Endogenous Price War Risks

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Abstract

We develop a general-equilibrium asset-pricing model with dynamic games of price competition. Price war risks arise endogenously from declines in long-run growth as firms’ incentive to undercut prices grow stronger with a worse growth outlook. The triggered price wars have amplification effects by narrowing profit margins. In industries with higher capacity of radical innovation, firms compete more fiercely for future market dominance. Their incentives for price undercutting are less responsive to long-run growth shocks, and they are more immune to price war risks and long-run growth shocks. Our results shed new light on the relationship between competition and equity returns.

Keywords: Product market frictions, Forward-looking competition, Long-run growth, Innovation similarity, Customer base, Self-fulfilling regime switching.

(JEL: G1, G3, O3, L1)
1 Introduction

A price war occurs when rival firms aggressively undercut prices to gain market shares. The risks of entering a price war are critical and concern investors, partly because product markets are highly concentrated, featuring rich strategic competition among leading firms (see, e.g., Autor et al., 2017; Loecker and Eeckhout, 2017). According to U.S. Census data, the top four firms within each four-digit Standard Industrial Classification (SIC) industry account for about 48% of that industry’s total revenue, and the top eight firms have a combined market share exceeding 60% (see Online Appendix Figure OA.6). However, little is known about which primitive forces drive price war risks or how price war risks systematically affect asset prices. This paper is the first to study endogenous price war risks and their asset pricing implications.

We document three stylized facts motivating our study. First, there is significant positive comovement between the average profit margin of industries and the long-run consumption growth rate (Panels A–D of Figure 1). Second, price war coverage by the media and analyst reports spikes during periods of low long-run growth (Panels E and F). Together, these two stylized facts suggest the existence of a systematic component in price war risks. Third, the pattern of long-run growth predicting one-year-ahead profit margins is more pronounced in industries with a low capacity for radical innovation (Panels G and H). These empirical findings raise three relevant questions: (1) What fundamentally drives price war risks at the aggregate level? (2) How is the heterogeneous exposure to price war risks determined across industries? (3) To what extent can price war risks amplify the asset pricing implications of long-run growth shocks by generating time-varying cash flow volatility?

1 The implications of price war risks on stock returns and profit margins have been extensively covered by the media and analysts. We list a few headline quotes in Online Appendix A and a few examples of analyst reports in Online Appendix B.

2 The average profit margin is the simple average of industries’ profit margins as in Machin and Van Reenen (1993), so the comovement is not because of a composition effect from time-varying industry size. We focus on the comovement between long-run growth and profit margins, instead of product markups, because profit margins are related directly to price war risks. Our stylized fact is consistent with the literature, which suggests that profit margins are strongly pro-cyclical (see, e.g., Machin and Van Reenen, 1993; Hall, 2012; Anderson, Rebelo and Wong, 2018). Although markups and profit margins are related, the empirical evidence on the cyclicality of markups is mixed, primarily because measuring markups is challenging (see Blanchard, 2009; Anderson, Rebelo and Wong, 2018). For example, Domowitz, Hubbard and Petersen (1986), Nekarda and Ramey (2011, 2013), Hall (2014), and Braun and Raddatz (2016) find that markups are pro-cyclical, whereas Bils (1987) and Chevalier and Scharfstein (1996) find markups to be countercyclical.
Our paper takes the first step toward answering these three questions in a unified framework. First, we develop a model showing that long-run growth shocks (as in Bansal and Yaron, 2004) can drive price war risks. Second, the model and the data show that an industry’s exposure to price war risks — and thus long-run growth shocks — is higher if the industry has a lower capacity for radical innovation. Our results shed new light on how long-run growth shocks are priced in the cross section (see, e.g., Bansal, Dittmar and Lundblad, 2005; Hansen, Heaton and Li, 2008; Bansal, Dittmar and Kiku, 2009; Malloy, Moskowitz and Vissing-Jørgensen, 2009; Ai, 2010; Kojien et al., 2010; Constantinides and Ghosh, 2011; Gârleanu, Panageas and Yu, 2012; Ai, Croce and Li, 2013; Kung and Schmid, 2015; Bansal, Kiku and Yaron, 2016; Dittmar and Lundblad, 2017; Gârleanu and Panageas, 2018). Third, we use the calibrated model to quantify the impact of price war risks. The model implies that endogenous price war risks can amplify the effects of long-run growth shocks by as high as 30% in terms of the equity premium, and more importantly, endogenous price war risks are crucial for long-run growth shocks to be priced in the cross section of industries sorted on the capacity for radical innovation.
Theoretically, what is a price war? A full-blown price war is a non-collusive price competition that serves as an enforcement device to sustain implicit collusion on prices (see, e.g., Friedman, 1971; Green and Porter, 1984; Porter, 1985; Abreu, Pearce and Stacchetti, 1986; Athey, Bagwell and Sanchirico, 2004; Sannikov and Skrzypacz, 2007). More broadly, a price war can also be a collusive price competition in which prices drop because of a decline in firms’ market power (see, e.g., Rotemberg and Saloner, 1986; Lambson, 1987; Haltiwanger and Harrington, 1991). Sufficient narrowing of profit margins triggers a full-blown price war in the equilibrium, featuring a regime shift from a collusive equilibrium to a non-collusive equilibrium. Thus, price war risks are also related inherently to the risk of jumping from a collusive regime to a non-collusive regime.

