Term Structure of Interest Rates with Short-Run and Long-Run Risks*

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April 6, 2016

Abstract

Bond returns are time-varying and predictable. What economic forces drive this variation? To answer this long-standing question, we propose a consumption-based model with recursive preferences, long-run risks, and inflation non-neutrality. Our model offers two important insights. First, our model matches well the upward-sloping U.S. Treasury yield curve. Second, consistent with our model's implication, variance risk premium based on the U.S. interest rate derivatives data emerges as a strong predictor for short-horizon Treasury excess returns, above and beyond the predictive power of other popular factors. In the model equilibrium, the variance risk premium is related to the short-run risks in the economy, while standard forward-rate-based factors are associated with long-run risks in the economy.

JEL Classification: G12, G13, G14

Keywords: Long-run risk, economic uncertainty, term structure of interest rates, bond risk premium, variance risk premium, predictability, interest rate derivatives

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*We are grateful to seminar participants at the financial mathematics seminar at Penn State University, internal Monetary and Financial Market Analysis section seminar at the Federal Reserve Board, Luxembourg School of Finance, European Investment Bank, and Banque de France as well as conference participants at the 6th Risk Management Conference in Mont-Tremblant, Canada, 2014 Midwest Finance Association Meetings, 2014 Annual Missouri Economics Conference, 2014 Darla Moore School of Business Fixed Income Conference, 2014 CEPR Asset Pricing Workshop at Gerzensee, 2015 International Conference of the French Finance Association, and 2016 Finance Down Under Conference for useful discussions and comments. We thank Dobri Dobrev, Antonio Gargano, Alex Hsu, Don Kim, Canlin Li, Jeremy Stein, Min Wei, and Ram Yamarthi for helpful discussions and comments. The opinions expressed in this paper are those of the authors and do not necessarily reflect those of others in the Federal Reserve System.

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Abstract

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1 Introduction

The failure of the expectations hypothesis first documented by Fama and Bliss (1987) and Campbell and Shiller (1991) has attracted enormous attention in the asset pricing literature over the past decades. Various plausible risk factors that appear to capture bond return predictability (forward spread (Fama and Bliss, 1987), forward rates factor (Cochrane and Piazzesi, 2005), jump risk measure (Wright and Zhou, 2009), hidden term structure factor (Duffee, 2011), and macroeconomic variables' factor(s) (Ludvigson and Ng, 2009; Huang and Shi, 2012), among many others) have been proposed. Despite this impressive progress the fundamental challenge for uncovering a particular economic mechanism behind bond return variation still stands out. Its resolution is equally important for market participants as well as for monetary policy makers. Our paper focuses on this issue.

To that extent we propose a stylized general equilibrium model. This model can be viewed as an extension of long-run risk models in Bansal and Yaron (2004, BY) and Bollerslev, Tauchen, and Zhou (2009, BTZ). Bansal and Yaron (2004) emphasize importance of the long-run risk in consumption growth for explaining the equity premium, while Bollerslev, Tauchen, and Zhou (2009) show that richer volatility dynamics in consumption growth can be successful in capturing future stock return predictability. Our model includes both long-run risk and certain nontrivial volatility dynamics in consumption growth. It generates a two-factor volatility structure for the endogenously determined bond risk premium, in which the factors are explicitly related to the underlying volatility dynamics of consumption growth where different volatility concepts load differently on the fundamental risk factors and capture separately short-run and long-run risks of Treasury excess returns. In particular, the difference between the risk-neutral and objective expectations of variation in interest rates, the factor that we term *the interest-rate variance risk premium (IRVRP)*, effectively isolates the *short-run* risk factor associated with the volatility-of-volatility of consumption growth. The *long-run* risk factor associated with volatility of consumption growth appears to be captured by Fama and Bliss (1987) forward spread.

Our main empirical findings exploit the informational content of the interest-rate variance risk premium constructed from the U.S. interest rate swaps and swaptions markets. Why do we use this data to measure the short-run risk? First, the interest rate derivatives markets represent the largest segment of the U.S. fixed-income market and are important tools for corporate treasurers, asset managers, and public institutions to hedge interest rate risk.[1] According to the Bank of International Settlements, as of June 2012, the outstanding notional value of interest rate swaps and swaptions exceeded $379 and $50 trillion, respectively, on a net basis. This outstanding notional value combined together is much larger than the $52 trillion of all exchange-traded interest rate futures and options, such as Treasury futures and futures options traded on Chicago Mercantile Exchange. In addition, a 2009 survey by the International Swaps and Derivatives Association reports that 88.3% of the Fortune Global 500 companies use swaps and swaptions for hedging interest rate risk. Last but not least, Dai and Singleton (2000) pointed to the similarities between the U.S. Treasury yields and swap rates and swaption prices. Thus, it appears likely that the interest rate derivatives markets can be informative for explaining variation in Treasury yields.[2]

Our first empirical finding connects time variation in the relatively short-horizon (one- to three-month) bond risk premiums to the variation in the interest-rate variance risk premium. The latter is driven entirely by the volatility-of-volatility factor with a positive loading predicted by our theory. Consistent with this prediction, interest-rate variance risk premium always loads positively on Treasury excess returns in the data and high(low) values of the interest-rate variance risk premium are associated with subsequent high(low) Treasury excess returns. Interest-rate variance risk premium alone explains a nontrivial share (40 to 50 percent) of the variation in one- and three-month Treasury excess returns, with the most

[1] Interest rate swaps play a central role in the whole financial system as swap rates reflect term financing rates of major financial institutions. In fact, the floating leg of a plain vanilla swap is usually tied to the 3-month LIBOR, which serves as a benchmark rate for corporate treasurers, mortgage lenders, and credit card agencies.

[2] Dai and Singleton noted, in particular, that "though the institutional structures of dollar swap and U.S. Treasury markets are different, some of the basic distributional characteristics of the associated yields are similar".

explanatory power concentrated at the quarterly horizon. Moreover, combined with other standard predictors such as forward spread or forward rate risk factors, IRVRP generates even higher bond return predictability than on its own and contributes nontrivially (about 20 percent) to standard predictors.

Our second empirical result is that the interest-rate variance risk premium has limited forecasting power for longer-maturity excess bond returns, while the forward spread is a more important factor that captures information related to the variation of the long-run risk factor. Thus, our findings are consistent with our model where both long-run and short-run risk factors drive bond risk premium in equilibrium.

Finally, our calibration exercise suggests that the model fits the upward-sloping nominal yield curve remarkably well. The key two factors in fitting the nominal yield curve are the presence of the long-run risk in the model and inflation non-neutrality. As such, the long-run risk state variable from the real side of the model affects nominal prices via inflation channel. We reasonably calibrate inflation process while we leave the real side model parameters like the ones in BY and BTZ. The most important feature of the inflation process is the negative correlation with consumption volatility shock, consistent with recent empirical findings (Piazzesi and Shneider, 2007; Campbell, Sunderam, and Viceira, 2013; Bansal and Shaliastovich, 2013). Without this feature, the nominal yield curve is downward-sloping.

It stands to reason that aggregate macro series, and consumption growth, in particular, are more volatile when investors are relatively more uncertain about economy growth prospects compared to the periods with relatively low volatility environment. Thus, our two-factor volatility structure of consumption risks can be linked to the studies that model economic uncertainty. The idea of economic uncertainty as a potential risk factor has gained attention recently, both for explaining variation in stock returns (Bollerslev, Tauchen, and Zhou, 2009; Bloom, 2009; Drechsler, 2013) and in bond returns (Wright, 2011; Bansal and Shaliastovich, 2013; Giacoletti, Laursen, and Singleton, 2015). The last two papers are especially relevant to our study. Bansal and Shaliastovich (2013) link bond excess return variation to a variation in volatility in real activity and inflation – variables they interpret as uncertainty - but they do not model uncertainty process explicitly. Giacoletti, Laursen, and Singleton (2015) find that dispersion of beliefs about future interest rates that can be loosely linked to investors' uncertainty about interest rates is distinct from information about the macroeconomy and can be useful in explaining variation in bond returns.

While the bond pricing empirical literature (Fama and Bliss, 1987; Campbell and Shiller, 1991) has documented predictability of long-horizon bond returns, bond predictability in the short run did not receive much attention until recently. A growing literature argues for the existence of the short-run and long-run risk components of the aggregate volatility to study the variation of stock returns (Adrian and Rosenberg, 2008; Christoffersen, Jacobs, Ornthanalai, and Wang, 2008; Branger, Rodrigues, and Schlag, 2011; Zhou and Zhu, 2012, 2013). Zhou (2009), in particular, emphasized importance of the short-run risk factor for short-horizon (such as one-month) stock returns. A recent related paper by Ghysels, Le, Park, and Zhu (2014) emphasises a short-run volatility component of bond yields as a useful predictor for future excess returns, as opposed to a long-run volatility component that does not predict Treasury excess returns. In the frame of this literature and to the best of our knowledge, our paper is the first that explores short-horizon bond return predictability and explains empirical findings within a structural two-factor volatility model framework. It appears that while the volatility-of-volatility of consumption growth (short-run risk factor) drives the variation in the short-horizon Treasury excess returns, the variation in long-horizon returns appears to be related to a different kind, possibly more to a longer-run growth factor. The long-run risk factor is important for matching the term structure of nominal interest rates.

The rest of the paper is organized as follows. Section 2 presents our long-run risk model with two different volatility types and inflation non-neutrality, and derives asset pricing implications of the model, Section 3 discusses the calibration of the U.S. Treasury yield curve implied by our model, Section 4 provides overview of the interest rate swaptions market and introduces the interest-rate variance risk premium, Section 5 describes all relevant data to our empirical exercise, Section 6 discusses empirical results, and Section 7 concludes.

2 Model and Asset Pricing

2.1 Preferences

We consider a discrete-time endowment economy with recursive preferences for early resolution of uncertainty introduced by Kreps and Porteus (1978), Epstein and Zin (1989), and Weil (1989):

$$U_t = \left[(1-\delta)C_t^{\frac{1-\gamma}{\theta}} + \delta\left(\mathrm{E}_t U_{t+1}^{1-\gamma}\right)^{\frac{1}{\theta}}\right]^{\frac{\theta}{1-\gamma}}, \quad (1)$$

where δ is the time discount factor, $\gamma \geq 0$ is the risk aversion parameter, $\psi \geq 0$ is the intertemporal elasticity of substitution (IES), and $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$. Preference for early resolution of uncertainty implies $\gamma > \frac{1}{\psi}$, which, in general, implies $\theta < 1$. We will assume throughout the paper that $\gamma > 1$ and $\psi > 1$, which implies $\theta < 0$ and refer to preference for early resolution of uncertainty as consistent with $\theta < 0$.[3] A special case of recursive preferences - expected utility - corresponds to the case of $\gamma = \frac{1}{\psi}$ $(\theta = 1)$.

Epstein and Zin (1989) show that the log-linearized form of the associated real stochastic discount factor m_t is given by:

$$m_{t+1} = \theta \ln \delta - \frac{\theta}{\psi} g_{t+1} + (\theta - 1) r_{c,t+1}, \quad (2)$$

where $g_{t+1} = \log\left(\frac{C_{t+1}}{C_t}\right)$ is the log growth of the aggregate consumption, $r_{c,t+1}$ is the log

[3] Bansal, Kiku, and Yaron (2012, BKY) discuss the wide range of regression-based estimates of the IES in the literature and their sensitivity to the presence of measurement errors. They argue that a better approach is undertaken in Bansal, Kiku, and Yaron (2007) and Hansen, Heaton, Lee, and Roussanov (2007) who use a large set of instruments to estimate conditional Euler equations for the real bond and find that the EIS is larger than one. Beeler and Campbell (2012) disagree in a sense that aggregate consumption growth does not appear to respond to the real risk-free rate fluctuations in a manner consistent with EIS being greater than one. They report, however, that their instrumental variables estimation approach of the BKY model yields the median estimates above 1.3.

return on an aggregate wealth portfolio that delivers aggregate consumption as its dividend each time period. Note that the return on wealth is different from the observed return on the market portfolio because aggregate consumption is not equal to aggregate dividends. Consequently, the return on wealth is not observable in the data. The nominal discount factor $m^{\$}_{t+1}$ is equal to the real discount factor minus expected inflation π_{t+1}:

$$m^{\$}_{t+1} = m_{t+1} - \pi_{t+1}. \tag{3}$$

2.2 Economy dynamics

To solve for the equilibrium asset prices we specify consumption and inflation dynamics. Consumption dynamics features time-varying consumption growth rate g_t and expected consumption growth rate x_t, time-varying volatility of consumption growth $\sigma^2_{g,t}$ and time-varying volatility-of-volatility of consumption growth q_t:

$$\begin{aligned} x_{t+1} &= \rho_x x_t + \phi_e \sigma_{g,t} z_{x,t+1}, \\ g_{t+1} &= \mu_g + x_t + \sigma_{g,t} z_{g,t+1}, \\ \sigma^2_{g,t+1} &= a_\sigma + \rho_\sigma \sigma^2_{g,t} + \sqrt{q_t} z_{\sigma,t+1}, \\ q_{t+1} &= a_q + \rho_q q_t + \phi_q \sqrt{q_t} z_{q,t+1}, \end{aligned} \tag{4}$$

where the parameters satisfy $a_\sigma > 0$, $a_q > 0$, $|\rho_\sigma| < 1$, $|\rho_q| < 1$ and $\phi_q > 0$. The vector of shocks $(z_{x,t+1},\ z_{g,t+1},\ z_{\sigma,t+1},\ z_{q,t+1})$ follows *i.i.d.* normal distribution with zero mean and unit variance and shocks are assumed to be uncorrelated among themselves. The second pair of equations in (4) is new compared to Bansal and Yaron (2004) and Bansal and Shaliastovich (2013). Stochastic volatility $\sigma^2_{g,t+1}$ represents time-varying economic uncertainty in consumption growth with time-varying volatility-of-volatility (vol-of-vol) measured by q_t.[4]

[4]Recent studies provided empirical support in favor of time-varying consumption growth volatility, e.g., Bekaert and Liu (2004), Bansal and Yaron (2005), Lettau, Ludvigson, and Wachter (2008), Bekaert, Engstrom, and Xing (2009), among others.

Since $\sigma_{g,t}^2$ directly affects variation in x_t, the predictable component in consumption growth, we will refer to $\sigma_{g,t}^2$ as the state variable that captures *the long-run risk*. The volatility-of-volatility process q_t can be thought of as the volatility risk or, *the short-run risk*. As we will see later, this terminology will be supported by our empirical findings.

In order for the real economy model (4) to have realistic implications for nominal bond risk premiums, we conjecture a fairly rich inflation process motivated by previous literature. Indeed, Bansal and Shaliastovich (2013) allow for expected inflation shocks to be correlated (negatively) with expected consumption growth, and Zhou (2011) allows for a vol-of-vol shock to affect inflation. We incorporate both of these features into expected inflation dynamics π_{t+1}:

$$\pi_{t+1} = a_\pi + \rho_\pi \pi_t + \phi_\pi z_{\pi,t+1} + \phi_{\pi g} \sigma_{g,t} z_{g,t+1} + \phi_{\pi\sigma} \sqrt{q_t} z_{\sigma,t+1}, \tag{5}$$

where ρ_π is a persistence and $\frac{a_\pi}{1-\rho_\pi}$ is the long-run mean of the inflation process. There are three shocks that drive inflation process: (1) a constant volatility part ϕ_π with an autonomous shock $z_{\pi,t+1}$; (2) a stochastic volatility part $\phi_{\pi\sigma}\sigma_{g,t}$ that works through consumption growth channel $z_{g,t+1}$; and (3) another stochastic volatility part $\phi_{\pi\sigma}\sqrt{q_t}$ that works through the volatility channel $z_{\sigma,t+1}$. Exogenous inflation shock $z_{\pi,t+1}$ does not generate inflation risk premium even in the presence of the time-varying volatility of this shock.[5] In contrast, the second and the third shocks generate inflation risk premium because real side shocks (stochastic volatility of consumption growth and uncertainty) affect inflation. In addition, since $\phi_{\pi g}$ and $\phi_{\pi\sigma}$ control inflation exposures to the growth and uncertainty risks, this process implicitly violates inflation neutrality in the short run, but not in the long run.[6]

[5]The inability of the expected inflation process with only one (autonomous) shock even with stochastic volatility to generate inflation risk premium is examined in Zhou (2009).

[6]There is no violation of inflation neutrality in the long run because unconditional expectation of inflation process (5) is $\mathrm{E}\pi_t = \frac{a_\pi}{1-\rho_\pi}$.

2.3 Pricing kernel

In equilibrium, the log wealth-consumption ratio z_t is affine in expected consumption growth x_t, stochastic volatility of consumption growth σ_t^2, and the vol-of-vol factor q_t:

$$z_t = A_0 + A_x x_t + A_\sigma \sigma_{g,t}^2 + A_q q_t. \tag{6}$$

Campbell and Shiller (1988) show that the return on this asset can be approximated as follows:

$$r_{c,t+1} = \kappa_0 + \kappa_1 z_{t+1} - z_t + g_{t+1}, \tag{7}$$

where $\kappa_0 = \ln((1+\exp\bar{z})) - \kappa_1 \bar{z}$, $\kappa_1 = \frac{\exp(\bar{z}))}{1+\exp(\bar{z})}$, and $\bar{z}$ is the average wealth-consumption ratio:

$$\bar{z} = A_0(\bar{z}) + A_\sigma(\bar{z})\bar{\sigma}^2 + A_q(\bar{z})\bar{q}. \tag{8}$$

The equilibrium loadings for (6) are derived in Appendix A.1:

$$\begin{aligned} A_x &= \frac{1-\frac{1}{\psi}}{1-\kappa_1\rho_x}, \\ A_\sigma &= \frac{1}{2\theta(1-\kappa_1\rho_\sigma)}\left[\left(\theta-\frac{\theta}{\psi}\right)^2 + (\theta\kappa_1 A_x \phi_e)^2\right], \\ A_q &= \frac{1-\kappa_1\rho_q - \sqrt{(1-\kappa_1\rho_q)^2 - \theta^2\kappa_1^4\phi_q^2 A_\sigma^2}}{\theta(\kappa_1\phi_q)^2}. \end{aligned} \tag{9}$$

As in Bansal and Yaron (2004), recursive preferences along with the early resolution of uncertainty are crucial in determining the sign of the equilibrium loadings of the state variables in our model. When the intertemporal elasticity of substitution $\psi > 1$, the intertemporal substitution effect dominates the wealth effect. In response to higher expected consumption growth, agents invest more and, consequently, wealth-consumption ratio increases. Therefore, the wealth-consumption ratio loading on the expected consumption growth is positive ($A_x > 0$) whereas loadings on the volatility and volatility-of-volatility of consumption growth are both negative ($A_\sigma < 0$ and $A_q < 0$) as in times of high volatility and/or uncertainty agents sell off risky assets driving the wealth-consumption ratio down.[7] The persistence of expected growth shock ρ_x and time-varying volatility ρ_σ magnify the effect of the changes in these state variables on the valuation ratio since investors perceive such macroeconomic changes as long-lasting. Contrary to that, persistence of the volatility-of-volatility, ρ_q, roughly cancels out in the A_q loading. This provides further support for interpretation of q_t as a state variable that captures relatively short-run economic risks.[8]

Using the solution for the wealth-consumption ratio we show in Appendix A.3 that the conditional mean of the stochastic discount factor m_{t+1} is linear in the fundamental state variables and the innovation in m_{t+1} pins down the fundamental sources and compensations for risks in the economy:

$$m_{t+1} - \mathrm{E}_t[m_{t+1}] = -\lambda_g \sigma_{g,t} z_{g,t+1} - \lambda_x \sigma_{g,t} z_{x,t+1} - \lambda_\sigma \sqrt{q_t} z_{\sigma,t+1} - \lambda_q \sqrt{q_t} z_{q,t+1}, \tag{10}$$

where the quantities of risks are time-varying volatility and volatility-of-volatility of consumption growth, $\sigma_{g,t}$ and $\sqrt{q_t}$, respectively, and λ_g, λ_x, λ_σ, λ_q represent the market prices of risk of consumption growth, expected consumption growth, volatility, and volatility-of-volatility:

$$\begin{aligned} \lambda_g &= \gamma, & \lambda_\sigma &= (1-\theta)\kappa_1 A_\sigma, \\ \lambda_x &= (1-\theta)\kappa_1 A_x \phi_e, & \lambda_q &= (1-\theta)\kappa_1 A_q \phi_q. \end{aligned} \tag{11}$$

The market price of consumption risk λ_g is equal to the coefficient of relative risk aversion γ. Other risk prices crucially depend on our preference assumptions. When agents have preference for early resolution of uncertainty ($\theta < 0$), the market price of expected consumption risk is positive: $\lambda_x > 0$. In this case, positive shocks to consumption and expected consump-

[7]The solution for A_q represents one of a pair of roots of a quadratic equation, but we pick the one presented in Eq. (9) as the more meaningful one. We elaborate on this choice in Section A.1.

