Quantile Impulse Response Functions

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January, 2016

Abstract

This article develops a new approach for analyzing the impact and transmission of market shocks to the quantiles of financial return series. Our framework accommodates a bivariate system of dynamic conditional quantiles of random variables where one of the random variables evolves exogenously to the system. We provide the statistical framework to define structural quantile shocks and the associated quantile impulse response functions (QIRFs). We also derive the asymptotic distribution of the QIRFs and provide finite sample evidence with a series of Monte Carlo experiments. The empirical part of the paper investigates the effect and transmission of US equity market shocks to a set of financial institutions.

Keywords: Quantile regression; VAR for VaR; Correlation.

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1 Introduction

The tail dynamics of random variables can be conveniently modelled using quantile regression (QR), introduced by Koenker and Bassett (1978) and extensively reviewed in Koenker (2005). Many researchers have applied this methodology to analyse time series data. Engle and Manganelli (2004) and Kim, Manganelli and White (2008, 2015) have introduced a class of models for estimation and inference on the quantiles of financial returns, the conditionally autoregressive value-at-risk (CAViaR) and the vector autoregression for value-at-risk (VAR for VaR) models.

QR is a semi-parametric method that allows one to capture tail specific dynamics which are fundamental for financial risk management. It remains, however, a univariate technique. When cast in a multivariate framework such as a VAR model, it remains unclear how to trace the transmission of shocks to one variable to the whole system. It is in fact unclear how to think about structural shocks in the first place in a multivariate quantile regression setup.

This paper proposes a new approach to define quantile structural shocks and to derive quantile impulse-response functions (QIRFs) associated with these shocks. To make things concrete, we proceed by setting up a model of the interaction between the market and individual banks returns, where the tail dynamics of each bank return is exposed to spillovers from the lagged market return. Given the quantile setup, a tail market shock is conveniently defined as a stock market return being equal to its quantile associated with a sufficiently small probability. We next define the impulse response function as the difference between the expected tail dynamics with and without the market tail shock. The structural nature of the shock is derived from the assumption that market shocks can contemporaneously affect individual banks returns, but not vice versa.

The first part of the paper introduces the system of the conditional quantiles and shows how the quantile shocks are related to the shocks of an analogous system modeled along a multivariate generalized autoregressive conditional heteroskedasticity (GARCH). In the second part we define the QIRFs and formalize the related asymptotic theory for inference on the parameters as well as on the confidence bands of the QIRFs.

The second part of the paper presents the empirical estimates of the model on a selected sample of US banks. We first show how the standardized quantile shocks are characterized by properties similar to those of the standardized residuals of a GARCH model. We next use these standardized quantile residuals to obtain the structural shocks of the model and derive the associated quantile impulse response functions. We observe important differences in the magnitude, persistence and transmission dynamics across different financial institutions.

The paper is organized as following. Section 2 defines the bivariate conditional quantile system and links it to the GARCH framework. Section 3 defines the QIRFs. Section 4 provides the asymptotic properties. Section 5 discusses the finite sample properties. Section 6 presents the empirical estimates. Section 7 briefly concludes. Some of technical proofs and analytical results are provided in the Appendix.

2 The model

In what follows we are interested in the tail dynamics of the market and individual institution specific returns represented by the sequences $\{y_{mt}\}_{t=1}^{T}$ and $\{y_{it}\}_{t=1}^{T}$. This section specifies the framework designed to explore the tail dynamics in terms of the conditional quantile functions as well as the impact of the conditional quantile specific shocks.

2.1 Conditional quantile shocks

Given the information set $\mathcal{F}_{t-1}$, generated by the sequence of past returns, and for some confidence level $\theta \in (0,1)$ we concentrate our attention on the following data generating processes for the market and individual returns processes

$$\begin{aligned} y_{mt} &= q_{\theta,y_{mt}} + q_{\theta,y_{mt}}\eta_{mt}, \\ y_{it} &= q_{\theta,y_{it}} + q_{\theta,y_{it}}\varepsilon_{it}, \end{aligned} \tag{1}$$

where the conditional quantile error terms are assumed to satisfy the following conditional quantile restrictions

$$\Pr(\eta_{mt} \leq 0) = \Pr(\varepsilon_{it} \leq 0) = \theta, \quad \text{a.s.} \tag{2}$$

The system of equations (1) - (2) specify the return processes in terms of quantile function $q_{\theta,y_{jt}}$ that occur with the probability level θ and the tail shocks $\{\eta_{mt}, \varepsilon_{it}\}$. We are specifically interested in those circumstances when the market specific shocks are zero, so that the losses expected at the probability rate θ are materialized, i.e. $y_{mt} = q_{\theta,y_{mt}}$. Furthermore, we are interested in the time profile of the impact both to individual and market tail dynamics.

In order to formalize our discussion the following theorem introduces the source of uncertainty of the system represented by the market and individual financial institution, that account for the unpredictable movements in the asset returns and determine the shape and tail dynamics of corresponding distributions. Then it summarizes the properties of the tail specific shocks $\{\eta_{mt}, \varepsilon_{it}\}$ and shows that the system (1) - (2) is the conditional quantile representation of the standard return process with the generalized autoregressive conditionally heteroskedastic (GARCH) volatility dynamics by Bollerslev (1986).

Theorem 1 *Let z_{mt} and η_{it} represent i.i.d.(0,1) over t and mutually independent structural market and institution specific shocks with the continuous elliptical distribution function $F(\cdot)$. Furthermore, let σ_{mt} and σ_{it} be the dynamic volatility components and the market and individual returns be given as*

$$y_{mt} = \sigma_{mt} z_{mt}, \quad y_{it} = \sigma_{it} z_{it}, \tag{3}$$

where z_{it} is i.i.d.(0,1) over t reduced form individual shock satisfying the following linear partial correlation relationship

$$z_{it} = \rho z_{mt} + \sqrt{1-\rho^2}\eta_{it}, \quad \rho \in [-1, 1]. \tag{4}$$

Then

(1) *the tail specific shocks are given as*

$$\eta_{mt} = -1 + \frac{1}{F_{z_m}^{-1}(\theta)} z_{mt}, \quad \text{and} \quad \varepsilon_{it} = \rho\frac{F_{z_m}^{-1}(\theta)}{F_{z_i}^{-1}(\theta)} - 1 + \rho\frac{F_{z_m}^{-1}(\theta)}{F_{z_i}^{-1}(\theta)}\eta_{mt} + \frac{\sqrt{1-\rho^2}}{F_{z_i}^{-1}(\theta)}\eta_{it},$$

(2) *and the quantile values are given as*

$$q_{\theta,\mathrm{sgn}(q_{\theta,y_{mt}})\eta_m} = q_{\theta,\mathrm{sgn}(q_{\theta,y_{it}})\varepsilon_i} = 0.$$

Theorem 1 facilitates to identify the structural tail shocks η_{mt}, η_{it} under the linear restriction (4) in terms of the relationship that can be conveniently rewritten as

$$\varepsilon_{it} = \rho_i + \rho_{im}\eta_{mt} + \sigma_i\eta_{it}, \tag{5}$$

where it is obvious that the structural market tail shocks η_{mt} have a contemporaneous impacts on individual tail shock ε_{it} but not vice versa. In subsequent section we will discuss how this specification allows estimating the parameters of the relationship using the ordinary least squares regression (OLS) and identifying the structural individual quantile specific shock η_{it}. Furthermore, under very general GARCH specification the tail shocks defined by (1) is a valid system of conditional quantiles and can be estimated separately using the QR approach by Kim, Manganelli and White (2015).

