	0.418^{b}	0.468^{a}	0.050	0.234	0.228	−0.005
	(0.200)	(0.177)	(0.086)	(0.187)	(0.178)	(0.131)	(0.146)	(0.140)	(0.126)
DCC-GARCH	0.487^{b}	0.525^{a}	0.038	0.369^{c}	0.412^{b}	0.042	0.282^{c}	0.206	−0.076
	(0.199)	(0.179)	(0.066)	(0.190)	(0.165)	(0.146)	(0.146)	(0.155)	(0.146)
CCC-GJR	0.074	0.199^{c}	0.125	0.094	0.194	0.100	0.018	0.098	0.080
	(0.155)	(0.116)	(0.147)	(0.151)	(0.123)	(0.154)	(0.097)	(0.154)	(0.118)
DCC-GJR	0.056	0.244^{b}	0.188	0.013	0.195^{b}	0.182	0.052	0.086	0.034
	(0.151)	(0.105)	(0.118)	(0.133)	(0.099)	(0.118)	(0.094)	(0.144)	(0.134)
RiskMetrics	0.094	−0.210	−0.304	0.017	−0.500^{a}	−0.517^{a}	−0.381	−0.415	−0.034
	(0.135)	(0.299)	(0.238)	(0.132)	(0.173)	(0.142)	(0.457)	(0.533)	(0.150)

(e) biweekly returns, direct forecasts

Test	Normal			Empirical			Student's t		
	1 vs. 3	1 vs. 8	3 vs. 8	1 vs. 3	1 vs. 8	3 vs. 8	1 vs. 3	1 vs. 8	3 vs. 8
CCC-GARCH	0.255	0.239	−0.017	0.063	0.463^{b}	0.400^{a}	0.150	0.176	0.026
	(0.165)	(0.211)	(0.096)	(0.211)	(0.198)	(0.129)	(0.140)	(0.207)	(0.141)
DCC-GARCH	0.292^{c}	0.240	−0.052	0.107	0.453^{b}	0.345^{b}	0.183	0.181	−0.003
	(0.155)	(0.208)	(0.114)	(0.226)	(0.193)	(0.166)	(0.129)	(0.199)	(0.157)
CCC-GJR	0.176	0.218	0.042	−0.039	0.284^{c}	0.322^{b}	−0.009	0.078	0.087
	(0.188)	(0.204)	(0.090)	(0.170)	(0.155)	(0.130)	(0.141)	(0.202)	(0.124)
DCC-GJR	0.206	0.238	0.032	−0.041	0.304^{b}	0.345^{a}	0.027	0.115	0.088
	(0.179)	(0.197)	(0.093)	(0.176)	(0.151)	(0.123)	(0.120)	(0.175)	(0.125)
RiskMetrics	−0.001	−0.856	−0.855^{b}	−0.514	−1.017^{c}	−0.502^{a}	−0.188	−0.922	−0.734^{c}
	(0.432)	(0.642)	(0.407)	(0.498)	(0.583)	(0.185)	(0.541)	(0.795)	(0.385)

See table note on previous page.

Table C.3: Results of the DM-tests for model choice

(a) Asset level

	Daily, iterated			Daily, scaled			Weekly, iterated			Weekly, scaled			Biweekly, direct		
	N	Emp.	t	N	Emp.	t	N	Emp.	t	N	Emp.	t	N	Emp.	t
GARCH vs.	0.380	0.342	0.357	0.180	0.067	0.116	0.487^b	0.513^b	0.421	0.529^b	0.474^b	0.463^c	0.247	0.405^a	0.341^c
GJR	(0.274)	(0.249)	(0.258)	(0.154)	(0.118)	(0.142)	(0.218)	(0.232)	(0.268)	(0.255)	(0.217)	(0.273)	(0.161)	(0.141)	(0.175)
GARCH vs.	0.302	0.139	0.181	0.186	0.061	0.144	0.021	−0.055	0.163	0.132	−0.153	0.175	−0.174	−0.249	-0.263^b
RiskMetrics	(0.238)	(0.150)	(0.339)	(0.146)	(0.098)	(0.247)	(0.167)	(0.170)	(0.292)	(0.203)	(0.195)	(0.299)	(0.172)	(0.171)	(0.130)
GJR vs.	−0.078	−0.203	−0.176	0.006	−0.006	0.028	-0.466^c	-0.568^c	−0.258	−0.397	-0.627^c	−0.288	-0.421^c	-0.654^a	-0.604^a
RiskMetrics	(0.164)	(0.200)	(0.131)	(0.156)	(0.139)	(0.212)	(0.256)	(0.319)	(0.161)	(0.308)	(0.343)	(0.179)	(0.245)	(0.244)	(0.177)

(b) Asset class level

	Daily, iterated			Daily, scaled			Weekly, iterated			Weekly, scaled			Biweekly, direct		
	N	Emp.	t	N	Emp.	t	N	Emp.	t	N	Emp.	t	N	Emp.	t
CCC-GARCH vs.	0.097	0.116	0.078	0.081	0.064	0.002	0.021	0.024	0.018	0.033	−0.056	−0.022	0.001	−0.010	0.005
DCC-GARCH	(0.087)	(0.080)	(0.077)	(0.067)	(0.070)	(0.054)	(0.035)	(0.044)	(0.046)	(0.036)	(0.038)	(0.042)	(0.010)	(0.025)	(0.029)
CCC-GARCH vs.	0.336^c	0.270	0.326	0.112	0.106	0.100	0.301	0.185	0.381	0.235	0.200	0.333	0.226^b	0.226^b	0.243^c
CCC-GJR	(0.183)	(0.175)	(0.225)	(0.095)	(0.079)	(0.095)	(0.187)	(0.186)	(0.236)	(0.163)	(0.153)	(0.208)	(0.113)	(0.095)	(0.139)
CCC-GARCH vs.	0.406	0.350	0.388	0.222^c	0.178^c	0.180	0.324	0.231	0.456^c	0.280	0.202	0.320	0.246^c	0.246^b	0.280
DCC-GJR	(0.248)	(0.235)	(0.268)	(0.134)	(0.105)	(0.111)	(0.214)	(0.236)	(0.272)	(0.210)	(0.185)	(0.210)	(0.130)	(0.118)	(0.178)
CCC-GARCH vs.	-0.493^b	-0.361^a	−0.576	-0.524^b	-0.717^b	−0.563	-0.625^c	-1.115^a	-0.462^c	-0.571^c	-1.121^a	-0.468^c	-1.269^c	-1.728^b	-1.361^b
RiskMetrics	(0.214)	(0.138)	(0.362)	(0.262)	(0.330)	(0.388)	(0.354)	(0.266)	(0.258)	(0.342)	(0.237)	(0.254)	(0.663)	(0.672)	(0.668)
DCC-GARCH vs.	0.239	0.155	0.247	0.031	0.042	0.098	0.280^c	0.161	0.363^c	0.202	0.256^c	0.355	0.225^b	0.236^b	0.239^c
CCC-GJR	(0.148)	(0.143)	(0.176)	(0.101)	(0.104)	(0.113)	(0.168)	(0.151)	(0.210)	(0.136)	(0.153)	(0.231)	(0.108)	(0.094)	(0.133)
DCC-GARCH vs.	0.308	0.235	0.310	0.141	0.114	0.178^c	0.303	0.207	0.438^c	0.247	0.258	0.342	0.245^b	0.256^b	0.275
DCC-GJR	(0.188)	(0.173)	(0.208)	(0.096)	(0.079)	(0.108)	(0.194)	(0.201)	(0.246)	(0.180)	(0.186)	(0.230)	(0.123)	(0.115)	(0.170)
DCC-GARCH vs.	-0.590^b	-0.476^a	−0.654	-0.606^c	-0.781^b	−0.564	-0.646^c	-1.139^a	-0.480^c	−0.603	-1.065^a	-0.446^c	-1.270^c	-1.718^b	-1.365^b
RiskMetrics	(0.275)	(0.184)	(0.424)	(0.310)	(0.380)	(0.409)	(0.373)	(0.297)	(0.280)	(0.368)	(0.235)	(0.244)	(0.670)	(0.677)	(0.673)
CCC-GJR vs.	0.069	0.080	0.062	0.110	0.072	0.080	0.023	0.046	0.075	0.045	0.001	−0.013	0.020	0.020	0.037
DCC-GJR	(0.091)	(0.106)	(0.064)	(0.075)	(0.077)	(0.059)	(0.041)	(0.058)	(0.047)	(0.052)	(0.041)	(0.035)	(0.024)	(0.034)	(0.057)
CCC-GJR vs.	-0.829^b	-0.631^b	−0.902	-0.636^b	-0.823^b	−0.662	-0.927^c	-1.300^a	-0.843^c	-0.805^c	-1.322^a	-0.801^c	-1.495^b	-1.954^a	-1.604^b
RiskMetrics	(0.369)	(0.260)	(0.550)	(0.322)	(0.333)	(0.434)	(0.517)	(0.430)	(0.469)	(0.480)	(0.364)	(0.440)	(0.752)	(0.739)	(0.786)
DCC-GJR vs.	-0.899^b	-0.711^b	−0.964	-0.747^b	-0.895^b	−0.743	-0.950^c	-1.346^a	-0.918^c	−0.851	-1.323^a	-0.789^c	-1.515^b	-1.974^a	-1.640^b
RiskMetrics	(0.436)	(0.323)	(0.595)	(0.372)	(0.392)	(0.463)	(0.542)	(0.478)	(0.502)	(0.526)	(0.394)	(0.441)	(0.771)	(0.766)	(0.824)

This table shows the results of the test that the expected value of the asymmetric tick loss function of Giacomini and Komunjer (2005) as in Equation (20) with $\alpha = 0.99$ for two methods are equal. The two methods differ in their model choice. We report the loss differential (multiplied by 100), calculated by substracting the loss of second method from the loss of the first one. We report standard errors of the average loss differential in parentheses, based on Newey and West (1987) with 14 leads and lags. We test that the average loss differential is zero based on the statistic proposed by Diebold and Mariano (1995), evaluated in the setting of Giacomini and White (2006). Superscripts ^{a},^{b},^{c} denote the rejection with a significance level of 1%, 5% and 10%.