To answer the three questions motivated by the stylized facts in Figure 1, we develop a general-equilibrium asset pricing model incorporating dynamic games of price competition among firms. We first introduce in Section 2 a baseline model of endogenous price war risks. The baseline model highlights the key mechanism: price war risks rise endogenously because of declines in long-run growth, from which the first stylized fact (in Panels A–D) emerges. In Section 3, we extend the baseline model by allowing full-blown price wars. This extended model highlights the self-fulfilling endogenous regime shifting from a collusive regime to a non-collusive regime. The probability of entering a full-blown price war endogenously increases as long-run growth declines, a result consistent with the second stylized fact (in Panels E and F). However, the model with full-blown price war risks cannot account for the third stylized fact (in Panels G and H) because, by design, cross-sectional heterogeneity does not exist across industries. In Section 4, we further extend to a full model by allowing firms to innovate and snatch their competitors’ customer base within each industry. Industries are heterogeneous in terms of their capacities for radical innovation. The full model allows us to analyze the cross-sectional implications of endogenous price war risks (as well as long-run growth shocks) for cash flows and stock returns. It offers a rich set of cross-sectional testable implications of the key mechanism. Most importantly, the full model sheds new light on the relationship between stock returns and industry competition: to appreciate the relationship of competition and stock returns, what matters more is the competition for future market shares (i.e., the future market structure), captured by the capacity for radical innovation, rather than the current market structure.

Our baseline model in Section 2 deviates from the standard Lucas-tree economy mainly in two aspects: (1) consumers have deep habits (see Ravn, Schmitt-Grohé and
Uribe, 2006; van Binsbergen, 2016) over firms’ products, and thus firms find it valuable to maintain their customer base; and (2) there is a continuum of industries, and each industry features a dynamic Bertrand duopoly with differentiated products and implicit price collusion. In our baseline model, duopolists can collude implicitly with each other to set high product prices and obtain high profit margins. Knowing that the competitor will honor the collusive price-setting agreement, a firm can boost its short-run revenue by undercutting prices to attract more customers; however, deviating from the collusive price-setting scheme may reduce revenue in the long run if the competitor finds out and punishes the firm. Following the literature (see, e.g., Green and Porter, 1984; Brock and Scheinkman, 1985; Rotemberg and Saloner, 1986), we adopt the non-collusive Nash equilibrium as the incentive-compatible punishment for deviation. The implicit collusive price levels depend on firms’ deviation incentives: a higher implicit collusive price can only be sustained by a lower deviation incentive, which is further shaped by firms’ tradeoff between short-term and long-term cash flows. In other words, higher collusive prices are more difficult to sustain when long-run growth is lower, because future punishment becomes less threatening when firms expect a persistent decline in aggregate consumption demand. As a result, collusive prices decline following negative long-run-growth shocks, generating endogenous price war risks.

In the extended model in Section 3, we augment the baseline model by adding one ingredient — imperfect monitoring. In particular, we assume that firms may need to incur a non-pecuniary cost to monitor their competitors for potential deviation. Imperfect monitoring has been argued to be a major channel through which full-blown price wars break out in equilibrium (see Green and Porter, 1984). With large negative long-run growth shocks, collusive prices decline significantly, and the benefit of collusion exceeds its cost. As a result, firms optimally abandon collusion, and the industry falls into a non-collusive equilibrium — a full-blown price war occurs. Importantly, a full-blown price war generates a significant downward jump in profit margins and amplifies the impact of long-run growth shocks. The endogenous equilibrium switching driven by

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3Tirole (1988, Chapter 6) builds oligopoly models with Bertrand price competition and obtains similar price war implications to those of the models of Cournot quantity competition (see Green and Porter, 1984; Rotemberg and Saloner, 1986).

4Even though explicit collusion is illegal in many countries including the US, Canada, and most of the EU because of antitrust laws, implicit collusion in the form of price leadership and tacit understanding still takes place. For example, Intel and AMD implicitly colluded on the prices of graphic cards and central processing units in the 2000s, although a price war was waged between the two companies in October 2018.
fundamental shocks is similar in spirit to that of Tsyvinski, Mukherji and Hellwig (2006), Angeletos, Hellwig and Pavan (2007), Bebchuk and Goldstein (2011), and Goldstein, Dow and Guembel (2017), among others. Further, the probability of jumping into a full-blown price war is endogenously time-varying and driven negatively by long-run growth shocks.