By focusing on the cases when $\rho_t = \rho$ for all t, our framework falls into the general class of the constant conditional correlation model by Bollerslev (1990). Similarly to Engle, Ito, and Lin (1990) and Lin (1997) we follow the structure, where the shock propagates through the dynamic volatility structure and not the time varying dynamic correlation structure. Since there are different possible ways how one can accommodate the dynamic transmission channels through the dependence structure among standardized residuals $\{z_{it}, z_{m,t}\}$, for example one can add lags of $z_{m,t}$ to the right hand side of the dependence structure in equation 4 or make the correlation coefficient time varying, here we focus on the parsimonious structure to build the foundations that can be extended in various different ways depending on the empirical task at hand.

2.2 Conditional quantile functions

In order to study the time variation in the tail, we need to introduce the processes for the volatilities σ_{mt} and σ_{it}. We introduce the specification that treats the market rate as the exogenous state process that drives the dynamics of the individual conditional quantile along with the individual specific effects. In particular, we assume the daily volatility components are defined by the linear GARCH(1,1) specification suggested by Taylor (1986) modified as following

$$\begin{aligned} \sigma_{mt} &= \alpha_m + \gamma_m \sigma_{mt-1} + \beta_m |y_{mt-1}|, \\ \sigma_{it} &= \alpha_i + \gamma_i \sigma_{it-1} + \beta_i |y_{it-1}| + \beta_{im} |y_{mt-1}|, \end{aligned} \tag{6}$$

where the conditional quantile functions are given as

$$\begin{aligned} q_{\theta,y_{mt}} &= \alpha_m(\theta) + \gamma_m q_{\theta,y_{mt-1}} + \beta_m(\theta) |y_{mt-1}|, \\ q_{\theta,y_{it}} &= \alpha_i(\theta) + \gamma_i q_{\theta,y_{it-1}} + \beta_i(\theta) |y_{it-1}| + \beta_{im}(\theta) |y_{mt-1}|, \end{aligned} \tag{7}$$

where $\alpha_m(\theta) = \alpha_m F_{z_m}^{-1}(\theta)$, $\beta_m(\theta) = \beta_m F_{z_m}^{-1}(\theta)$, $\alpha_i(\theta) = \alpha_i F_{z_i}^{-1}(\theta)$, $\beta_{im}(\theta) = \beta_{im} F_{z_i}^{-1}(\theta)$, $\beta_i(\theta) = \beta_i F_{z_i}^{-1}(\theta)$.

The linear specification for the volatility components are computationally challenging compared to the standard quadratic specification. However, it is less sensitive to the extreme realizations of daily returns. Moreover, this specification incorporates the symmetric impact of lagged observations on the volatility, aiming at considering the positive impact of high and low realizations on the volatility and therefore on the tail dynamics.

3 Impulse response analysis

This presents the framework for analyzing the dynamic propagation of the tail shock trough the system. We first introduce the Quantile Impulse Response Function (QIRF) for uncovering the expected future propagation of the shock.

We are interested in the time profile of the effect of tail shocks to the system. In particular, at a point in time t, we are interested in expected transmission of the shock given the current and past realizations. The following definition summarizes our approach.

Definition 1 (QIRF) *Let $\theta \in (0,1)$. Then for $j \in \{m, i\}$ the impact of the shock to the market quantile at t can be evaluated as*

$$QIRF_{y_j}(h, \theta | \eta_{mt} = 0, \mathcal{F}_{t-1}) = E_{t-1}[q_{\theta, y_{jt+h}} | \eta_{mt} = 0] - E_{t-1}[q_{y_{\theta, jt+h}}], \quad \text{for} \quad h = 1, 2, \ldots,$$

where given the past information $\mathcal{F}_{t-1}$, we are interested in the expected impact and the transmission of the market tail shock.[1]

Intuitively, this definition based on the information available at time $t-1$ aims at understanding, to how differently the system would have behaved vis-à-vis the expected, i.e. average behavior, if the system was subject to the tail shock at time t.[2]

[1] In subsequent discussions for simplicity we suppress the time subscript from the expectation operator.

[2] For comparison to the standard impulse response analysis, following the exposition in Hamilton (1994) Section 11.4, consider any process written as the simple VAR(1) process, $Y_t = \Phi Y_{t-1} + V_t$, with eigenvalues of Φ lie inside the unit circle. Then as $h \to \infty$, $Y_{t+h} = \Phi^{h+1} Y_{t-1} + V_{t+n} + \cdots + \Phi^h V_t$ will have MA(∞)

In what follows we make use of the empirical fact that VaR is typically negative for the quantiles values of 1% and 5%. Therefore, we can reconsider the system of quantiles (1) as one with the time varying parameters as following

$$\begin{aligned} q_{\theta,y_{mt+1}} &= \alpha_m(\theta) + \gamma_{mt}(\theta) q_{\theta,y_{mt}}, \\ q_{\theta,y_{it+1}} &= \alpha_i(\theta) + \gamma_{it}(\theta) q_{\theta,y_{it}} + \beta_{imt}(\theta) q_{\theta,y_{mt}}, \end{aligned}$$

where the parameters are functionals of the market and individual tail shocks $\gamma_{mt}(\theta) = [\gamma_m - \beta_m(\theta)|1+\eta_{mt}|]$, $\gamma_{it}(\theta) = [\gamma_i - \beta_i(\theta)|1+\varepsilon_{it}|]$ and $\beta_{imt}(\theta) = -\beta_{im}(\theta)|1+\eta_{mt}|$. The system can be conveniently written in a matrix form as

$$\begin{pmatrix} q_{\theta,y_{mt+1}} \\ q_{\theta,y_{it+1}} \end{pmatrix} = \begin{pmatrix} \alpha_m(\theta) \\ \alpha_i(\theta) \end{pmatrix} + \begin{pmatrix} \gamma_{mt}(\theta) & 0 \\ \beta_{imt}(\theta) & \gamma_{it}(\theta) \end{pmatrix} \begin{pmatrix} q_{\theta,y_{mt}} \\ q_{\theta,y_{it}} \end{pmatrix},$$

or somewhat more compactly as

$$\boldsymbol{q}_{\theta,\boldsymbol{y}_{it+1}} = \boldsymbol{\alpha}_i(\theta) + \boldsymbol{\Gamma}_{it}(\theta) \boldsymbol{q}_{\theta,\boldsymbol{y}_{it}},$$

where

$$\boldsymbol{\Gamma}_{it}(\theta) \equiv \begin{pmatrix} \gamma_m - \beta_m(\theta)|1+\eta_{mt}| & 0 \\ -\beta_{im}(\theta)|1+\eta_{mt}| & \gamma_i - \beta_i(\theta)|1+\varepsilon_{it}| \end{pmatrix}.$$

For some h, the foregone expression conveniently yields the following result

$$\boldsymbol{q}_{\theta,\boldsymbol{y}_{it+h}} = \tilde{\boldsymbol{q}}_{\boldsymbol{y}_{it+h}}(\theta) + \prod_{j=1}^{h-1} \boldsymbol{\Gamma}_{it+j}(\theta) \left[\boldsymbol{\Gamma}_{it}(\theta) \boldsymbol{q}_{\theta,\boldsymbol{y}_{it}} \right],$$

where $\tilde{\boldsymbol{q}}_{\boldsymbol{y}_{it+h}}(\theta) \equiv \left[\boldsymbol{I} + \sum_{l=1}^{h-1} \prod_{j=l}^{h-1} \boldsymbol{\Gamma}_{it+j}(\theta) \right] \boldsymbol{\alpha}_i(\theta)$.