Table C.3: Results of the DM-tests for model choice – *continued*

(c) Portfolio level

	Daily, iterated			Daily, scaled			Weekly, iterated			Weekly, scaled			Biweekly, direct		
	N	Emp.	t	N	Emp.	t	N	Emp.	t	N	Emp.	t	N	Emp.	t
CCC-GARCH vs.	−0.042	−0.014	−0.076	−0.058	−0.085	−0.085	0.002	−0.020	−0.017	−0.007	−0.048	0.049^{c}	0.036	0.045	0.034
DCC-GARCH	(0.064)	(0.058)	(0.082)	(0.080)	(0.091)	(0.092)	(0.031)	(0.035)	(0.037)	(0.021)	(0.038)	(0.027)	(0.043)	(0.055)	(0.050)
CCC-GARCH vs.	0.190	0.129	0.165	0.006	−0.067	−0.023	0.182	0.093	0.319	0.108	0.150	0.248	0.168	0.304^{a}	0.182
CCC-GJR	(0.236)	(0.206)	(0.201)	(0.122)	(0.140)	(0.112)	(0.244)	(0.188)	(0.245)	(0.234)	(0.194)	(0.218)	(0.115)	(0.106)	(0.121)
CCC-GARCH vs.	0.120	0.048	0.201	−0.037	−0.101	0.038	0.173	0.060	0.252	0.091	0.070	0.281	0.198	0.301^{b}	0.218
DCC-GJR	(0.186)	(0.166)	(0.217)	(0.133)	(0.157)	(0.118)	(0.233)	(0.190)	(0.245)	(0.236)	(0.184)	(0.235)	(0.140)	(0.127)	(0.164)
CCC-GARCH vs.	−0.569	−0.536	−0.564	−0.469	−0.528	−0.536	−0.380^{c}	−0.705^{b}	−0.506^{c}	−0.269	−0.555^{b}	−0.440	−0.431	−0.826	−0.601
RiskMetrics	(0.392)	(0.412)	(0.414)	(0.412)	(0.389)	(0.410)	(0.217)	(0.303)	(0.283)	(0.211)	(0.225)	(0.281)	(0.500)	(0.657)	(0.518)
DCC-GARCH vs.	0.231	0.143	0.241	0.064	0.018	0.062	0.180	0.113	0.336	0.116	0.199	0.199	0.131	0.259^{a}	0.149
CCC-GJR	(0.271)	(0.228)	(0.241)	(0.157)	(0.140)	(0.150)	(0.228)	(0.168)	(0.227)	(0.225)	(0.170)	(0.206)	(0.100)	(0.098)	(0.100)
DCC-GARCH vs.	0.161	0.062	0.277	0.021	−0.016	0.123	0.171	0.080	0.269	0.098	0.118	0.233	0.161	0.256^{b}	0.185
DCC-GJR	(0.204)	(0.170)	(0.252)	(0.122)	(0.130)	(0.141)	(0.216)	(0.169)	(0.226)	(0.226)	(0.159)	(0.223)	(0.114)	(0.105)	(0.137)
DCC-GARCH vs.	−0.527	−0.522	−0.488	−0.411	−0.443	−0.451	−0.382^{c}	−0.686^{b}	−0.489	−0.261	−0.506^{b}	−0.488^{c}	−0.467	−0.871	−0.634
RiskMetrics	(0.349)	(0.377)	(0.349)	(0.345)	(0.309)	(0.331)	(0.224)	(0.320)	(0.300)	(0.213)	(0.238)	(0.285)	(0.528)	(0.703)	(0.542)
CCC-GJR vs.	−0.070	−0.081	0.036	−0.043	−0.034	0.060	−0.008	−0.032	−0.067^{b}	−0.018	−0.081^{b}	0.034	0.030	−0.002	0.036
DCC-GJR	(0.097)	(0.090)	(0.042)	(0.082)	(0.054)	(0.049)	(0.037)	(0.045)	(0.032)	(0.021)	(0.033)	(0.030)	(0.043)	(0.036)	(0.052)
CCC-GJR vs.	−0.759	−0.665	−0.729	−0.475	−0.461	−0.514	−0.562	−0.798^{c}	−0.825^{c}	−0.377	−0.705^{c}	−0.687	−0.598	−1.129	−0.783
RiskMetrics	(0.591)	(0.572)	(0.548)	(0.448)	(0.358)	(0.440)	(0.424)	(0.471)	(0.491)	(0.403)	(0.377)	(0.462)	(0.532)	(0.704)	(0.583)
DCC-GJR vs.	−0.689	−0.584	−0.765	−0.432	−0.427	−0.574	−0.554	−0.766^{c}	−0.758	−0.359	−0.624^{c}	−0.721	−0.628	−1.127	−0.819
RiskMetrics	(0.509)	(0.496)	(0.562)	(0.382)	(0.327)	(0.431)	(0.406)	(0.463)	(0.488)	(0.400)	(0.360)	(0.476)	(0.561)	(0.730)	(0.618)

See table note on previous page.

Table C.4: Results of the DM-tests for the distribution choice

(a) daily returns, iterated forecasts

Asset split	Normal vs. t			Normal vs. Empirical			Empirical vs. t		
	1	3	8	1	3	8	1	3	8
CCC-GARCH	0.038	0.024	0.051	0.068	0.034	0.103	−0.030	−0.010	−0.052
	(0.039)	(0.052)	(0.041)	(0.077)	(0.083)	(0.091)	(0.063)	(0.059)	(0.068)
DCC-GARCH		−0.011	0.032		0.062	0.122		−0.073	−0.089
		(0.036)	(0.034)		(0.094)	(0.092)		(0.079)	(0.076)
CCC-GJR	0.015	−0.001	0.041	0.030	−0.027	0.037	−0.015	0.026	0.003
	(0.020)	(0.046)	(0.093)	(0.053)	(0.059)	(0.080)	(0.047)	(0.063)	(0.076)
DCC-GJR		0.105	0.033		−0.037	0.048		0.142^{b}	−0.015
		(0.086)	(0.060)		(0.060)	(0.078)		(0.071)	(0.051)
RiskMetrics	−0.083	0.029	−0.032	−0.096	0.067	0.236	0.013	−0.038	−0.267
	(0.209)	(0.038)	(0.174)	(0.062)	(0.066)	(0.201)	(0.227)	(0.055)	(0.324)

(b) daily returns, scaled forecasts

Asset split	Normal vs. t			Normal vs. Empirical			Empirical vs. t		
	1	3	8	1	3	8	1	3	8
CCC-GARCH	0.140^{c}	0.126	0.187	0.265	0.176	0.242	−0.125	−0.050	−0.054
	(0.080)	(0.089)	(0.116)	(0.188)	(0.148)	(0.171)	(0.121)	(0.068)	(0.063)
DCC-GARCH		0.099	0.108		0.149	0.224		−0.050	−0.117
		(0.078)	(0.084)		(0.136)	(0.172)		(0.073)	(0.097)
CCC-GJR	0.076	0.097	0.175	0.152	0.102	0.235^{c}	−0.075	−0.006	−0.060
	(0.057)	(0.077)	(0.116)	(0.110)	(0.102)	(0.140)	(0.065)	(0.077)	(0.046)
DCC-GJR		0.200	0.145		0.112	0.197		0.089	−0.053
		(0.128)	(0.091)		(0.109)	(0.140)		(0.081)	(0.059)
RiskMetrics	0.098	0.059	0.149	0.140	0.117	0.049	−0.042	−0.058	0.100
	(0.250)	(0.091)	(0.169)	(0.143)	(0.158)	(0.210)	(0.123)	(0.091)	(0.139)

(c) weekly returns, iterated forecasts

Asset split	Normal vs. t			Normal vs. Empirical			Empirical vs. t		
	1	3	8	1	3	8	1	3	8
CCC-GARCH	0.258^{c}	0.133	0.032	0.214	0.354	0.364	0.044	−0.221	−0.332
	(0.136)	(0.087)	(0.044)	(0.166)	(0.285)	(0.233)	(0.092)	(0.219)	(0.217)
DCC-GARCH		0.114	0.028		0.332	0.366		−0.219	−0.338
		(0.096)	(0.037)		(0.287)	(0.239)		(0.219)	(0.214)
CCC-GJR	0.191	0.270^{c}	0.112	0.239	0.265	0.247	−0.048	0.005	−0.135
	(0.186)	(0.141)	(0.085)	(0.185)	(0.236)	(0.213)	(0.107)	(0.156)	(0.147)
DCC-GJR		0.211	0.163^{c}		0.241	0.270		−0.030	−0.106
		(0.156)	(0.090)		(0.241)	(0.232)		(0.162)	(0.158)
RiskMetrics	0.399	0.007	0.195	0.138	0.029	−0.126	0.261	−0.022	0.321
	(0.435)	(0.048)	(0.159)	(0.125)	(0.192)	(0.372)	(0.353)	(0.182)	(0.283)

This table shows the results of the test that the expected value of the asymmetric tick loss function of Giacomini and Komunjer (2005) as in Equation (20) with $\alpha = 0.99$ for two methods are equal. The two methods differ in the distribution of the innovations. We consider the normal, empirical and Student's t-distributions. We report the loss differential (multiplied by 100), calculated by substracting the loss of second method from the loss of the first one. We report standard errors of the average loss differential in parentheses, based on Newey and West (1987) with 14 leads and lags. We test that the average loss differential is zero based on the statistic proposed by Diebold and Mariano (1995), evaluated in the setting of Giacomini and White (2006). Superscripts ^{a},^{b},^{c} denote the rejection with a significance level of 1%, 5% and 10%.

Table C.4: Results of the DM-tests for the distribution choice – ***continued***

(d) weekly returns, scaled forecasts

	Normal vs. t			Normal vs. Empirical			Empirical vs. t		
Asset split	1	3	8	1	3	8	1	3	8
CCC-GARCH	0.445[b]	0.184	0.181[c]	0.535	0.458	0.510	−0.090	−0.274	−0.329
	(0.195)	(0.127)	(0.096)	(0.348)	(0.373)	(0.330)	(0.194)	(0.263)	(0.243)
DCC-GARCH		0.240[c]	0.126[c]		0.417	0.421		−0.177	−0.295
		(0.133)	(0.070)		(0.373)	(0.289)		(0.252)	(0.236)
CCC-GJR	0.379[c]	0.323[c]	0.279[c]	0.480	0.500	0.476	−0.101	−0.177	−0.197
	(0.215)	(0.185)	(0.146)	(0.308)	(0.339)	(0.292)	(0.129)	(0.206)	(0.158)
DCC-GJR		0.375[c]	0.221[b]		0.437	0.432		−0.063	−0.211
		(0.199)	(0.106)		(0.317)	(0.278)		(0.166)	(0.184)
RiskMetrics	0.488	0.013	0.283	0.250	0.172	−0.041	0.238	−0.159	0.324
	(0.481)	(0.072)	(0.210)	(0.256)	(0.335)	(0.466)	(0.291)	(0.284)	(0.310)

(e) biweekly returns, direct forecasts

	Normal vs. t			Normal vs. Empirical			Empirical vs. t		
Asset split	1	3	8	1	3	8	1	3	8
CCC-GARCH	0.196[c]	0.091	0.133[c]	−0.132	−0.325	0.092	0.329[c]	0.416	0.042
	(0.115)	(0.114)	(0.076)	(0.278)	(0.357)	(0.299)	(0.183)	(0.280)	(0.235)
DCC-GARCH		0.088	0.137[b]		−0.317	0.080		0.405	0.056
		(0.107)	(0.069)		(0.369)	(0.298)		(0.285)	(0.240)
CCC-GJR	0.290[c]	0.106	0.150	0.025	−0.189	0.091	0.265[b]	0.295	0.059
	(0.164)	(0.168)	(0.095)	(0.235)	(0.331)	(0.248)	(0.134)	(0.220)	(0.163)
DCC-GJR		0.112	0.167		−0.221	0.091		0.333	0.076
		(0.168)	(0.110)		(0.324)	(0.258)		(0.211)	(0.163)
RiskMetrics	0.108	−0.079	0.042	−0.207	−0.721[a]	−0.368	0.315	0.641[a]	0.410
	(0.236)	(0.094)	(0.095)	(0.250)	(0.252)	(0.393)	(0.196)	(0.242)	(0.378)

See table note on previous page.