In the full model in Section 4, we incorporate the competition role of innovation. Firms can snatch their competitors’ customer base and dominate their industries through successful radical innovations. A successful radical innovation creates products sufficiently distinctive from existing ones, allowing firms to disrupt the market and rapidly grab substantial market shares. The capacity for radical innovation is a fundamental and persistent industry characteristic; it is the only ex-ante heterogeneity in the model. The full model sheds new light on industries’ heterogeneous exposure to price war risks and thus long-run growth shocks. The model and the data show that firms in industries with a higher capacity for radical innovation (see, e.g., Jaffe, 1986; Christensen, 1997; Manso, 2011; Kelly et al., 2018) are more immune to price war risks. Intuitively, in such industries, the market structure is more likely to experience dramatic changes and to become highly concentrated in the future. Firms in such industries find it more difficult to collude with each other because they all rationally expect a highly concentrated product market to emerge, so the punishment for deviation becomes less threatening. As a result, these industries feature low collusive prices regardless of long-run growth rates, and their profit margins are less responsive to long-run growth fluctuations. By contrast, in industries with a lower capacity for radical innovation, the market structure is relatively stable, making the punishment for deviation more threatening. As a result, firms are more eager to maintain their existing customer base and collude to set higher prices. Their profit margins are more sensitive to fluctuations in long-run growth, and these industries are more exposed to price war risks and long-run growth shocks.

Empirically, our full model has the following testable cross-sectional implications. First, in industries with a lower capacity for radical innovation, profit margins are higher and decrease to a greater extent after negative long-run growth shocks. Second, these industries are more exposed to long-run growth shocks, and investors demand higher (risk-adjusted) expected excess returns on their equity. To test these predictions, we first construct an innovation similarity measure based on US patenting activities from 1976 to 2017 to capture the capacity for radical innovation across industries. In light

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5A prominent example involves Apple Inc., a company that disrupted the mobile phone market by cobbling together an amazing touch screen with a user-friendly interface.
of previous studies (see, e.g., Jaffe, 1986; Bloom, Schankerman and Van Reenen, 2013), our innovation similarity measure is constructed based on the technology classifications of firms’ patents within industries. An industry is associated with a higher innovation similarity measure if the patents produced by firms within the industry have more similar technology classifications, and such an industry is expected to have a lower capacity for radical innovation.

Consistent with our theory, we find that the average profit margin of industries comoves positively with the long-run growth rate. In the cross section, we show that profit margins are more exposed to long-run growth shocks in industries with a lower capacity for radical innovation. By exploiting detailed product-level data, we find that product prices are more exposed to long-run growth shocks in industries with a lower capacity for radical innovation. In particular, our event-type study shows these industries to have been more likely to engage in price wars in response to the Lehman crash in September 2008, a time when the US economy experienced a prominent negative long-run growth shock according to the estimation of Schorfheide, Song and Yaron (2018). Finally, we find that the stock returns and dividend growth of industries with a lower capacity for radical innovation are more exposed to long-run growth shocks. These industries have higher (risk-adjusted) expected excess returns.

**Related Literature.** Our paper contributes to the burgeoning literature on the intersection between industrial organization, marketing, and finance (see, e.g., Phillips, 1995; Kovenock and Phillips, 1997; Allen and Phillips, 2000; Aghion et al., 2005; Morellec and Zhdanov, 2005; Hou and Robinson, 2006; Zhdanov, 2007; Morellec and Zhdanov, 2008; Carlin, 2009; Aguerrevere, 2009; Hoberg and Phillips, 2010; Hackbarth and Miao, 2012; Phillips and Zhdanov, 2013; Carlson et al., 2014; Hackbarth, Mathews and Robinson, 2014; Hoberg, Phillips and Prabhala, 2014; Bustamante, 2015; Weber, 2015; Hoberg and Phillips, 2016; Loualiche, 2016; Bustamante and Donangelo, 2017; Corhay, 2017; Corhay, Kung and Schmid, 2017; Andrei and Carlin, 2018; D’Acunto et al., 2018; Dong, Massa and Zaldokas, 2018; Yang, 2018; Dou and Ji, 2018; Dou et al., 2018; Hackbarth and Taub, 2018; Roussanov, Ruan and Wei, 2018; Zhdanov and Morellec, 2019). In a closely related paper, Corhay, Kung and Schmid (2017) develop a novel general-equilibrium production-based

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6There is also a strand of literature that studies the asset pricing implications of imperfect competition in the market micro-structure setting (see, e.g., Christie and Schultz, 1994; Biais, Martimort and Rochet, 2000; Atkeson, Eisfeldt and Weill, 2015; Liu and Wang, 2018).
asset pricing model to understand the endogenous relation between markups and stock returns amid strategic competition among firms. Their model implies that industries with higher markups are associated with higher expected returns. Our model yields a similar implication through firms’ dynamic strategic competition. We show that industries with a lower capacity for radical innovation have higher equilibrium profit margins and are more exposed to price war risks and long-run growth shocks. Theoretically, our paper advances the literature by developing a general-equilibrium model incorporated with dynamic games, in which price war risks arise endogenously from declines in long-run growth.