Given the *i.i.d.* property of quantile specific residuals over time, the expectation of the expression follows naturally to be as

$$\mathrm{E}\, \boldsymbol{q}_{\theta,\boldsymbol{y}_{it+h}} = \mathrm{E}\, \tilde{\boldsymbol{q}}_{\boldsymbol{y}_{it+h}}(\theta) + (\mathrm{E}\, \boldsymbol{\Gamma}_{it+j}(\theta))^{h-1} \left[\mathrm{E}\, \boldsymbol{\Gamma}_{it}(\theta) \boldsymbol{q}_{\theta,\boldsymbol{y}_{it}} \right],$$

where $(\mathrm{E}\, \boldsymbol{\Gamma}_{it+j}(\theta))$ is the expected value of matrix $\boldsymbol{\Gamma}_{it+j}$ for some $j = 1, \ldots, h-1$ and can be consistently estimated by its empirical counterpart.

representation. By setting $\{V_{t+2}, \ldots, V_{t+h}\}$, Y_{t-1} and all elements of V_t besides v_{jt} to zero, the element $\partial y_{it+h} / \partial v_{jt} \delta$ will give the impulse response as a function of h. Furthermore, this approach can be formalized as $I_Y(h, \delta, \mathcal{F}_{t-1}) \equiv \mathrm{E}(Y_{t+h} | v_{jt} = \delta, \mathcal{F}_{t-1}) - \mathrm{E}(Y_{t+n} | \mathcal{F}_{t-1})$.

For notational simplicity consider the following moments $\mu_m = \mathrm{E}[|1+\eta_{mt}|]$, $\mu_i = \mathrm{E}[|1+\varepsilon_{it}|]$ and $\tilde{\mu}_i = \mathrm{E}\,|1+\rho_i(\theta)+\sigma_i(\theta)\eta_{it}|$. Then the vector QIRF (VQIRF) can be obtained as

$$\mathbf{VQIRF}(h,\theta|\eta_{mt}=0,\mathcal{F}_{t-1}) \;=\; (\mathrm{E}\,\boldsymbol{\Gamma}_{it}(\theta))^{h-1}\,\mathrm{E}\left[\boldsymbol{\Gamma}_{it}(\theta|\eta_{mt}=0)-\boldsymbol{\Gamma}_{it}(\theta)\right]\boldsymbol{q}_{\theta,\boldsymbol{y}_{it}},$$

where the time t implication of the tail shock is given as

$$\mathrm{E}\left[\boldsymbol{\Gamma}_{it}(\theta|\eta_{mt}=0)-\boldsymbol{\Gamma}_{it}(\theta)\right] = \begin{pmatrix} \beta_m(\theta)(\mu_m-1) & 0 \\ \beta_{im}(\theta)(\mu_m-1) & \beta_i(\theta)(\mu_i-\tilde{\mu}_i) \end{pmatrix}.$$

We will explore the asymptotic properties of this estimator in the following sections. But before, in order to facilitate the empirical analysis we impose the following stability condition.

Assumption 1 *For any $\theta \in (0,1)$ the matrix $\mathrm{E}\,\boldsymbol{\Gamma}_{it}(\theta)$ has eigenvalues less than one in absolute value.*

Our definition of the impulse response function bears important information for analyzing systemic stability and resilience to the market shocks. In particular, given the market value of particular financial institution, the resilience to the market shock can be well characterized by the number of consecutive periods required for getting back to the time t level. In particular we are interested in

$$\hat{h} \quad \text{s.t} \quad \mathbf{VQIRF}(\hat{h},\theta|\eta_{mt}=0,\mathcal{F}_{t-1}) \approx 0, \tag{8}$$

where $\hat{h}$ will typically measure the expected number of days the system requires to reach back to the level it has been at t.

4 Asymptotic distribution of the QIRF

In this section we derive the asymptotic distribution of the QIRF using the delta method. In what follows, for $\theta \in (0,1)$ we consider the vector of conditional quantile parameters $\boldsymbol{\beta}(\theta) = (\boldsymbol{\beta}_m(\theta)', \boldsymbol{\beta}_i(\theta)')' = (\alpha_m(\theta), \gamma_m, \beta_m(\theta), \alpha_i(\theta), \gamma_i, \beta_i(\theta), \beta_{im}(\theta))'$. Given the conditional quantile restriction in (7), the parameters of the model $\beta_j(\theta)$, $j \in (i, m)$ solve the following asymmetric loss problem by Koenker and Basset (1978)

$$\hat{\beta}_j(\theta) = \arg\min \frac{1}{T} \sum_{t=1}^{T} \left[\theta - I\left(u_{y_{jt}} < 0\right)\right] u_{y_{jt}}, \tag{9}$$

where we use $u_{y_{jt}} = y_{jt} - q_{\theta, y_{jt}}$, $j \in \{m, i\}$. Then the standardized quantile specific residuals can be recovered as $\hat{u}_{y_{jt}} / \hat{q}_{\theta, y_{jt}}(\theta|\mathcal{F}_{t-1})$ for $j \in \{m, i\}$.

Note that the conditional quantiles in (7) can be viewed as the special case of the model considered in White, Kim and Manganelli (2015). The following Corollary summarizes the properties of the quantile regression estimator $\hat{\boldsymbol{\beta}}_j(\theta), j \in \{m, i\}$.

Corollary 1 (Corollary to Kim, Manganelli and White (2015)) *Suppose assumptions 1-6 of* **KMW** *hold, then the QR estimator $\hat{\boldsymbol{\beta}}_j(\tau)$ of model $j \in \{m, i\}$ is consistent and asymptotically normal with the following asymptotic distribution*

$$\sqrt{T}(\hat{\boldsymbol{\beta}}_j(\theta) - \boldsymbol{\beta}_j(\theta)) \sim \mathcal{N}\left(0, \theta(1-\theta)\boldsymbol{D}_{j1}(\theta)^{-1}\boldsymbol{D}_{j0}(\theta)\boldsymbol{D}_{j1}(\theta)^{-1}\right),$$

with the positive definite matrices given as

$$\begin{aligned} \lim_{T\to\infty} T^{-1} \textstyle\sum_{t=1}^{T} \nabla_{\boldsymbol{\beta}_j} \hat{q}_{\theta, y_{jt}} \nabla_{\boldsymbol{\beta}_j} \hat{q}'_{\theta, y_{jt}} &= \boldsymbol{D}_{j0}(\theta), \\ \lim_{T\to\infty} T^{-1} \textstyle\sum_{t=1}^{T} f_{jt}(\hat{q}_{\theta, y_{jt}}) \nabla_{\boldsymbol{\beta}_j} \hat{q}_{\theta, y_{jt}} \nabla_{\boldsymbol{\beta}_j} \hat{q}'_{\theta, y_{jt}} &= \boldsymbol{D}_{j1}(\theta), \end{aligned}$$

where f_{jt} is the continuous density function.

The Appendix A.2 outlines the relation of the corollary to the framework by **KWM** and discusses how the covariance matrix $\hat{\Omega}_j(\theta), \theta \in (0,1), j \in (i, m)$ could be constructed. The following theorem summarizes the asymptotic properties of the impulse response function $\mathbf{VQIRF}(h, \theta|\eta_{mt} = 0, \mathcal{F}_{t-1}), \theta \in (0,1)$.

Theorem 2 *Suppose the result of the Corollary 1 holds and $\mathbf{VQIRF}(h, \theta|\eta_{mt} = 0, \mathcal{F}_{t-1})$, is continuously differentiable function with respect to parameters $\boldsymbol{\beta}(\theta)$ for all $\theta \in (0,1)$. Then*

$$\sqrt{T}[\mathbf{VQIRF}(\hat{\boldsymbol{\beta}}(\theta)) - \mathbf{VQIRF}(\boldsymbol{\beta}(\theta))] \sim \mathcal{N}\left(\mathbf{0}, \nabla_{\boldsymbol{\beta}(\theta)}\mathbf{VQIRF}(\boldsymbol{\beta}(\theta))\boldsymbol{\Omega}(\theta)\nabla_{\boldsymbol{\beta}(\theta)}\mathbf{VQIRF}(\boldsymbol{\beta}(\theta))'\right),$$

where

$$\boldsymbol{\Omega}(\theta) = \theta(1-\theta)\begin{pmatrix} \boldsymbol{D}_{m1}(\theta)^{-1}\boldsymbol{D}_{m0}(\theta)\boldsymbol{D}_{m1}(\theta)^{-1} & \mathbf{O}_{3\times4} \\ \mathbf{O}_{4\times3} & \boldsymbol{D}_{i1}(\theta)^{-1}\boldsymbol{D}_{i0}(\theta)\boldsymbol{D}_{i1}(\theta)^{-1} \end{pmatrix}.$$

The result of the Theorem 2 can be used to approximate the confidence bands for the QIRFs.