Table C.5: Results of the DM-tests for the forecasting method

(a) daily returns

Asset split	Normal 1	Normal 3	Normal 8	Empirical 1	Empirical 3	Empirical 8	Student's t 1	Student's t 3	Student's t 8
CCC-GARCH	−0.037	-0.120^{c}	−0.044	0.161	0.022	0.095	0.065	−0.018	0.092
	(0.074)	(0.071)	(0.062)	(0.139)	(0.061)	(0.110)	(0.073)	(0.034)	(0.108)
DCC-GARCH		-0.137^{c}	−0.060		−0.050	0.043		−0.027	0.016
		(0.071)	(0.047)		(0.054)	(0.098)		(0.031)	(0.068)
CCC-GJR	−0.236	−0.304	-0.268^{b}	−0.114	−0.174	−0.070	−0.175	−0.206	−0.133
	(0.181)	(0.225)	(0.132)	(0.107)	(0.192)	(0.098)	(0.146)	(0.165)	(0.088)
DCC-GJR		−0.277	-0.227^{c}		−0.128	−0.078		−0.181	−0.116
		(0.214)	(0.137)		(0.157)	(0.095)		(0.169)	(0.101)
RiskMetrics	−0.153	−0.020	-0.075^{b}	0.083	0.029	-0.261^{c}	0.029	0.009	0.106
	(0.098)	(0.050)	(0.038)	(0.084)	(0.084)	(0.150)	(0.049)	(0.029)	(0.094)

(b) weekly returns

Asset split	Normal 1	Normal 3	Normal 8	Empirical 1	Empirical 3	Empirical 8	Student's t 1	Student's t 3	Student's t 8
CCC-GARCH	-0.168^{c}	-0.105^{c}	-0.079^{c}	0.154	−0.001	0.067	0.019	−0.054	0.070
	(0.087)	(0.056)	(0.042)	(0.152)	(0.054)	(0.080)	(0.040)	(0.041)	(0.054)
DCC-GARCH		-0.115^{b}	−0.067		−0.030	−0.012		0.012	0.030
		(0.057)	(0.043)		(0.058)	(0.031)		(0.030)	(0.035)
CCC-GJR	−0.126	−0.178	-0.145^{c}	0.115	0.056	0.083^{b}	0.062	−0.125	0.022
	(0.108)	(0.110)	(0.082)	(0.132)	(0.068)	(0.041)	(0.056)	(0.078)	(0.048)
DCC-GJR		-0.188^{c}	−0.123		0.008	0.038		−0.025	−0.066
		(0.107)	(0.080)		(0.072)	(0.037)		(0.067)	(0.053)
RiskMetrics	−0.057	0.006	−0.024	0.055	0.150	0.061	0.032	0.012	0.063
	(0.071)	(0.031)	(0.020)	(0.115)	(0.141)	(0.113)	(0.058)	(0.023)	(0.053)

This table shows the results of the test that the expected value of the asymmetric tick loss function of Giacomini and Komunjer (2005) as in Equation (20) with $\alpha = 0.99$ for two methods are equal. The two methods differ in their forecast construction. We consider iterated and scaled forecasts. We report the loss differential (multiplied by 100), calculated by substracting the loss of second method from the loss of the first one. We report standard errors of the average loss differential in parentheses, based on Newey and West (1987) with 14 leads and lags. We test that the average loss differential is zero based on the statistic proposed by Diebold and Mariano (1995), evaluated in the setting of Giacomini and White (2006). Superscripts ^{a},^{b} ,^{c} denote the rejection with a significance level of 1%, 5% and 10%.

Table C.6: Empirical coverage of the VaR forecasts, $\alpha = 95\%$

(a) daily returns, iterated forecasts

	Normal			Empirical			Student's t		
Asset split	1	3	8	1	3	8	1	3	8
CCC-	5.60	5.78	6.43^b	5.56	5.34	6.07^c	5.54	5.95^c	6.47^b
GARCH	(0.74)	(0.73)	(0.79)	(0.73)	(0.70)	(0.76)	(0.73)	(0.75)	(0.79)
DCC-		5.88	6.23^c		5.50	5.92		6.07^c	6.37^b
GARCH		(0.74)	(0.76)		(0.73)	(0.74)		(0.76)	(0.77)
CCC-	4.84	5.16	5.68	4.86	4.80	5.48	5.04	5.08	5.82
GJR	(0.65)	(0.66)	(0.71)	(0.65)	(0.63)	(0.69)	(0.66)	(0.64)	(0.71)
DCC-		5.02	5.64		4.84	5.46		5.14	5.74
GJR		(0.66)	(0.70)		(0.65)	(0.69)		(0.66)	(0.69)
RiskMetrics	5.22	5.34	6.21^c	5.60	5.48	5.70	3.17^a	5.40	5.95
	(0.70)	(0.73)	(0.80)	(0.73)	(0.73)	(0.76)	(0.56)	(0.74)	(0.78)

(b) daily returns, scaled forecasts

	Normal			Empirical			Student's t		
Asset split	1	3	8	1	3	8	1	3	8
CCC-	5.62	5.90	6.35^b	5.64	5.56	6.07^c	5.76	6.23^c	6.77^b
GARCH	(0.73)	(0.73)	(0.78)	(0.73)	(0.71)	(0.75)	(0.74)	(0.77)	(0.80)
DCC-		5.95	6.29^b		5.42	6.01^c		6.19^c	6.59^b
GARCH		(0.75)	(0.77)		(0.71)	(0.75)		(0.76)	(0.78)
CCC-	5.60	5.60	6.11^c	5.54	5.40	5.92^c	5.82	6.01^c	6.45^b
GJR	(0.68)	(0.68)	(0.72)	(0.68)	(0.67)	(0.71)	(0.70)	(0.71)	(0.74)
DCC-		5.72	6.15^c		5.52	5.84		5.99^c	6.33^b
GJR		(0.69)	(0.73)		(0.68)	(0.70)		(0.70)	(0.73)
RiskMetrics	5.36	5.44	6.19^c	5.62	5.52	5.97	3.60^a	5.58	6.29^c
	(0.71)	(0.74)	(0.80)	(0.73)	(0.74)	(0.79)	(0.59)	(0.75)	(0.80)

(c) weekly returns, iterated forecasts

	Normal			Empirical			Student's t		
Asset split	1	3	8	1	3	8	1	3	8
CCC-	6.09^c	6.07^c	5.99^c	5.54	5.04	5.18	6.45^b	6.27^c	6.21^c
GARCH	(0.77)	(0.77)	(0.76)	(0.73)	(0.68)	(0.70)	(0.81)	(0.79)	(0.78)
DCC-		6.11^c	6.05^c		5.08	5.16		6.33^b	6.17^c
GARCH		(0.77)	(0.77)		(0.68)	(0.69)		(0.79)	(0.77)
CCC-	6.01^c	5.64	5.78	5.68	4.96	4.98	5.99^c	5.76	5.94^c
GJR	(0.75)	(0.71)	(0.73)	(0.72)	(0.66)	(0.66)	(0.75)	(0.73)	(0.73)
DCC-		5.76	5.78		4.84	5.10		5.86	5.95^c
GJR		(0.71)	(0.71)		(0.65)	(0.67)		(0.72)	(0.73)
RiskMetrics	5.52	5.26	5.52	5.50	5.02	4.66	5.72	5.22	5.54
	(0.74)	(0.72)	(0.73)	(0.73)	(0.70)	(0.67)	(0.75)	(0.72)	(0.74)

See Table 3.

Table C.6: Results for the unconditional coverage test, $\alpha = 95\%$ – *continued*

(d) weekly returns, scaled forecasts

	Normal			Empirical			Student's t		
Asset split	1	3	8	1	3	8	1	3	8
CCC-GARCH	6.13^c	5.95	5.97^c	5.54	4.88	5.06	6.43^b	6.37^b	6.25^c
	(0.78)	(0.75)	(0.76)	(0.73)	(0.65)	(0.69)	(0.80)	(0.79)	(0.78)
DCC-GARCH		6.07^c	6.01^c		4.96	4.94		6.39^b	6.27^c
		(0.75)	(0.76)		(0.66)	(0.67)		(0.78)	(0.78)
CCC-GJR	6.15^c	5.92	6.03^c	5.60	4.92	5.00	6.19^c	6.01^c	6.19^c
	(0.75)	(0.73)	(0.74)	(0.71)	(0.65)	(0.65)	(0.75)	(0.73)	(0.75)
DCC-GJR		6.01^c	5.97^c		4.82	5.14		6.21^c	6.23^c
		(0.73)	(0.73)		(0.64)	(0.66)		(0.74)	(0.75)
RiskMetrics	5.60	5.32	5.54	5.50	4.86	4.72	5.84	5.40	5.72
	(0.74)	(0.73)	(0.73)	(0.74)	(0.69)	(0.68)	(0.76)	(0.73)	(0.75)

(e) biweekly returns, direct forecasts

	Normal			Empirical			Student's t		
Asset split	1	3	8	1	3	8	1	3	8
CCC-GARCH	6.43^b	6.03^c	6.19^c	5.97^c	4.84	4.92	6.81^b	6.35^b	6.49^b
	(0.79)	(0.76)	(0.78)	(0.74)	(0.67)	(0.69)	(0.83)	(0.78)	(0.79)
DCC-GARCH		6.21^c	6.25^c		4.84	4.82		6.49^b	6.55^b
		(0.77)	(0.78)		(0.68)	(0.67)		(0.79)	(0.79)
CCC-GJR	6.57^b	6.19^c	6.25^b	6.21^c	5.02	5.14	6.75^b	6.53^b	6.49^b
	(0.78)	(0.75)	(0.76)	(0.74)	(0.67)	(0.68)	(0.79)	(0.77)	(0.76)
DCC-GJR		6.17^c	6.23^c		5.04	4.98		6.39^b	6.59^b
		(0.75)	(0.76)		(0.67)	(0.67)		(0.76)	(0.78)
RiskMetrics	6.03^c	5.40	6.03^c	6.05^c	5.72	6.03^c	6.47^b	5.70	6.33^c
	(0.78)	(0.73)	(0.80)	(0.77)	(0.74)	(0.76)	(0.79)	(0.74)	(0.82)

(f) Number of significant violations

	significance level			
Temporal aggregation	1%	5%	10%	tests
Daily, iterated	0	3	8	39
Daily, scaled	0	6	17	39
Weekly, iterated	0	2	14	39
Weekly, scaled	0	3	18	39
Biweekly, direct	8	18	28	39

	significance level			
Model	1%	5%	10%	tests
GARCH, uv.	2	5	7	15
GJR, uv.	3	3	7	15
RiskMetrics, uv.	3	3	3	15
CCC-GARCH	0	7	19	30
DCC-GARCH	0	7	19	30
CCC-GJR	0	4	12	30
DCC-GJR	0	3	12	30
RiskMetrics, mv.	0	0	6	30

	significance level			
Portfolio aggregation	1%	5%	10%	tests
1	8	11	17	45
3	0	7	24	75
8	0	14	44	75

	significance level			
Distribution	1%	5%	10%	tests
Normal	3	7	34	65
Empirical	2	3	8	65
Student's t	3	22	43	65

See table note on previous page.