Our paper is also related to a growing literature that studies the implications of innovation on asset prices (see, e.g., Li, 2011; Gârleanu, Kogan and Panageas, 2012; Gârleanu, Panageas and Yu, 2012; Hirshleifer, Hsu and Li, 2013; Kung and Schmid, 2015; Kumar and Li, 2016; Hirshleifer, Hsu and Li, 2017; Kogan et al., 2017; Corhay, Kung and Schmid, 2017; Dou, 2017; Fitzgerald et al., 2017; Kogan, Papanikolaou and Stoffman, 2018; Kogan et al., 2018). One of the key results from the work of Corhay, Kung and Schmid (2017) is that extensive and intensive margins of innovation endogenously drive volatility risks and long-run growth shocks. This result deepens our understanding of the economic origins of the fluctuations in risk premia. Our model abstracts away from the growth effect of innovation (see, e.g., Acemoglu et al., 2018) and focuses on the competition role of innovation. We contribute to this literature by showing that industries with a higher capacity for radical innovation are less exposed to price war risks and are associated with lower (risk-adjusted) expected excess returns. Importantly, we emphasize that the capacity for radical innovation provides forward-looking information on the degree of competition in the product market, complementing the traditional static measures of competition such as the Herfindahl-Hirschman Index (HHI) and the product similarity measure (see Hoberg and Phillips, 2016).

Our paper also contributes in two ways to the macroeconomics and industrial organization literature on implicit collusion and price wars in dynamic oligopoly industries (see, e.g., Stigler, 1964; Green and Porter, 1984; Rotemberg and Saloner, 1986; Haltiwanger and Harrington, 1991; Rotemberg and Woodford, 1991; Staiger and Wolak, 1992; Bagwell and Staiger, 1997; Athey, Bagwell and Sanchirico, 2004; Opp, Parlour and Walden, 2014). First, we analyze the asset pricing implications of price war risks. Second, the model and the data show that the exposure to price war risks varies across industries with different capacities for radical innovation.
Finally, our paper is situated in the vast literature on cross-sectional asset pricing (see, e.g., Cochrane, 1991; Berk, Green and Naik, 1999; Gomes, Kogan and Zhang, 2003; Pastor and Stambaugh, 2003; Ait-Sahalia, Parker and Yogo, 2004; Lustig and Van Nieuwerburgh, 2005; Nagel, 2005; Yogo, 2006; Lustig and Verdelhan, 2007; Campbell, Hilscher and Szilagyi, 2008; Livdan, Sapriza and Zhang, 2009; Gomes and Schmid, 2010; Garlappi and Yan, 2011; Lustig, Roussanov and Verdelhan, 2011; Papanikolaou, 2011; Belo and Lin, 2012; Ai and Kiku, 2013; Kogan and Papanikolaou, 2013; Belo, Lin and Bazdresch, 2014; Donangelo, 2014; Kogan and Papanikolaou, 2014; Gandhi and Lustig, 2015; Hackbarth and Johnson, 2015; Herskovic et al., 2016; Tsai and Wachter, 2016; Dou, 2017; Kojen, Lustig and Van Nieuwerburgh, 2017; Kozak, Nagel and Santosh, 2017; Ai et al., 2018; Belo, Lin and Yang, 2018; Gomes and Schmid, 2018; Gu, Hackbarth and Johnson, 2018). Nagel (2013) provides a comprehensive survey. We show that the exposure to price war risks varies across industries with different capacities for radical innovation. Our paper is related particularly to the works investigating the cross-sectional stock return implications of firms’ fundamental characteristics through intangible capital (see, e.g., Ai, Croce and Li, 2013; Eisfeldt and Papanikolaou, 2013; Belo, Lin and Vitorino, 2014; Dou et al., 2018). Another closely related paper is van Binsbergen (2016), which studies the implications of multiple goods for asset prices based on the framework of deep habits developed by Ravn, Schmitt-Grohé and Uribe (2006). Monopolistically competitive firms offer individual varieties of goods. The demand elasticities of each monopolist are time-varying and proportional to the consumption surplus ratio for the monopolist’s good. Firms with low demand elasticities are more sensitive to aggregate shocks because of low operating flexibility on cash flows.

2 The Baseline Model

The economy contains a continuum of industries indexed by $i \in I \equiv [0, 1]$. Each industry $i$ is a duopoly, consisting of two all-equity firms indexed by $j \in F \equiv \{1, 2\}$. We label a generic firm with $ij$, referring to firm $j$ in industry $i$, and its competitor by $i\overline{j}$. All firms are owned by a continuum of atomistic homogeneous households. Firms produce differentiated goods and set prices strategically to maximize shareholder value.