[8]Bansal, Kiku, and Yaron (2012) check that their approximate solutions are very accurate when compared against numerical solutions, used, e.g., in Binsbergen, Brandt, and Koijen (2012).

tion cause risk premium to decrease as agents buy risky assets and drive wealth-consumption ratio up. On the contrary, market prices of risk of volatility and volatility-of-volatility are negative ($\lambda_\sigma < 0$ and $\lambda_q < 0$): Consistent with the so-called leverage effect, in response to either type of volatility positive shock agents sell risky assets and drive wealth-consumption ratio down and volatility risk premiums up. It is worth noting that these effects are not based on the statistical linkages between return and volatility (as the endowment and volatility shocks are uncorrelated) but arise endogenously in the equilibrium. In the absence of the early resolution of uncertainty ($\gamma = \frac{1}{\psi}$ and $\theta = 1$) neither expected consumption, volatility, or volatility-of-volatility compensate investors for these risks.

2.4 Asset prices

We focus in this paper on the nominal yield curve and nominal bond return predictability. Hence in this section we provide the model solutions for the nominal quantities in our economy.[9]

Nominal risk-free rate. The nominal risk-free rate is the negative of the (log) price of the nominal one-period bond. Thus, it is equal to the real risk-free rate plus inflation compensation. The closed form for the nominal risk-free rate is derived in Appendix A.3:

$$\begin{aligned}
r^{\$}_{f,t} = & -\theta \ln \delta + \gamma \mu_g + a_\pi - (\theta - 1)[\kappa_0 + (\kappa_1 - 1)A_0 + \kappa_1(A_\sigma a_\sigma + A_q a_q)] - \frac{1}{2}\phi_\pi^2 \\
& + [\gamma - (\theta - 1)A_x(\kappa_1 \rho_x - 1)]\, x_t \\
& + \left[-(\theta - 1)A_\sigma(\kappa_1 \rho_\sigma - 1) - \frac{1}{2}\gamma^2 - \frac{1}{2}(\theta - 1)^2(\kappa_1 A_x \phi_e)^2 - \frac{1}{2}\phi_{\pi g}^2 - \gamma \phi_{\pi g}\right] \sigma_{g,t}^2 \qquad (12) \\
& + \left[-(\theta - 1)A_q(\kappa_1 \rho_q - 1) - \frac{1}{2}(\theta - 1)^2 \kappa_1^2 (A_\sigma^2 + A_q^2 \phi_q^2) - \frac{1}{2}\phi_{\pi\sigma}^2 + (\theta - 1)\kappa_1 A_\sigma \phi_{\pi\sigma}\right] q_t \\
& + \rho_\pi \pi_t .
\end{aligned}$$

Since inflation is not an autonomous process, it affects loadings on σ_t^2 and q_t in (12) via additional terms, related to $\phi_{\pi g}$ and $\phi_{\pi\sigma}$ coefficients, respectively, besides having a direct

[9] The corresponding real quantities are can be computed similarly and available upon request.

effect on the nominal rates, $\rho_\pi \pi_t$. This results in inflation non-neutrality, which means that inflation affects future real growth in the economy.

Nominal $n-$period bond price. A general recursion for solving for the $n-$period nominal bond price is as follows:

$$P_t^{\$,n} = \mathrm{E}_t \left[M_{t+1}^{\$} P_{t+1}^{\$,n-1} \right]. \tag{13}$$

We assume that the (log) price of the $n-$period nominal bond $p_t^{\$,n}$ follows an affine representation of the real state variables x_t, σ_t^2, q_t and inflation π_t:

$$p_t^{\$,n} = B_0^{\$,n} + B_1^{\$,n} x_t + B_2^{\$,n} \sigma_t^2 + B_3^{\$,n} q_t + B_4^{\$,n} \pi_t. \tag{14}$$

We solve for the nominal bond state loadings $B_i^{\$,n}$, $i = 0, \ldots, 4$ using initial conditions $B_i^{\$,0} = 0$, $i = 0, \ldots, 4$ (since $p_t^{\$,0} = 0$) and the above recursion, see Appendix A.4.

Nominal bond risk premium. Nominal bond risk premium $\mathrm{brp}_t^{\$,n}$ is given by the negative of covariance between the nominal pricing kernel $m_{t+1}^{\$,n-1}$ and the nominal bond price $p_t^{\$,n}$ (see Appendix A.5 for details):

$$\begin{aligned}
\mathrm{brp}_t^{\$,n} &= -\mathrm{Cov}_t \left[m_{t+1}^{\$}, p_{t+1}^{\$,n-1} \right] \\
&= \left[(\gamma + \phi_{\pi g}) B_4^{\$,n-1} \phi_{\pi g} - (\theta - 1)\kappa_1 A_x B_1^{\$,n-1} \phi_e^2 \right] \sigma_{g,t}^2 \\
&- \left[((\theta - 1)\kappa_1 A_\sigma - \phi_{\pi\sigma})(B_2^{\$,n-1} + B_4^{\$,n-1} \phi_{\pi\sigma}) + (\theta - 1)\kappa_1 A_q B_3^{\$,n-1} \phi_q^2 \right] q_t \\
&+ B_4^{\$,n-1} \phi_\pi^2 \\
&\equiv \beta_1^{\$,n-1} \sigma_{g,t}^2 + \beta_2^{\$,n-1} q_t + B_4^{\$,n-1} \phi_\pi^2.
\end{aligned} \tag{15}$$

Bond risk premium (15) is driven by two volatility factors: consumption volatility factor $\sigma_{g,t}^2$ and volatility-of-volatility factor q_t.[10] The effect of the long-run risk captured by A_x

[10]The third constant term provides a correction for inflation risk through ϕ_π, due to the autonomous inflation shock π_t.

equilibrium loading on the wealth-consumption ratio amplifies the overall contribution of the consumption risk, $\sigma_{g,t}$. This effect is absent in Zhou (2011) and Mueller, Vedolin, and Zhou (2011), and thus, makes it more difficult to explain the upward sloping term structure of the nominal yield curve. The two volatility factors $\sigma^2_{g,t}$ and q_t are inherently latent. While consumption volatility risk $\sigma^2_{g,t}$ represents the classic risk-return tradeoff and is the standard factor in consumption-based models, q_t factor did not receive a lot of attention with the exception of Bollerslev, Tauchen, and Zhou (2009) paper. The next section demonstrates how q_t factor can be empirically isolated from $\sigma^2_{g,t}$ factor.

Nominal bond variance risk premium. Bollerslev, Tauchen, and Zhou (2009) show that the equity variance risk premium —the difference in expectations of the equity variance under risk-neutral and physical measures —is driven entirely by vol-of-vol factor q_t and is a useful predictor of time variation in aggregate stock returns. Motivated by this result, we derive the nominal bond variance risk premium (NVRP) — the difference in expectations of the bond return variance under risk-neutral and actual measures. We show in this section that the NVRP also loads only on the q_t factor.

By definition, nominal bond variance risk premium is given by the covariance of the nominal bond return variance $\sigma^2_{r^\$,t+1}$ with the nominal stochastic discount factor $m^\$_{t+1}$:

$$\mathrm{E}^{\mathbb{Q}}_t\left[\sigma^2_{r^\$,t+1}\right] - \mathrm{E}_t\left[\sigma^2_{r^\$,t+1}\right] = \mathrm{Cov}_t\left[\sigma^2_{r^\$,t+1}, m^\$_{t+1}\right]. \tag{16}$$

In terms of the model parameters, the nominal bond variance risk premium is given by:

$$\begin{aligned}
&\mathrm{NVRP}_t\left[\sigma^2_{r^\$,t+1}\right] = \mathrm{E}^{\mathbb{Q}}_t\left[\sigma^2_{r^\$,t+1}\right] - \mathrm{E}_t\left[\sigma^2_{r^\$,t+1}\right] = \\
&(\theta-1)\kappa_1\left\{(A_\sigma - \phi\pi\sigma)\left[\left(B_1^{\$,n-2}\phi_e\right)^2 + \left(B_4^{\$,n-2}\phi_{\pi g}\right)^2\right] + \right. \\
&\left. A_q\phi_q^2\left[\left(B_2^{\$,n-2} + B_4^{\$,n-2}\phi_{\pi\sigma}\right)^2 + \left(B_3^{\$,n-2}\phi_q\right)^2\right]\right\} q_t.
\end{aligned} \tag{17}$$

Appendix A.6 provides derivation details for (17). A first and central observation here is that the time variation in the NVRP is solely due to the time variation in q_t state variable. If volatility-of-volatility is constant, $q_t = q$, the variance risk premium reduces to a constant $(\theta - 1)\kappa_1 A_\sigma(1 + \kappa_1^2 A_x^2 \phi_e^2)q$ contrary to empirical evidence that the variance risk premium is time-varying (e.g., BTZ). A second observation is that although consumption growth risk $\sigma_{g,t}^2$ does not affect variance risk premium directly it still has an indirect effect on it. If consumption volatility $\sigma_{g,t}^2$ is not stochastic then the wealth-consumption ratio equilibrium loadings $A_\sigma = 0$ and $A_q = 0$ by construction and variance risk premium is zero. A third observation is that if there is no recursive preference ($\theta = 1$) then variance risk premium is zero by construction. Lastly, positivity of the variance risk premium is guaranteed by negative θ along with negative A_σ and A_q.

3 Calibration

In this section we discuss calibration of the nominal yield curve implied by our model (4) and inflation process (5). We consider two benchmark cases, Bansal and Yaron (2004, BY) and Bollerslev, Tauchen, and Zhou (2009, BTZ). Compared to BY, BTZ incorporate the time-varying vol-of-vol factor, but in the absence of the long-run risk channel. We differ from BTZ in two aspects: (1) we have the long-run risk state variable in the real side model; and (2) we model inflation process in order to derive implications for the nominal bond prices. We present all three models' parameters (BY, BTZ, and ours) in Table 1.

Panel A provides calibration values for the real economy dynamics. We set preference parameters $\delta = 0.997$, $\gamma = 8$, and $\psi = 1.5$.[11] Consumption growth parameters $\mu_g = 0.0015$, $\rho_x = 0.979$, $\phi_e = 0.001$ are consistent with BY (and BTZ except for $\rho_x = \phi_e \equiv 0$). Volatility persistence $\rho_\sigma = 0.978$ is the same as in BY and BTZ, and $a_\sigma = (1 - \rho_\sigma)\mathrm{E}\sigma$ is set so that the unconditional expectation $\mathrm{E}\sigma_t^2 = 0.0234^2$, which is slightly higher than in BY and BTZ because we find that this value matches better the nominal yield curve in the model. We set the expected volatility-of-volatility level $\mathrm{E}q = a_q(1 - \rho_q)^{-1} = 10^{-9}$ so that $a_q = 2^{-10}$ given

[11]BY and BTZ use $\gamma = 10$, but in our model slightly lower value of γ works reasonably well.

$\rho_q = 0.8$. In addition, $\phi_q = 10^{-4}$. Our choice of ρ_σ and ρ_q is broadly consistent with the estimates of Bollerslev, Xu, and Zhou (2013), who find that the long-run risk (proxied by $\sigma^2_{g,t}$) is more persistent than the short-run risk (proxied by q_t). Thus, the calibrated model is connected with the next empirical section where we show that these two types of risks in the bond premia are disentangled.

Panel B provides calibration parameters for the expected inflation process. We set the average annualized annualized inflation rate $\mathrm{E}\pi = 2\%$ and persistence parameter $\rho_\pi = 0.95$ in accordance with the current Fed's inflation target and Great Moderation period overall.[12] Implied a_π on a monthly basis is equal to $\mathrm{E}\pi(1-\rho_\pi) = 0.02/12 \times 0.05 = 8 \times 10^{-5}$. The total unconditional variance of the inflation process (5) is given by:

$$\mathrm{Var}(\pi) = \frac{1}{1-\rho_\pi^2}\left(\phi_\pi^2 + \phi_{\pi g}^2 \mathrm{E}\sigma^2 + \phi_{\pi\sigma}^2 \mathrm{E}q\right). \tag{18}$$

We calibrate variance-related parameters of (18) so that the total annualized unconditional inflation volatility is 2%. Since $\rho_\pi = 0.95$, $\mathrm{E}\sigma^2 = 0.0234^2$, $\mathrm{E}q = 10^{-9}$, the term in parentheses in (18) on a monthly basis is: $\phi_\pi^2 + \phi_{\pi g}^2 \times 0.0234^2 + \phi_{\pi\sigma}^2 \times 10^{-9} = 0.02^2/12 \times (1 - 0.95^2) = 3.25 \times 10^{-6}$. Further, we assume that the first (autonomous) shock contributes one half to the total variance while the other two shocks contribute equally to the remaining half of the total variance of the inflation process.[13] Thus, the contribution of the first shock to the total inflation variance is $0.5 \times 3.25 \times 10^{-6} = 1.625 \times 10^{-6}$, implying $\phi_\pi = 0.0013$. The contribution of the second and third shocks are equal to each other and to $0.25 \times 3.25 \times 10^{-6} = 8.125 \times 10^{-7}$. Therefore, the implied $\phi_{\pi g} = (8.125 \times 10^{-7}/0.0234^2)^{1/2} = -0.0385$. The negative sign of $\phi_{\pi g}$ is motivated by previous empirical findings (Piazzesi and Shneider, 2007; Campbell, Sunderam, and Viceira, 2013; Bansal and Shaliastovich, 2013). In particular, Bansal and

[12] These numbers may be justified by the data after 1980s and especially after 2008, when Fed launched unprecedented measures of accommodative monetary policy, namely, quantitative easing. Our expected inflation rate is lower than the one in Bansal and Shaliastovich (2013), who set it at 3.61% (see their Table 5).

[13] Equal distribution of variance among the shocks results in slight overshooting of the model-implied interest rates levels relative to those in the sample.

Shaliastovich (2013) use SPF survey data for one-year ahead consensus inflation forecast over 1969 - 2010 sample and a latent factor for the expected consumption growth to estimate relationship between the two. They find that expected inflation negatively affects future consumption growth thus suggesting non-neutrality of inflation. Last, the implied $\phi_{\pi\sigma} = (8.125 \times 10^{-7}/10^{-9})^{1/2} = 28.5$.

Figure 1 reports our calibration results. Both panels show the average nominal yield curve out to 10 years (blue solid line) in January 1991 - December 2010 sample period and the calibrated nominal yield curve (red dashed line) implied by our model (Panel A) and by our modified model in the absence of the long-run risk channel x_t (Panel B). It is obvious from Panel A that our model matches very well the levels of the nominal yields and captures the slope the yield curve too. The 1-, 5-, and 10-year model-implied yields are 3.71%, 5.14% and 5.58% relative to observed yields of 4%, 4.95%, and 5.65% at corresponding maturities. Figure on Panel B shows that without long-run risk the model is not successful in fitting the nominal upward-sloping yield curve as it generates downward-sloping yield curve, even with the presence of time-varying economic uncertainty.[14] To understand the effect of the long-run risk factor better, it is useful to write down the nominal yields as an affine combination of state variables:

$$y_t^{\$,n} = -\frac{1}{n}\left[B_0^{\$,n} + B_1^{\$,n} x_t + B_2^{\$,n}\sigma_t^2 + B_3^{\$,n} q_t + B_4^{\$,n}\pi_t\right], \tag{19}$$

where $B_1^{\$,n}$, $B_2^{\$,n}$, $B_3^{\$,n}$, $B_4^{\$,n}$ are model-implied nominal bond price loadings provided in Appendix A.4. The equilibrium nominal yield loadings are plotted in Figure 2.[15] In our model, nominal yields hedge expected consumption and inflation risks. As the top left panel of Figure 2 shows, nominal yields increase when expected consumption is high because $-B_1^{\$,n} > 0$, and the effect is stronger for higher n. Intuitively, a negative shock to expected consump-

[14] Bansal and Shaliastovich (2013) fit the term structure of interest rates for the short- and intermediate-term yields (up to five years only), whereas our model quantitatively matches the level and slope of the nominal term structure from one- to ten-year interest rates.

[15] Nominal yield loadings are nominal bond price loadings with a negative sign.

tion drives bond prices up and yields down and a positive shock to expected consumption drives bond prices down and yields up. The same effect is obvious for expected inflation as $-B_4^{\$,n} > 0$ and this loading is also increasing with maturity (bottom right panel). The top right panel shows the effect of consumption volatility shock on nominal yields. Corresponding loading $-B_3^{\$,n}$ manifests negative correlation of expected inflation and consumption growth that we discussed above. Initial effect of both positive consumption volatility and volatility-of-volatility shocks on yields is negative although this effect mean-reverts in the long-run. Given that the steady state values of consumption volatility and volatility-of-volatility processes are relatively small, long-run risk has the largest effect on nominal yields and helps to fit the upward-sloping nominal yields curve dramatically. Therefore, the slope of the term structure of interest rates appears to be tightly linked to the slow-moving predictable component in consumption growth.

4 Empirical Measurements

The theoretical model outlined in Section 2 suggests that the interest-rate variance risk premium, the difference between the market's risk-neutral expectation of future interest rate return variation and the objective expectation of the future interest rate variation, can serve as a useful predictor for the Treasury bond premiums. To measure the interest-rate variance risk premium and investigate this conjecture, we rely on the new nonparametric "model-free" variation concept derived from the interest rate swaptions market (for the risk-neutral expectation of the future interest rate return variation) and from the interest rate swap market (for the expectation of the future interest rate return variation under the physical measure). To that extent, we first provide an overview of the mechanics of the interest rate swaptions (Section 4.1), then discuss the construction of swaption-implied variance (Section 4.2), and, finally, outline the construction of the realized interest rate swap variance (Section 4.3).

4.1 Interest rate swaptions

Consider a forward start fixed versus floating interest rate swap with a start date T_m and maturity date T_n. The fixed annuity payments are made on a pre-specified set of dates, $T_{m+1} < T_{m+2} < \cdots < T_n$, with the intervals equally spaced by δ, which equals six months in the U.S. swaption market. The floating payments tied to the three-month LIBOR are made quarterly at $T_{m+1} - \delta/2$, T_{m+1}, $T_{m+2} - \delta/2$, T_{m+2}, $\cdots$, $T_n - \delta/2$, and T_n.[16]

At time T_m, the value of the floating leg equals par, and the time–t value of the floating leg is $D(t, T_m)$, where $D(t, T_m)$ is the time–t price of a zero-coupon bond maturing at time T_m. The time–t value of the fixed leg is equal to $D(t, T_n) + A_{m,n}(t)$, where $A_{m,n}(t) \equiv \sum_{j=m+1}^{n} D(t, T_j)$ is the present value of an annuity associated with the fixed leg of the forward swap contract, also known as the "price value of the basis point" (PVBP) of a swap. The time-t forward swap rate, $S_{m,n}(t)$, is the rate on the fixed leg that makes the present value of the swap contract equal to zero at t:

$$S_{m,n}(t) = \frac{D(t, T_m) - D(t, T_n)}{A_{m,n}(t)}. \tag{20}$$

This forward swap rate becomes the spot swap rate $S_{m,n}(T_m)$ at time T_m.

A swaption gives its holder the right but not the obligation to enter into an interest rate swap either as a fixed leg (payer swaption) or as a floating leg (receiver swaption) with a pre-specified fixed coupon rate. The underlying security of a swaption is a forward start interest rate swap contract. For example, let T_m be the expiration date of the swaption, K be the coupon rate on the swap, and T_n be the final maturity date of the swap. The payer swaption allows the holder to enter into a swap at time T_m with a remaining term of $T_n - T_m$ and to pay the fixed annuity of K. At time t, this swaption is usually called a $(T_m - t)$ into $(T_n - T_m)$ payer swaption, also known as a $(T_m - t)$ by $(T_n - T_m)$ payer swaption, where $(T_m - t)$ is the option maturity and $(T_n - T_m)$ is the tenor of the underlying swap. Because

[16] We assume that both the fixed and floating legs pay \$1 principal at T_n.

the value of the floating leg will be par at time T_m, the payer swaption is equivalent to a put option on a bond with a coupon rate K and a remaining maturity of $T_n - T_m$, where the strike of this put option is \$1. Similarly, the receiver swaption is equivalent to a call option on the same coupon bond with the strike price of \$1.