Table C.7: Results of the dynamic quantile tests, $\alpha = 95\%$

(a) daily returns, iterated forecasts

Asset split	Normal 1	Normal 3	Normal 8	Empirical 1	Empirical 3	Empirical 8	Student's t 1	Student's t 3	Student's t 8
CCC-	−0.028	−0.035	−0.078	−0.041	−0.056	−0.108	−0.033	−0.020	−0.072
GARCH	(0.135)	(0.125)	(0.129)	(0.146)	(0.134)	(0.142)	(0.134)	(0.131)	(0.131)
DCC-		−0.029	−0.052		−0.041	−0.110		−0.027	−0.055
GARCH		(0.124)	(0.128)		(0.137)	(0.125)		(0.135)	(0.122)
CCC-	−0.024	0.037	−0.012	−0.030	−0.020	−0.070	−0.014	0.022	−0.023
GJR	(0.101)	(0.131)	(0.122)	(0.115)	(0.128)	(0.125)	(0.106)	(0.123)	(0.117)
DCC-		0.030	−0.022		−0.018	−0.045		0.027	0.001
GJR		(0.124)	(0.111)		(0.122)	(0.122)		(0.125)	(0.115)
RiskMetrics	0.014	0.143	0.181	0.005	0.058	0.137	−0.080	0.086	0.135
	(0.132)	(0.182)	(0.182)	(0.135)	(0.147)	(0.166)	(0.137)	(0.161)	(0.165)

(b) daily returns, scaled forecasts

Asset split	Normal 1	Normal 3	Normal 8	Empirical 1	Empirical 3	Empirical 8	Student's t 1	Student's t 3	Student's t 8
CCC-	−0.006	−0.039	−0.054	0.015	−0.067	−0.053	−0.003	−0.012	−0.061
GARCH	(0.125)	(0.127)	(0.132)	(0.138)	(0.125)	(0.134)	(0.126)	(0.131)	(0.125)
DCC-		−0.072	−0.043		−0.080	−0.028		−0.045	−0.017
GARCH		(0.114)	(0.126)		(0.114)	(0.123)		(0.111)	(0.121)
CCC-	0.024	−0.003	−0.009	0.039	−0.032	−0.018	0.021	0.014	−0.010
GJR	(0.114)	(0.130)	(0.118)	(0.124)	(0.122)	(0.120)	(0.110)	(0.132)	(0.116)
DCC-		−0.011	−0.011		−0.028	0.000		0.013	0.020
GJR		(0.114)	(0.109)		(0.107)	(0.113)		(0.121)	(0.118)
RiskMetrics	0.034	0.132	0.186	0.031	0.180	0.139	−0.071	0.115	0.144
	(0.133)	(0.171)	(0.185)	(0.133)	(0.202)	(0.171)	(0.125)	(0.163)	(0.167)

(c) weekly returns, iterated forecasts

Asset split	Normal 1	Normal 3	Normal 8	Empirical 1	Empirical 3	Empirical 8	Student's t 1	Student's t 3	Student's t 8
CCC-	0.226	0.123	0.063	0.164	0.107	0.029	0.209	0.152	0.094
GARCH	(0.194)	(0.170)	(0.164)	(0.179)	(0.158)	(0.156)	(0.195)	(0.174)	(0.167)
DCC-		0.111	0.053		0.143	−0.001		0.121	0.089
GARCH		(0.163)	(0.160)		(0.158)	(0.148)		(0.161)	(0.162)
CCC-	0.152	0.130	0.046	0.110	0.107	0.024	0.228	0.100	0.065
GJR	(0.150)	(0.147)	(0.138)	(0.157)	(0.152)	(0.138)	(0.185)	(0.142)	(0.141)
DCC-		0.160	0.087		0.107	0.066		0.118	0.092
GJR		(0.156)	(0.137)		(0.152)	(0.141)		(0.150)	(0.142)
RiskMetrics	0.294	0.262	0.262	0.300	0.262	0.404[b]	0.239	0.220	0.243
	(0.184)	(0.180)	(0.165)	(0.187)	(0.170)	(0.178)	(0.184)	(0.168)	(0.160)

See Table 4.

Table C.7: Results for the dynamic quantile test, $\alpha = 95\%$ – *continued*

(d) weekly returns, scaled forecasts

	Normal			Empirical			Student's t		
Asset split	1	3	8	1	3	8	1	3	8
CCC-GARCH	0.212	0.091	0.056	0.192	0.152	0.067	0.188	0.075	0.065
	(0.185)	(0.151)	(0.158)	(0.168)	(0.155)	(0.151)	(0.184)	(0.147)	(0.156)
DCC-GARCH		0.098	0.047		0.122	0.118		0.117	0.096
		(0.145)	(0.156)		(0.140)	(0.156)		(0.151)	(0.157)
CCC-GJR	0.189	0.058	0.053	0.188	0.008	0.059	0.166	0.075	0.056
	(0.164)	(0.135)	(0.138)	(0.165)	(0.123)	(0.132)	(0.163)	(0.137)	(0.137)
DCC-GJR		0.073	0.070		0.011	0.066		0.097	0.093
		(0.138)	(0.136)		(0.117)	(0.129)		(0.143)	(0.141)
RiskMetrics	0.275	0.246^{c}	0.291^{c}	0.285^{c}	0.221^{b}	0.285^{c}	0.227	0.152	0.224
	(0.176)	(0.135)	(0.172)	(0.164)	(0.109)	(0.148)	(0.175)	(0.118)	(0.154)

(e) biweekly returns, direct forecasts

	Normal			Empirical			Student's t		
Asset split	1	3	8	1	3	8	1	3	8
CCC-GARCH	0.246	0.288	0.199	0.282^{b}	0.188	0.197	0.207	0.278	0.195
	(0.203)	(0.186)	(0.183)	(0.183)	(0.167)	(0.181)	(0.201)	(0.198)	(0.190)
DCC-GARCH		0.281	0.252		0.250	0.311^{c}		0.253	0.220
		(0.187)	(0.186)		(0.194)	(0.184)		(0.191)	(0.190)
CCC-GJR	0.472	0.414^{b}	0.284	0.391^{c}	0.332^{c}	0.241	0.317	0.345^{c}	0.249
	(0.242)	(0.206)	(0.195)	(0.221)	(0.201)	(0.179)	(0.210)	(0.209)	(0.181)
DCC-GJR		0.420^{c}	0.282		0.319	0.234		0.357^{c}	0.265
		(0.216)	(0.196)		(0.203)	(0.172)		(0.204)	(0.188)
RiskMetrics	0.380	0.425^{b}	0.572^{b}	0.274^{a}	0.409^{b}	0.694^{a}	0.476^{c}	0.361^{c}	0.569^{a}
	(0.205)	(0.189)	(0.232)	(0.180)	(0.165)	(0.180)	(0.228)	(0.185)	(0.219)

(f) Negative versus positive coefficients

Temporal aggregation	neg.	sign. neg.	pos.	sign. pos.
Daily, iterated	26	0	13	0
Daily, scaled	23	0	16	0
Weekly, iterated	1	0	38	1
Weekly, scaled	0	0	39	5
Biweekly, direct	0	0	39	16

Model	neg.	sign. neg.	pos.	sign. pos.
GARCH, uv.	5	0	10	1
GJR, uv.	3	0	12	1
RiskMetrics, uv.	2	0	13	3
CCC-GARCH	12	0	18	0
DCC-GARCH	13	0	17	1
CCC-GJR	9	0	21	3
DCC-GJR	6	0	24	2
RiskMetrics, mv.	0	0	30	11

Portfolio aggregation	neg.	sign. neg.	pos.	sign. pos.
1	10	0	35	5
3	18	0	57	10
8	22	0	53	7

Distribution	neg.	sign. neg.	pos.	sign. pos.
Normal	17	0	48	6
Empirical	18	0	47	11
Student's t	15	0	50	5

See table note on previous page.

Table C.8: Average asymmetric tick loss, $\alpha = 95\%$

(a) daily returns, iterated forecasts

	Normal			Empirical			Student's t		
Asset split	1	3	8	1	3	8	1	3	8
CCC-GARCH	18.26	17.77	17.85	18.30	17.83	17.88	18.23	17.81	17.87
DCC-GARCH		17.89	17.73		17.94	17.75		17.90	17.76
CCC-GJR	17.94	17.69	17.52	18.03	17.78	17.62	17.99	17.67	17.50
DCC-GJR		17.76	17.42		17.86	17.50		17.65	17.43
RiskMetrics	18.05	18.31	18.65	18.12	18.27	18.49	18.75	18.23	18.48

(b) daily returns, scaled forecasts

	Normal			Empirical			Student's t		
Asset split	1	3	8	1	3	8	1	3	8
CCC-GARCH	18.15	17.74	17.76	18.17	17.75	17.70	18.14	17.83	17.77
DCC-GARCH		17.83	17.65		17.89	17.65		17.85	17.71
CCC-GJR	17.95	17.68	17.50	18.00	17.72	17.50	17.95	17.69	17.54
DCC-GJR		17.75	17.42		17.79	17.43		17.66	17.51
RiskMetrics	18.04	18.28	18.66	18.08	18.46	18.40	18.37	18.22	18.51

(c) weekly returns, iterated forecasts

	Normal			Empirical			Student's t		
Asset split	1	3	8	1	3	8	1	3	8
CCC-GARCH	19.50	19.04	18.93	19.38	18.96	18.83	19.30	19.01	19.03
DCC-GARCH		19.06	18.89		18.97	18.80		19.03	18.95
CCC-GJR	18.95	18.89	18.64	18.91	18.88	18.61	18.82	18.73	18.68
DCC-GJR		18.91	18.66		18.91	18.65		18.74	18.63
RiskMetrics	19.54	19.26	19.57	19.49	19.41	19.90	19.41	19.25	19.54

(d) weekly returns, scaled forecasts

	Normal			Empirical			Student's t		
Asset split	1	3	8	1	3	8	1	3	8
CCC-GARCH	19.51	19.03	18.93	19.40	18.96	18.82	19.32	19.01	18.95
DCC-GARCH		19.03	18.88		19.02	18.78		19.00	18.95
CCC-GJR	18.89	18.85	18.64	18.84	18.85	18.52	18.76	18.73	18.63
DCC-GJR		18.89	18.62		18.90	18.53		18.71	18.62
RiskMetrics	19.53	19.24	19.56	19.57	19.48	19.72	19.41	19.26	19.55

(e) biweekly returns, direct forecasts

	Normal			Empirical			Student's t		
Asset split	1	3	8	1	3	8	1	3	8
CCC-GARCH	19.65	19.44	19.37	19.56	19.37	19.33	19.50	19.44	19.36
DCC-GARCH		19.41	19.33		19.33	19.34		19.42	19.34
CCC-GJR	19.40	19.16	19.07	19.21	19.09	19.04	19.15	19.24	19.04
DCC-GJR		19.15	19.06		19.06	19.02		19.22	19.07
RiskMetrics	19.93	19.80	20.72	19.87	19.90	20.98	19.84	19.81	20.79

See Table 5.