Households are homogeneous and have stochastic differential utility as in the work of Duffie and Epstein (1992a,b), defined recursively as follows:
\[ U_0 = \mathbb{E}_0 \left[ \int_0^\infty f(C_t, U_t)dt \right], \quad (2.1) \]

where

\[ f(C_t, U_t) = \beta U_t \frac{1 - \gamma}{1 - \frac{1}{\psi}} \left[ \frac{C_t^{\frac{1}{1-\gamma}}}{(1 - \gamma)U_t^{\frac{1}{1-\gamma}}} - 1 \right]. \quad (2.2) \]

This preference is a continuous-time version of the recursive preferences proposed by Kreps and Porteus (1978), Epstein and Zin (1989), and Weil (1990). The felicity function \( f(C_t, U_t) \) is an aggregator over the current consumption rate \( C_t \) of the final consumption good and future utility level \( U_t \). Coefficient \( \beta \) is the subjective discount rate, \( \gamma \) is the relative risk aversion parameter for one-period consumption, and \( \psi \) is the elasticity of intertemporal substitution (EIS) for deterministic consumption paths.

The final consumption good \( C_t \) is determined by a two-layer aggregation. Following the functional form of relative deep habits (see Ravn, Schmitt-Grohé and Uribe, 2006)\(^7\), industry \( i \)'s composite \( C_{i,t} \) is determined by aggregating firm-level differentiated goods

\[ C_{i,t} = \left( \sum_{j \in F} \left( \frac{M_{ij,t}}{M_{i,t}} \right)^{\frac{1}{\eta}} \right)^{\frac{\eta}{\eta-1}}, \quad (2.3) \]

where parameter \( \eta > 1 \) captures the elasticity of substitution among goods produced in the same industry. \( M_{ij,t}/M_{i,t} \) captures the relative deep habits of firm \( j \) in industry \( i \), where \( M_{i,t} \) is defined as \( M_{i,t} = \sum_{j \in F} M_{ij,t} \).

Further, the demand for the final consumption good \( C_t \) is determined through the aggregation of industry composites

\[ C_t = \left( \int_0^1 M_{i,t}^{\frac{1}{\epsilon}} C_{i,t}^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}, \quad (2.4) \]

where parameter \( \epsilon > 1 \) captures the elasticity of substitution among industry composites. Consistent with the literature (see, e.g., Atkeson and Burstein, 2008; Corhay, Kung and

\(^7\)The specification of relative deep habits is inspired by the habit formation of Abel (1990), which features catching up with the Joneses. The habit formation arises endogenously from the pecuniary externality of the competition over scarce resources (see DeMarzo, Kaniel and Kremer, 2007, 2008). The key difference between the formation of relative deep habits and the formation of habits is that in the former, agents form habits over individual varieties of goods as opposed to a composite consumption good.
Schmid, 2017), we assume $\eta \geq \epsilon > 1$, meaning that products within the same industry are more substitutable. For example, the elasticity of substitution between the Apple iPhone and the Samsung Galaxy is much higher than that between a cell phone and coffee.

### 2.1 Customer Base

The habit coefficient $M_{ij,t}$ in equation (2.3) is persistent over time, which can be interpreted as customer inertia to firm $j$’s products (see Klemperer, 1995). From the firm’s perspective, the customer inertia $M_{ij,t}$ can be viewed as its customer base, as $M_{ij,t}$ determines the demand for its products $C_{ij,t}$.

**Demand Curves.** Let $P_{i,t}$ denote the price of industry $i$’s composite. Given $P_{i,t}$ and $C_{i,t}$, we obtain $C_{i,t}$ by solving a standard expenditure minimization problem:

$$C_{i,t} = \left( \frac{P_{i,t}}{P_t} \right)^{-\epsilon} M_{i,t} C_t, \quad \text{with} \quad P_t = \left( \int_0^1 M_{i,t} P_{i,t}^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}},$$

(2.5)

where $P_t$ is the price index for the final consumption good. Without loss of generality, we normalize $P_t \equiv 1$ so that the final consumption good is the numeraire. Next, given $C_{i,t}$, the demand for firm $j$’s good is

$$C_{ij,t} = \left( \frac{P_{ij,t}}{P_{i,t}} \right)^{-\eta} P_{i,t}^{-\epsilon} M_{ij,t} C_t, \quad \text{with} \quad P_{i,t} = \left[ \sum_{j \in F} \left( \frac{M_{ij,t}}{M_{i,t}} \right) P_{ij,t}^{1-\eta} \right]^{\frac{1}{1-\eta}}.$$

(2.6)

In equation (2.6), the demand for firm $j$’s good increases with $M_{ij,t}$. Thus, it is natural to think of $M_{ij,t}$ as capturing firm $j$’s customer base and $M_{i,t}$ as capturing industry $i$’s total customer base. Moreover, equation (2.6) implies that firm $j$ has more influence on the price index $P_{i,t}$ when the share of its customer base $M_{ij,t}/M_{i,t}$ is higher. Thus, firm $j$ has the incentive to accumulate $M_{ij,t}$ to increase demand and gain market power.