Let $\mathcal{P}_{m,n}(t;K)$ and $\mathcal{R}_{m,n}(t;K)$ denote the time-t value of a European payer and receiver swaption, respectively, expiring at T_m with strike K on a forward start swap for the time period between T_m and T_n. At the option expiration date T_m, the payer swaption has a payoff of

$$[1 - D(T_m, T_n) - K A_{m,n}(T_m)]^+ = A_{m,n}(T_m)\,[S_{m,n}(T_m) - K]^+\,,$$

where Eq. (20) evaluated at T_m is used. Therefore, the time-t ($< T_m$) price of this payer swaption is given by

$$\begin{aligned}\mathcal{P}_{m,n}(t;K) &= \mathbb{E}_t^{\mathbb{Q}}\left\{e^{-\int_t^{T_m} r(s)ds} A_{m,n}(T_m)\,[S_{m,n}(T_m) - K]^+\right\} \\ &= A_{m,n}(t)\,\mathbb{E}_t^{\mathbb{A}^{m,n}}\left\{[S_{m,n}(T_m) - K]^+\right\}, \qquad (21)\end{aligned}$$

where $\mathbb{Q}$ is the risk-neutral measure and $\mathbb{A}^{m,n}$ is the annuity measure with $A_{m,n}(t)$ as the numeraire. That is, the Radon-Nikodym derivative of the annuity measure with respect to the risk-neutral measure is $\frac{d\mathbb{A}^{m,n}}{d\mathbb{Q}} = e^{-\int_t^{T_m} r(s)ds}\frac{A_{m,n}(T_m)}{A_{m,n}(t)}$. Similarly, the time-$t$ price of the receiver swaption is given by

$$\mathcal{R}_{m,n}(t;K) = A_{m,n}(t)\,\mathbb{E}_t^{\mathbb{A}^{m,n}}\left\{[K - S_{m,n}(T_m)]^+\right\}. \qquad (22)$$

We note from (21) and (22) that a swaption is tied to two sources of uncertainty: (i) the forward swap rate $S_{m,n}(t)$, and (ii) the swap's PVBP realized at time T_m, $A_{m,n}(T_m)$. The change of measure from $\mathbb{Q}$ to $\mathbb{A}^{m,n}$ allows us to focus on the risk of $S_{m,n}(t)$ and facilitates the pricing of swaptions.

4.2 Measure of swaption-implied variance

Variance swaps on equities allow one to hedge the risk of the realized variance of stock returns. The *variance contract on swap rates* that we develop below allows us to hedge the risk of the realized variance of interest rate swap rates. At time t, the short leg promises to pay the long leg at T_m:

$$A_{m,n}(T_m)\left[\left(\ln\frac{S_{m,n}(t+\Delta)}{S_{m,n}(t)}\right)^2+\left(\ln\frac{S_{m,n}(t+2\Delta)}{S_{m,n}(t+\Delta)}\right)^2+\cdots+\left(\ln\frac{S_{m,n}(T_m)}{S_{m,n}(T_m-\Delta)}\right)^2\right], \quad (23)$$

the product of the realized variance of the log forward swap rate $\log S_{m,n}(t)$ over $[t, T_m]$ and the PVBP $A_{m,n}(T_m)$. In return, the long leg pays the short leg $A_{m,n}(T_m) \times \mathbb{VP}_{m,n}(t)$ at T_m, where $\mathbb{VP}_{m,n}(t)$ is determined at time t such that the value of the contract equals zero at initiation. We refer to $\mathbb{VP}_{m,n}(t)$ as the variance price of the forward swap rate.

The *variance contract on swap rates* uses the sum of squared log changes to measure the realized variance of forward swap rates over $[t, T_m]$. Similar to the payoff of a swaption, the payoff of the *variance contract on swap rates* depends on the realized variance of forward swap rates as well as an annuity discount factor. This design makes it convenient to obtain the variance price $\mathbb{VP}_{m,n}(t)$ by a change of the risk-neutral measure to the corresponding annuity measure. It also makes it easier to replicate the variance contract using swaptions given the similar payoff structures. The variance price $\mathbb{VP}_{m,n}(t)$ is the $\mathbb{A}^{m,n}$-expectation of the quadratic variation of the forward swap rate $S_{m,n}(t)$ over $[t, T_m]$.

Similar to the equity variance swap whose payoff can be replicated with a portfolio of out-of-the-money equity options, the time-varying payoff of the *variance contract on swap rates* can be replicated with a portfolio of out-of-the-money swaptions written on $S_{m,n}(t)$. In particular, generalizing the algorithm used by CBOE in constructing VIX, we have

$$\mathbb{IV}_{m,n}(t) \equiv \frac{2}{A_{m,n}(t)}\left\{\int_{K>S_{m,n}(t)}\frac{1}{K^2}\mathcal{P}_{m,n}(t;K)dK+\int_{K<S_{m,n}(t)}\frac{1}{K^2}\mathcal{R}_{m,n}(t;K)dK\right\}, \quad (24)$$

where $T_m - t$ is the time-to-maturity for the variance swap. As observed from (24), this replication portfolio contains positions in out-of-the-money swaptions with a weight that is inversely proportional to the squares of their strikes. A similar replication portfolio based on equity options has been employed in the literature to construct model-free implied volatility measures (Bollerslev, Tauchen, and Zhou, 2009; Carr and Wu, 2009).

4.3 Interest-rate realized variance

In order to quantify the actual return variation of interest rates, we follow the methodology of Bollerslev, Tauchen, and Zhou (2009). Let p_t^n be the logarithmic price of the n-period zero-coupon bond at time t. The realized variation over the discrete time interval $[t-1, t]$ can then be measured in a "model-free" way as follows:

$$RV_t = \sum_{i=1}^{M} \left[p^n(t-1+\frac{i}{M}) - p^n(t-1+\frac{i-1}{M}) \right]^2 \rightarrow \text{Return variation}(t-1, t), \qquad (25)$$

where the convergence to the return variation relies on $M \rightarrow \infty$,that is, on an increasing number of within-the-period price observations. Prior literature (Andersen, Bollerslev, Diebold, and Ebens, 2001; Andersen, Bollerslev, Diebold, and Labys, 2001; Barndorff-Nielsen and Shephard, 2004; Wright and Zhou, 2009; Bollerslev, Tauchen, and Zhou, 2009) demonstrates that this "model-free" realized variance measure based on high-frequency intraday data results in more accurate ex post observations of the true (unobserved) return variation compared to more traditional sample variances based on daily or coarser frequency returns. In addition, RV_t measure provides a nonparametric empirical analog to $\sigma_{r,t}^2$ that appears in the definition of the variance risk premium (16).

4.4 Interest-rate variance risk premium (IRVRP)

The interest-rate variance risk premium is defined as the difference between the market's risk-neutral expectation of the future swap rate variation over [t, T_m] under the annuity measure $\mathbb{A}^{m,n}$ and the corresponding expectation under the physical measure over the same time interval

$$\text{IRVRP}_{m,n}(t) \equiv \text{IV}_{m,n}(t) - \text{RV}_{m,n}(t). \tag{26}$$

5 Data and Estimates

In this section we discuss the data, construction, and the estimates of the IRVRP, then describe Treasury yield data and other control variables used in predictive regressions in Section 6.

5.1 Estimates of the interest-rate variance risk premium

We use interest rate swaptions and swap market data for computing implied and realized variances of interest rates, respectively. We compute then the IRVRP as a difference between the implied variance and (the expectation of) the realized variance.

5.1.1 Construction of the implied variance of interest rates

To construct the implied variance (24) we need two ingredients: First, we need to construct $A_{m,n}(t)$ and $S_{m,n}(t)$, the PVBP of a swap and the forward swap rate curves, respectively; Second, we need to obtain $\mathcal{P}_{m,n}(t;K)$ and $\mathcal{R}_{m,n}(t;K)$, payer and receiver swaption prices, respectively. We discuss the construction of these objects below.

To construct $A_{m,n}(t)$ and $S_{m,n}(t)$ curves, we first obtain daily LIBOR rates with maturities of 3, 6, 9, and 12 months, as well as daily 2-, 3-, 4-, 5-, 7-, 10-, 15-, 20-, 25-, 30-, and 35-year spot swap rates between February 11, 2002 and January 31, 2013 from J.P. Morgan. February 11, 2002 is the first day on which we have the intraday swap quotes from Barclays Capital that we need for constructing the proxy for the realized variance of interest rates. We bootstrap the swap rates to first obtain daily zero-coupon curves. Then we construct the PVBP curve $A_{m,n}(t)$ and the forward swap rate curve $S_{m,n}(t)$ up to 35 years according to (20).[17]

We focus on 1-, 3-, and 12-month expiry swaptions for the 10-year tenor in line with the frequencies of our predictive regressions. The market convention is to quote swaption prices in terms of their log-normal implied volatility based on Black (1976) formula.[18]

We combine daily observations of (European) swaption prices from J.P. Morgan and Barclays Capital, two of the largest inter-dealer brokers in interest rate derivatives markets. The swaption prices from J.P. Morgan are available starting from June 1, 1993 with five strikes, namely, at-the-money-forward (ATMF), ATMF $\pm$ 100, and ATMF $\pm$ 50 basis points. The swaption prices from Barclays are available from December 1, 2004 with thirteen strikes, namely, ATMF, ATMF $\pm$ 200, ATMF $\pm$ 150, ATMF $\pm$ 100, ATMF $\pm$ 75, ATMF $\pm$ 50, and ATMF $\pm$ 25 basis points. In our empirical analysis, we use swaption prices from J.P. Morgan from February 11, 2002 through December 1, 2004 and those from Barclays after December 1, 2004.[19]

To obtain swaption payer $\mathcal{P}_{m,n}(t;K)$ and receiver $\mathcal{R}_{m,n}(t;K)$ prices on a continuum of strikes as requested by Eq. (24), we follow Carr and Wu (2009) and Du and Kapadia (2012) and interpolate implied volatilities across the range of observed strikes and use implied volatility of the lowest (highest) available strike to replace those of the strikes below (above). We further generate 200 implied volatility points that are equally spaced over a strike range with moneyness between $0.9 \times S_{m,n}(t)$ and $1.1 \times S_{m,n}(t)$, where $S_{m,n}(t)$ is the current forward swap rate on each day. This implied volatility/strike grid together with PVBP and forward swap curves allows us to compute the empirical counterpart for the implied variance (24).

[17]We first use a standard cubic spline algorithm to interpolate the swap rates at semiannual intervals from one year to 35 years. We then solve for the zero curve by bootstrapping the interpolated par curve with swap rates as par bond yields. The day count convention is 30/360 for the fixed leg, and Actual/360 for the floating leg.

[18]Many market participants think in terms of normal (or absolute or basis point) implied volatilities–the volatility parameter that, plugged into the normal pricing formula, matches a given price–as they are more uniform across the swaption grid and more stable over time than log-normal implied volatilities.

[19]All our empirical results remain unchanged if we use only the J.P. Morgan swaption data rather than a combination of the J.P. Morgan and Barclays Capital swaption data. Recall that we start our sample from February 11, 2002, the first day when we have intraday swap rates data available from Barclays Capital, the data that we use for constructing realized variance of interest rates.

5.1.2 Construction of the realized variance of interest rates

Next, we construct the "model-free" realized variance of interest rates. To do this, we use intraday 10-year interest rate swap quotes from February 11, 2002 and January 31, 2013 obtained from Barclays Capital. The data cover the period from 8:20 to 15:00 New York time each day for a maximum of 80 five-minute observations per day.[20] Ideally, the sampling frequency for the computation of the realized variance should go to infinity. However, in practice high-frequency data is affected by a number of microstructure issues such as price discreteness, bid-ask spreads, and nonsynchronous trading effects. A number of studies (Andersen, Bollerslev, Diebold, and Labys, 2000; Hansen and Lunde, 2006; Bollerslev, Tauchen, and Zhou, 2009), suggest that a five-minute sampling frequency provides a reasonable choice. Hence, we compute five-minute continuously compounded returns on a hypothetical 10-year zero-coupon bond as a difference between the 10-year swap rates at the five-minute intervals. We then square these differences to obtain the "model-free" proxy for the daily realized variance RV_t given by Eq. (25).

IRVRP given by Eqs. (17) and (26) is forward-looking and requests the expectation of the realized variance of interest rates. In contrast to the "model-free" measures of implied and realized variances constructed above, we need an explicit forecasting model for the RV_t. While numerous forecasting models have been proposed in the literature (see, e.g., Andersen, Bollerslev, Christoffersen, and Diebold (2006)), here we follow Bollerslev, Tauchen, and Zhou (2009) and rely on the *heterogeneous autoregressive volatility model of realized volatility* (HAR-RV), suggested by Andersen, Bollerslev, and Diebold (2007) and Corsi (2009). The model is simple enough to implement yet it appears to produce highly accurate volatility forecast as the above authors have shown. It aims to parsimoniously capture the long memory behavior of volatility by incorporating daily, weekly, and monthly realized variances into the

[20]In this set-up we follow Wright and Zhou (2009) who used this time period to estimate bond variance and jump risk premia in the U.S. Treasury futures market.

one-month ahead variance forecast:

$$RV_{t+22,mon} = \alpha + \beta_D RV_t + \beta_W RV_{t,week} + \beta_M, RV_{t,mon} + \epsilon_{t+22,mon}, \tag{27}$$

where $RV_{t,mon} = \frac{1}{22}\sum_{j=0}^{21} RV_{t-j}$ and $RV_{t,week} = \frac{1}{5}\sum_{j=0}^{4} RV_{t-j}$ are the monthly and weekly realized variances, respectively.

5.1.3 Empirical measure of the interest-rate variance risk premium

The forward-looking IRVRP is computed then as a difference between the expectations of interest rate variances under the $\mathbb{Q}$ and $\mathbb{P}$ measures as defined in Eq. (26). Figures 3, 4, and 5 plot implied variance, expected variance, and resulting interest-rate variance risk premium. First result from these figures is that the variance risk premium is almost everywhere positive suggesting that market participants seek compensation for variance exposure. Second, market variance risk premium increased dramatically during the NBER recession, represented by the shaded blue bar on each chart.[21] Such an increase general captures the spirit of increased uncertainty amid recessions: this is a reason that we loosely refer to the variance risk premium a as a compensation for uncertainty.[22] In addition, swaption-based IRVRP increased notably amid European financial crisis in the second half of 2011.

5.2 Treasury yield data

In our empirical exercise we use Fama-Bliss data set of monthly zero-coupon Treasury yield data from CRSP to compute excess returns for two to five-year Treasury bonds from February 2002 to January 2013. We denote the $h-$period log return on a $\tau-$year zero-coupon note with the log price $p_t^{(\tau)}$ as $r_{t+h}^{(\tau)} = p_{t+h}^{(\tau-h)} - p_t^{(\tau)}$, and the $h-$period log return in excess of the $h-$period risk-free rate $y_t^{(h)}$ as $rx_{t+h}^{(\tau)} = r_{t+h}^{(\tau)} - y_t^{(h)}$. In our application we consider $h = 1$, 3,

[21]There was only one NBER recession in our sample period.

[22]Bloom (2009) provides a similar argument about the relationship between uncertainty and volatility.

and 12 months.[23] The summary statistics of the Treasury excess returns is presented in Panel A of Table 2. A notable difference between one-year and one-month returns is that the latter are much less persistent than the former.

5.3 Other predictive variables

In addition to the variance risk premium, we include two other variables in our predictive regressions that are shown to capture variation in long-horizon bond returns. First, we use the classical Fama and Bliss (1987) predictor, the forward spread factor, defined as the spread between the forward rate of a particular maturity and the risk-free rate. Second, we use Cochrane and Piazzesi (2005) factor that is an affine combination of forward rates. Both variables are computed using Fama-Bliss data set, downloaded from CRSP and confined to our sample.

Panel B of Table 2 summarizes statistics for the predictive variables. The forward spread is extremely persistent, with AR(1) coefficients between 0.92 and 0.97. IRVRP has persistence coefficient of 0.87, while Cochrane-Piazzesi factor is the least persitent factor, with AR(1) coefficient of 0.72. According to Panel A, 1-year Treasury excess returns have persistence similar to that of forward spread factors, while 1-month excess returns of Treasury bonds are less persistent and closer in persistence to IRVRP.

6 Empirical Results

In this section we discuss how well the interest-rate variance risk premium – endogenous proxy for economic uncertainty in our model – predicts Treasury excess returns. To assess

[23]Note that maturities $\tau - 1$ month and $\tau - 3$ months, with $\tau = 2, \ldots, 5$ years are not in the Fama-Bliss data set. Therefore, the prices of these securities are obtained via a linear interpolation of adjacent maturity securities' prices.

its predictability content, we run the following regressions:

$$rx_{t+h}^{(\tau)} = \beta_0^{(\tau)} + \beta_1^{(\tau)}(h) IRVRP_t^{(m,n)} + \beta_2^{(\tau)}(h) FS_t^{(\tau)} + CP_t + \epsilon_{t+h}^{(\tau)}, \tag{28}$$

where $rx_{t+h}^{(\tau)}$ is the $h-$period excess return on a $\tau-$year Treasury synthetic zero-coupon note, $IRVRP_t^{(m,n)}$ is the interest-rate variance risk premium that corresponds to the n-period swaption expiry on the m-period forward swap rate, $FS_t^{(\tau)}$ is $(\tau)-$maturity Fama-Bliss forward spread, and CP_t is Cochrane-Piazzesi forward-rate factor. Excess returns are computed using Fama-Bliss discount bond data set. For each bond maturity ($\tau = 2, 3, 4, 5$ years), and each return horizon ($h = 1, 3, 12$ months) we run univariate regressions as well as joint regressions on a subset of factors and also run a kitchen-sink-type regression using all three factors. $IRVRP_t^{(m,n)}$ is constructed in Section 5.1 using U.S. interest rate derivatives data. In our empirical exercise we use forward swap tenor $m = 10$ years, and swaption expiries $n = 1, 3, 12$ months. The relatively short life period of the interest rate derivatives markets and intraday swap data availability limit the sample period of our regressions.

6.1 Predictability with the interest-rate variance risk premium

We report empirical results for three different bond holding period return (HPR) horizons and for interest-rate variance risk premiums based on three different swaption expiries. Thus, we have nine tables in total. Tables 3, 4, and 5 report regression results 1-, 3-, and 12-month HPR for the variance risk premium based on the 1-month expriry swaptions on the 10-year forward swap rate. Likewise, tables 6, 7, and 8 report regression results for 1-, 3-, and 12-month HPR for the variance risk premium based on the 3-month expriry swaptions on the 10-year forward swap rate. Finally, tables 9, 10, and 11 report regression results for the same HPR horizons for the variance risk premium based on the 12-month expiry swaptions on the 10-year forward swap rate. In all these tables we present univariate results for the variance risk premium, forward spread, and Cochrane-Piazzesi factors, as well as joint regressions for two and three Treasury excess return predictors. Four panels in each table correspond to 2-, 3-, 4-, and 5-year maturity of Treasury synthetic zero-coupon notes.

6.1.1 Results with 1-month expiry swaption IRVRP

In this section we discuss results of Tables 3, 4, and 5. As Table 3 shows, excess returns for all maturities load positively and significantly on the 1-month expiry swaption-based variance risk premium, with solid Newey-West corrected t-statistics above or around 4. The adjusted R^2 of these univariate regressions (column 2) varies from 43 to 25 percent and declines with bond maturity. Nevertheless, even for a 5-year maturity, swaption-based variance risk premium alone explains 25 percent of excess return variation. In addition, IRVRP remains significant in the presence of Fama-Bliss factor (column 5) and Cochrane-Piazzessi factor (column 6), and highly significant in the multi-variate regression with both factors (column 7), where all three factors are included. IRVRP in the predictive regressions appears to add a nontrivial forecasting power: for example, when it is added to CP factor, the regression's adjusted R^2 increases from 14 percent to 45 percent for 2-year returns; when it is added to Fama-Bliss factor, the adjusted R^2 increases from 37 percent to 71 percent. Such an impressive marginal increase in R^2 of about 30 percentage points due to added IRVRP declines with maturity. For 5-year excess returns, marginal increase due to IRVRP is 6 and 17 percentage points when added to the forward spread and Cochrane-Piazzesi factors, respectively (Panel D). Nevertheless, such a marginal increase in explanatory power appears to be economically significant for all maturities. As a comparison, Bollerslev, Tauchen, and Zhou (2009) find that the equity variance risk premium adds six percentage points in explanatory power in addition to the log of price-earning ratio that by itself explains seven percentage points of variation in future quarterly stock returns (see Table 4 in their published RFS version). As such, this table confirms our intuition that the interest-rate-derivatives-based variance risk premium appears to contain useful information for predicting Treasury excess returns beyond that contained in the standard predictors. Moreover, these results show that, in a univariate setting, variance risk premium has higher explanatory power for short-horizon Treasury excess returns at all maturities compared to Cochrane-Piazzesi factor and higher explanatory power than Fama-Bliss factor at a 2-year maturity.