Table C.9: Results of the Diebold-Mariano tests, $\alpha = 95\%$

Method A	Method B	# of tests	neg.	sign. neg. at 1%	5%	10%	pos.	sign. pos. at 1%	5%	10%
(a) Temporal Aggregation										
Daily, iterated	Weekly, iterated	39	39	0	13	37	0	0	0	0
Daily, iterated	Biweekly, direct	39	39	2	29	38	0	0	0	0
Daily, scaled	Weekly, scaled	39	39	0	11	36	0	0	0	0
Daily, scaled	Biweekly, direct	39	39	1	20	39	0	0	0	0
Weekly, iterated	Biweekly, direct	39	39	2	12	24	0	0	0	0
Weekly, scaled	Biweekly, direct	39	39	2	14	21	0	0	0	0
(b) Portfolio Aggregation										
Asset level	Asset class level	75	58	0	5	8	17	0	0	2
Asset level	Portfolio level	75	63	4	15	27	12	0	0	4
Asset class level	Portfolio level	75	59	3	7	13	16	0	2	5
(c) Model choice										
CCC-GARCH	CCC-GJR	45	0	0	0	0	45	0	2	14
CCC-GARCH	DCC-GJR	30	3	0	0	0	27	0	0	6
DCC-GARCH	DCC-GJR	30	0	0	0	0	30	0	1	7
DCC-GARCH	CCC-GJR	30	0	0	0	0	30	0	2	5
CCC-GARCH	DCC-GARCH	30	12	0	0	0	18	0	0	0
CCC-GJR	DCC-GJR	30	13	0	0	0	17	0	0	0
CCC-GARCH	RiskMetrics	45	41	4	9	13	4	0	0	0
DCC-GARCH	RiskMetrics	30	30	3	9	12	0	0	0	0
CCC-GJR	RiskMetrics	45	45	3	13	23	0	0	0	0
DCC-GJR	RiskMetrics	30	30	2	9	15	0	0	0	0
(d) Distribution choice										
Normal	Empirical	65	31	0	1	2	34	1	1	1
Normal	Student's t	65	30	1	1	2	35	0	1	2
Empirical	Student's t	65	25	0	1	1	40	1	1	2
(e) Forecast Aggregation										
Daily, iterated	Daily, scaled	39	10	0	1	1	29	1	1	1
Weekly, iterated	Weekly, scaled	39	12	0	0	0	27	0	0	3

See Table C.9.

Table C.10: Results of the DM-test for temporal aggregation, $\alpha = 95\%$

(a) daily vs. weekly returns, iterated forecasts

	Normal			Empirical			Student's t		
Asset split	1	3	8	1	3	8	1	3	8
CCC-GARCH	-1.248^{b}	-1.266^{c}	-1.085^{c}	-1.085^{b}	-1.128^{c}	-0.951^{c}	-1.065^{b}	-1.207^{c}	-1.161^{c}
	(0.508)	(0.673)	(0.618)	(0.467)	(0.592)	(0.576)	(0.492)	(0.687)	(0.663)
DCC-GARCH		-1.170^{c}	-1.162^{c}		-1.029^{c}	-1.044^{c}		-1.128^{c}	-1.194^{c}
		(0.604)	(0.658)		(0.529)	(0.595)		(0.609)	(0.689)
CCC-GJR	-1.006^{b}	-1.201^{c}	-1.115^{c}	-0.879^{b}	-1.100^{b}	-0.996^{c}	-0.828^{c}	-1.063^{c}	-1.175^{c}
	(0.503)	(0.631)	(0.593)	(0.447)	(0.534)	(0.539)	(0.438)	(0.595)	(0.626)
DCC-GJR		-1.155^{b}	-1.244^{c}		-1.053^{b}	-1.148^{b}		-1.085^{c}	-1.204^{c}
		(0.588)	(0.646)		(0.488)	(0.573)		(0.574)	(0.657)
RiskMetrics	-1.492^{b}	-0.948^{c}	-0.926	-1.366^{b}	-1.138^{b}	-1.404^{b}	-0.658	-1.020^{c}	-1.058^{c}
	(0.744)	(0.523)	(0.738)	(0.683)	(0.479)	(0.671)	(0.717)	(0.554)	(0.587)

(b) daily vs. weekly returns, scaled forecasts

	Normal			Empirical			Student's t		
Asset split	1	3	8	1	3	8	1	3	8
CCC-GARCH	-1.353^{b}	-1.284^{c}	-1.168^{c}	-1.226^{b}	-1.206^{c}	-1.116^{c}	-1.177^{b}	-1.181	-1.178^{c}
	(0.592)	(0.722)	(0.683)	(0.490)	(0.641)	(0.621)	(0.564)	(0.719)	(0.709)
DCC-GARCH		-1.201^{c}	-1.232^{c}		-1.128^{c}	-1.129^{c}		-1.149^{c}	-1.234^{c}
		(0.654)	(0.725)		(0.580)	(0.650)		(0.652)	(0.732)
CCC-GJR	-0.940^{c}	-1.164^{c}	-1.137^{c}	-0.839^{c}	-1.127^{b}	-1.020^{c}	-0.813^{c}	-1.042^{c}	-1.090^{c}
	(0.492)	(0.625)	(0.620)	(0.454)	(0.553)	(0.534)	(0.423)	(0.584)	(0.630)
DCC-GJR		-1.134^{b}	-1.202^{c}		-1.111^{b}	-1.096^{b}		-1.050^{c}	-1.118^{c}
		(0.570)	(0.648)		(0.499)	(0.553)		(0.551)	(0.646)
RiskMetrics	-1.484^{b}	-0.960^{c}	-0.901	-1.489^{b}	-1.015^{b}	-1.322^{b}	-1.044	-1.041^{c}	-1.038^{c}
	(0.745)	(0.530)	(0.734)	(0.712)	(0.471)	(0.637)	(0.691)	(0.553)	(0.581)

(c) daily vs. biweekly returns, iterated forecasts

	Normal			Empirical			Student's t		
Asset split	1	3	8	1	3	8	1	3	8
CCC-GARCH	-1.393^{a}	-1.669^{b}	-1.521^{b}	-1.259^{a}	-1.539^{b}	-1.452^{b}	-1.266^{b}	-1.638^{c}	-1.488^{b}
	(0.504)	(0.842)	(0.731)	(0.429)	(0.693)	(0.663)	(0.500)	(0.845)	(0.751)
DCC-GARCH		-1.518^{b}	-1.603^{b}		-1.386^{b}	-1.591^{b}		-1.511^{b}	-1.581^{b}
		(0.759)	(0.775)		(0.607)	(0.690)		(0.756)	(0.786)
CCC-GJR	-1.453^{b}	-1.473^{c}	-1.541^{b}	-1.182^{b}	-1.311^{c}	-1.429^{b}	-1.160^{b}	-1.572^{c}	-1.544^{b}
	(0.591)	(0.853)	(0.736)	(0.487)	(0.695)	(0.664)	(0.541)	(0.829)	(0.759)
DCC-GJR		-1.394^{c}	-1.644^{b}		-1.199^{c}	-1.519^{b}		-1.565^{b}	-1.643^{b}
		(0.781)	(0.798)		(0.616)	(0.693)		(0.795)	(0.796)
RiskMetrics	-1.880^{b}	-1.497^{c}	-2.073^{c}	-1.745^{b}	-1.634^{b}	-2.488^{b}	-1.091	-1.581^{c}	-2.311^{b}
	(0.814)	(0.809)	(1.121)	(0.714)	(0.745)	(1.095)	(0.863)	(0.832)	(1.093)

See Table C.1.

Table C.10: Results of the DM-test for temporal aggregation, $\alpha = 95\%$ – *continued*

(d) daily vs. biweekly returns, scaled forecasts

Asset split	Normal 1	Normal 3	Normal 8	Empirical 1	Empirical 3	Empirical 8	Student's t 1	Student's t 3	Student's t 8
CCC-GARCH	-1.496^b	-1.698^c	-1.612^b	-1.386^a	-1.614^b	-1.631^b	-1.360^b	-1.610^c	-1.581^c
	(0.592)	(0.909)	(0.815)	(0.499)	(0.768)	(0.732)	(0.572)	(0.895)	(0.830)
DCC-GARCH		-1.578^c	-1.684^c		-1.438^b	-1.696^b		-1.567^c	-1.625^c
		(0.826)	(0.863)		(0.667)	(0.764)		(0.816)	(0.844)
CCC-GJR	-1.449^b	-1.479^c	-1.563^b	-1.205^b	-1.369^c	-1.548^b	-1.203^b	-1.548^c	-1.500^c
	(0.593)	(0.864)	(0.775)	(0.521)	(0.727)	(0.698)	(0.547)	(0.833)	(0.784)
DCC-GJR		-1.398^c	-1.644^b		-1.263^c	-1.585^b		-1.553^b	-1.564^c
		(0.783)	(0.823)		(0.653)	(0.715)		(0.792)	(0.807)
RiskMetrics	-1.887^b	-1.521^c	-2.062^c	-1.786^b	-1.439^c	-2.579^b	-1.474^c	-1.596^c	-2.278^b
	(0.819)	(0.813)	(1.118)	(0.750)	(0.741)	(1.121)	(0.825)	(0.826)	(1.081)

(e) weekly vs. biweekly returns, iterated vs direct forecasts

Asset split	Normal 1	Normal 3	Normal 8	Empirical 1	Empirical 3	Empirical 8	Student's t 1	Student's t 3	Student's t 8
CCC-GARCH	−0.145	−0.403	-0.436^b	−0.174	-0.411^c	-0.501^a	−0.201	-0.431^c	-0.327^b
	(0.164)	(0.263)	(0.192)	(0.209)	(0.210)	(0.185)	(0.173)	(0.242)	(0.166)
DCC-GARCH		−0.349	-0.441^b		-0.357^c	-0.547^a		−0.383	-0.387^b
		(0.255)	(0.197)		(0.215)	(0.192)		(0.245)	(0.175)
CCC-GJR	-0.447^b	−0.272	-0.425^b	-0.303^c	−0.210	-0.433^b	-0.332^c	-0.509^c	-0.369^c
	(0.207)	(0.348)	(0.212)	(0.172)	(0.303)	(0.207)	(0.199)	(0.298)	(0.191)
DCC-GJR		−0.239	-0.399^c		−0.145	-0.372^c		−0.480	-0.438^b
		(0.335)	(0.222)		(0.295)	(0.217)		(0.292)	(0.196)
RiskMetrics	-0.388^c	−0.548	-1.147^b	−0.379	−0.496	-1.084^c	-0.433^c	−0.561	-1.253^b
	(0.207)	(0.355)	(0.505)	(0.235)	(0.351)	(0.607)	(0.246)	(0.345)	(0.626)

(f) weekly vs. biweekly returns, scaled vs direct forecasts

Asset split	Normal 1	Normal 3	Normal 8	Empirical 1	Empirical 3	Empirical 8	Student's t 1	Student's t 3	Student's t 8
CCC-GARCH	−0.143	−0.414	-0.444^b	−0.160	-0.408^c	-0.515^a	−0.183	−0.429	-0.404^b
	(0.167)	(0.278)	(0.202)	(0.224)	(0.234)	(0.191)	(0.178)	(0.267)	(0.188)
DCC-GARCH		−0.377	-0.452^b		−0.310	-0.566^a		−0.418	-0.391^b
		(0.273)	(0.211)		(0.218)	(0.200)		(0.258)	(0.187)
CCC-GJR	-0.510^b	−0.314	-0.426^c	-0.366^c	−0.243	-0.527^b	-0.389^c	−0.505	-0.409^b
	(0.206)	(0.371)	(0.225)	(0.202)	(0.305)	(0.241)	(0.213)	(0.313)	(0.205)
DCC-GJR		−0.264	-0.442^c		−0.152	-0.489^b		−0.503	-0.445^b
		(0.355)	(0.241)		(0.305)	(0.237)		(0.307)	(0.215)
RiskMetrics	-0.403^c	−0.561	-1.161^b	−0.298	−0.424	-1.257^b	-0.430^c	−0.556	-1.240^b
	(0.212)	(0.352)	(0.507)	(0.267)	(0.356)	(0.622)	(0.239)	(0.343)	(0.624)

See table note on previous page.