5 Monte Carlo Evidence

This section aims at exploring the robustness of the finite sample properties of the QR for the individual conditional quantile $q_{y_{it}}$ from the system (7) vis-à-vis the different correlation structure and error distribution.

5.1 Setting

In what comes we consider the data generating process as in volatility system (6) with two *i.i.d.* normal and *i.i.d.* Student-t distribution with 4 degrees of freedom samples of size $T \in \{500, 1000\}$. Furthermore, we consider the following set of parameters

DGP 1: $\alpha_m = \alpha_i = 0.1$, $\gamma_m = \gamma_i = 0.5$, $\beta_m = \beta_i = 0.3$, $\beta_{im} = 0.3$.

DGP 2: $\alpha_m = \alpha_i = 0.1$, $\gamma_m = \gamma_i = 0.8$, $\beta_m = \beta_i = 0.1$, $\beta_{im} = 0.1$.

Following Engle (2002), we set the following correlation coefficients

Constant: $\rho \in \{0.3, 0.5, 0.9\}$.

Sine: $\rho_t = 0.5 + 0.4 \cos(2\pi t/200)$.

Fast sine: $\rho_t = 0.5 + 0.4 \cos(2\pi t/20)$.

Step: $\rho_t = 0.9 - 0.5 \cos(t > 500)$.

Ramp: $\rho_t = \mod(t/200)$.

We simulate the system of All results are based on 100 replications of the DGPs using the GARCH system of equations (6). Since, the market quantile follows standard CAViaR specification, here we only focus on the performance of the institution conditional quantile $q_{y_{it}}$.

First, we report the bias and root mean square error (RMSE) based on the difference between Monte Carlo estimates and the true values. The quantile specific true values are calculated as $b_i \hat{F}_{z_l}^{-1}(\theta)$, $b_i \in \{\alpha_i, \beta_i, \beta_{im}\}$ where $\hat{F}_{z_l}^{-1}(\theta)$ is the empirical quantile of true errors $\{z_{l1}, \ldots, z_{lt}\}$ for l-th Monte Carlo iteration. Then, for the QIRFs we report true value as well as Monte Carlo two standard error bands and estimated QIRFs.

5.2 Results

[**In Progress**]

6 Impact of the US equity market tail shocks

The recent financial crisis triggered numerous works in the field of systemic risk (See Adrian and Brunnermeier (2009), Engle and Brownlees (2005), Acharya et al. (2010) among others.). From the prospectives of the systemic risk, institutions that are overly sensitive to the market movements tend to be important for the system stability. In this empirical application we examine the sensitivity of the set of the US based financial institutions to the equity market shock.

6.1 Data description and computational issues

The data is obtained from Datastream. We use the daily closing equity prices for the set of US based financial institutions and the market and transform it into the continuously compounded log returns covering the period from 3 January 2000 to 12 December 2014, constituting the sample of 3913 observations. We use last 500 observations for out-of-sample analysis. Table 1 reports the set of institutions as well as descriptive statistics for them.

[**Insert Table 1 around here**]

We use the Nelder-Mead (NM) simplex algorithm for solving computational problems in the spirit of Engle and Manganelli (2004). In particular, we generate $10^4 \times 1$ vector of $i.i.d. \mathcal{U}(-1, 1)$ random variables and select first 1000 that minimizes the objective function. Then for each of those initial parameter values, we run NM algorithm and select the optimal estimates.[3] Furthermore, we make sure that the data at hand closely matches the GARCH

[3]All calculations are done either using Stata or Matlab.

framework in (7) by recovering the serially uncorrelated series $\hat{y}_{jt}, j \in \{i, m\}$ by fitting autoregressive filter of certain order. We fit AR(6) for the S&P 500 series and use residual for modeling purposes.

6.2 Specification and correlation tests

If equations in (1) represents the true model, the conditional quantile restriction holds $\Pr(y_{jt} - q_{\theta,y_{jt}} < 0) = \theta, j \in \{i, m\}, t = 1, \ldots, T$. We use the in-sample and out of sample dynamic quantile (DQ) tests by Engle and Manganelli (2004) for testing whether the conditional quantile restriction holds or not. For this purpose we use the one step ahead forecast values of the conditional quantile function and the fitted values of $\hat{\text{Hit}}_{y_{jt}}(\theta) = I(y_{jt} - \hat{q}_{\theta,y_{jt}}) - \theta, j \in \{i, m\}, t = 1, \ldots, T$.

Furthermore, we test the presence of serial correlation in level and squared tail specific shocks $\hat{\varepsilon}_{it}$ and $\hat{\eta}_{mt}$ for the empirical justification of the model and use them to run the following regression

$$\hat{\varepsilon}_{it} = \left[\rho \frac{F_{z_m}^{-1}(\theta)}{F_{z_i}^{-1}(\theta)} - 1\right] + \rho \frac{F_{z_m}^{-1}(\theta)}{F_{z_i}^{-1}(\theta)} \hat{\eta}_{mt} + \frac{\sqrt{1-\rho^2}}{F_{z_i}^{-1}(\theta)} \hat{\eta}_{it},$$

where since $\frac{F_{z_m}^{-1}(\theta)}{F_{z_i}^{-1}(\theta)} \neq 0$ for any $\theta \in (0, 1)$ then the following t statistics of the hypothesis $H_0 : \rho \frac{F_{z_m}^{-1}(\theta)}{F_{z_i}^{-1}(\theta)} = 0$ will be equivalently conceived as the $H_0 : \rho = 0$. Since, the value of T we are working with is high, we can treat the generated regressors as the data, therefore it can be expected that the t-statistics is well defined within our framework.

6.3 Results

Table 2 reports the parameters of the conditional quantile functions at $\theta = 1\%$ rarity level for the market rate captured by the S&P 500 and the set of individual financial institutions such as J.P. Morgan, Wells Fargo, U.S. Bancorp, American Express and Goldman Sachs.

[**Insert Table 2 around here**]

Findings are pretty much in line with the existing literature on the time dynamics of VaR. In particular, VaR tends to be a persistent process with the parameter values being in the vicinity of 0.9 uniformly across all series considered. Furthermore, we find the significant loading on the lagged absolute value of the market rate for J.P. Morgan, U.S. Bancorp and American Express hinting at the sensitivity to the market dynamics.

[**Insert Figure 1 around here**]

Figure 1 plots the time dynamics of the conditional VaR. The severe spikes are observed around the period of the recent financial crisis. This evidence is commensurate with the amplified uncertainty that has been prevailing during the periods following the default of Lehman Brothers.

[**Insert Table 3 around here**]

Table 3 reports the OLS estimates for the linear relationship in equation (5). We notice the significantly high loading on the structural market tail shock uniformly for each institution. Therefore indicating at the presence of high degree co-movement between the reduced form individual tail shocks and the structural market shock.

[**Insert Tables 4 and 5 around here**]

Tables 4 and 5 report the statistical properties of the standardized conditional and unconditional quantile shocks. We use the θ-th empirical quantile in order to calculate the standardized unconditional quantile residual. As it is clearly seen from the Ljung-Box statistics we account the most of the serial correlation by considering the dynamic conditional quantile model.

[**Insert Table 6 around here**]

The values of the matrix $\hat{\mathrm{E}}\boldsymbol{\Gamma}_t$ reported in Table 6 show that the shock to the market rate is persistent with the diagonal elements being close to unity. We expect that the initial impact will have a persistent effect on the system.