Regression results in Table 4 (3-month holding period Treasury excess returns) are broadly similar to Table 3. IRVRP is statistically significant at the 1% level in all four panels in the univariate regressions and not subsumed by Fama-Bliss and Cochrane-Piazzesi factors for Treasury 2, 3, and 4-year Treasury notes. However, IRVRP loses its significance in the presence of the Fama-Bliss factor for the 5-year (longest-maturity considered) Treasury note (Panel D) but it withstands its significance and is not subsumed by the Cochrane-Piazzesi factor. Additional insight from Panel D contends that the forward spread factor better captures short-horizon returns variation of the long-horizon bonds compared to Cochrane-Piazzesi factor, at least in our sample. The overall explanatory power of the IRVRP is smaller compared to that in Table 3, but it still adds some reasonable amount to either forward spread or Cochrane-Piazzesi factor in most of the reported regressions in this table.

Regression results in Table 5 (one-year Treasury excess returns) are notably weaker than in either Table 3 or Table 4, with the IRVRP insignificant for all maturities and with the adjusted R^2 of around 7 percent in Panel A (2-year Treasury notes) and almost negligible in Panel D (5-year Treasury notes). This result further supports the conjecture that the IRVRP's content appears to be fairly short-lived but important for capturing variations in Treasury nominal securities over the short horizons.[24]

[24] In our sample forward spread and Cochrane-Piazzesi factors have considerably lower R^2's than in the original papers. This peculiarity is due to the sample period of our study that is confined by the availability of the interest-rate derivatives data. This sample period is affected to a large extent by the zero-lower bound period and QE-related events. Outside of the ZLB period and QE events, forward spread still retains its predictive power for one-year Treasury excess returns.

6.1.2 Results with 3-month expiry swaption IRVRP

Tables 6, 7, and 8 present results with IRVRP constructed from the 3-month expiry of the interest rate swaptions. Table 6 highlights the strongest result in our paper. The IRVRP in univariate regressions is highly significant in explaining monthly returns of Treasury securities with Newey-West corrected t-stats around 6. Alone, IRVRP appears to explain from one half (2-year Treasury notes, Panel A) to roughly one third (5-year Treasury notes, Panel D) of excess return variation. IRVRP also explains notably more of return variation than the forward spread factor (50 versus 37 percent, Panel A) and considerably more than Cochrane-Piazzesi factor (14 percent, Panel A). Additional marginal explanatory power of 3-month IRVRP ranges across maturities from 40 to 8 percentage points (when added to the forward spread factor) and from 40 to 22 percentage points (when added to CP factor). Also, the statistical significance of the IRVRP is not subsumed by either of the two predictors considered at any maturity and remains significant at the 1 percent level of statistical significance.

Table 7 is similar to Table 6. IRVRP is statistically significant at the 1 percent level for all maturities in univariate regressions and the adjusted R^2 ranges from 38 to 10 percentage points based on the note maturity. It further adds nontrivial marginal explanatory power to other factors. Yet, Panel D shows that the IRVRP partly loses its significance to 10 percent level or slightly lower when added to the forward spread factor (Panel D, column 5) and in the joint regression (Panel D, column 7).

Results of table 8 in general mirror those of Table 5. Overall, tables 6, 7, and 8 further strengthen our conjecture that the IRVRP is economically important for short-horizon returns generally for all considered Treasury note maturities.

6.1.3 Results with 12-month expiry swaption IRVRP

Results based on the 12-month expiry swaptions (Tables 9, 10, and 11) generally repeat previous results. 12-month swaption expiry IRVRP is still highly significant at the 1 percent level and is not subsumed by other predictors when applied to monthly and quarterly (short-run) Treasury excess returns. It loses its explanatory power for annual (long-run) Treasury excess returns. This last set of reported results allows us to further argue that regardless of the swaption expiry, IRVRP is the robust factor in explaining short-run Treasury excess returns as opposed to long-run Treasury excess returns, where other economic forces appear to be at work.

To summarize, empirical results discussed in Sections 6.1.1 - 6.1.3 convey, that based on the adjusted R^2 from regressions, the 3-month swaption-expiry-based IRVRP appears to be the strongest candidate among the three IRVRPs (that correspond to 1-, 3-, and 12-month swaption expiries) that captures useful information from interest rate derivatives market for explaining and predicting the Treasury excess returns.

6.2 Some additional checks

In addition to the interest-rate variance risk premium, we have used two alternative specifications of the variance risk premium in regressions (28).

First, we replaced the IRVRP with the equity VRP since the latter one has been shown to be a useful predictor of future stock returns (Bollerslev, Tauchen, and Zhou, 2009). We found that, irrespective of the holding period horizon, equity variance risk premium is only marginally relevant for predicting Treasury excess returns. Mueller, Vedolin, and Zhou (2011) also found that equity variance risk premium (joint with CP factor) explains rather a small fraction of variation in Treasury bond returns. The difference between explanatory powers of equity and interest-rate variance risk premiums in predicting bond returns might in part be explained by the differences in traders' clientele in the equity and derivatives markets as well as the persistence profile of the variance risk premiums in both markets: the interest-rate variance risk premium appears to be much more persistent (see Panel B of Table 2) than the equity variance risk premium.[25]

[25]In our sample, AR(1) coefficient for the equity VRP is 0.28.

Second, we run regressions with the IRVRP defined as implied variance minus the realized variance, with both concepts directly observable at time t. The results are not materially different although just a touch weaker (adjusted R^2 is lower by two or three percentage points compared to the IRVRP that uses the $\mathbb{P}$-expectation of the realized variance).[26]

7 Conclusion

We study bond pricing implications in the context of the long-run risk asset-pricing model with two types of volatilities and inflation non-neutrality. The model is promising in explaining first and second moments of the Treasury market returns.

The contribution of our paper to the fixed-income asset pricing literature is two-fold. First, our reasonably calibrated version of the model with long-run risks matches well the upward-sloping yield curve out to ten years, and the long-run risk appears to be the main driving force behind this result. Second, the interest-rate variance risk premium constructed from interest rate derivatives markets drives short-horizon (one- and three-month) Treasury excess returns, while other popular predictive variables, such as Fama-Bliss forward spread or Cochrane-Piazzessi forward-rate factor drive variation in longer-maturity Treasury excess returns. In our model time-varying bond risk premium is driven by two volatility factors (volatility of consumption and volatility-of-volatility of consumption) whereas variance risk premium loads entirely on the vol-of-vol factor. Since variance risk premium explains a significant part in variation in short-horizon Treasury excess returns, we interpret vol-of-vol factor as a short-run risk factor. Since the forward-rate-related factors appear to explain time-variation in long-horizon Treasury excess returns, we interpret these factors as related to the long-run risk factor, the second factor that drives the variation in bond risk premium in our model. Thus, our model and empirical findings provide useful insights on different volatility risks. These insights should be useful for market participants and monetary policy makers alike.

[26] Results discussed in this section are not reported but readily available upon request.

A Appendix

A.1 Solution for the consumption-wealth ratio coefficients

Euler equation imposes equilibrium restrictions on the asset prices:

$$\mathrm{E}[\exp(m_{t+1} + r_{t+1})] = 1. \tag{29}$$

Since this Euler equation should hold for any asset, it should also hold for the wealth-consumption ratio z_t. Thus, the return on this asset $r_{c,t+1}$ should satisfy (29). Using it and the wealth return equation (7), obtain:

$$\mathrm{E}_t\left[\exp(m_{t+1} + r_{c,t+1})\right] = \mathrm{E}_t\left[\exp\left(\theta \ln\delta - \frac{\theta}{\psi} g_{t+1} + \theta r_{c,t+1}\right)\right] = 1, \tag{30}$$

or, in log-linearized dynamics:

$$\mathrm{E}_t\left[\theta \ln\delta - \frac{\theta}{\psi} g_{t+1} + \theta r_{c,t+1}\right] + \frac{1}{2}\mathrm{Var}_t\left[\theta \ln\delta - \frac{\theta}{\psi} g_{t+1} + \theta r_{c,t+1}\right] = 0. \tag{31}$$

Substituting out $r_{c,t+1}$ in terms of z_t dynamics (6) and consumption growth g_{t+1}, we can solve for the equilibrium wealth-consumption ratio loadings A_0, A_x, A_{σ^2}, A_q:

$$\begin{aligned}
&\mathrm{E}_t[\theta \ln\delta - \frac{\theta}{\psi}(\mu_g + x_t + \sigma_{g,t} z_{g,t+1}) + \theta(\kappa_0 + \kappa_1(A_0 + A_x x_{t+1} + A_\sigma \sigma^2_{g,t+1} + A_q q_{t+1}) - \\
&A_0 - A_x x_t - A_\sigma \sigma^2_{g,t} - A_q q_t + \mu_g + x_t + \sigma_{g,t} z_{g,t+1})] + \\
&\frac{1}{2}\mathrm{Var}_t[\theta \ln\delta - \frac{\theta}{\psi}(\mu_g + x_t + \sigma_{g,t} z_{g,t+1}) + \theta(\kappa_0 + \kappa_1(A_0 + A_x x_{t+1} + A_\sigma \sigma^2_{g,t+1} + A_q q_{t+1}) - \\
&A_0 - A_x x_t - A_\sigma \sigma^2_{g,t} - A_q q_t + \mu_g + x_t + \sigma_{g,t} z_{g,t+1})] = 0.
\end{aligned} \tag{32}$$

To solve for A_0, set constant terms under the expectation in (32) equal to zero:

$$\begin{aligned} &\theta \ln \delta + \theta(\kappa_0 + \kappa_1(A_0 + A_\sigma a_\sigma + A_q a_q)) - A_0 + \left(\theta - \frac{\theta}{\psi}\right)\mu_g = 0 \quad \Rightarrow \\ &A_0 = \frac{1}{1-\kappa_1}\left[\ln \delta + \kappa_0 + \kappa_1(A_\sigma a_\sigma + A_q a_q) + \left(1 - \frac{1}{\psi}\right)\mu_g\right]. \end{aligned} \tag{33}$$

To solve for A_x, match the terms in front of x_t:

$$-\frac{\theta}{\psi} + \theta(\kappa_1 A_x \rho_x - A_x + 1) = 0 \quad \Rightarrow \quad A_x = \frac{1 - \frac{1}{\psi}}{1 - \kappa_1 \rho_x}. \tag{34}$$

To solve for A_σ, match the terms in front of $\sigma^2_{g,t}$:

$$\begin{aligned} &(\theta\kappa_1 A_\sigma \rho_\sigma - \theta A_\sigma)\sigma^2_{g,t} + \frac{1}{2}\mathrm{Var}_t\left[-\frac{\theta}{\psi}\sigma^2_{g,t} z_{g,t+1} + \theta\kappa_1 A_x \phi_e \sigma_{g,t} z_{x,t+1} + \theta\sigma_{g,t} z_{g,t+1}\right] = \\ &\theta A_\sigma(\kappa_1\rho_\sigma - 1)\sigma^2_{g,t} + \frac{1}{2}\mathrm{Var}_t\left[\left(\theta - \frac{\theta}{\psi}\right)\sigma_{g,t} z_{g,t+1} + \theta\kappa_1 A_x \phi_e \sigma_{g,t} z_{x,t+1}\right] = 0 \quad \Rightarrow \\ &\theta A_\sigma(\kappa_1\rho_\sigma - 1) + \frac{1}{2}\left[\left(\theta - \frac{\theta}{\psi}\right)^2 + (\theta\kappa_1 A_x \phi_e)^2\right] = 0 \quad \Rightarrow \\ &A_\sigma = \frac{1}{2\theta(1-\kappa_1\rho_\sigma)}\left[\left(\theta - \frac{\theta}{\psi}\right)^2 + (\theta\kappa_1 A_x \phi_e)^2\right]. \end{aligned} \tag{35}$$

To solve for A_q, match the terms in front of q_t and set equal to zero:

$$\begin{aligned} &(\theta\kappa_1 A_q \rho_q - \theta A_q)q_t + \frac{1}{2}\mathrm{Var}_t[\theta\kappa_1 A_\sigma \sqrt{q_t} z_{\sigma_{t+1}} + \theta\kappa_1 A_q(\rho_q q_t + \phi_q \sqrt{q_t} z_{q_{t+1}}) - \theta A_q q_t] = \\ &\theta A_q(\kappa_1\rho_q - 1)q_t + \frac{1}{2}\mathrm{Var}(\theta\kappa_1 A_\sigma \sqrt{q_t} z_{\sigma_{t+1}} + \theta\kappa_1 A_q \phi_q \sqrt{q_t} z_{q_{t+1}}) = 0 \quad \Longrightarrow \\ &\frac{1}{2}(\theta\kappa_1\phi_q)^2 A_q^2 + \theta(\kappa_1\rho_q - 1)A_q + \frac{1}{2}(\theta\kappa_1 A_\sigma)^2 = 0 \quad \text{or, equivalently,} \\ &(\theta\kappa_1\phi_q)^2 A_q^2 + 2\theta(\kappa_1\rho_q - 1)A_q + (\theta\kappa_1 A_\sigma)^2 = 0. \end{aligned} \tag{36}$$

The solution for A_q represents the solution to a quadratic equation and is given by:

$$A_q^{\pm} = \frac{1 - \kappa_1\rho_q \pm \sqrt{(1-\kappa_1\rho_q)^2 - (\theta\kappa_1^2\phi_q A_\sigma)^2}}{\theta(\kappa_1\phi_q)^2}. \tag{37}$$

As Tauchen (2011) notes, a "positive" root A_q^+ has an unfortunate property $\lim_{\phi_q \to 0} \phi_q^2 A_q^+ \neq 0$, which is, essentially, a violation of the transversality condition in this setting: though uncertainty q_t vanishes with $\phi_q \to 0$, the effect of it on prices is not. Therefore, we choose A_q^- root as a viable solution for A_q:

$$A_q = \frac{1 - \kappa_1 \rho_q - \sqrt{(1 - \kappa_1 \rho_q)^2 - \theta^2 \kappa_1^4 \phi_q^2 A_\sigma^2}}{\theta (\kappa_1 \phi_q)^2}. \tag{38}$$

To insure that the determinant in (38) is positive, we also impose a constraint on ϕ_q —the magnitude of the shock $z_{q,t+1}$:

$$\phi_q^2 \leq \frac{(1 - \kappa_1 \rho_q)^2}{\theta^2 \kappa_1^4 A_\sigma^2}. \tag{39}$$

A.2 Solution for the real pricing kernel and the real risk-free rate

Using the solutions for A's obtained in A.1, we solve for the expected value $\mathrm{E}_t[m_{t+1}]$ and the variance $\mathrm{Var}_t[m_{t+1}]$ of the real pricing kernel m_{t+1}:

$$\begin{aligned}
&\mathrm{E}_t[m_{t+1}] = \theta \ln \delta - \frac{\theta}{\psi} \mathrm{E}_t[g_{t+1}] + (\theta - 1) \mathrm{E}_t[r_{c,t+1}] = \\
&= \theta \ln \delta - \frac{\theta}{\psi} (\mu_g + x_t) + (\theta - 1) \mathrm{E}_t (\kappa_0 + \kappa_1 z_{t+1} + g_t - z_t) \\
&= \theta \ln \delta - \frac{\theta}{\psi} (\mu_g + x_t) + (\theta - 1) [\kappa_0 + \kappa_1 (A_0 + A_x \rho_x x_t + A_\sigma (a_\sigma + \rho_\sigma \sigma_{g,t}^2) + A_q (a_q + \rho_q q_t)) \\
&+ \mu_g + x_t - A_0 - A_x x_t - A_\sigma \sigma_{g,t}^2 - A_q q_t] \\
&= \theta \ln \delta + \underbrace{\left((\theta - 1) - \frac{\theta}{\psi} \right)}_{-\gamma} \mu_g + (\theta - 1)[\kappa_0 + (\kappa_1 - 1) A_0 + \kappa_1 (A_\sigma a_\sigma + A_q a_q)] \\
&- \frac{\theta}{\psi} x_t + (\theta - 1)[(A_x (\kappa_1 \rho_x - 1) + 1) x_t + A_\sigma (\kappa_1 \rho_\sigma - 1) \sigma_{g,t}^2 + A_q (\kappa_1 \rho_q - 1) q_t] \\
&= \theta \ln \delta - \gamma (\mu_g + x_t) + (\theta - 1)[\kappa_0 + (\kappa_1 - 1) A_0 + \kappa_1 (A_\sigma a_\sigma + A_q a_q)] \\
&+ (\theta - 1)[A_x (\kappa_1 \rho_x - 1) x_t + A_\sigma (\kappa_1 \rho_\sigma - 1) \sigma_{g,t}^2 + A_q (\kappa_1 \rho_q - 1) q_t].
\end{aligned} \tag{40}$$

and

$$\begin{aligned}
\text{Var}_t[m_{t+1}] &= \text{Var}_t\left[\theta \ln \delta - \frac{\theta}{\psi} g_{t+1} + (\theta-1) r_{c,t+1}\right] \\
&= \text{Var}_t\left[-\frac{\theta}{\psi} g_{t+1} + (\theta-1)[\kappa_1(A_0 + A_x x_{t+1} + A_\sigma \sigma^2_{g,t+1} + A_q q_{t+1}) + g_{t+1}]\right] \\
&= \text{Var}_t\left[\left((\theta-1) - \frac{\theta}{\psi}\right)\sigma_{g,t} z_{g,t+1} + (\theta-1)\kappa_1(A_x \phi_e \sigma_{g,t} z_{x,t+1} + A_\sigma \sqrt{q_t} z_{\sigma,t+1} + A_q \phi_q \sqrt{q_t} z_{q,t+1})\right] \\
&= \gamma^2 \sigma^2_{g,t} + (\theta-1)^2 \kappa_1^2 \left[A_x^2 \phi_e^2 \sigma^2_{g,t} + (A_\sigma^2 + A_q^2 \phi_q^2) q_t\right].
\end{aligned} \tag{41}$$

The real risk-free rate is the negative of the (log) real pricing kernel with the Jensen's correction. Using the solutions (40) and (41) for the real pricing kernel, the model-implied real risk-free rate is given by:

$$\begin{aligned}
r_{f,t} = -p_t^1 &= -\text{E}_t[m_{t+1}] - \frac{1}{2}\text{Var}_t[m_{t+1}] \\
&= -\theta \ln \delta + \gamma \mu_g - (\theta-1)[\kappa_0 + (\kappa_1 - 1)A_0 + \kappa_1(A_\sigma a_\sigma + A_q a_q)] \\
&+ [\gamma - (\theta-1) A_x(\kappa_1 \rho_x - 1)]\, x_t \\
&+ \left[-(\theta-1)A_\sigma(\kappa_1 \rho_\sigma - 1) - \frac{1}{2}(\theta-1)^2 \kappa_1^2 A_x^2 \phi_e^2 - \frac{1}{2}\gamma^2\right] \sigma^2_{g,t} \\
&+ \left[-(\theta-1)A_q(\kappa_1 \rho_q - 1) - \frac{1}{2}(\theta-1)^2 \kappa_1^2 (A_\sigma^2 + A_q^2 \phi_q^2)\right] q_t.
\end{aligned} \tag{42}$$

Note that the time variation of the risk-free rate (42) crucially depends on the assumption of the preference for early resolution of uncertainty ($\theta < 0$). Without it ($\theta = 1$) the time variation in the risk-free rate depends only on the variation of the predictable consumption growth component x_t (long-run risk) and equals to: $r_{f,t} = -\ln \delta + \gamma(\mu_g + x_t) - \frac{1}{2}\gamma^2 \sigma^2_{g,t}$. Moreover, in the absence of the long-run risk, it is nearly constant (ignoring time-varying Jensen's inequality correction): $r_{f,t} = -\ln \delta + \gamma \mu_g - \frac{1}{2}\gamma^2 \sigma^2_{g,t}$. In the steady state the real risk-free rate can be written as:

$$r_f = -[c_0\ c_1\ c_2\ c_3] \times [1\ \text{E}_x\ \text{E}_{\sigma^2}\ \text{E}_q]'. \tag{43}$$

where steady-state loadings c_i, $i = 0, \dots, 3$ are given by:

$$\begin{aligned} c_0 &= \theta \ln \delta - \gamma \mu_g + (\theta - 1)[\kappa_0 + (\kappa_1 - 1)A_0 + \kappa_1(A_\sigma a_\sigma + A_q a_q)], \\ c_1 &= -\gamma + (\theta - 1)A_x(\kappa_1 \rho_x - 1), \\ c_2 &= \frac{1}{2}\gamma^2 + \frac{1}{2}(\theta - 1)^2 \kappa_1^2 A_x^2 \phi_e^2 + (\theta - 1)A_\sigma(\kappa_1 \rho_\sigma - 1), \\ c_3 &= \frac{1}{2}(\theta - 1)^2 \kappa_1^2 (A_\sigma^2 + A_q^2 \phi_q^2) + (\theta - 1)A_q(\kappa_1 \rho_q - 1). \end{aligned} \tag{44}$$