Table C.11: Results of the DM-tests for portfolio aggregation, $\alpha = 95\%$

(a) daily returns, iterated forecasts

Asset split	Normal			Empirical			Student's t		
	1 vs. 3	1 vs. 8	3 vs. 8	1 vs. 3	1 vs. 8	3 vs. 8	1 vs. 3	1 vs. 8	3 vs. 8
CCC-GARCH	0.486	0.407	−0.078	0.468	0.418	−0.050	0.429	0.366	−0.062
	(0.338)	(0.268)	(0.210)	(0.329)	(0.280)	(0.197)	(0.347)	(0.285)	(0.217)
DCC-GARCH	0.366	0.527^{c}	0.161	0.354	0.546^{c}	0.192	0.330	0.478	0.149
	(0.267)	(0.303)	(0.156)	(0.257)	(0.312)	(0.169)	(0.276)	(0.318)	(0.171)
CCC-GJR	0.255	0.420^{c}	0.165	0.247	0.411^{c}	0.165	0.321	0.489^{b}	0.168
	(0.250)	(0.230)	(0.192)	(0.245)	(0.236)	(0.193)	(0.254)	(0.247)	(0.182)
DCC-GJR	0.189	0.527^{b}	0.338^{b}	0.169	0.529^{b}	0.360^{b}	0.338	0.563^{b}	0.224
	(0.189)	(0.252)	(0.163)	(0.178)	(0.252)	(0.177)	(0.230)	(0.272)	(0.173)
RiskMetrics	−0.258	−0.597^{c}	−0.338^{c}	−0.147	−0.373	−0.226	0.516	0.268	−0.248
	(0.315)	(0.346)	(0.205)	(0.313)	(0.316)	(0.199)	(0.443)	(0.580)	(0.211)

(b) daily returns, scaled forecasts

Asset split	Normal			Empirical			Student's t		
	1 vs. 3	1 vs. 8	3 vs. 8	1 vs. 3	1 vs. 8	3 vs. 8	1 vs. 3	1 vs. 8	3 vs. 8
CCC-GARCH	0.412	0.396	−0.016	0.416	0.470	0.054	0.308	0.366	0.058
	(0.327)	(0.273)	(0.203)	(0.358)	(0.309)	(0.205)	(0.331)	(0.284)	(0.201)
DCC-GARCH	0.323	0.506^{c}	0.183	0.278	0.523	0.244	0.293	0.429	0.136
	(0.252)	(0.303)	(0.161)	(0.276)	(0.332)	(0.172)	(0.258)	(0.298)	(0.182)
CCC-GJR	0.265	0.446^{c}	0.181	0.283	0.508^{b}	0.224	0.254	0.402	0.148
	(0.252)	(0.242)	(0.177)	(0.258)	(0.253)	(0.192)	(0.256)	(0.252)	(0.173)
DCC-GJR	0.197	0.531^{b}	0.334^{c}	0.210	0.573^{b}	0.362^{b}	0.283	0.441	0.157
	(0.181)	(0.258)	(0.171)	(0.188)	(0.253)	(0.174)	(0.219)	(0.270)	(0.174)
RiskMetrics	−0.241	−0.614^{c}	−0.373^{c}	−0.384	−0.323	0.061	0.149	−0.147	−0.296
	(0.310)	(0.343)	(0.197)	(0.327)	(0.309)	(0.226)	(0.416)	(0.573)	(0.225)

(c) weekly returns, iterated forecasts

Asset split	Normal			Empirical			Student's t		
	1 vs. 3	1 vs. 8	3 vs. 8	1 vs. 3	1 vs. 8	3 vs. 8	1 vs. 3	1 vs. 8	3 vs. 8
CCC-GARCH	0.467^{b}	0.570^{a}	0.103	0.426	0.552^{b}	0.126	0.287	0.271^{c}	−0.017
	(0.215)	(0.178)	(0.124)	(0.262)	(0.223)	(0.121)	(0.177)	(0.163)	(0.169)
DCC-GARCH	0.444^{b}	0.612^{a}	0.169^{c}	0.410	0.587^{b}	0.177^{c}	0.267	0.350^{b}	0.082
	(0.213)	(0.179)	(0.098)	(0.263)	(0.230)	(0.091)	(0.183)	(0.150)	(0.136)
CCC-GJR	0.060	0.311^{c}	0.250^{c}	0.025	0.294^{c}	0.269^{c}	0.086	0.142	0.056
	(0.220)	(0.160)	(0.139)	(0.209)	(0.166)	(0.143)	(0.168)	(0.200)	(0.158)
DCC-GJR	0.040	0.289^{c}	0.248^{b}	−0.006	0.260^{c}	0.266^{b}	0.081	0.186	0.105
	(0.211)	(0.152)	(0.115)	(0.208)	(0.157)	(0.127)	(0.169)	(0.167)	(0.140)
RiskMetrics	0.285	−0.030	−0.315	0.081	−0.411	−0.492^{b}	0.155	−0.131	−0.286
	(0.242)	(0.347)	(0.254)	(0.260)	(0.365)	(0.222)	(0.355)	(0.473)	(0.247)

See Table C.2

Table C.11: Results of the DM-test for portfolio aggregation, $\alpha = 95\%$ – *continued*

(d) weekly returns, scaled forecasts

	Normal			Empirical			Student's t		
Asset split	1 vs. 3	1 vs. 8	3 vs. 8	1 vs. 3	1 vs. 8	3 vs. 8	1 vs. 3	1 vs. 8	3 vs. 8
CCC-GARCH	0.481^b	0.581^a	0.100	0.437^c	0.580^a	0.144	0.304	0.365^b	0.061
	(0.224)	(0.178)	(0.124)	(0.264)	(0.215)	(0.112)	(0.189)	(0.156)	(0.162)
DCC-GARCH	0.475^b	0.626^a	0.152	0.377	0.620^a	0.243^a	0.320^c	0.372^b	0.052
	(0.226)	(0.186)	(0.097)	(0.246)	(0.223)	(0.080)	(0.187)	(0.149)	(0.137)
CCC-GJR	0.040	0.249	0.209	−0.005	0.326^c	0.331^a	0.025	0.125	0.100
	(0.220)	(0.165)	(0.142)	(0.220)	(0.189)	(0.116)	(0.162)	(0.202)	(0.157)
DCC-GJR	0.003	0.268^c	0.266^b	−0.061	0.316^c	0.377^a	0.047	0.136	0.089
	(0.209)	(0.155)	(0.118)	(0.222)	(0.182)	(0.123)	(0.160)	(0.169)	(0.140)
RiskMetrics	0.283	−0.031	−0.314	0.090	−0.157	−0.246	0.152	−0.142	−0.293
	(0.246)	(0.357)	(0.250)	(0.210)	(0.335)	(0.216)	(0.350)	(0.467)	(0.246)

(e) biweekly returns, direct forecasts

	Normal			Empirical			Student's t		
Asset split	1 vs. 3	1 vs. 8	3 vs. 8	1 vs. 3	1 vs. 8	3 vs. 8	1 vs. 3	1 vs. 8	3 vs. 8
CCC-GARCH	0.210	0.280	0.070	0.188	0.225	0.037	0.057	0.145	0.087
	(0.221)	(0.236)	(0.092)	(0.305)	(0.368)	(0.122)	(0.174)	(0.223)	(0.110)
DCC-GARCH	0.241	0.317	0.076	0.227	0.213	−0.014	0.085	0.164	0.079
	(0.215)	(0.220)	(0.092)	(0.310)	(0.354)	(0.136)	(0.159)	(0.196)	(0.096)
CCC-GJR	0.236	0.332	0.096	0.118	0.164	0.046	−0.091	0.105	0.196^c
	(0.253)	(0.227)	(0.114)	(0.269)	(0.302)	(0.115)	(0.202)	(0.225)	(0.108)
DCC-GJR	0.248	0.336	0.088	0.152	0.192	0.040	−0.067	0.080	0.146
	(0.248)	(0.223)	(0.118)	(0.264)	(0.299)	(0.125)	(0.180)	(0.206)	(0.099)
RiskMetrics	0.125	−0.789	-0.914^c	−0.036	-1.117^b	-1.080^b	0.026	−0.952	-0.978^c
	(0.419)	(0.579)	(0.470)	(0.469)	(0.564)	(0.523)	(0.499)	(0.739)	(0.574)

See table note on previous page.

Table C.12: Results of the DM-tests for model choice, $\alpha = 95\%$

(a) Asset level

	Daily, iterated			Daily, scaled			Weekly, iterated			Weekly, scaled			Biweekly, direct		
	N	Emp.	t	N	Emp.	t	N	Emp.	t	N	Emp.	t	N	Emp.	t
GARCH vs.	0.312	0.271	0.245	0.206	0.166	0.194	0.554[c]	0.477[c]	0.482[c]	0.619[b]	0.553[b]	0.558[c]	0.252	0.347	0.351[c]
GJR	(0.250)	(0.222)	(0.242)	(0.192)	(0.212)	(0.194)	(0.291)	(0.265)	(0.287)	(0.303)	(0.275)	(0.310)	(0.211)	(0.236)	(0.208)
GARCH vs.	0.207	0.176	−0.516	0.112	0.090	−0.226	−0.037	−0.105	−0.109	−0.019	−0.172	−0.093	−0.280	−0.310	−0.341[b]
RiskMetrics	(0.233)	(0.179)	(0.406)	(0.161)	(0.142)	(0.308)	(0.234)	(0.236)	(0.201)	(0.235)	(0.255)	(0.184)	(0.251)	(0.276)	(0.168)
GJR vs.	−0.105	−0.094	−0.761[a]	−0.094	−0.075	−0.421	−0.591[c]	−0.582	−0.591[b]	−0.638[c]	−0.725[c]	−0.651[b]	−0.532	−0.657[c]	−0.692[b]
RiskMetrics	(0.185)	(0.192)	(0.292)	(0.189)	(0.189)	(0.289)	(0.351)	(0.380)	(0.282)	(0.376)	(0.409)	(0.303)	(0.359)	(0.363)	(0.281)