**Endogenous Price Elasticity.** The short-run price elasticity of demand for firm $j$ is

$$\frac{\partial \ln C_{ij,t}}{\partial \ln P_{ij,t}} = s_{ij,t} \frac{\partial \ln C_{i,t}}{\partial \ln P_{i,t}} + (1 - s_{ij,t}) \frac{\partial \ln (C_{ij,t}/C_{i,t})}{\partial \ln (P_{ij,t}/P_{i,t})} = s_{ij,t} \epsilon + (1 - s_{ij,t}) \eta,$$

(2.7)
where $s_{ij,t}$ is the (revenue) market share of firm $j$ in industry $i$:

$$s_{ij,t} = \frac{P_{ij,t}C_{ij,t}}{P_{i,t}C_{i,t}} = \left(\frac{P_{ij,t}}{P_{i,t}}\right)^{1-\eta} \frac{M_{ij,t}}{M_{i,t}}. \tag{2.8}$$

Equation (2.7) shows that the short-run price elasticity of demand is given by the average of the within-industry elasticity of substitution $\eta$ and the between-industry elasticity of substitution $\epsilon$ weighted by the firm’s market share. Although $\eta$ and $\epsilon$ are constant, the short-run elasticity $\ln C_{ij,t}/\ln P_{ij,t}$ is time-varying given the two-layer competition.

Depending on $s_{ij,t}$, firm $j$’s short-run price elasticity of demand lies in $[\epsilon, \eta]$. On the one hand, when firm $j$’s market share $s_{ij,t}$ is small, within-industry competition becomes more relevant, so firm $j$’s price elasticity of demand depends more on $\eta$. In the extreme case of $s_{ij,t} = 0$, firm $j$ becomes atomistic and takes the industry price index $P_{i,t}$ as given. As a result, firm $j$’s price elasticity of demand is exactly $\eta$. On the other hand, when $s_{ij,t}$ is large, between-industry competition becomes more relevant and thus firm $j$’s price elasticity of demand depends more on $\epsilon$. In the extreme case of $s_{ij,t} = 1$, firm $j$ becomes the monopoly in industry $i$ and its price elasticity of demand is exactly $\epsilon$.

Each firm’s price has a non-negligible effect on the price index of the duopoly industry. The magnitude of this effect is determined by $s_{ij,t}$. Thus, when setting prices, each firm internalizes the effect of its own price on $P_{i,t}$, which in turn determines the demand for the industry’s goods given the between-industry elasticity of substitution $\epsilon$. If a continuum of firms exist in each industry, as in standard monopolistic competition models, each firm is atomistic and has no influence on the industry’s price index. Between-industry competition would have no impact on the firm’s price elasticity of demand.

**Dynamics of Customer Base.** Firms can attract consumers through undercutting prices or offering discounts. Lowering prices temporarily can have a persistent effect on increasing the firm’s demand due to consumption inertia, information frictions, and switching costs. Lower prices can attract new customers to buy the firm’s products, and some are likely to be satisfied and to become “loyal” to the firm. To capture this idea, following Phelps and Winter (1970) and Ravn, Schmitt-Grohé and Uribe (2006), we model the evolution of firm $j$’s customer base as

$$dM_{ij,t} = -\delta M_{ij,t}dt + z(C_{ij,t}/C_t)dt, \tag{2.9}$$
where parameter \( z \geq 0 \) determines the speed of customer base accumulation. Intuitively, a lower price \( P_{ij,t} \) increases the current demand \( C_{ij,t} \), allowing the firm to accumulate more customer base over \([t, t + dt]\). Parameter \( \delta > 0 \) captures customer base depreciation.

The firm’s price-setting decision depends on the value of \( z \) and its customer base \( M_{ij,t} \). To elaborate, if \( z = 0 \), the firm’s optimal price-setting decision is static, chosen to maximize current profits. If \( z > 0 \), the firm’s price-setting decision becomes dynamic, capturing the tradeoff between increasing contemporaneous profits by setting higher prices to exploit the existing customer base \( M_{ij,t} \) and increasing future profits by setting lower prices to accumulate more customer base for the future (see, e.g., Chevalier and Scharfstein, 1996; Gilchrist et al., 2017; Dou and Ji, 2018). Consistent with the empirical evidence, the customer base \( M_{ij,t} \) is sticky, which implies that the long-run price elasticity of demand is higher (but not by much) than the short-run elasticity (see, e.g., Rotemberg and Woodford, 1991; Gilchrist et al., 2017). In other words, coefficient \( z \) needs to be a small positive value to capture the sticky property of the customer base.