[**Insert Figure 2 around here**]

Figure 2 reports the QIRF for the set of financial institutions in response to the market quantile shock captured by the value of the S&P 500 return being equal to its quantile value. We accompany our plots by two standard deviation confidence bands. There are several comments in line. First, some set of financial institutions tend to react with a higher amplitude compared to the market rate. For example J.P. Morgan overreacts to the market shock as well as the impact tends to be more persistent compared to the market rate. Second, as we have seen in the table 2, some institutions as Goldman Sachs and Wells Fargo have insignificant loading on the lagged absolute value of the market rate. This fact is well expressed in the QIRFs, because the impact is modest and most of the transmission period it coincides the zero line. The remaining institutions tend to display higher significant responses.

7 Conclusion

This article proposed a unified framework for definition and measurement of the tail shocks within the context of financial returns. We consider the set up based on the canonical GARCH framework and developed the comprehensive statistical framework for systemic analysis of the impact ad transmission of the severe market downturns. The empirical application demonstrates the relevance of the proposed approach for practical purposes. We delegate the various important extensions withing the context of the systemic risk to the future research.

Appendices

A Proofs

A.1 Proof of Theorem 1

Proof 1 (Proof of Theorem 1) *The result can be established using straightforward probabilistic manipulations, by noticing that the linear combination of elliptically distributed random variables variables is elliptical.*

Part 1 *Recall, that the market* $\{y_{mt}\}_{t=1}^{T}$ *and individual return series* $\{y_{it}\}_{t=1}^{T}$ *obey the following constant correlation setting*

$$y_{mt} = \sigma_{mt} z_{mt}, \quad y_{it} = \sigma_{it} z_{it}, \tag{10}$$

with the following relationship among the random components

$$z_{it} = \rho z_{mt} + \sqrt{1-\rho^2}\eta_{it}, \quad \rho \in [0,1]. \tag{11}$$

Then the conditional quantiles of the market and individual asset returns are results of the following conditional quantile restriction, a.s.,

$$\Pr(\eta_{mt} \leq 0) = \Pr(\varepsilon_{it} \leq 0) = \theta,$$

where $q_{\theta,y_{mt}} = \sigma_{y_{mt}} F_{z_m}^{-1}(\theta)$ *and* $q_{\theta,y_{it}}(\theta|\mathcal{F}_{t-1}) = \sigma_{y_{it}} F_{z_i}^{-1}(\theta)$ *and the reduced form standardized market quantile shocks* $\eta_{mt}(\theta)$ *satisfy*

$$\begin{aligned} 1+\eta_{mt} &= 1+\tfrac{y_{mt}-q_{\theta,y_{mt}}}{q_{\theta,y_{mt}}} \\ &= \tfrac{1}{F_{z_m}^{-1}(\theta)} z_{mt}. \end{aligned} \tag{12}$$

Furthermore, the standardized individual conditional quantile shock is defined as

$$\begin{aligned} 1+\varepsilon_{it} &= 1+\tfrac{y_{it}-q_{\theta,y_{it}}}{q_{\theta,y_{it}}} \\ &= \tfrac{1}{F_{z_i}^{-1}(\theta)} z_{it} \\ &= \tfrac{1}{F_{z_i}^{-1}(\theta)}\left(\rho z_{mt} + \sqrt{1-\rho^2}\eta_{it}\right) \\ &= \rho\tfrac{F_{z_m}^{-1}(\theta)}{F_{z_i}^{-1}(\theta)}(1+\eta_{mt}) + \tfrac{\sqrt{1-\rho^2}}{F_{z_i}^{-1}(\theta)}\eta_{it}. \end{aligned}$$

Part 2 *It is straightforward to claim that the structural market and individual reduced form shocks satisfy the following relation*

$$(1+\eta_{mt})F^{-1}(\theta) \sim i.i.d.\,(0,1)\,,$$

and

$$(1+\varepsilon_{it})F^{-1}(\theta) \sim i.i.d.\,(0,1)\,,$$

Since $\varepsilon_{it}(\theta)$ *is elliptically distributed with the distribution function* $F(\cdot)$ *the quantile value* $(1+\varepsilon_{it})F^{-1}(\theta)$ *can be approximated as*

$$\mathrm{E}(\rho z_{mt} + \sqrt{1-\rho^2}\eta_{it}|\mathcal{F}_{t-1}) + \sqrt{\mathrm{var}(\rho z_{mt} + \sqrt{1-\rho^2}\eta_{it}|\mathcal{F}_{t-1})}F^{-1}(\theta) = F^{-1}(\theta).$$

We note the following important feature for θ*, for which the conditional quantile is negative, namely* $\Pr(y_t - q_{\theta,y_t} \leq q_{\theta,y_t}\eta_{mt}) = \Pr((y_t - q_{\theta,y_t})/q_{\theta,y_t} \leq -\eta_{mt})$*. Then, we have to have quantile* $-\eta_{mt} = 0$ *.*

A.2 Proof of the Corollary 1

Proof 2 *The model we consider in this paper is nested in one considered by* **KWM***. Therefore, the corollary directly flows from the Theorem 2 in* **KWM***.*

Furthermore, the estimator of the variance covariance matrix is based on the Powell's (1986) approach. In particular, the components of $\hat{\boldsymbol{\Omega}}_{jT}(\theta), j \in \{i,m\}$ can be estimated as

$$\hat{\boldsymbol{H}}_{jT}(\theta) = \frac{1}{2Th_T}\sum_{t=1}^{T} I(|y_{jt} - \hat{q}_{\theta,y_{jt}}| \leq h_T)\nabla_{\boldsymbol{\beta}_j(\theta)}\hat{q}_{\theta,y_{jt}}\nabla_{\boldsymbol{\beta}_j(\theta)}\hat{q}'_{\theta,y_{jt}},$$

and

$$\hat{\boldsymbol{J}}_{jT}(\theta) = \frac{1}{T}\sum_{t=1}^{T}\left(\theta - I(y_{jt} \leq \hat{q}_{\theta,y_{jt}})\right)^2 \nabla_{\boldsymbol{\beta}_j(\theta)}\hat{q}_{\theta,y_{jt}}\nabla_{\boldsymbol{\beta}_j(\theta)}\hat{q}'_{\theta,y_{jt}},$$

where h_T is the bandwidth satisfying $h_T \to 0$ and $h_T^2 T \to \infty$. In this paper we follow Machado and Santos Silva (2013) for bandwidth selection, and use the following estimate

$$\hat{h}_T = \hat{\kappa}_T[\phi^{-1}(\theta + c_T) - \phi^{-1}(\theta + c_T)]$$

where

$$c_T = T^{-1/3}(\phi^{-1}(1-0.025)^{2/3}\left(\frac{1.5(\phi(\Phi^{-1}(\theta)))}{2(\Phi^{-1}(\theta))^2+1}\right)^{1/3},$$

where $\phi(\cdot)$ and $\Phi(\cdot)$ are standard normal cumulative distribution and probability density functions and $\hat{\kappa}_T$ is the median absolute deviation of θ-th conditional quantile residuals.