A.3 Solution for the nominal risk-free rate

Similarly to the real risk-free rate (42), the nominal risk-free rate is the negative of the (log) nominal pricing kernel with the Jensen's correction:

$$\begin{aligned} r_{f,t}^{\$} &= -\mathrm{E}_t\left[m_{t+1}^{\$}\right] - \frac{1}{2}\mathrm{Var}_t\left[m_{t+1}^{\$}\right] \\ &= -\mathrm{E}_t\left[m_{t+1} - \pi_{t+1}\right] - \frac{1}{2}\mathrm{Var}_t\left[m_{t+1}\right] - \frac{1}{2}\mathrm{Var}_t\left[\pi_{t+1}\right] + \mathrm{Cov}_t[m_{t+1}, \pi_{t+1}] \\ &= r_{f,t} + \mathrm{E}_t[\pi_{t+1}] - \frac{1}{2}\mathrm{Var}_t[\pi_{t+1}] + \mathrm{Cov}_t[m_{t+1}, \pi_{t+1}] \\ &= r_{f,t} + a_\pi + \rho_\pi \pi_t - \frac{1}{2}[\phi_\pi^2 + \phi_{\pi g}^2 \sigma_{g,t}^2 + \phi_{\pi\sigma}^2 q_t] + \mathrm{Cov}_t[m_{t+1}, \pi_{t+1}]. \end{aligned} \tag{45}$$

We need to compute the last term in (45) to complete the expression for the nominal risk-free rate in closed form:

$$\mathrm{Cov}_t[m_{t+1}, \pi_{t+1}] = \mathrm{E}_t[(m_{t+1} - \mathrm{E}_t m_{t+1}) \times (\pi_{t+1} - \mathrm{E}_t \pi_{t+1})]. \tag{46}$$

The unexpected components of the pricing kernel m_{t+1} and inflation π_{t+1} are given by:

$$\begin{aligned} m_{t+1} - \mathrm{E}_t[m_{t+1}] &= -\gamma \sigma_{g,t} z_{g,t+1} + (\theta - 1)\kappa_1(A_x \phi_e z_{x,t+1} + A_\sigma \sqrt{q_t} z_{\sigma,t+1} + A_q \phi_q \sqrt{q_t} z_{q,t+1}), \\ \pi_{t+1} - \mathrm{E}_t[\pi_{t+1}] &= \phi_\pi z_{\pi,t+1} + \phi_{\pi g} \sigma_{g,t} z_{g,t+1} + \phi_{\pi\sigma} \sqrt{q_t} z_{\sigma,t+1}, \end{aligned} \tag{47}$$

which implies for (46):

$$\mathrm{E}_t[(m_{t+1} - \mathrm{E}_t m_{t+1}) \times (\pi_{t+1} - \mathrm{E}_t \pi_{t+1})] = -\gamma \phi_{\pi g} \sigma_{g,t}^2 + (\theta - 1)\kappa_1 A_\sigma \phi_{\pi\sigma} q_t. \tag{48}$$

Combining together (42), (45), and (48), we obtain the closed-form expression for the nominal risk-free rate in terms of the model parameters and state variables:

$$\begin{aligned} r_{f,t}^{\$} = & -\theta \ln \delta + \gamma \mu_g + a_\pi - (\theta - 1)[\kappa_0 + (\kappa_1 - 1)A_0 + \kappa_1(A_\sigma a_\sigma + A_q a_q)] - \frac{1}{2}\phi_\pi^2 \\ & + [\gamma - (\theta - 1)A_x(\kappa_1 \rho_x - 1)]\, x_t \\ & + \left[-(\theta - 1)A_\sigma(\kappa_1 \rho_\sigma - 1) - \frac{1}{2}\gamma^2 - \frac{1}{2}(\theta - 1)^2(\kappa_1 A_x \phi_e)^2 - \frac{1}{2}\phi_{\pi g}^2 - \gamma \phi_{\pi g}\right] \sigma_{g,t}^2 \\ & + \left[-(\theta - 1)A_q(\kappa_1 \rho_q - 1) - \frac{1}{2}(\theta - 1)^2 \kappa_1^2 (A_\sigma^2 + A_q^2 \phi_q^2) - \frac{1}{2}\phi_{\pi\sigma}^2 + (\theta - 1)\kappa_1 A_\sigma \phi_{\pi\sigma}\right] q_t \\ & + \rho_\pi \pi_t. \end{aligned} \tag{49}$$

The nominal steady-state risk-free rate can be expressed similarly to that of the real risk-free rate:

$$r_f^{\$} = -[c_0^{\$}\ c_1^{\$}\ c_2^{\$}\ c_3^{\$}\ c_4^{\$}] \times [1\ \mathrm{E}_x\ \mathrm{E}_{\sigma^2}\ \mathrm{E}_\pi]', \tag{50}$$

where the nominal risk-free rate loadings $c_i^{\$}$, $i = 0, \ldots, 4$ are related to the real risk-free rate loadings c_i, $i = 0, \ldots, 3$ as:

$$\begin{aligned} c_0^{\$} &= c_0 - a_\pi + \frac{1}{2}\phi_\pi^2, \\ c_1^{\$} &= c_1, \\ c_2^{\$} &= c_2 + \frac{1}{2}\phi_{\pi g}^2 + \gamma \phi_{\pi g}, \\ c_3^{\$} &= c_3 + \frac{1}{2}\phi_{\pi\sigma}^2 - (\theta - 1)\kappa_1 A_\sigma \phi_{\pi\sigma}, \\ c_4^{\$} &= \rho_\pi. \end{aligned} \tag{51}$$

A.4 Solution for the nominal $n-$period bond price

The nominal $n-$period bond (log) price $p_t^{\$,n}$ is given by:

$$p_t^{\$,n} = \mathrm{E}_t[m_{t+1}^{\$}] + \frac{1}{2}\mathrm{Var}_t[m_{t+1}^{\$}] + \mathrm{E}_t[p_{t+1}^{\$,n-1}] + \frac{1}{2}\mathrm{Var}_t[p_{t+1}^{\$,n-1}] + \mathrm{Cov}_t[m_{t+1}^{\$}, p_{t+1}^{\$,n-1}]. \tag{52}$$

The first and the second terms in (52) are known from the nominal risk-free rate calculations (45). The last three terms can be computed using an affine pricing conjecture:

$$p_t^{\$,n} = B_0^{\$,n} + B_1^{\$,n} x_t + B_2^{\$,n} \sigma_t^2 + B_3^{\$,n} q_t + B_4^{\$,n} \pi_t. \tag{53}$$

Then the expected nominal bond price is:

$$\begin{aligned} \mathrm{E}_t\left[p_{t+1}^{\$,n-1}\right] &= B_0^{\$,n-1} + B_1^{\$,n-1}\rho_x x_t + B_2^{\$,n-1}(a_\sigma + \rho_\sigma \sigma_{g,t}^2) \\ &+ B_3^{\$,n-1}(a_q + \rho_q q_t) + B_4^{\$,n-1}(a_\pi + \rho_\pi \pi_t) \\ &= \left[B_0^{\$,n-1} + B_2^{\$,n-1} a_\sigma + B_3^{\$,n-1} a_q + B_4^{\$,n-1} a_\pi\right] \\ &+ B_1^{\$,n-1}\rho_x x_t + B_2^{\$,n-1}\rho_\sigma \sigma_{g,t}^2 + B_3^{\$,n-1}\rho_q q_t + B_4^{\$,n-1}\rho_\pi \pi_t. \end{aligned} \tag{54}$$

The shock to the nominal bond price is given by:

$$\begin{aligned} p_{t+1}^{\$,n-1} - \mathrm{E}_t\left[p_{t+1}^{\$,n-1}\right] &= B_1^{\$,n-1}\phi_e \sigma_{g,t} z_{x,t+1} + B_2^{\$,n-1}\sqrt{q_t} z_{\sigma,t+1} + B_3^{\$,n-1}\phi_q \sqrt{q_t} z_{q,t+1} \\ &+ B_4^{\$,n-1}[\phi_\pi z_{\pi,t+1} + \phi_{\pi g}\sigma_{g,t} z_{g,t+1} + \phi_{\pi\sigma}\sqrt{q_t} z_{\sigma,t+1}]. \end{aligned} \tag{55}$$

Thus, the variance of the nominal bond price is given by:

$$\begin{aligned} \mathrm{Var}_t[p_{t+1}^{\$,n-1}] = \mathrm{E}_t\left[p_{t+1}^{\$,n-1} - \mathrm{E}_t\left[p_{t+1}^{\$,n-1}\right]\right]^2 &= \left[(B_1^{\$,n-1}\phi_e)^2 + (B_4^{\$,n-1}\phi_{\pi g})^2\right]\sigma_{g,t}^2 \\ + \left[\left(B_2^{\$,n-1} + B_4^{\$,n-1}\phi_{\pi\sigma}\right)^2 + \left(B_3^{\$,n-1}\phi_q\right)^2\right] q_t &+ \left(B_4^{\$,n-1}\phi_\pi\right)^2. \end{aligned} \tag{56}$$

Lastly, covariance between between the nominal pricing kernel and the nominal bond price equals to:

$$\mathrm{Cov}_t\left[m^{\$}_{t+1}, p^{\$,n-1}_{t+1}\right] = \mathrm{E}_t\left[\left(m^{\$}_{t+1} - \mathrm{E}_t m^{\$}_{t+1}\right) \times \left(p^{\$,n-1}_{t+1} - \mathrm{E}_t p^{\$,n-1}_{t+1}\right)\right], \tag{57}$$

where the shock to the nominal pricing kernel in terms of state variables is:

$$\begin{aligned} & m^{\$}_{t+1} - \mathrm{E}_t m^{\$}_{t+1} = m_{t+1} - \mathrm{E}_t m_{t+1} - (\pi_{t+1} - \mathrm{E}_t \pi_{t+1}) \\ & = -\gamma \sigma_{g,t} z_{g,t+1} + (\theta - 1)\kappa_1 \left(A_x \phi_e \sigma_{g,t} z_{x,t+1} + A_\sigma \sqrt{q_t} z_{\sigma,t+1} + A_q \phi_q \sqrt{q_t} z_{q,t+1}\right) \\ & - \phi_\pi z_{\pi,t+1} - \phi_{\pi g} \sigma_{g,t} z_{g,t+1} - \phi_{\pi\sigma} \sqrt{q_t} z_{\sigma,t+1}, \end{aligned} \tag{58}$$

and the shock to the nominal bond price, $p^{\$,n-1}_{t+1} - \mathrm{E}_t p^{\$,n-1}_{t+1}$, is given in (55). Thus, a final expression for a covariance term in (52) is:

$$\begin{aligned} & \mathrm{Cov}_t\left[m^{\$}_{t+1}, p^{\$,n-1}_{t+1}\right] = \left[(\theta - 1)\kappa_1 A_x B^{\$,n-1}_1 \phi_e^2 - (\gamma + \phi_{\pi g}) B^{\$,n-1}_4 \phi_{\pi g}\right] \sigma_{g,t} \\ & + \left[((\theta - 1)\kappa_1 A_\sigma - \phi_{\pi\sigma})(B^{\$,n-1}_2 + B^{\$,n-1}_4 \phi_{\pi\sigma}) + (\theta - 1)\kappa_1 A_q B^{\$,n-1}_3 \phi_q^2 q_t\right] q_t \\ & - B^{\$,n-1}_4 \phi_\pi^2. \end{aligned} \tag{59}$$

Combining together (45), (54), (56), and (59), the solution for the nominal $n-$period bond price is:

$$
\begin{aligned}
B_0^{\$,n} &= c_0 - a_\pi + \left[B_0^{\$,n-1} + B_2^{\$,n-1}a_\sigma + B_3^{\$,n-1}a_q + B_4^{\$,n-1}a_\pi\right] + \frac{1}{2}\phi_\pi^2\left(B_4^{\$,n-1} - 1\right)^2 \\
B_1^{\$,n} &= c_1 + B_1^{\$,n-1}\rho_x \\
B_2^{\$,n} &= B_2^{\$,n-1}\rho_\sigma + (\theta-1)A_\sigma(\kappa_1\rho_\sigma - 1) + \frac{1}{2}(\gamma+\phi_{\pi g})^2 + \frac{1}{2}\phi_e^2\left[(\theta-1)\kappa_1 A_x + B_1^{\$,n-1}\right]^2 \\
&\quad + \frac{1}{2}(B_4^{\$,n-1}\phi_{\pi g})^2 - (\gamma+\phi_{\pi g})B_4^{\$,n-1}\phi_{\pi g} \\
B_3^{\$,n} &= B_3^{\$,n-1}\rho_q + (\theta-1)A_q(\kappa_1\rho_q - 1) + \frac{1}{2}\left[(\theta-1)\kappa_1 A_\sigma + B_2^{\$,n-1} + \phi_{\phi\sigma}\left(B^{\$,n-1} - 1\right)\right]^2 \\
&\quad + \frac{1}{2}\left[(\theta-1)\kappa_1 A_q + B_3^{\$,n-1}\right]^2\phi_q^2 \\
B_4^{\$,n} &= \phi_\pi\left(B^{\$,n-1} - 1\right).
\end{aligned}
\tag{60}
$$

A.5 Solution for the nominal bond risk premium

Let $rx_{t+1}^{\$,n-1} = p_{t+1}^{\$,n-1} - p_t^{\$,n} - r_{f,t}^{\$}$ be the bond excess return on an $n-$period nominal bond holding one period from time t to $t+1$. Then its expected value, or nominal bond risk premium, $\mathrm{brp}_t^{\$,n}$, is given by the negative of covariance between the nominal pricing kernel $m_{t+1}^{\$,n-1}$ and the nominal bond price $p_t^{\$,n}$, which is the negative of eq. (59):

$$
\begin{aligned}
\mathrm{brp}_t^{\$,n} &= -\mathrm{Cov}_t\left[m_{t+1}^{\$}, p_{t+1}^{\$,n-1}\right] \\
&= \left[(\gamma+\phi_{\pi g})B_4^{\$,n-1}\phi_{\pi g} - (\theta-1)\kappa_1 A_x B_1^{\$,n-1}\phi_e^2\right]\sigma_{g,t}^2 \\
&- \left[((\theta-1)\kappa_1 A_\sigma - \phi_{\pi\sigma})(B_2^{\$,n-1} + B_4^{\$,n-1}\phi_{\pi\sigma}) + (\theta-1)\kappa_1 A_q B_3^{\$,n-1}\phi_q^2\right] q_t \\
&+ B_4^{\$,n-1}\phi_\pi^2 \\
&\equiv \beta_1^{\$,n-1}\sigma_{g,t}^2 + \beta_2^{\$,n-1}q_t + B_4^{\$,n-1}\phi_\pi^2.
\end{aligned}
\tag{61}
$$

A.6 Solution for the nominal bond variance risk premium

Define $\sigma^2_{r^\$,t} = \mathrm{Var}_t(r^{\$,n}_{t,t+1})$, where $r^{\$,n}_{t,t+1} = p^{\$,n-1}_{t+1} - p^{\$,n}_t$, so $\sigma^2_{r^\$,t} = \mathrm{Var}_t\left(p^{\$,n-1}_{t+1}\right)$. Similarly, $\sigma^2_{r^\$,t+1} = \mathrm{Var}_{t+1}\left(p^{\$,n-2}_{t+2}\right)$. We need the conditional variance at time $t+1$ because time-t conditional variance is known and therefore, variance risk premium is constant. To derive the nominal bond variance risk premium we start with the affine pricing conjecture (53):

$$p^{\$,n}_t = B^{\$,n}_0 + B^{\$,n}_1 x_t + B^{\$,n}_2 \sigma^2_t + B^{\$,n}_3 q_t + B^{\$,n}_4 \pi_t. \tag{62}$$

Therefore:

$$\begin{aligned}
&\sigma^2_{r^\$,t+1} = \mathrm{Var}_{t+1}\left[p^{\$,n-2}_{t+2}\right] = \mathrm{E}_t\left[p^{\$,n-2}_{t+2} - \mathrm{E}_t\left[p^{\$,n-2}_{t+2}\right]\right]^2 = \\
&\left[\left(B^{\$,n-2}_1\phi_e\right)^2 + \left(B^{\$,n-2}_4\phi_{\pi g}\right)^2\right]\sigma^2_{g,t+1} + \left[\left(B^{\$,n-2}_2 + B^{\$,n-2}_4\phi_{\pi\sigma}\right)^2 + \left(B^{\$,n-2}_3\phi_q\right)^2\right] q_{t+1} \\
&+ \left(B^{\$,n-2}_4\phi_\pi\right)^2,
\end{aligned} \tag{63}$$

and its expectation equals to:

$$\begin{aligned}
\mathrm{E}_t\left[\sigma^2_{r^\$,t+1}\right] &= \left[\left(B^{\$,n-2}_1\phi_e\right)^2 + \left(B^{\$,n-2}_4\phi_{\pi g}\right)^2\right](a_\sigma + \rho_\sigma\sigma^2_{g,t}) \\
&+ \left[\left(B^{\$,n-2}_2 + B^{\$,n-2}_4\phi_{\pi\sigma}\right)^2 + \left(B^{\$,n-2}_3\phi_q\right)^2\right](a_q + \rho_q q_t) + \left(B^{\$,n-2}_4\phi_\pi\right)^2.
\end{aligned} \tag{64}$$

The nominal variance risk premium is defined as the difference in expectations of the variance under risk-neutral $\mathbb{Q}$ and actual measures, which is given by the covariance between the variance of the nominal bond price and the nominal stochastic discount factor $m^\$_{t+1}$:

$$\begin{aligned}
\mathrm{E}^{\mathbb{Q}}_t\left[\sigma^2_{r^\$,t+1}\right] - \mathrm{E}_t\left[\sigma^2_{r^\$,t+1}\right] &= \mathrm{Cov}_t\left[\sigma^2_{r^\$,t+1}, m^\$_{t+1}\right] \\
&= \mathrm{E}_t\left[\left(\sigma^2_{r^\$,t+1} - \mathrm{E}_t\sigma^2_{r^\$,t+1}\right) \times \left(m^\$_{t+1} - \mathrm{E}_t m^\$_{t+1}\right)\right].
\end{aligned} \tag{65}$$

The unexpected part of the nominal pricing kernel is given by (58) and the unexpected part of the variance of the nominal bond price is given by:

$$\sigma^2_{r^\$,t+1} - \mathrm{E}_t \sigma^2_{r^\$,t+1} = \left[\left(B_1^{\$,n-2} \phi_e \right)^2 + \left(B_4^{\$,n-2} \phi_{\pi g} \right)^2 \right] \sqrt{q_t} z_{\sigma,t+1} + \left[\left(B_2^{\$,n-2} + B_4^{\$,n-2} \phi_{\pi\sigma} \right)^2 + \left(B_3^{\$,n-2} \phi_q \right)^2 \right] \phi_q \sqrt{q_t} z_{q,t+1}. \tag{66}$$

Taking the expectation of the product of (58) and (66), we obtain the the nominal bond variance risk premium (65):

$$\mathrm{NVRP}_t \left[\sigma^2_{r^\$,t+1} \right] = \mathrm{E}_t^{\mathbb{Q}} \left[\sigma^2_{r^\$,t+1} \right] - \mathrm{E}_t \left[\sigma^2_{r^\$,t+1} \right] = (\theta - 1)\kappa_1 \left\{ (A_\sigma - \phi \pi \sigma) \left[\left(B_1^{\$,n-2} \phi_e \right)^2 + \left(B_4^{\$,n-2} \phi_{\pi g} \right)^2 \right] + A_q \phi_q^2 \left[\left(B_2^{\$,n-2} + B_4^{\$,n-2} \phi_{\pi\sigma} \right)^2 + \left(B_3^{\$,n-2} \phi_q \right)^2 \right] \right\} q_t. \tag{67}$$

References

Adrian, T., and J. Rosenberg, 2008, "Stock Returns and Volatility: Pricing the Short-Run and Long-Run Components of Market Risk," *Journal of Finance*, 63, 2997–3030.