**Consumption Risks for the Long Run.** We directly model the dynamics of aggregate consumption demand \( C_t \). Thus, our model essentially incorporates product market frictions into a Lucas-tree model (see Lucas, 1978) with homogeneous agents and complete financial markets. Many extensions of the basic Lucas-tree model have been developed in the literature by incorporating frictions, market incompleteness, and multi-asset features.\(^8\) Our Lucas-tree economy has multiple sectors whose shares in the whole economy are determined endogenously in equilibrium. Specifically, \( C_t \) evolves according to

\[
\frac{dC_t}{C_t} = \theta_t dt + \sigma_c dZ_{c,t}, \quad \text{where} \quad d\theta_t = \kappa(\bar{\theta} - \theta_t) dt + \phi_{\theta} \sigma_c dZ_{\theta,t}. \tag{2.10}
\]

The consumption growth rate contains a persistent component \( \theta_t \), which determines

the conditional expectation of consumption growth (see, e.g., Kandel and Stambaugh, 1991, for empirical evidence). The parameter $\bar{\theta}$ captures the average long-run growth rate. The parameter $\kappa$ determines the persistence of the expected growth rate process. The parameter $\varphi_\theta$ determines the exposure to long-run growth shocks. The standard Brownian motions $dZ_{c,t}$ and $dZ_{\theta,t}$ are independent. Unlike other models with long-run growth shocks, the key feature of our model is that firm-level demand is endogenous and depends on strategic competition.

**Stochastic Discount Factors in Equilibrium.** The stochastic discount factor (SDF) $\Lambda_t$ is

$$\Lambda_t = \exp \left[ \int_0^t f_U(C_s, U_s) ds \right] f_C(C_t, U_t).$$

(2.11)

The SDF evolves according to

$$\frac{d\Lambda_t}{\Lambda_t} = -r_t dt - \gamma \sigma_c dZ_{c,t} - \frac{\gamma - \psi^{-1}}{h + \kappa} \varphi_\theta \sigma_c dZ_{\theta,t},$$

(2.12)

where $r_t$ is the interest rate given by

$$r_t = \beta + \psi^{-1} \theta_t - \frac{1}{2} \gamma (1 + \psi^{-1}) \sigma_c^2 - \frac{1}{2} \frac{(1 - \psi^{-1})(\gamma - \psi^{-1})}{(h + \kappa)^2} \varphi_\theta^2 \sigma_c^2.$$  

(2.13)

In equations (2.12) and (2.13), $h = \exp \left( \ln C/W \right)$ is the long-run deterministic steady-state consumption-wealth ratio determined in equilibrium. It suffices to solve the equilibrium by solving $h$. The detailed derivations are given in Online Appendix G.

### 2.2 Price-setting Dynamic Games

**Firm Production and Cash Flows.** The marginal cost for a firm to produce a flow of goods is $\omega$ with $\omega > 0$. That is, the firm incurs cost with intensity $\omega Y_{ij,t}$ in producing a flow of goods with intensity $Y_{ij,t}$ over $[t, t + dt]$. Given $C_{ij,t}$ and $P_{ij,t}$, firm $j$’s optimal net profits over $[t, t + dt]$ are

$$dE_{ij,t} = \max_{Y_{ij,t} \geq 0} \left[ P_{ij,t} \min \{ Y_{ij,t}, C_{ij,t} \} - \omega Y_{ij,t} \right] dt.$$  

(2.14)
In equilibrium, the firm would never produce more than the demand $C_{ij,t}$ because production is costly. Therefore, the firm finds it optimal to choose $P_{ij,t} > \omega$ and produce up to $C_{ij,t}$ in equilibrium, and the optimal net profits (2.14) can be written as

$$dE_{ij,t} = (P_{ij,t} - \omega) C_{ij,t} dt,$$

with $P_{ij,t} > \omega$. (2.15)

All net profits are paid out as dividends, as the model has no financial friction. We do not explicitly model firms’ entry or exit. In fact, most entries and exits in the data are associated with small firms (see, e.g., Haltiwanger, 2012; Tian, 2018), while our model focuses on the major players in an industry. Moreover, our model emphasizes the cyclicality of profit margins driven by firms’ endogenous time-varying collusion incentive, while evidence on the cyclicality of business startups is mixed given the existence of countervailing forces (see, e.g., Parker, 2009; Fairlie, 2013). The data do not suggest strong cyclical patterns in firms’ entry rates (see, e.g., Stangler and Kedrosky, 2010).

Substituting equation (2.6) into equation (2.15) gives

$$dE_{ij,t} = \Pi_{ij}(P_{ij,t}, \bar{P}_{ij,t}) M_{ij,t} dt,$$

where $\Pi_{ij}(P_{ij,t}, \bar{P}_{ij,t})$ is the locally deterministic profit rate defined by

$$\Pi_{ij}(P_{ij,t}, \bar{P}_{ij,t}) \equiv (P_{ij,t} - \omega) \left( \frac{P_{ij,t}}{\bar{P}_{ij,t}} \right)^{-\eta} P_{ij,t}^\epsilon C_t,$$

where $P_{ij,t}$ is defined in (2.6). (2.17)

Equation (2.17) shows that $\Pi_{ij}(P_{ij,t}, \bar{P}_{ij,t})$ depends on competitor $\bar{j}$’s product price $P_{ij,t}$ through the industry’s price index $\bar{P}_{ij,t}$, which reflects the direct externality of firm $\bar{j}$’s decisions. For example, if firm $\bar{j}$ sets a lower price $P_{ij,t}$, the price index $\bar{P}_{ij,t}$ will drop, increasing firm $j$’s price elasticity of demand. This will motivate firm $j$ to set a lower price $P_{ij,t}$, so the two firms’ price-setting decisions exhibit strategic complementarity.