Furthermore, consider the following analytic derivatives

$$\frac{\partial}{\partial \alpha_m(\theta)} q_{\theta,y_{mt}} = \begin{cases} 1 + \gamma_m \frac{\partial}{\partial \alpha_m(\theta)} q_{\theta,y_{mt-1}}, & \text{for} \quad t \geq 2, \\ 0, & \text{for} \quad t = 1, \end{cases}$$

$$\frac{\partial}{\partial \gamma_m} q_{\theta,y_{mt}} = \begin{cases} q_{\theta,y_{mt-1}} + \gamma_m \frac{\partial}{\partial \gamma_m} q_{\theta,y_{mt-1}}, & \text{for} \quad t \geq 2, \\ 0, & \text{for} \quad t = 1, \end{cases}$$

$$\frac{\partial}{\partial \beta_m(\theta)} q_{\theta,y_{mt}} = \begin{cases} |y_{mt-1}| + \gamma_m \frac{\partial}{\partial \beta_m(\theta)} q_{\theta,y_{mt-1}}, & \text{for} \quad t \geq 2, \\ 0, & \text{for} \quad t = 1, \end{cases}$$

$$\frac{\partial}{\partial \alpha_i(\theta)} q_{\theta,y_{it}} = \begin{cases} 1 + \gamma_i \frac{\partial}{\partial \alpha_i(\theta)} q_{\theta,y_{it-1}}, & \text{for} \quad t \geq 2, \\ 0, & \text{for} \quad t = 1, \end{cases}$$

$$\frac{\partial}{\partial \gamma_i} q_{y_{it}} = \begin{cases} q_{\theta,y_{it-1}} + \gamma_i \frac{\partial}{\partial \gamma_i} q_{\theta,y_{it-1}}, & \text{for} \quad t \geq 2, \\ 0, & \text{for} \quad t = 1, \end{cases}$$

$$\frac{\partial}{\partial \beta_i(\theta)} q_{\theta,y_{it}} = \begin{cases} |y_{it-1}| + \gamma_i \frac{\partial}{\partial \beta_i(\theta)} q_{\theta,y_{it-1}}, & \text{for} \quad t \geq 2, \\ 0, & \text{for} \quad t = 1, \end{cases}$$

$$\frac{\partial}{\partial \beta_{im}(\theta)} q_{y_{it}} = \begin{cases} |y_{mt-1}| + \gamma_i \frac{\partial}{\partial \beta_{im}(\theta)} q_{y_{it-1}}, & \text{for} \quad t \geq 2, \\ 0, & \text{for} \quad t = 1. \end{cases}$$

Or somewhat more compactly, for $j \in \{i, m\}$ the following Jacobian terms are given as

$$\nabla_{\boldsymbol{\beta}_j(\theta)} \hat{q}_{\theta,y_{jt}} = \begin{cases} \boldsymbol{z}_{jt} + \gamma_j \nabla_{\boldsymbol{\beta}_j(\theta)} \hat{q}_{\theta,y_{\theta,jt-1}} & \text{for} \quad t \geq 2, \\ 0 & \text{for} \quad t = 1, \end{cases}$$

where results follows due to the fact that the CAViaR filter is initialized using the empirical quantile of first T_0 observations and $\boldsymbol{z}'_{mt} = (1, q_{\theta,y_{mt-1}}, |y_{mt-1}|)$ and $\boldsymbol{z}'_{it} = (1, q_{\theta,y_{it-1}}, |y_{it-1}|, |y_{mt-1}|)$.[4]

[4]In empirical application we set $T_0 = 100$.

A.3 Proof of Theorem 2

Proof 3 *The proof follows from the standard application of the delta method. We note that the gradient term* $\nabla_{\boldsymbol{\beta}(\theta)}\mathbf{VQIRF}(\boldsymbol{\beta}(\theta))$ *is nonstochastic and the result follows by using the mean value expansion.*[5]

Here we provide the analytic expressions for the gradient. In what follows for simplicity we set $\bar{\boldsymbol{\Gamma}}_i(\theta) = \mathrm{E}\,\boldsymbol{\Gamma}_{it}(\theta)$ Note that for some parameter $\beta_{i1}(\theta)$ from the parameter $\boldsymbol{\beta}_i(\theta)$, the following holds

$$\frac{\partial}{\partial\beta_{i1}(\theta)}\left(\bar{\boldsymbol{\Gamma}}_i(\theta)\right)^h = \frac{\partial}{\partial\beta_{i1}(\theta)}\bar{\boldsymbol{\Gamma}}_i(\theta)\left(\bar{\boldsymbol{\Gamma}}_i(\theta)\right)^{h-1} + \bar{\boldsymbol{\Gamma}}_i(\theta)\frac{\partial}{\partial\beta_{i1}(\theta)}\left(\bar{\boldsymbol{\Gamma}}_i(\theta)\right)^{h-1}, \quad j = 2,\ldots,h,$$

[5] See, for example, Theorem 4.36 in White (2001).

where

$$\begin{aligned}
\frac{\partial}{\partial \alpha_m(\theta)} \operatorname{E} \boldsymbol{\Gamma}_{it}(\theta) &= \mathbf{0}, \\
\frac{\partial}{\partial \gamma_m} \operatorname{E} \boldsymbol{\Gamma}_{it}(\theta) &= \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \\
\frac{\partial}{\partial \beta_m(\theta)} \operatorname{E} \boldsymbol{\Gamma}_{it}(\theta) &= \begin{pmatrix} -\mu_m & 0 \\ 0 & 0 \end{pmatrix}, \\
\frac{\partial}{\partial \alpha_i(\theta)} \operatorname{E} \boldsymbol{\Gamma}_{it}(\theta) &= \mathbf{0}, \\
\frac{\partial}{\partial \gamma_i} \operatorname{E} \boldsymbol{\Gamma}_{it}(\theta) &= \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \\
\frac{\partial}{\partial \beta_i(\theta)} \operatorname{E} \boldsymbol{\Gamma}_{it}(\theta) &= \begin{pmatrix} 0 & 0 \\ 0 & -\mu_i \end{pmatrix}, \\
\frac{\partial}{\partial \beta_{im}(\theta)} \operatorname{E} \boldsymbol{\Gamma}_{it}(\theta) &= \begin{pmatrix} 0 & 0 \\ -\mu_m & 0 \end{pmatrix}. \\
\frac{\partial}{\partial \alpha_m(\theta)} \operatorname{E} \left[\boldsymbol{\Gamma}_{it}(\theta|\eta_{mt}=0) - \boldsymbol{\Gamma}_{it}(\theta)\right] &= \mathbf{0}, \\
\frac{\partial}{\partial \gamma_m} \operatorname{E} \left[\boldsymbol{\Gamma}_{it}(\theta|\eta_{mt}=0) - \boldsymbol{\Gamma}_{it}(\theta)\right] &= \mathbf{0}, \\
\frac{\partial}{\partial \beta_m(\theta)} \operatorname{E} \left[\boldsymbol{\Gamma}_{it}(\theta|\eta_{mt}=0) - \boldsymbol{\Gamma}_{it}(\theta)\right] &= \begin{pmatrix} (\mu_m - 1) & 0 \\ 0 & 0) \end{pmatrix}, \\
\frac{\partial}{\partial \alpha_i(\theta)} \operatorname{E} \left[\boldsymbol{\Gamma}_{it}(\theta|\eta_{mt}=0) - \boldsymbol{\Gamma}_{it}(\theta)\right] &= \mathbf{0}, \\
\frac{\partial}{\partial \gamma_i} \operatorname{E} \left[\boldsymbol{\Gamma}_{it}(\theta|\eta_{mt}=0) - \boldsymbol{\Gamma}_{it}(\theta)\right] &= \mathbf{0}, \\
\frac{\partial}{\partial \beta_i(\theta)} \operatorname{E} \left[\boldsymbol{\Gamma}_{it}(\theta|\eta_{mt}=0) - \boldsymbol{\Gamma}_{it}(\theta)\right] &= \begin{pmatrix} 0 & 0 \\ 0 & (\mu_i - \tilde{\mu}_i) \end{pmatrix}, \\
\frac{\partial}{\partial \beta_{im}(\theta)} \operatorname{E} \left[\boldsymbol{\Gamma}_{it}(\theta|\eta_{mt}=0) - \boldsymbol{\Gamma}_{it}(\theta)\right] &= \begin{pmatrix} 0 & 0 \\ (\mu_m - 1) & 0) \end{pmatrix}.
\end{aligned}$$

B Finite sample evidence

[**In Progress**]

C Empirical results

Table 1: Summary statistics of data.