(b) Asset class level

	Daily, iterated			Daily, scaled			Weekly, iterated			Weekly, scaled			Biweekly, direct		
	N	Emp.	t	N	Emp.	t	N	Emp.	t	N	Emp.	t	N	Emp.	t
CCC-GARCH vs.	0.119	0.127	0.112	0.109	0.053	0.063	0.042	0.035	0.079	0.045	0.039	0.007	0.037	−0.012	0.020
DCC-GARCH	(0.107)	(0.108)	(0.118)	(0.111)	(0.104)	(0.107)	(0.052)	(0.058)	(0.070)	(0.053)	(0.054)	(0.053)	(0.047)	(0.052)	(0.052)
CCC-GARCH vs.	0.324[c]	0.264	0.368[c]	0.256[c]	0.203	0.230	0.294	0.219	0.354[c]	0.286	0.298	0.318	0.304[c]	0.286[c]	0.312[c]
CCC-GJR	(0.190)	(0.169)	(0.195)	(0.147)	(0.132)	(0.156)	(0.189)	(0.175)	(0.208)	(0.184)	(0.192)	(0.203)	(0.173)	(0.155)	(0.168)
CCC-GARCH vs.	0.431[c]	0.381[c]	0.441[c]	0.340[c]	0.269	0.269	0.272	0.185	0.398	0.306	0.288	0.328	0.308[c]	0.314[c]	0.287
DCC-GJR	(0.257)	(0.217)	(0.259)	(0.196)	(0.164)	(0.199)	(0.214)	(0.195)	(0.248)	(0.218)	(0.208)	(0.241)	(0.183)	(0.169)	(0.196)
CCC-GARCH vs.	−0.797[a]	−0.615[c]	−0.614	−0.899[a]	−0.703[b]	−0.740	−0.638[c]	−1.068[a]	−0.511	−0.632[c]	−0.909[a]	−0.600[c]	−1.349[b]	−1.652[b]	−1.437[b]
RiskMetrics	(0.300)	(0.330)	(0.400)	(0.345)	(0.339)	(0.475)	(0.361)	(0.378)	(0.316)	(0.379)	(0.348)	(0.341)	(0.587)	(0.715)	(0.652)
DCC-GARCH vs.	0.205	0.137	0.256	0.146	0.150	0.167	0.252	0.184	0.275[c]	0.241	0.259	0.311[c]	0.267[c]	0.298[b]	0.293[b]
CCC-GJR	(0.156)	(0.162)	(0.165)	(0.143)	(0.147)	(0.164)	(0.159)	(0.148)	(0.165)	(0.153)	(0.166)	(0.180)	(0.152)	(0.147)	(0.139)
DCC-GARCH vs.	0.312[c]	0.254	0.329[c]	0.231[c]	0.215[c]	0.206	0.230	0.150	0.319	0.261	0.249	0.321	0.271[c]	0.326[b]	0.267[c]
DCC-GJR	(0.184)	(0.158)	(0.183)	(0.133)	(0.121)	(0.149)	(0.179)	(0.163)	(0.197)	(0.179)	(0.177)	(0.212)	(0.158)	(0.157)	(0.160)
DCC-GARCH vs.	−0.916[a]	−0.742[b]	−0.726	−1.008[a]	−0.756[b]	−0.803	−0.680[c]	−1.102[a]	−0.590[c]	−0.677	−0.948[b]	−0.607[c]	−1.386[b]	−1.640[b]	−1.456[b]
RiskMetrics	(0.323)	(0.330)	(0.448)	(0.388)	(0.366)	(0.504)	(0.391)	(0.416)	(0.351)	(0.412)	(0.378)	(0.367)	(0.623)	(0.725)	(0.690)
CCC-GJR vs.	0.107	0.118	0.074	0.085	0.065	0.039	−0.022	−0.034	0.044	0.020	−0.010	0.011	0.004	0.028	−0.026
DCC-GJR	(0.102)	(0.102)	(0.105)	(0.092)	(0.088)	(0.103)	(0.043)	(0.039)	(0.057)	(0.047)	(0.045)	(0.055)	(0.023)	(0.039)	(0.049)
CCC-GJR vs.	−1.121[a]	−0.879[b]	−0.982[c]	−1.154[a]	−0.906[b]	−0.970[c]	−0.932[c]	−1.286[b]	−0.864[c]	−0.918[c]	−1.208[b]	−0.918[c]	−1.653[b]	−1.938[b]	−1.749[b]
RiskMetrics	(0.416)	(0.426)	(0.544)	(0.439)	(0.416)	(0.582)	(0.491)	(0.504)	(0.458)	(0.512)	(0.477)	(0.479)	(0.709)	(0.779)	(0.768)
DCC-GJR vs.	−1.228[a]	−0.996[b]	−1.055[c]	−1.239[a]	−0.971[b]	−1.009[c]	−0.910[c]	−1.253[b]	−0.909[c]	−0.938[c]	−1.197[b]	−0.928[c]	−1.657[b]	−1.965[b]	−1.723[b]
RiskMetrics	(0.457)	(0.432)	(0.587)	(0.470)	(0.420)	(0.610)	(0.513)	(0.521)	(0.486)	(0.543)	(0.492)	(0.512)	(0.719)	(0.801)	(0.793)

See Table C.3.

Table C.12: Results of the DM-tests for model choice, $\alpha = 95\%$ – *continued*

(c) Portfolio level

	Daily, iterated			Daily, scaled			Weekly, iterated			Weekly, scaled			Biweekly, direct		
	N	Emp.	t	N	Emp.	t	N	Emp.	t	N	Emp.	t	N	Emp.	t
CCC-GARCH vs.	−0.120	−0.114	−0.099	−0.089	−0.137	−0.015	−0.023	−0.016	−0.020	−0.007	−0.060	0.017	0.031	0.039	0.028
DCC-GARCH	(0.103)	(0.102)	(0.114)	(0.105)	(0.107)	(0.109)	(0.038)	(0.036)	(0.049)	(0.038)	(0.048)	(0.050)	(0.042)	(0.063)	(0.045)
CCC-GARCH vs.	0.081	0.049	0.137	0.059	0.033	0.141	0.147	0.076	0.281	0.178	0.112	0.279	0.278	0.277	0.204
CCC-GJR	(0.182)	(0.169)	(0.188)	(0.159)	(0.161)	(0.161)	(0.219)	(0.222)	(0.217)	(0.226)	(0.217)	(0.223)	(0.192)	(0.179)	(0.165)
CCC-GARCH vs.	0.015	−0.029	0.154	−0.009	−0.040	0.170	0.127	0.045	0.276	0.140	0.055	0.301	0.290	0.311	0.228
DCC-GJR	(0.169)	(0.157)	(0.187)	(0.166)	(0.169)	(0.169)	(0.219)	(0.225)	(0.235)	(0.227)	(0.234)	(0.242)	(0.209)	(0.197)	(0.186)
CCC-GARCH vs.	−0.537	−0.439	−0.428	−0.541	−0.709	−0.385	−0.219	−0.449	−0.241	−0.217	−0.519	−0.245	−0.365	−0.535	−0.371
RiskMetrics	(0.401)	(0.431)	(0.419)	(0.450)	(0.501)	(0.460)	(0.326)	(0.346)	(0.350)	(0.339)	(0.357)	(0.366)	(0.493)	(0.565)	(0.482)
DCC-GARCH vs.	0.201	0.163	0.236	0.148	0.170	0.156	0.170	0.092	0.301	0.185	0.171	0.262	0.247	0.239	0.176
CCC-GJR	(0.242)	(0.227)	(0.246)	(0.213)	(0.213)	(0.214)	(0.215)	(0.219)	(0.210)	(0.228)	(0.223)	(0.212)	(0.180)	(0.168)	(0.153)
DCC-GARCH vs.	0.135	0.085	0.253	0.080	0.098	0.185	0.150	0.061	0.296	0.147	0.115	0.284	0.259	0.273	0.200
DCC-GJR	(0.192)	(0.173)	(0.219)	(0.165)	(0.171)	(0.184)	(0.212)	(0.221)	(0.222)	(0.223)	(0.234)	(0.225)	(0.190)	(0.175)	(0.165)
DCC-GARCH vs.	−0.417	−0.325	−0.329	−0.452	−0.572	−0.370	−0.196	−0.433	−0.221	−0.211	−0.459	−0.262	−0.395	−0.573	−0.399
RiskMetrics	(0.346)	(0.375)	(0.366)	(0.388)	(0.429)	(0.408)	(0.332)	(0.349)	(0.360)	(0.339)	(0.351)	(0.370)	(0.505)	(0.592)	(0.499)
CCC-GJR vs.	−0.066	−0.078	0.017	−0.068	−0.073	0.029	−0.020	−0.031	−0.005	−0.038	−0.056	0.022	0.012	0.034	0.024
DCC-GJR	(0.095)	(0.099)	(0.075)	(0.098)	(0.096)	(0.082)	(0.037)	(0.039)	(0.045)	(0.039)	(0.043)	(0.053)	(0.041)	(0.050)	(0.050)
CCC-GJR vs.	−0.619	−0.488	−0.566	−0.600	−0.742	−0.526	−0.366	−0.525	−0.522	−0.396	−0.630	−0.524	−0.643	−0.812	−0.575
RiskMetrics	(0.512)	(0.530)	(0.513)	(0.524)	(0.570)	(0.525)	(0.474)	(0.511)	(0.494)	(0.498)	(0.522)	(0.512)	(0.570)	(0.620)	(0.572)
DCC-GJR vs.	−0.553	−0.410	−0.583	−0.532	−0.670	−0.555	−0.346	−0.494	−0.517	−0.358	−0.574	−0.546	−0.655	−0.846	−0.599
RiskMetrics	(0.463)	(0.478)	(0.504)	(0.468)	(0.516)	(0.509)	(0.465)	(0.503)	(0.503)	(0.493)	(0.529)	(0.529)	(0.585)	(0.640)	(0.586)

See table note on previous page.