Andersen, T. G., T. Bollerslev, P. Christoffersen, and F. X. Diebold, 2006, "Volatility and Correlation Forecasting," in *Handbook of Financial Forecasting*, ed. by G. Elliott, C. W. J. Granger, and A. Timmermann. Elsevier, Amsterdam.

Andersen, T. G., T. Bollerslev, and F. X. Diebold, 2007, "Roughting it up: Including Jump Components in the Measurement, Modeling, and Forecasting of Return Volatility," *Review of Economics and Statistics*, 89, 701–720.

Andersen, T. G., T. Bollerslev, F. X. Diebold, and H. Ebens, 2001, "The Distribution of Realized Stock Return Volatility," *Journal of Financial Economics*, 61, 43–76.

Andersen, T. G., T. Bollerslev, F. X. Diebold, and P. Labys, 2000, "Great Realizations," *Risk*, 50, 331–367.

Andersen, T. G., T. Bollerslev, F. X. Diebold, and T. Labys, 2001, "The Distribution of Realized Exchange Rate Volatility," *Journal of the American Statistical Association*, 96, 42–55.

Bansal, R., D. Kiku, and A. Yaron, 2007, "Risks for the Long-run: Estimation and Inference," University of Pennsylavania, working paper.

——— , 2012, "An Empirical Evaluation of the Long-Run Risks Model for Asset Prices," *Critical Finance Review*, 1, 183–221.

Bansal, R., and I. Shaliastovich, 2013, "A Long-Run Risks Explanation of Predictability Puzzles in Bond and Currency Markets," *Review of Financial Studies*, 26(1), 1–33.

Bansal, R., and A. Yaron, 2004, "Risks for the Long Run: A Potential Resolution of Asset Pricing Puzzles," *Journal of Finance*, 59(4), 1481–1509.

——— , 2005, "Interpretable Asset Markets?," *European Economic Review*, 49, 531–560.

Barndorff-Nielsen, O., and N. Shephard, 2004, "Power and Bipower Variation with Stochastic Volatility and Jumps," *Journal of Financial Econometrics*, 2, 1–48.

Beeler, J., and J. Campbell, 2012, "The Long-run Risks Model and Aggregate Asset Prices: An Empirical Assessment," *Critical Finance Review*, 1, 183–221.

Bekaert, G., E. Engstrom, and Y. Xing, 2009, "Risk, Uncertainty, and Asset Prices," *Journal of Financial Economics*, 91(1), 59–82.

Bekaert, G., and J. Liu, 2004, "Conditioning Information and Variance Bounds on Pricing Kernels," *Review of Financial Studies*, 17, 339–78.

Binsbergen, J. H. V., M. Brandt, and R. S. Koijen, 2012, "On the Timing and Pricing of Dividends," *American Economic Review*, 102(4), 1596–1618.

Black, F., 1976, "The Pricing of Commodity Contracts," *Journal of Financial Economics*, 3, 16–179.

Bloom, N., 2009, "The Impact of Uncertainty Shocks," *Econometrica*, 77(3), 623–685.

Bollerslev, T., G. Tauchen, and H. Zhou, 2009, "Expected Stock Returns and Variance Risk Premia," *Review of Financial Studies*, 22(11), 4463–4492.

Bollerslev, T., L. Xu, and H. Zhou, 2013, "Stock Return and Cash Flow Predictability: The Role of Volatility Risk," *Journal of Econometrics, forthcoming*.

Branger, N., P. Rodrigues, and C. Schlag, 2011, "Long-Run Risks in Economic Uncertainty and Implications for Equity Premium Puzzles," Working paper, `http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2021070`.

Campbell, J. Y., and R. J. Shiller, 1988, "The Dividend-price Ratio and Expectations of Future Dividends and Discount Factors," *Review of Financial Studies*, 1, 195–228.

Campbell, J. Y., and R. J. Shiller, 1991, "Yield Spreads and Interest Rate Movements: A Bird's Eye View," *Review of Economic Studies*, 58, 495–514.

Campbell, J. Y., A. Sunderam, and L. Viceira, 2013, "Inflation Bets or Deflation Hedges? The Changing Risks of Nominal Bonds," Working Paper, Harvard University.

Carr, P., and L. Wu, 2009, "Variance Risk Premiums," *Review of Financial Studies*, 22, 1311–1341.

Christoffersen, P., K. Jacobs, C. Ornthanalai, and Y. Wang, 2008, "Option Valuation with Long-Run and Short-Run Volatility Components," *Journal of Financial Economics*, 90, 272–297.

Cochrane, J., and M. Piazzesi, 2005, "Bond Risk Premia," *American Economic Review*, 95, 138–160.

Corsi, F., 2009, "A Simple Approximate Long-Memory Model of Realized Volatility," *Journal of Financial Econometrics*, 7(2), 174–196.

Dai, Q., and K. Singleton, 2000, "Specification Analysis of Affine Term Structure Models," *Journal of Finance*, LV, 1943–1978.

Drechsler, I., 2013, "Uncertainty, Time-Varying Fear, and Asset Prices," *Journal of Finance*, 68, 1843–1889.

Du, J., and N. Kapadia, 2012, "The Tail in the Volatility Index," Working Paper.

Duffee, G. R., 2011, "Information in (and not in) the Term Structure," *Review of Financial Studies*, 24(9), 2895–2934.

Epstein, L., and S. Zin, 1989, "Substitution, Risk Aversion and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework," *Econometrica*, 57, 937–969.

Fama, E., and R. Bliss, 1987, "The Information in Long-Maturity Forward Rates," *The American Economic Review*, 77(4), 680–692.

Ghysels, E., A. Le, S. Park, and H. Zhu, 2014, "Risk and Return Trade-off in the U.S. Treasury Market," SSRN library: http://ssrn.com/abstract=2403483.

Giacoletti, M., K. T. Laursen, and K. J. Singleton, 2015, "Learning, Dispersion of Beliefs, and Risk Premiums in an Arbitrage-free Term Structure Model," Stanford University Graduate School of Business Research Paper No. 15-45.

Hansen, L. P., J. Heaton, J. Lee, and N. Roussanov, 2007, "Intertemporal Substitution and Risk Aversion," in *Handbook of Econometrics*, ed. by J. J. Heckman, and E. Leamer. Elsevier, The Netherlands, vol. 6.

Hansen, P. R., and A. Lunde, 2006, "Realized Variance and Market Microstructure Noise," *Journal of Business and Economic Statistics*, 24, 127–161.

Huang, J.-Z., and Z. Shi, 2012, "Determinants of Bond Risk Premia," Working paper, Penn State University.

Kreps, D. M., and E. L. Porteus, 1978, "Temporal Resolution of Uncertainty and Dynamic Choice Theory," *Econometrica*, 46, 185–200.

Lettau, M., S. Ludvigson, and J. Wachter, 2008, "The Declining Equity Premium: What Role Does Macroeconomic Risk Play?," *Review of Financial Studies*, 21, 1653–2991687.

Ludvigson, S., and S. Ng, 2009, "Macro Factors in Bond Risk Premia," *Review of Financial Studies*, 22, 5027–5067.

Mueller, P., A. Vedolin, and H. Zhou, 2011, "Short-run Bond Risk Premia," Working paper, London School of Economics and Federal Reserve Board.

Newey, W. K., and K. D. West, 1987, "A Simple Positive Semi-definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix," *Econometrica*, 55(3), 703–708.

Piazzesi, M., and M. Shneider, 2007, "Equilibrium Yield Curves," in *NBER Macroeconomics Annual*. MIT Press, Cambridge, vol. 21, pp. 389–472.

Tauchen, G., 2011, "Stochastic Volatility in General Equilibrium," *Quarterly Journal of Finance*, 01(04), 707–731.

Weil, P., 1989, "The Equity Premium Puzzle and the Riskfree Rate Puzzle," *Journal of Monetary Economics*, 24, 401–421.

Wright, J. H., 2011, "Term Premia and Inflation Uncertainty: Empirical Evidence from International Panel Dataset," *American Economic Review*, 101, 1514–1534.

Wright, J. H., and H. Zhou, 2009, "Bond Risk Premia and Realized Jump Risk," *Journal of Banking and Finance*, 33(12), 2333 – 2345.

Zhou, G., and Y. Zhu, 2012, "Volatility Trading: What is the Role of the Long-Run Volatility Component?," *Journal of Financial and Quantitative Analysis*, 47(2), 273–307.

——— , 2013, "The Long-run Risks Model: What Differences Can an Extra Volatility Factor Make?," Working paper, http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1403869.

Zhou, H., 2009, "Variance Risk Premia, Asset Predictability Puzzles, and Macroeconomic Uncertainty," Working paper, http://ssrn.com/abstract=1400049.

——— , 2011, "Term Structure of Interest Rates with Inflation Uncertainty," Technical appendix, Federal Reserve Board.

Table 1: Model Calibration

This table reports the calibrated parameters used in previous studies and in our paper. Column "BY" refers to the choice of parameters in Bansal and Yaron (2004), column "BTZ" – to that in Bollerslev, Tauchen, and Zhou (2009), and column "GSZ" refers to our choice of parameters.

Type	Parameters	BY	BTZ	GSZ
Panel A: Real economy dynamics				
	δ	0.997	0.997	0.997
Preferences	γ	10	10	8
	ψ	1.5	1.5	1.5
	μ_g	0.0015	0.0015	0.0015
	ρ_x	0.979	0	0.979
Endowment	ϕ_e	0.001	0	0.001
	a_σ	1.20463e-05	1.20463e-05	1.20463e-05
	ρ_σ	0.978	0.978	0.978
	a_q		2e-07	2e-10
Uncertainty	ρ_q		0.8	0.8
	ϕ_q		0.001	0.0001
Panel B: Inflation dynamics				
Constant	a_π			8.33333e-05
Persistence	ρ_π			0.95
Autonomous	ϕ_π			0.0013
Consumption	$\phi_{\pi g}$			-0.0385
Uncertainty	$\phi_{\pi\sigma}$			28.5044
Panel C: Campbell-Shiller constants				
	κ_0	0.3251	0.3251	0.3251
	κ_1	0.9	0.9	0.9

Table 2: Summary Statistics

This table presents summary statistics for the data used in the study. Panel A presents a summary statistics for the annualized 1-month and 1-year Treasury excess returns for maturities 2 to 5 years; Panel B reports the summary statistics for the forward-rate based predictors and the 3-month interest rate swaption expiry based variance risk premium. In Panel B FS_j, $j = 1, \ldots, 4$ refers to Fama-Bliss j-year forward spreads, CP – to Cochrane-Piazzesi factor. IV is the implied variance derived from interest swaption market, RV is the expected value of the realized variance derived from the intraday quotes of the interest rate swap market. IRVRP is the interest-rate variance risk premium. Sample period is April 2002 to January 2013, at monthly frequency. Treasury excess returns, forward spreads, and Cochrane-Piazzesi factor are computed using Fama-Bliss Treasury Bond data set from CRSP. Interest rate swaptions data is from J.P. Morgan and Barclays Capital, and intraday interest rate swap quotes data is from Barclays Capital.

Panel A: Summary statistics of Treasury excess returns

	1-month excess returns				1-year excess returns			
	2yr	3yr	4yr	5yr	2yr	3yr	4yr	5yr
Mean	-1.37	-1.30	-1.24	-1.17	0.61	1.46	2.35	3.16
Max	0.93	1.69	3.61	4.84	3.03	6.37	9.34	11.79
Min	-5.18	-5.50	-5.79	-6.03	-1.54	-2.55	-3.62	-4.63
Std. Dev.	1.71	1.84	2.04	2.24	1.10	2.09	2.96	3.68
Skewness	-0.78	-0.64	-0.39	-0.23	0.18	0.10	0.02	-0.05
Kurtosis	2.19	2.12	2.19	2.32	2.40	2.51	2.35	2.28
AR(1) coeff	0.93	0.84	0.72	0.63	0.91	0.89	0.88	0.87

Panel B: Summary statistics for IRVRP and forward-rate based predictors

	FS_2	FS_3	FS_4	FS_5	CP	IV	RV	$IRVRP$
Mean	0.37	0.91	1.47	1.95	1.89	3.16	1.16	2.01
Max	1.44	2.50	3.44	4.32	4.17	9.70	3.43	6.27
Min	-0.70	-0.61	-0.54	-0.43	-0.76	0.66	0.52	-0.00
Std. Dev.	0.53	0.88	1.18	1.49	0.98	1.83	0.54	1.59
Skewness	0.07	0.07	-0.03	-0.15	-0.39	0.78	1.80	0.72
Kurtosis	2.19	1.81	1.75	1.69	2.88	3.48	6.93	2.47
AR(1) coeff	0.92	0.95	0.95	0.97	0.72	0.92	0.75	0.87

Table 3: 1m HPR Bond Predictability with 10y1m Interest-rate VRP

This table presents regression results for the following regression: $rx_{t+h}^{(\tau)} = \beta_0^{(\tau)} + \beta_1^{(\tau)}(h)IRVRP_t + \sum_{j=1}^{2} \beta_j^{(\tau)}(h)F_{t,j} + \epsilon_{t+h}^{(\tau)}$, where $rx_{t+h}^{(\tau)}$ are excess returns on Treasury notes, $h = 1$ month and $\tau = 2, \ldots, 5$ years. $IRVRP_t$ is the interest-rate variance risk premium derived from interest rate derivatives markets, $F_{t,j}$, $j = 1, 2$ is the forward spread (FS) and Cochrane-Piazzesi (CP) factors. t-statistics in parentheses are calculated using Newey and West (1987) standard errors. Adjusted R2 are given in percentage points. Treasury excess returns are computed using Fama-Bliss data set, obtained from CRSP. Interest rate swaptions and swaps data are from J.P. Morgan and Barclays Capital. Sample is from April 2002 to January 2013, monthly frequency.

(1)	(2)	(3)	(4)	(5)	(6)	(7)
Panel A: maturity = 2 years						
Const	-0.026	-0.021	-0.027	-0.031	-0.032	-0.032
	(-6.56)	(-6.46)	(-3.27)	(-12.72)	(-4.82)	(-9.76)
IRVRP	0.004			0.003	0.003	0.003
	(4.65)			(5.85)	(4.40)	(5.56)
FS		1.983		1.732		1.688
		(4.98)		(7.87)		(7.01)
CP			0.689		0.361	0.090
			(2.16)		(1.43)	(0.62)
Adj. R2	42.62	37.35	14.16	71.00	45.88	70.95
Panel B: maturity = 3 years						
Const	-0.025	-0.026	-0.027	-0.034	-0.032	-0.035
	(-6.06)	(-7.11)	(-3.25)	(-13.14)	(-4.68)	(-10.31)
IRVRP	0.004			0.003	0.003	0.003
	(4.49)			(4.53)	(4.16)	(4.39)
FS		1.407		1.194		1.167
		(5.89)		(8.22)		(7.82)
CP			0.730		0.412	0.082
			(2.26)		(1.58)	(0.59)
Adj. R2	36.07	44.63	13.82	67.11	39.76	66.97
Panel C: maturity = 4 years						
Const	-0.024	-0.029	-0.027	-0.037	-0.032	-0.039
	(-5.39)	(-8.10)	(-3.21)	(-16.80)	(-4.41)	(-13.83)
IRVRP	0.004			0.003	0.003	0.003
	(3.96)			(5.67)	(3.58)	(4.99)
FS		1.159		1.018		0.989
		(7.36)		(9.25)		(8.63)
CP			0.765		0.460	0.151
			(2.33)		(1.68)	(1.06)
Adj. R2	27.74	44.58	12.25	61.16	31.43	61.26
Panel D: maturity = 5 years						
Const	-0.024	-0.033	-0.028	-0.036	-0.032	-0.038
	(-5.30)	(-10.44)	(-3.15)	(-14.57)	(-4.38)	(-11.16)
IRVRP	0.004			0.002	0.003	0.002
	(4.40)			(2.80)	(3.97)	(2.58)
FS		1.066		0.905		0.876
		(9.03)		(7.94)		(7.38)
CP			0.826		0.510	0.167
			(2.40)		(1.74)	(1.01)
Adj. R2	24.92	49.30	11.73	55.03	28.63	55.07

Table 4: 3m HPR Bond Predictability with 10y1m Interest-rate VRP

This table presents regression results for the following regression: $rx^{(\tau)}_{t+h} = \beta^{(\tau)}_0 + \beta^{(\tau)}_1(h)IRVRP_t + \sum_{j=1}^{2}\beta^{(\tau)}_j(h)F_{t,j} + \epsilon^{(\tau)}_{t+h}$, where $rx^{(\tau)}_{t+h}$ are excess returns on Treasury notes, $h = 3$ months and $\tau = 2, \ldots, 5$ years. $IRVRP_t$ is the interest-rate variance risk premium derived from interest-rate derivatives markets, $F_{t,j}$, $j = 1, 2$ is the forward spread (FS) and Cochrane-Piazzesi (CP) factors. t-statistics in parentheses are calculated using Newey and West (1987) standard errors. Adjusted R2 are given in percentage points. Treasury excess returns are computed using Fama-Bliss data set, obtained from CRSP. Interest rate swaptions and swaps data are from J.P. Morgan and Barclays Capital. Sample is from April 2002 to January 2013, monthly frequency.

(1)	(2)	(3)	(4)	(5)	(6)	(7)
Panel A: maturity = 2 years						
Const	-0.020	-0.017	-0.022	-0.025	-0.026	-0.026
	(-5.43)	(-5.12)	(-2.88)	(-7.70)	(-3.99)	(-7.06)
IRVRP	0.003			0.003	0.003	0.003
	(4.31)			(5.21)	(4.05)	(4.79)
FS		1.608		1.396		1.346
		(4.08)		(4.91)		(4.41)
CP			0.618		0.328	0.112
			(2.02)		(1.31)	(0.69)
Adj. R2	32.40	28.06	12.37	53.39	35.14	53.33
Panel B: maturity = 3 years						
Const	-0.018	-0.020	-0.021	-0.026	-0.024	-0.027
	(-3.93)	(-5.10)	(-2.45)	(-6.44)	(-3.23)	(-6.44)
IRVRP	0.003			0.002	0.003	0.002
	(3.75)			(3.60)	(3.37)	(3.33)
FS		1.288		1.124		1.103
		(4.78)		(4.95)		(4.71)
CP			0.646		0.375	0.069
			(1.90)		(1.25)	(0.38)
Adj. R2	19.18	32.12	8.61	42.61	21.31	42.17
Panel C: maturity = 4 years						
Const	-0.015	-0.024	-0.019	-0.028	-0.022	-0.030
	(-2.62)	(-5.33)	(-2.04)	(-6.02)	(-2.49)	(-6.15)
IRVRP	0.003			0.002	0.002	0.002
	(2.58)			(2.84)	(2.19)	(2.43)
FS		1.157		1.066		1.046
		(5.40)		(5.26)		(5.08)
CP			0.658		0.426	0.110
			(1.72)		(1.20)	(0.49)
Adj. R2	9.41	30.20	5.46	34.15	10.98	33.70
Panel D: maturity = 5 years						
Const	-0.014	-0.026	-0.018	-0.027	-0.021	-0.028
	(-2.23)	(-4.87)	(-1.69)	(-4.97)	(-2.12)	(-4.67)
IRVRP	0.003			0.001	0.002	0.001
	(2.83)			(1.00)	(2.44)	(0.91)
FS		1.068		0.993		0.980
		(5.31)		(4.51)		(4.34)
CP			0.710		0.452	0.083
			(1.59)		(1.07)	(0.30)
Adj. R2	8.10	29.17	4.41	29.28	9.20	28.69

Table 5: 12m HPR Bond Predictability with 10y1m Interest-rate VRP

This table presents regression results for the following regression: $rx_{t+h}^{(\tau)} = \beta_0^{(\tau)} + \beta_1^{(\tau)}(h)IRVRP_t + \sum_{j=1}^{2}\beta_j^{(\tau)}(h)F_{t,j} + \epsilon_{t+h}^{(\tau)}$, where $rx_{t+h}^{(\tau)}$ are excess returns on Treasury notes, $h = 12$ months and $\tau = 2, \ldots, 5$ years. $IRVRP_t$ is the interest-rate variance risk premium derived from interest rate derivatives markets, $F_{t,j}$, $j = 1, 2$ is the forward spread (FS) and Cochrane-Piazzesi (CP) factors. t-statistics in parentheses are calculated using Newey and West (1987) standard errors. Adjusted R2 are given in percentage points. Treasury excess returns are computed using Fama-Bliss data set, obtained from CRSP. Interest rate swaptions and swaps data are from J.P. Morgan and Barclays Capital. Sample is from April 2002 to January 2013, monthly frequency.