	Mean	Std. Dev.	Max	Min	Skewness	Kurtosis
S&P 500	0.0000	1.3134	10.2282	-9.1455	-0.3411	10.2934
JPM	-0.0003	2.7799	22.3848	-23.2257	0.2653	14.4935
GS	0.0163	2.5841	23.4164	-21.0459	0.2986	13.9036
WFC	0.0180	2.6994	28.3386	-27.2104	0.8443	26.3367
AXP	0.0081	2.5083	18.7607	-19.3595	0.0002	11.4304
USB	0.0130	2.4336	20.5734	-20.0514	-0.0541	15.5924

Note: Statistics for S&P 500 is presented after applying AR(6) filter. The sample ranges from January 10, 2000, to January 29, 2013.

Table 2: Estimates and test statistics for the dynamic conditional quantile functions.

	S&P500		JPM	GS	WFC	AXP	USB
	−0.0756		−0.0917	-0.0319	0.0026	-0.0270	-0.0422
$\alpha_m(\theta)$	(0.0281)	$\alpha_i(\theta)$	(0.0439)	(0.0194)	(0.0142)	(0.0172)	(0.0188)
	{0.0071}		{0.0365}	{0.0999}	{0.8526}	{0.1159}	{0.0247}
	0.9050		0.8512	0.9368	0.9550	0.9286	0.8876
γ_m	(0.0195)	γ_i	(0.0296)	(0.0164)	(0.0086)	(0.0144)	(0.0159)
	{0.0000}		{0.0000}	{0.0000}	{0.0000}	{0.0000}	{0.0000}
	−0.2444		−0.2453	-0.1671	-0.1188	-0.1575	-0.2735
$\alpha_m(\theta)$	(0.0402)	$\beta_i(\theta)$	(0.0502)	(0.0473)	(0.0215)	(0.0457)	(0.0327)
	{0.0000}		{0.0000}	{0.0004}	{0.0000}	{0.0006}	{0.0000}
			−0.4012	-0.0269	-0.0552	-0.1192	-0.1151
		$\beta_{im}(\theta)$	(0.2198)	(0.0564)	(0.0372)	(0.0524)	(0.0608)
			{0.0680}	{0.6341}	{0.1377}	{0.0230}	{0.0584}
$\mathcal{RQ}$	128.9529		267.3816	262.8442	221.5332	239.3424	229.8153
Hits in-sample	0.0000		-0.0003	0.0000	-0.0003	0.0000	0.0000
Hits out-of-sample	-0.0020		-0.0060	-0.0060	0.0000	-0.0060	0.0040
$\mathcal{DQ}$ in-sample	0.7138		0.4741	0.2936	0.2444	0.4160	0.2581
$\mathcal{DQ}$ out-of-sample	0.1489		0.5643	0.8355	0.0484	0.8278	0.0000

Note: All calculations are done at $\theta = 1\%$ level. Standard deviations are reported in parentheses and p values are in braces.

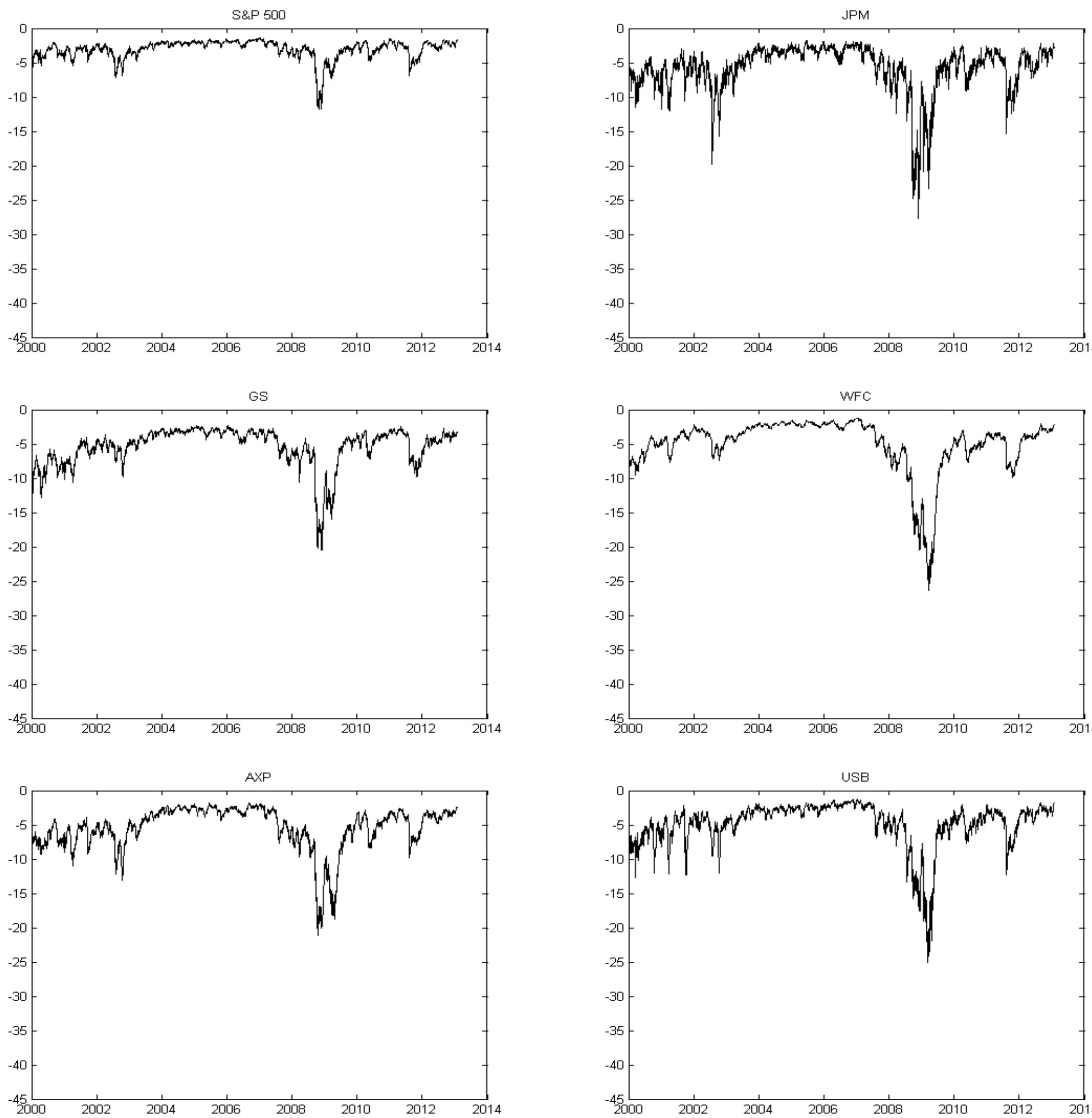

Figure 1: 1% Estimated conditional quantile functions using specification in (7).

Table 3: Estimates and test statistics for the reduced form standardized conditional quantile shock.

	JPM	GS	WFC	AXP	USB
	−.2126	-0.1946	−0.2766	-.2312	-.2933
$\rho_i(\theta)$	(.0154)	(0.0185)	(0.0174)	(.0174)	(.0171)
	{0.0000}	{0.0000}	{0.000}	{0.000}	{0.000}
	.7858	0.8099	.7268	.7709	.7080
$\rho_{im}(\theta)$	(.0147)	(0.0174)	(0.0169)	(0.0165)	(0.0161)
	{0.0000}	{0.0000}	{0.0000}	{0.0000}	{0.000}
$\sigma_{im}(\theta)$	.2674	0.3002	0.2981	.2706	.30237
$\mathcal{R}^2$	0.5460	0.5035	0.4530	0.5306	0.4330
$\mathcal{RSS}$	243.4728	306.8068	302.5208	249.386	311.3111

Note: Parameters of the equation (5) are obtained using the ordinary least squares estimator.