Table C.13: Results of the DM-test for the distribution choice, $\alpha = 95\%$

(a) daily returns, iterated forecasts

	Normal vs. t			Normal vs. Emp			Emp. vs. t		
Asset split	1	3	8	1	3	8	1	3	8
CCC-GARCH	0.023	−0.034	−0.019	−0.041	−0.058	−0.030	0.063	0.024	0.011
	(0.037)	(0.048)	(0.037)	(0.075)	(0.083)	(0.093)	(0.078)	(0.102)	(0.103)
DCC-GARCH		−0.013	−0.026		−0.053	−0.022		0.039	−0.004
		(0.054)	(0.044)		(0.082)	(0.081)		(0.112)	(0.102)
CCC-GJR	−0.044	0.021	0.025	−0.082	−0.090	−0.091	0.038	0.112	0.115
	(0.032)	(0.034)	(0.042)	(0.107)	(0.088)	(0.114)	(0.102)	(0.098)	(0.121)
DCC-GJR		0.105	−0.009		−0.103	−0.080		0.207^{c}	0.071
		(0.065)	(0.047)		(0.098)	(0.096)		(0.120)	(0.122)
RiskMetrics	-0.700^{a}	0.074	0.164	−0.072	0.040	0.152	-0.629^{b}	0.035	0.012
	(0.261)	(0.047)	(0.201)	(0.093)	(0.068)	(0.110)	(0.309)	(0.086)	(0.236)

(b) daily returns, scaled forecasts

	Normal vs. t			Normal vs. Emp			Emp. vs. t		
Asset split	1	3	8	1	3	8	1	3	8
CCC-GARCH	0.014	-0.091^{c}	−0.016	−0.016	−0.012	0.058	0.029	−0.079	−0.074
	(0.026)	(0.053)	(0.037)	(0.044)	(0.067)	(0.056)	(0.054)	(0.108)	(0.077)
DCC-GARCH		−0.016	−0.063		−0.060	0.002		0.044	−0.064
		(0.054)	(0.053)		(0.062)	(0.051)		(0.102)	(0.084)
CCC-GJR	0.002	−0.009	−0.042	−0.056	−0.038	0.006	0.058	0.029	−0.047
	(0.020)	(0.038)	(0.043)	(0.040)	(0.056)	(0.036)	(0.041)	(0.085)	(0.065)
DCC-GJR		0.089	−0.088		−0.042	−0.014		0.131	−0.074
		(0.063)	(0.054)		(0.053)	(0.040)		(0.083)	(0.073)
RiskMetrics	−0.324	0.065	0.143	−0.037	-0.180^{b}	0.254^{a}	−0.288	0.245^{a}	−0.111
	(0.206)	(0.044)	(0.207)	(0.065)	(0.079)	(0.097)	(0.253)	(0.091)	(0.257)

(c) weekly returns, iterated forecasts

	Normal vs. t			Normal vs. Emp			Emp. vs. t		
Asset split	1	3	8	1	3	8	1	3	8
CCC-GARCH	0.205^{c}	0.025	−0.095	0.122	0.081	0.104	0.083	−0.055	−0.199
	(0.114)	(0.060)	(0.061)	(0.089)	(0.150)	(0.141)	(0.144)	(0.175)	(0.190)
DCC-GARCH		0.029	−0.058		0.088	0.097		−0.060	−0.154
		(0.066)	(0.065)		(0.150)	(0.143)		(0.178)	(0.189)
CCC-GJR	0.134	0.160	−0.035	0.045	0.010	0.029	0.088	0.150	−0.063
	(0.112)	(0.102)	(0.051)	(0.100)	(0.112)	(0.101)	(0.132)	(0.141)	(0.132)
DCC-GJR		0.174	0.031		−0.001	0.017		0.175	0.014
		(0.109)	(0.047)		(0.109)	(0.099)		(0.138)	(0.122)
RiskMetrics	0.134	0.003	0.033	0.055	−0.149	−0.326	0.079	0.153	0.359
	(0.280)	(0.067)	(0.132)	(0.045)	(0.093)	(0.224)	(0.275)	(0.129)	(0.238)

See Table C.4.

Table C.13: Results for the DM-test the distribution choice, $\alpha = 95\%$– *continued*

(d) weekly returns, scaled forecasts

	Normal vs. t			Normal vs. Emp			Emp. vs. t		
Asset split	1	3	8	1	3	8	1	3	8
CCC-GARCH	0.190	0.012	−0.027	0.111	0.066	0.110	0.079	−0.054	−0.137
	(0.118)	(0.063)	(0.053)	(0.152)	(0.180)	(0.169)	(0.169)	(0.204)	(0.202)
DCC-GARCH		0.036	−0.064		0.013	0.104		0.022	−0.169
		(0.062)	(0.065)		(0.167)	(0.168)		(0.193)	(0.211)
CCC-GJR	0.128	0.113	0.005	0.045	−0.000	0.122	0.083	0.114	−0.117
	(0.115)	(0.108)	(0.045)	(0.095)	(0.140)	(0.152)	(0.124)	(0.151)	(0.167)
DCC-GJR		0.173	−0.004		−0.019	0.092		0.192	−0.096
		(0.116)	(0.050)		(0.143)	(0.138)		(0.152)	(0.156)
RiskMetrics	0.116	−0.015	0.006	−0.042	-0.235^{c}	−0.167	0.157	0.220	0.173
	(0.273)	(0.064)	(0.134)	(0.081)	(0.129)	(0.230)	(0.302)	(0.164)	(0.242)

(e) biweekly returns, direct forecasts

	Normal vs. t			Normal vs. Emp			Emp. vs. t		
Asset split	1	3	8	1	3	8	1	3	8
CCC-GARCH	0.149	−0.003	0.014	0.094	0.072	0.039	0.056	−0.075	−0.025
	(0.117)	(0.123)	(0.070)	(0.168)	(0.217)	(0.222)	(0.215)	(0.274)	(0.272)
DCC-GARCH		−0.006	−0.003		0.080	−0.010		−0.086	0.006
		(0.121)	(0.071)		(0.234)	(0.212)		(0.285)	(0.260)
CCC-GJR	0.249^{b}	−0.078	0.022	0.189	0.072	0.021	0.060	−0.149	0.001
	(0.113)	(0.163)	(0.066)	(0.144)	(0.195)	(0.177)	(0.164)	(0.261)	(0.210)
DCC-GJR		−0.066	−0.008		0.093	0.045		−0.159	−0.052
		(0.153)	(0.080)		(0.198)	(0.191)		(0.253)	(0.226)
RiskMetrics	0.089	−0.010	−0.074	0.064	−0.097	−0.263	0.025	0.088	0.190
	(0.224)	(0.147)	(0.254)	(0.147)	(0.122)	(0.196)	(0.202)	(0.187)	(0.269)

See table note on previous page.

Table C.14: Results of the DM-test for the forecasting method, $\alpha = 95\%$

(a) daily returns

	Normal			Empirical			Student's t		
Asset split	1	3	8	1	3	8	1	3	8
CCC-GARCH	0.102	0.029	0.091	0.128	0.075	0.179	0.093	−0.028	0.093
	(0.102)	(0.077)	(0.095)	(0.121)	(0.125)	(0.142)	(0.095)	(0.075)	(0.088)
DCC-GARCH		0.059	0.081		0.052	0.105		0.056	0.044
		(0.078)	(0.097)		(0.119)	(0.126)		(0.078)	(0.068)
CCC-GJR	−0.004	0.006	0.022	0.022	0.059	0.119	0.043	−0.024	−0.044
	(0.090)	(0.084)	(0.072)	(0.125)	(0.101)	(0.116)	(0.093)	(0.095)	(0.077)
DCC-GJR		0.004	−0.000		0.064	0.066		−0.012	−0.079
		(0.079)	(0.067)		(0.110)	(0.106)		(0.091)	(0.078)
RiskMetrics	0.007	0.025	−0.011	0.042	-0.195^{b}	0.091	0.383^{a}	0.015	−0.032
	(0.029)	(0.023)	(0.015)	(0.097)	(0.093)	(0.090)	(0.082)	(0.038)	(0.063)

(b) weekly returns, scaled forecasts

	Normal			Empirical			Student's t		
Asset split	1	3	8	1	3	8	1	3	8
CCC-GARCH	−0.003	0.011	0.009	−0.014	−0.003	0.015	−0.018	−0.002	0.076^{c}
	(0.031)	(0.028)	(0.028)	(0.110)	(0.078)	(0.058)	(0.030)	(0.041)	(0.044)
DCC-GARCH		0.028	0.011		−0.047	0.019		0.035	0.005
		(0.031)	(0.031)		(0.066)	(0.058)		(0.032)	(0.035)
CCC-GJR	0.063	0.042	0.001	0.062	0.032	0.094	0.057	−0.004	0.040
	(0.058)	(0.049)	(0.046)	(0.089)	(0.074)	(0.070)	(0.056)	(0.054)	(0.043)
DCC-GJR		0.025	0.042		0.007	0.118^{c}		0.023	0.007
		(0.050)	(0.046)		(0.067)	(0.066)		(0.052)	(0.047)
RiskMetrics	0.016	0.013	0.014	−0.081	−0.073	0.173^{c}	−0.003	−0.006	−0.013
	(0.021)	(0.013)	(0.014)	(0.075)	(0.078)	(0.104)	(0.037)	(0.019)	(0.027)

See Table C.5.

Table C.15: Models removed from the Model Confidence Set, $\alpha = 95\%$

(a) daily returns, iterated forecasts

	Normal			Empirical			Student's t		
Asset split	1	3	8	1	3	8	1	3	8
CCC-GARCH									
DCC-GARCH									
CCC-GJR		0.079							
DCC-GJR									
RiskMetrics							0.095		

(b) daily returns, iterated forecasts

	Normal			Empirical			Student's t		
Asset split	1	3	8	1	3	8	1	3	8
CCC-GARCH									
DCC-GARCH									
CCC-GJR									
DCC-GJR									
RiskMetrics	0.078								

(c) weekly returns, iterated forecasts

	Normal			Empirical			Student's t		
Asset split	1	3	8	1	3	8	1	3	8
CCC-GARCH	0.055		0.054	0.054		0.049	0.061		0.059
DCC-GARCH			0.072			0.062			0.075
CCC-GJR	0.064	0.083	0.055	0.06	0.051	0.052	0.069	0.081	0.049
DCC-GJR									
RiskMetrics	0.055			0.055			0.048		

This table presents the MCS p-value with which a model has been removed from the Model Confidence Set. We follow the procedure of Hansen et al. (2011, Sec. 3.1.2), with the tick loss function in (20), a confidence level for VaR of 95% and a significance level for the MCS construction of 10%. The procedure starts with the complete set of 195 methods. Panel (f) presents the number of methods with a particular choice that have been removed from ("Out") and are maintained in ("In") the Model Confidence Set.

Table C.15: Models removed from the Model Confidence Set, $\alpha = 95\%$ – *continued*

(d) weekly returns, scaled forecasts

	Normal			Empirical			Student's t		
Asset split	1	3	8	1	3	8	1	3	8
CCC-GARCH		0.068	0.069		0.057	0.059		0.069	0.072
DCC-GARCH		0.062	0.063		0.054	0.057		0.069	0.067
CCC-GJR		0.075	0.069		0.059	0.059		0.075	0.075
DCC-GJR		0.069	0.064		0.056	0.059		0.075	0.072
RiskMetrics	0.089	0.086	0.078	0.055	0.056	0.059	0.079		

(e) biweekly returns, direct forecasts

	Normal			Empirical			Student's t		
Asset split	1	3	8	1	3	8	1	3	8
CCC-GARCH	0.049	0.062	0.073	0.046	0.049	0.058	0.053	0.059	0.075
DCC-GARCH		0.062	0.072		0.049	0.055		0.059	0.072
CCC-GJR	0.05	0.075	0.093	0.052	0.055	0.066	0.057	0.06	
DCC-GJR		0.072	0.092		0.055	0.064		0.059	0.076
RiskMetrics	0.049	0.075	0.055	0.046	0.065	0.049	0.046	0.075	0.058

(f) Number of removed and maintained methods

Temporal aggr.	Out	In	Total	Model	Out	In	Total
Daily iterated	2	37	39	CCC-GARCH	21	24	45
Daily scaled	1	38	39	DCC-GARCH	15	15	30
Weekly iterated	21	18	39	CCC-GJR	24	21	45
Weekly scaled	31	8	39	DCC-GJR	12	18	30
Biweekly direct	38	1	39	RiskMetrics	21	24	45
Portfolio aggr.	**Out**	**In**	**Total**	**Distribution**	**Out**	**In**	**Total**
Portfolio level	23	22	45	Normal	33	32	65
Asset class level	33	42	75	Empirical	31	34	65
Asset level	37	38	75	Student's t	29	36	65

See table note on the previous page.