(1)	(2)	(3)	(4)	(5)	(6)	(7)
Panel A: maturity = 2 years						
Const	0.004	0.007	0.007	0.006	0.007	0.007
	(1.28)	(2.96)	(2.15)	(1.80)	(1.94)	(1.86)
IRVRP	0.000			0.001	0.001	0.001
	(0.72)			(1.20)	(1.04)	(1.25)
FS		-0.534		-0.592		-0.558
		(-1.62)		(-1.87)		(-1.74)
CP			-0.107		-0.169	-0.081
			(-0.81)		(-1.22)	(-0.54)
Adj. R2	-0.07	6.59	-0.09	7.98	0.89	7.48
Panel B: maturity = 3 years						
Const	0.011	0.015	0.017	0.013	0.016	0.017
	(1.88)	(2.49)	(2.66)	(1.89)	(2.46)	(2.29)
IRVRP	0.001			0.001	0.001	0.001
	(0.67)			(1.07)	(1.00)	(1.22)
FS		-0.242		-0.331		-0.262
		(-0.60)		(-0.85)		(-0.68)
CP			-0.208		-0.320	-0.252
			(-0.83)		(-1.21)	(-0.93)
Adj. R2	-0.20	0.21	-0.02	0.84	0.80	1.00
Panel C: maturity = 4 years						
Const	0.020	0.018	0.026	0.017	0.026	0.024
	(2.40)	(1.86)	(2.84)	(1.56)	(2.65)	(2.05)
IRVRP	0.001			0.000	0.001	0.001
	(0.43)			(0.30)	(0.71)	(0.63)
FS		0.230		0.205		0.277
		(0.52)		(0.48)		(0.67)
CP			-0.253		-0.374	-0.451
			(-0.67)		(-0.96)	(-1.26)
Adj. R2	-0.58	-0.02	-0.28	-0.87	-0.24	-0.00
Panel D: maturity = 5 years						
Const	0.025	0.021	0.032	0.021	0.030	0.027
	(2.59)	(1.77)	(2.66)	(1.65)	(2.48)	(2.03)
IRVRP	0.002			0.001	0.002	0.001
	(0.81)			(0.32)	(0.99)	(0.52)
FS		0.411		0.358		0.422
		(0.98)		(0.79)		(0.96)
CP			-0.090		-0.300	-0.442
			(-0.18)		(-0.60)	(-1.00)
Adj. R2	0.50	2.14	-0.92	1.34	0.07	1.49

Table 6: 1m HPR Bond Predictability with 10y3m Interest-rate VRP

This table presents regression results for the following regression: $rx^{(\tau)}_{t+h} = \beta^{(\tau)}_0 + \beta^{(\tau)}_1(h)IRVRP_t + \sum_{j=1}^{2}\beta^{(\tau)}_j(h)F_{t,j} + \epsilon^{(\tau)}_{t+h}$, where $rx^{(\tau)}_{t+h}$ are excess returns on Treasury notes, $h = 1$ month and $\tau = 2, \ldots, 5$ years. $IRVRP_t$ is the interest-rate variance risk premium derived from interest rate derivatives markets, $F_{t,j}$, $j = 1, 2$ is the forward spread (FS) and Cochrane-Piazzesi (CP) factors. t-statistics in parentheses are calculated using Newey and West (1987) standard errors. Adjusted R2 are given in percentage points. Treasury excess returns are computed using Fama-Bliss data set, obtained from CRSP. Interest rate swaptions and swaps data are from J.P. Morgan and Barclays Capital. Sample is from April 2002 to January 2013, monthly frequency.

(1)	(2)	(3)	(4)	(5)	(6)	(7)
Panel A: maturity = 2 years						
Const	-0.029	-0.021	-0.027	-0.034	-0.035	-0.036
	(-7.57)	(-6.46)	(-3.27)	(-16.23)	(-5.91)	(-13.65)
IRVRP	0.008			0.007	0.007	0.007
	(6.44)			(11.38)	(6.48)	(10.52)
FS		1.983		1.699		1.639
		(4.98)		(9.03)		(7.86)
CP			0.689		0.382	0.121
			(2.16)		(1.64)	(1.03)
Adj. R2	50.21	37.35	14.16	77.53	54.10	77.73
Panel B: maturity = 3 years						
Const	-0.028	-0.026	-0.027	-0.037	-0.035	-0.038
	(-7.00)	(-7.11)	(-3.25)	(-17.17)	(-5.69)	(-14.43)
IRVRP	0.008			0.006	0.007	0.006
	(6.12)			(8.74)	(6.10)	(8.32)
FS		1.407		1.185		1.153
		(5.89)		(10.23)		(9.60)
CP			0.730		0.429	0.097
			(2.26)		(1.77)	(0.88)
Adj. R2	42.75	44.63	13.82	73.63	47.02	73.62
Panel C: maturity = 4 years						
Const	-0.027	-0.029	-0.027	-0.039	-0.035	-0.041
	(-6.32)	(-8.10)	(-3.21)	(-18.97)	(-5.33)	(-17.94)
IRVRP	0.008			0.006	0.007	0.006
	(5.55)			(9.66)	(5.42)	(8.11)
FS		1.159		0.984		0.948
		(7.36)		(10.79)		(9.94)
CP			0.765		0.471	0.181
			(2.33)		(1.83)	(1.50)
Adj. R2	33.67	44.58	12.25	64.52	37.76	64.86
Panel D: maturity = 5 years						
Const	-0.027	-0.033	-0.028	-0.038	-0.036	-0.040
	(-6.12)	(-10.44)	(-3.15)	(-16.50)	(-5.25)	(-13.28)
IRVRP	0.008			0.005	0.007	0.004
	(5.80)			(4.70)	(5.71)	(4.37)
FS		1.066		0.877		0.843
		(9.03)		(8.81)		(8.01)
CP			0.826		0.526	0.176
			(2.40)		(1.90)	(1.15)
Adj. R2	29.44	49.30	11.73	57.95	33.61	58.07

Table 7: 3m HPR Bond Predictability with 10y3m Interest-rate VRP

This table presents regression results for the following regression: $rx_{t+h}^{(\tau)} = \beta_0^{(\tau)} + \beta_1^{(\tau)}(h)IRVRP_t + \sum_{j=1}^{2} \beta_j^{(\tau)}(h)F_{t,j} + \epsilon_{t+h}^{(\tau)}$, where $rx_{t+h}^{(\tau)}$ are excess returns on Treasury notes, $h = 3$ months and $\tau = 2, \ldots, 5$ years. $IRVRP_t$ is the interest-rate variance risk premium derived from interest rate derivatives markets, $F_{t,j}$, $j = 1, 2$ is the forward spread (FS) and Cochrane-Piazzesi (CP) factors. t-statistics in parentheses are calculated using Newey and West (1987) standard errors. Adjusted R2 are given in percentage points. Treasury excess returns are computed using Fama-Bliss data set, obtained from CRSP. Interest rate swaptions and swaps data are from J.P. Morgan and Barclays Capital. Sample is from April 2002 to January 2013, monthly frequency.

(1)	(2)	(3)	(4)	(5)	(6)	(7)
Panel A: maturity = 2 years						
Const	-0.023	-0.017	-0.022	-0.027	-0.028	-0.028
	(-6.15)	(-5.12)	(-2.88)	(-8.43)	(-4.74)	(-8.41)
IRVRP	0.006			0.006	0.006	0.005
	(5.68)			(8.19)	(5.62)	(7.35)
FS		1.608		1.361		1.305
		(4.08)		(5.06)		(4.51)
CP			0.618		0.328	0.123
			(2.02)		(1.39)	(0.83)
Adj. R2	38.11	28.06	12.37	58.00	40.99	58.07
Panel B: maturity = 3 years						
Const	-0.021	-0.020	-0.021	-0.028	-0.026	-0.029
	(-4.42)	(-5.10)	(-2.45)	(-6.80)	(-3.75)	(-7.19)
IRVRP	0.006			0.005	0.005	0.005
	(4.66)			(5.42)	(4.41)	(4.89)
FS		1.288		1.109		1.089
		(4.78)		(5.20)		(4.90)
CP			0.646		0.371	0.064
			(1.90)		(1.30)	(0.38)
Adj. R2	23.00	32.12	8.61	45.95	25.18	45.53
Panel C: maturity = 4 years						
Const	-0.018	-0.024	-0.019	-0.030	-0.024	-0.031
	(-3.09)	(-5.33)	(-2.04)	(-6.08)	(-2.87)	(-6.38)
IRVRP	0.006			0.004	0.005	0.004
	(3.44)			(3.61)	(3.14)	(3.13)
FS		1.157		1.034		1.015
		(5.40)		(5.30)		(5.09)
CP			0.658		0.404	0.110
			(1.72)		(1.18)	(0.51)
Adj. R2	12.70	30.20	5.46	35.61	14.11	35.17
Panel D: maturity = 5 years						
Const	-0.017	-0.026	-0.018	-0.029	-0.024	-0.030
	(-2.63)	(-4.87)	(-1.69)	(-5.03)	(-2.44)	(-4.76)
IRVRP	0.006			0.003	0.005	0.002
	(3.45)			(1.63)	(3.18)	(1.56)
FS		1.068		0.959		0.948
		(5.31)		(4.57)		(4.36)
CP			0.710		0.437	0.066
			(1.59)		(1.07)	(0.24)
Adj. R2	10.43	29.17	4.41	30.17	11.46	29.57

Table 8: 12m HPR Bond Predictability with 10y3m Interest-rate VRP

This table presents regression results for the following regression: $rx^{(\tau)}_{t+h} = \beta^{(\tau)}_0 + \beta^{(\tau)}_1(h)IRVRP_t + \sum_{j=1}^{2}\beta^{(\tau)}_j(h)F_{t,j} + \epsilon^{(\tau)}_{t+h}$, where $rx^{(\tau)}_{t+h}$ are excess returns on Treasury notes, $h = 12$ months and $\tau = 2, \ldots, 5$ years. $IRVRP_t$ is the interest-rate variance risk premium derived from interest rate derivatives markets, $F_{t,j}$, $j = 1, 2$ is the forward spread (FS) and Cochrane-Piazzesi (CP) factors. t-statistics in parentheses are calculated using Newey and West (1987) standard errors. Adjusted R2 are given in percentage points. Treasury excess returns are computed using Fama-Bliss data set, obtained from CRSP. Interest rate swaptions and swaps data are from J.P. Morgan and Barclays Capital. Sample is from April 2002 to January 2013, monthly frequency.

(1)	(2)	(3)	(4)	(5)	(6)	(7)
Panel A: maturity = 2 years						
Const	0.004	0.007	0.007	0.006	0.006	0.007
	(1.13)	(2.96)	(2.15)	(1.59)	(1.73)	(1.69)
IRVRP	0.001			0.001	0.001	0.001
	(0.58)			(1.13)	(0.85)	(1.16)
FS		-0.534		-0.602		-0.572
		(-1.62)		(-1.92)		(-1.81)
CP			-0.107		-0.158	-0.072
			(-0.81)		(-1.16)	(-0.49)
Adj. R2	-0.27	6.59	-0.09	7.96	0.48	7.38
Panel B: maturity = 3 years						
Const	0.011	0.015	0.017	0.013	0.016	0.016
	(1.64)	(2.49)	(2.66)	(1.71)	(2.21)	(2.11)
IRVRP	0.001			0.002	0.002	0.002
	(0.62)			(1.03)	(0.90)	(1.15)
FS		-0.242		-0.342		-0.275
		(-0.60)		(-0.89)		(-0.73)
CP			-0.208		-0.310	-0.239
			(-0.83)		(-1.20)	(-0.91)
Adj. R2	-0.19	0.21	-0.02	0.95	0.72	1.03
Panel C: maturity = 4 years						
Const	0.018	0.018	0.026	0.016	0.024	0.023
	(2.03)	(1.86)	(2.84)	(1.44)	(2.38)	(1.94)
IRVRP	0.002			0.001	0.003	0.002
	(0.63)			(0.52)	(0.91)	(0.82)
FS		0.230		0.174		0.244
		(0.52)		(0.41)		(0.60)
CP			-0.253		-0.399	-0.460
			(-0.67)		(-1.07)	(-1.33)
Adj. R2	-0.08	-0.02	-0.28	-0.58	0.48	0.41
Panel D: maturity = 5 years						
Const	0.023	0.021	0.032	0.020	0.028	0.026
	(2.15)	(1.77)	(2.66)	(1.51)	(2.22)	(1.91)
IRVRP	0.004			0.002	0.005	0.003
	(1.11)			(0.72)	(1.34)	(0.93)
FS		0.411		0.289		0.359
		(0.98)		(0.66)		(0.83)
CP			-0.090		-0.348	-0.465
			(-0.18)		(-0.73)	(-1.08)
Adj. R2	1.76	2.14	-0.92	1.92	1.54	2.22

Table 9: 1m HPR Bond Predictability with 10y12m Interest-rate VRP

This table presents regression results for the following regression: $rx^{(\tau)}_{t+h} = \beta^{(\tau)}_0 + \beta^{(\tau)}_1(h)IRVRP_t + \sum_{j=1}^{2}\beta^{(\tau)}_j(h)F_{t,j} + \epsilon^{(\tau)}_{t+h}$, where $rx^{(\tau)}_{t+h}$ are excess returns on Treasury notes, $h = 1$ month and $\tau = 2, \ldots, 5$ years. $IRVRP_t$ is the interest-rate variance risk premium derived from interest rate derivatives markets, $F_{t,j}$, $j = 1, 2$ is the forward spread (FS) and Cochrane-Piazzesi (CP) factors. t-statistics in parentheses are calculated using Newey and West (1987) standard errors. Adjusted R2 are given in percentage points. Treasury excess returns are computed using Fama-Bliss data set, obtained from CRSP. Interest rate swaptions and swaps data are from J.P. Morgan and Barclays Capital. Sample is from April 2002 to January 2013, monthly frequency.

(1)	(2)	(3)	(4)	(5)	(6)	(7)
Panel A: maturity = 2 years						
Const	-0.021	-0.021	-0.027	-0.029	-0.034	-0.036
	(-5.20)	(-6.46)	(-3.27)	(-9.77)	(-4.84)	(-11.72)
IRVRP	0.012			0.013	0.012	0.013
	(4.23)			(7.82)	(4.65)	(7.92)
FS		1.983		2.094		1.885
		(4.98)		(8.05)		(7.30)
CP			0.689		0.697	0.381
			(2.16)		(2.61)	(2.67)
Adj. R2	23.40	37.35	14.16	65.55	38.27	69.50
Panel B: maturity = 3 years						
Const	-0.020	-0.026	-0.027	-0.035	-0.034	-0.040
	(-4.88)	(-7.11)	(-3.25)	(-14.64)	(-4.77)	(-17.13)
IRVRP	0.012			0.014	0.012	0.014
	(4.07)			(10.43)	(4.52)	(10.56)
FS		1.407		1.510		1.396
		(5.89)		(11.05)		(12.22)
CP			0.730		0.739	0.301
			(2.26)		(2.70)	(3.54)
Adj. R2	19.72	44.63	13.82	71.44	34.19	73.43
Panel C: maturity = 4 years						
Const	-0.019	-0.029	-0.027	-0.036	-0.034	-0.042
	(-4.51)	(-8.10)	(-3.21)	(-12.86)	(-4.66)	(-17.78)
IRVRP	0.012			0.012	0.012	0.012
	(3.83)			(7.34)	(4.28)	(7.67)
FS		1.159		1.152		1.059
		(7.36)		(11.79)		(12.65)
CP			0.765		0.773	0.396
			(2.33)		(2.75)	(3.18)
Adj. R2	15.95	44.58	12.25	60.55	28.75	63.48
Panel D: maturity = 5 years						
Const	-0.019	-0.033	-0.028	-0.038	-0.035	-0.043
	(-4.19)	(-10.44)	(-3.15)	(-16.64)	(-4.54)	(-16.50)
IRVRP	0.012			0.011	0.012	0.011
	(3.61)			(6.30)	(4.08)	(6.72)
FS		1.066		1.032		0.961
		(9.03)		(10.96)		(10.61)
CP			0.826		0.835	0.312
			(2.40)		(2.78)	(2.44)
Adj. R2	13.12	49.30	11.73	59.62	25.33	60.86

Table 10: 3m HPR Bond Predictability with 10y12m Interest-rate VRP

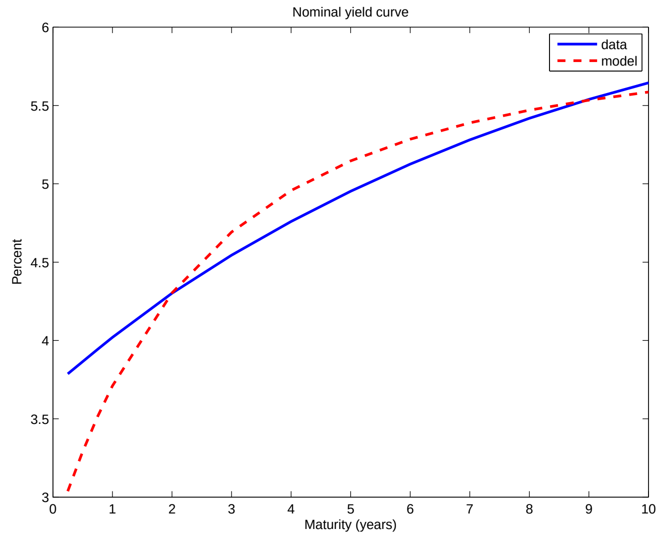
This table presents regression results for the following regression: $rx_{t+h}^{(\tau)} = \beta_0^{(\tau)} + \beta_1^{(\tau)}(h)IRVRP_t + \sum_{j=1}^{2} \beta_j^{(\tau)}(h)F_{t,j} + \epsilon_{t+h}^{(\tau)}$, where $rx_{t+h}^{(\tau)}$ are excess returns on Treasury notes, $h = 3$ months and $\tau = 2, \ldots, 5$ years. $IRVRP_t$ is the interest-rate variance risk premium derived from interest rate derivatives markets, $F_{t,j}$, $j = 1, 2$ is the forward spread (FS) and Cochrane-Piazzesi (CP) factors. t-statistics in parentheses are calculated using Newey and West (1987) standard errors. Adjusted R2 are given in percentage points. Treasury excess returns are computed using Fama-Bliss data set, obtained from CRSP. Interest rate swaptions and swaps data are from J.P. Morgan and Barclays Capital. Sample is from April 2002 to January 2013, monthly frequency.