Table 4: Summary statistics of the standardized conditional quantile shocks.

	Mean	Median	$q_{-\eta_t(0.01)}$	$Q(\eta_t)$	$P>Q(\eta_t)$	$Q(\eta_t^2)$	$P>Q(\eta_t^2)$
S&P500	-1.0012	-1.0254	0.000	8.3005	0.4047	13.191	0.1055
JPM	-.9993	-1.0000	0.0000	5.0884	0.7481	7.235	0.5115
WFC	-1.0043	-1.0000	0.0000	27.317	0.0006	20.159	0.0098
GS	-1.0055	-1.0000	0.0000	5.59	0.6930	9.1664	0.3285
AXP	-1.0031	-1.0000	0.0000	17.08	0.0293	31.266	0.0001
USB	-1.0022	-1.0000	0.0000	7.3857	0.4956	11.659	0.1671

Note: Following arguments of Tsay (2005) results of the autocorrelation tests are reported using the lag order $\tilde{p} \approx \ln(T)$, where T is the sample size.

Table 5: Summary statistics of the standardized unconditional quantile shocks.

	Mean	Median	$q_{-\eta_t(0.01)}$	$Q(\eta_t)$	$P>Q(\eta_t)$	$Q(\eta_t^2)$	$P>Q(\eta_t^2)$
S&P500	-1.0000	-1.0100	.4006	8.9478	0.3467	4.443	0.8151
JPM	-1.0000	-1.0000	-.0866	45.621	0.0000	95.312	0.0000
WFC	-1.0025	-1.0000	-.012596	85.514	0.0000	146.5	0.0000
GS	-1.0012	-1.0000	.4923	25.995	0.0011	10.684	0.2203
AXP	-1.0009	-1.0000	.1247	31.038	0.0001	35.635	0.0000
USB	-1.0019	-1.0000	-.1339	39.423	0.0000	129.61	0.0000

Note: The unconditional quantile specific residual is constructed as $(y_t - q_{y_t}(\theta))/q_{y_t}(\theta)$, where $q_{y_t}(\theta)$ is the empirical quantile. Following arguments of Tsay (2005) results of the autocorrelation tests are reported using the lag order $\tilde{p} \approx \ln(T)$, where T is the sample size.

Table 6: Estimated transmission dynamics.

	JPM		GS		WFC		AXP		USB	
$\hat{\mathrm{E}}\boldsymbol{\Gamma}_t$	0.9731	0.0000	0.9731	0.0000	0.9731	0.0000	0.9731	0.0000	0.9731	0.0000
	0.1118	0.9225	0.0075	0.9892	0.0154	0.9900	0.0332	0.9745	0.0321	0.9689

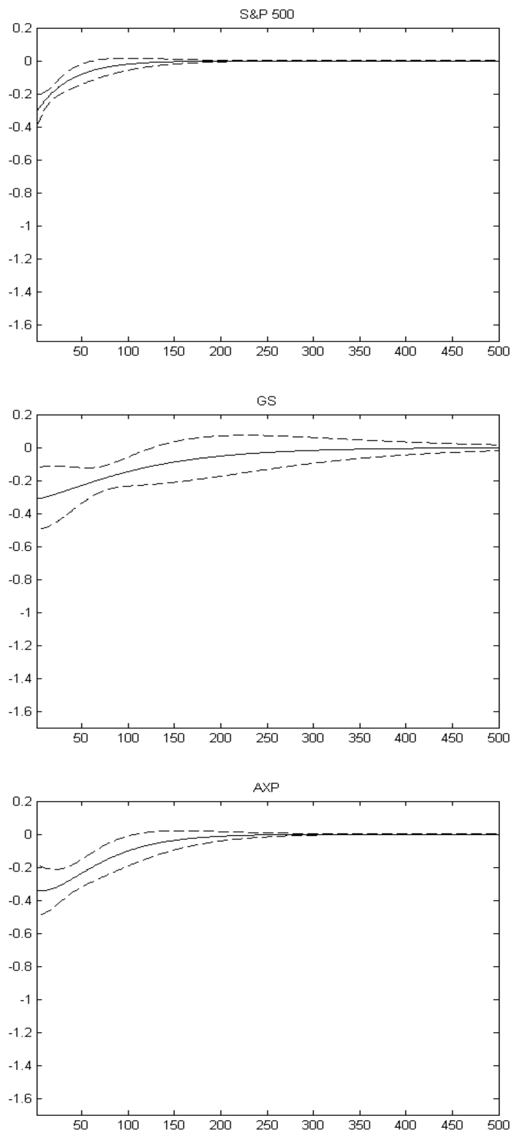

Figure 2: Quantile impulse response functions. All responses are triggered by the shock to market rate calculated as the value at which the market rate equals to 1% Estimated conditional quantile value.

References

[1] Acharya, V., Pedersen, L., Philippon, T., Richardson, M., 2010. Measuring systemic risk. Technical Report. Department of Finance, NYU.

[2] Adrian, T., Brunnermeier, M., 2009. CoVaR. Manuscript, Princeton University.

[3] Bollerslev T., 1986. Generalized autoregressive conditional heteroskedasticity. Journal of Econometrics 31, 307-327.

[4] Bollerslev T., 1990. Modeling the coherence in short-run nominal exchange rates: a multivariate generalized ARCH model. Review of Economics and Statistics 72, 498-505.

[5] Brownlees, C.T., Engle, R.F., 2010. Volatility, correlation and tails for systemic risk measurement. Manuscript, Stern School of Business, New York University.

[6] Engle R. F., 2002. Dynamic conditional correlation-a simple class of multivariate GARCH models. Journal of Business and Economic Statistics 20, 339-350.

[7] Engle, R.F., Manganelli, S., 2004. CAViaR: conditional autoregressive value at risk by regression quantiles. Journal of Business and Economic Statistics 22, 367-381.

[8] Engle R.F., Ito T., Lin W.-L., 1990. Meteor showers or heat waves? Heteroskedastic intra-daily volatility in the foreign exchange market. Econometrica 58, 525-542.

[9] Gallant, A.R., Rossi, P.E., Tauchen, G., 1993. Nonlinear dynamic structures. Econometrica 61, 871-908.

[10] Hamilton, J.D., 1994. Time Series Analysis. Princeton University Press, Princeton.

[11] Koenker, R., 2005. Quantile Regression. Cambridge University Press, Cambridge.

[12] Koenker, R., Bassett, G., 1978. Regression quantiles. Econometrica 46, 33-50.

[13] Lin, W.-L., 1997. Impulse Response Function for Conditional Volatility in GARCH Models. Journal of Business and Economic Statistics 15, 15-25.

[14] Machado, J.A.F., Silva, J.M.C.S., 2013. Quantile regression and heteroskedasticity, Discussion Paper, University of Essex, Department of Economics.

[15] Sims, C. A., 1980. Macroeconomics and Reality, Econometrica. 48, 1-48.

[16] Powell, J. L., 1986. Censored Regression Quantiles. Journal of Econometrics 32, 143-155.

[17] Taylor, S., 1986. Modelling Financial Time Series. Wiley, New York.

[18] Tsay, R. S., 2005. Analysis of Financial Time Series. 2nd Ed. Hoboken, NJ: John Wiley and Sons, Inc.

[19] White, H., 2001. Asymptotic Theory for Econometricians. Academic Press, San Diego.

[20] White, H., Kim, T.H., Manganelli, S., 2015. VAR for VaR: Measuring tail dependence using multivariate regression quantiles. Journal of Econometrics 187, 169-188.

[21] White, H., Kim, T.H., Manganelli, S., 2008. Modeling autoregressive conditional skewness and kurtosis with multi-quantile CAViaR. In: Russell, J., Watson, M. (Eds.), Volatility and Time Series Econometrics: A Festschrift in Honor of Robert F. Engle.