(1)	(2)	(3)	(4)	(5)	(6)	(7)
Panel A: maturity = 2 years						
Const	-0.016	-0.017	-0.022	-0.023	-0.027	-0.029
	(-4.42)	(-5.12)	(-2.88)	(-6.55)	(-4.01)	(-7.76)
IRVRP	0.010			0.011	0.010	0.011
	(3.99)			(6.39)	(4.28)	(6.37)
FS		1.608		1.679		1.503
		(4.08)		(5.22)		(4.61)
CP			0.618		0.588	0.333
			(2.02)		(2.21)	(2.02)
Adj. R2	17.86	28.06	12.37	48.86	29.24	51.96
Panel B: maturity = 3 years						
Const	-0.014	-0.020	-0.021	-0.028	-0.026	-0.031
	(-3.34)	(-5.10)	(-2.45)	(-6.57)	(-3.34)	(-7.75)
IRVRP	0.010			0.011	0.010	0.011
	(3.48)			(6.41)	(3.68)	(6.29)
FS		1.288		1.357		1.280
		(4.78)		(6.03)		(5.69)
CP			0.646		0.617	0.211
			(1.90)		(2.02)	(1.44)
Adj. R2	11.09	32.12	8.61	47.02	19.02	47.44
Panel C: maturity = 4 years						
Const	-0.012	-0.024	-0.019	-0.029	-0.024	-0.033
	(-2.52)	(-5.33)	(-2.04)	(-6.02)	(-2.78)	(-6.68)
IRVRP	0.011			0.010	0.010	0.010
	(3.08)			(4.34)	(3.23)	(4.36)
FS		1.157		1.140		1.086
		(5.40)		(6.10)		(5.81)
CP			0.658		0.627	0.245
			(1.72)		(1.80)	(1.25)
Adj. R2	7.60	30.20	5.46	37.22	12.59	37.47
Panel D: maturity = 5 years						
Const	-0.011	-0.026	-0.018	-0.030	-0.024	-0.032
	(-1.93)	(-4.87)	(-1.69)	(-5.22)	(-2.33)	(-5.21)
IRVRP	0.011			0.009	0.011	0.009
	(2.70)			(3.30)	(2.82)	(3.34)
FS		1.068		1.032		1.002
		(5.31)		(5.45)		(5.01)
CP			0.710		0.679	0.135
			(1.59)		(1.63)	(0.51)
Adj. R2	5.74	29.17	4.41	33.00	9.76	32.56

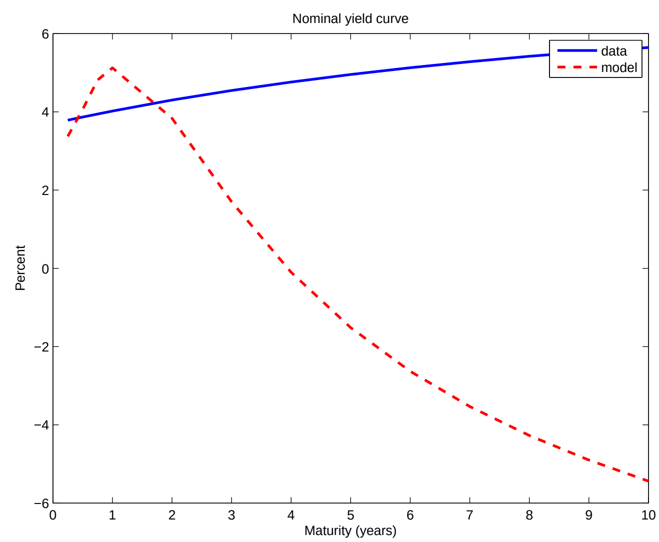
Table 11: 12m HPR Bond Predictability with 10y12m Interest-rate VRP

This table presents regression results for the following regression: $rx_{t+h}^{(\tau)} = \beta_0^{(\tau)} + \beta_1^{(\tau)}(h)IRVRP_t + \sum_{j=1}^{2}\beta_j^{(\tau)}(h)F_{t,j} + \epsilon_{t+h}^{(\tau)}$, where $rx_{t+h}^{(\tau)}$ are excess returns on Treasury notes, $h = 12$ months and $\tau = 2, \ldots, 5$ years. $IRVRP_t$ is the interest-rate variance risk premium derived from interest rate derivatives markets, $F_{t,j}$, $j = 1, 2$ is the forward spread (FS) and Cochrane-Piazzesi (CP) factors. t-statistics in parentheses are calculated using Newey and West (1987) standard errors. Adjusted R2 are given in percentage points. Treasury excess returns are computed using Fama-Bliss data set, obtained from CRSP. Interest rate swaptions and swaps data are from J.P. Morgan and Barclays Capital. Sample is from April 2002 to January 2013, monthly frequency.

(1)	(2)	(3)	(4)	(5)	(6)	(7)
Panel A: maturity = 2 years						
Const	0.005	0.007	0.007	0.007	0.007	0.007
	(1.77)	(2.96)	(2.15)	(2.31)	(1.84)	(1.80)
IRVRP	0.001			0.001	0.001	0.001
	(0.31)	( )		(0.40)	(0.36)	(0.40)
FS		-0.534		-0.536		-0.528
		(-1.62)		(-1.64)		(-1.59)
CP			-0.107		-0.109	-0.015
			(-0.81)		(-0.83)	(-0.10)
Adj. R2	-0.87	6.59	-0.09	5.84	-0.93	4.91
Panel B: maturity = 3 years						
Const	0.012	0.015	0.017	0.014	0.016	0.017
	(2.40)	(2.49)	(2.66)	(1.99)	(2.28)	(2.10)
IRVRP	0.002			0.002	0.002	0.002
	(0.55)			(0.60)	(0.62)	(0.64)
FS		-0.242		-0.248		-0.193
		(-0.60)		(-0.62)		(-0.48)
CP			-0.208		-0.215	-0.151
			(-0.83)		(-0.87)	(-0.63)
Adj. R2	-0.60	0.21	-0.02	-0.35	-0.57	-0.90
Panel C: maturity = 4 years						
Const	0.019	0.018	0.026	0.016	0.024	0.022
	(2.80)	(1.86)	(2.84)	(1.51)	(2.37)	(1.84)
IRVRP	0.006			0.006	0.007	0.006
	(1.13)			(1.12)	(1.22)	(1.19)
FS		0.230		0.189		0.273
		(0.52)		(0.44)		(0.66)
CP			-0.253		-0.271	-0.370
			(-0.67)		(-0.75)	(-1.23)
Adj. R2	0.54	-0.02	-0.28	0.20	0.37	0.59
Panel D: maturity = 5 years						
Const	0.025	0.021	0.032	0.019	0.027	0.024
	(3.14)	(1.77)	(2.66)	(1.45)	(2.17)	(1.74)
IRVRP	0.012			0.010	0.012	0.010
	(1.75)			(1.71)	(1.80)	(1.78)
FS		0.411		0.334		0.415
		(0.98)		(0.82)		(1.01)
CP			-0.090		-0.123	-0.358
			(-0.18)		(-0.27)	(-0.90)
Adj. R2	2.37	2.14	-0.92	3.39	1.52	3.22



(a) With LRR component



(b) No LRR component

Figure 1: The model-implied nominal yield curve

The figure plots the average zero-coupon nominal Treasury yield curve as observed in the data using the sample of January 1991 - December 2010 monthly data as the solid blue line in both Panels (a) and (b). The figure also plots the model-implied yield curve with the long-run risk component (Panel (a)) and without the long-run risk component (Panel (b)) as the dashed red line.

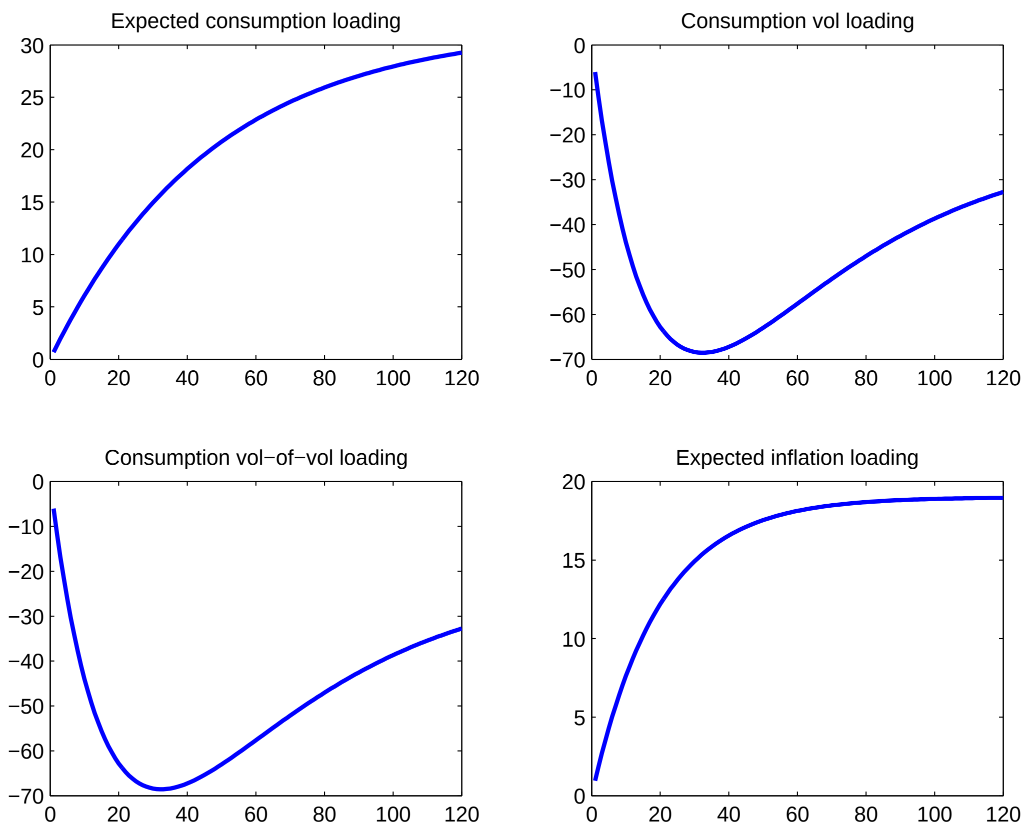


Figure 2: Equilibrium nominal bond yield loadings

The figure plots the model-implied nominal bond yield loadings on expected consumption growth (top left panel), consumption volatility (top right panel), consumption volatility-of-volatility (bottom left panel), and expected inflation (bottom right panel). Maturity on horizontal axes is in months.

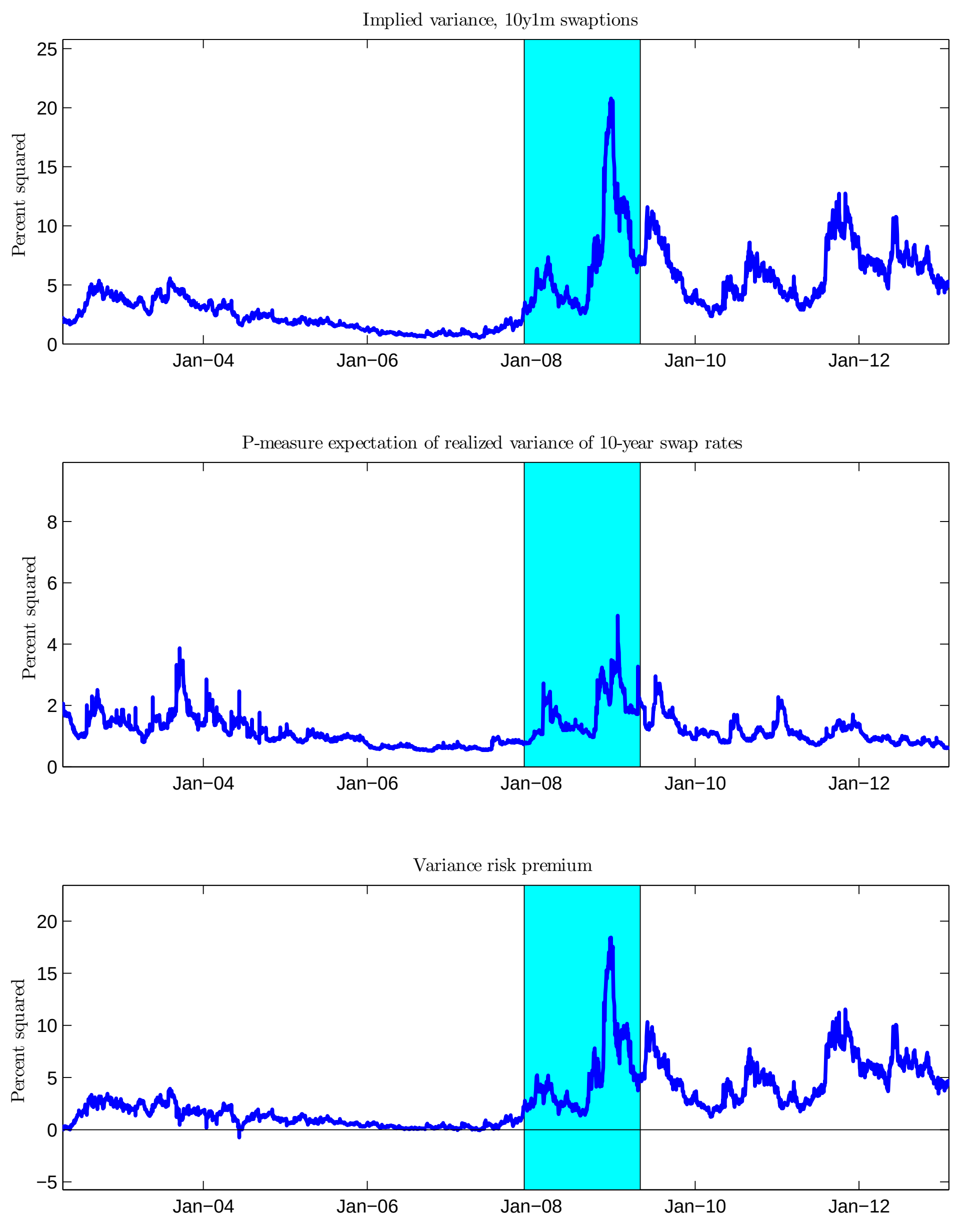


Figure 3: Interest-rate variance risk premium, 1-month horizon

This figure plots implied variance (top panel), expected variance (middle panel), and their difference, variance risk premium (bottom panel). Implied variance is derived from the one-month expiry swaptions on the 10-year forward swap rate. Realized variance is derived from the intraday ten-year swap rate data. Blue shaded bar indicates NBER recession. Data source: J.P. Morgan and Barclays Capital. Sample period is from April 2002 to February 2013.

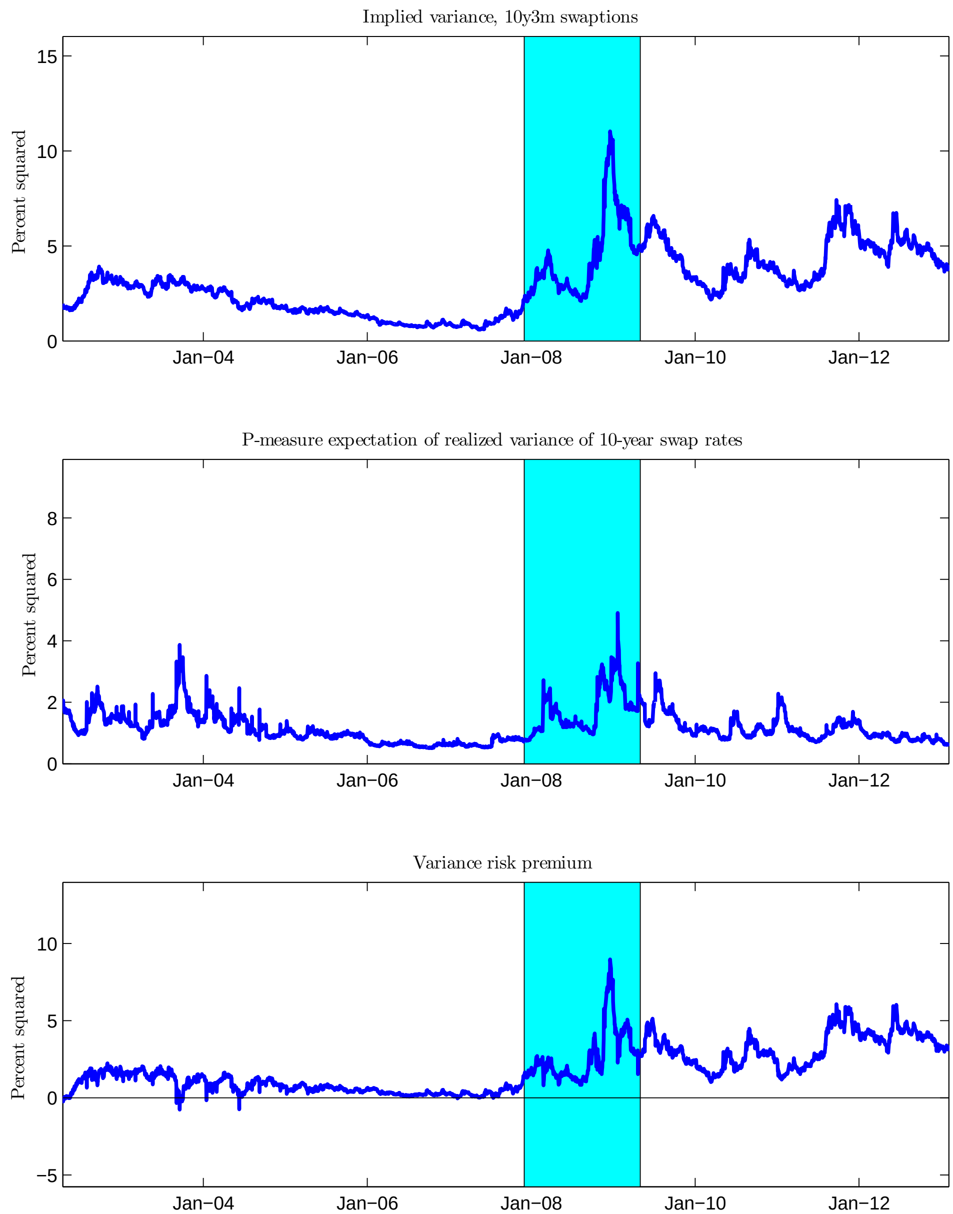


Figure 4: Interest-rate variance risk premium, 3-month horizon

This figure plots implied variance (top panel), expected variance (middle panel), and their difference, variance risk premium (bottom panel). Implied variance is derived from the three-month expiry swaptions on the 10-year forward swap rate. Realized variance is derived from the intraday ten-year swap rate data. Blue shaded bar indicates NBER recession. Data source: J.P. Morgan and Barclays Capital. Sample period is from April 2002 to February 2013.

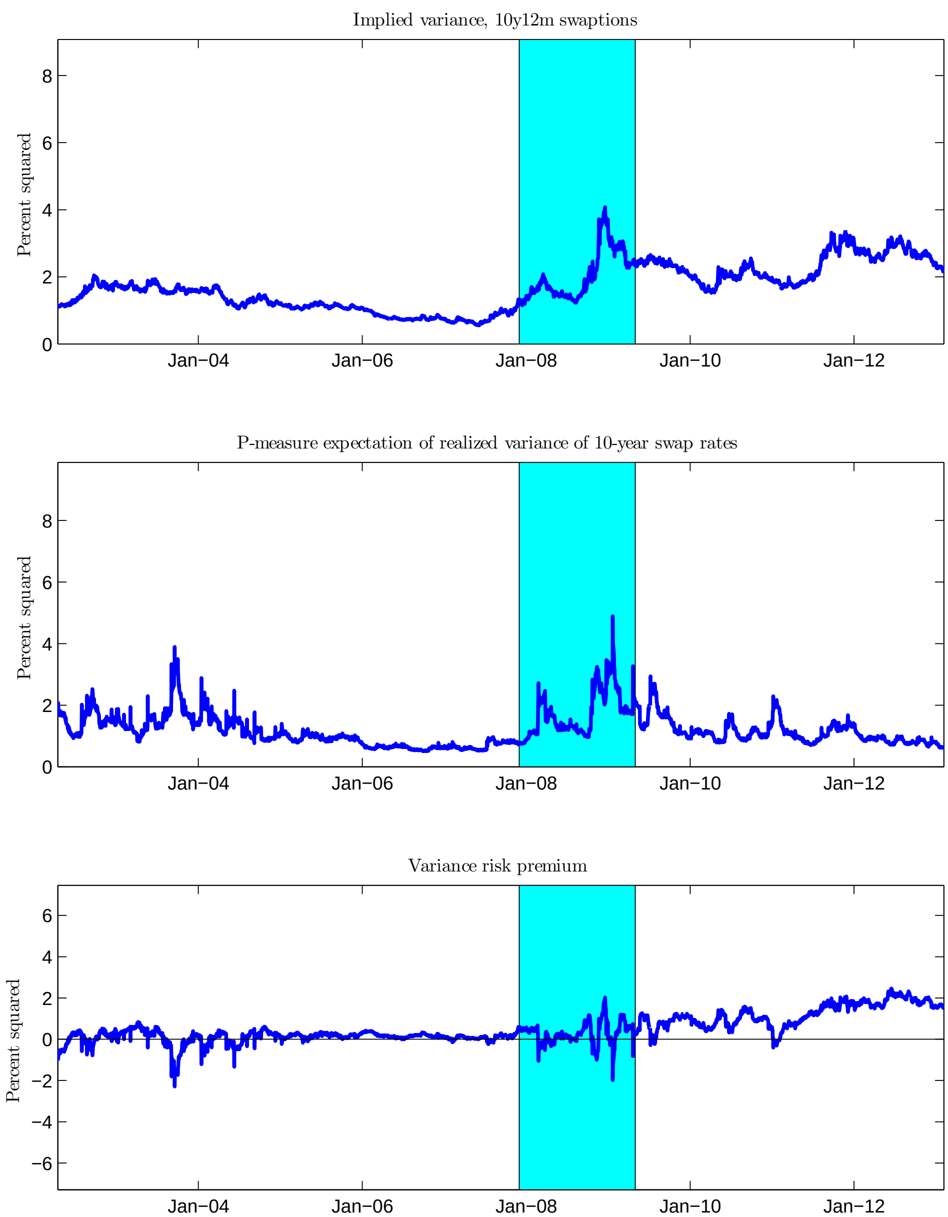


Figure 5: Interest-rate variance risk premium, 12-month horizon

This figure plots implied variance (top panel), expected variance (middle panel), and their difference, variance risk premium (bottom panel). Implied variance is derived from the twelve-month expiry swaptions on the 10-year forward swap rate. Realized variance is derived from the intraday ten-year interest rate swaps data. Blue shaded bar indicates NBER recession. Data source: J.P. Morgan and Barclays Capital. Sample period is from April 2002 to February 2013.