**Markov Perfect Nash Equilibrium.** The duopolists in the same industry play a dynamic game (see Friedman, 1971), in which the stage games of setting prices are played continuously and repeated infinitely with exogenous and endogenous state variables varying over time. Formally, a subgame perfect Nash equilibrium for the dynamic game consists of a collection of price-setting strategies that constitute a Nash equilibrium for every history of the game. We do not consider all such equilibria, only Markov perfect
Nash equilibria that allow for collusive pricing arrangements enforced by punishment schemes. All strategies are allowed to depend upon both “payoff-relevant” physical states \(x_{i,t} = \{M_{i1,t}, M_{i2,t}, C_t, \theta_t\}\) in state space \(X\), as in Maskin and Tirole (1988a,b), and a set of indicator functions that track whether any firms have deviated from a collusive price-setting agreement in the past, as in Fershtman and Pakes (2000, Page 212).\(^9\)

Particularly, there exists a non-collusive equilibrium, which is the repetition of one-shot Nash equilibrium and thus is Markov perfect. Meanwhile, there also exist multiple Markov perfect collusive equilibria in which price-setting strategies are sustained by conditional punishment strategies.\(^10\)

**Non-collusive Equilibrium.** Non-collusive equilibrium is characterized by price-setting scheme \(P^N_i(\cdot) = (P^N_{i1}(\cdot), P^N_{i2}(\cdot))\), a pair of functions defined in state space \(X\), such that each firm \(j\) chooses price \(P_{ij,t} \equiv P_{ij}(x_{i,t})\) to maximize shareholder value \(V^N_{ij,t} \equiv V^N_{ij}(x_{i,t})\), under the assumption that its competitor \(\bar{j}\) will set the one-shot Nash-equilibrium price \(P^N_{ij,t} \equiv P^N_{ij}(x_{i,t})\). Following the recursive formulation in dynamic games for characterizing Markov perfect Nash equilibrium (see, e.g., Pakes and McGuire, 1994; Ericson and Pakes, 1995; Maskin and Tirole, 2001), optimization problems can be formulated recursively by Hamilton-Jacobi-Bellman (HJB) equations:

\[
0 = \max_{P_{ij,t}} \Lambda_t \Pi_{ij}(P_{ij,t}, P^N_{ij,t})M_{ij,t}dt + \mathbb{E}_t \left[ d(\Lambda_t V^N_{ij,t}) \bigg| P_{ij,t}, P^N_{ij,t} \right]. \tag{2.18}
\]

The solutions to the coupled HJB equations give the non-collusive equilibrium prices \(P^N_{ij,t}\) with \(j = 1,2\), which are chosen based on intertemporal tradeoff considerations because \(P^N_{ij,t}\) determines the continuation value \(V^N_{ij,t+dt}\) by altering the customer base \(M_{ij,t+dt}\).

**Collusive Equilibrium.** In collusive equilibrium, firms “implicitly collude” on setting higher prices to gain higher profit margins, with any deviation triggering a switch to non-collusive Nash equilibrium. The collusion is “implicit” in the sense that it can be enforced without relying on legal contracts. Each firm is deterred from breaking the collusion agreement because provoking fierce non-collusive competition is a credible

---

\(^9\)For notational simplicity, we omit the indicator states of historical deviations.

\(^{10}\)In the industrial organization and macroeconomics literature, this equilibrium is called collusive equilibrium or collusion (see, e.g., Green and Porter, 1984; Rotemberg and Saloner, 1986). Game theorists generally call it the equilibrium of repeated game (see Fudenberg and Tirole, 1991) in order to distinguish it from one-shot Nash equilibrium (i.e., our non-collusive equilibrium).
threat. Consider a generic collusive equilibrium in which firms follow a collusive price-setting scheme. Both firms can costlessly observe the other’s product price, so that price deviation can be detected and punished. The assumption of perfect information follows the work of Rotemberg and Saloner (1986), Haltiwanger and Harrington (1991), Staiger and Wolak (1992), and Bagwell and Staiger (1997).

In particular, if one firm deviates from collusive equilibrium, the competitor will implement a punishment strategy with probability \( \xi dt \) over \([t, t + dt]\). The punishment is to set non-collusive prices in the future forever. Setting non-collusive prices is considered a punishment strategy because the industry will switch to non-collusive equilibrium, which features the lowest profit margins.\(^{11}\) We use the idiosyncratic Poisson process \( N_{ij,t} \) to characterize whether a firm can successfully implement a punishment strategy. One interpretation of \( N_{ij,t} \) is that, with \( 1 - \xi dt \) probability, the deviator can persuade its competitor not to enter the non-collusive Nash equilibrium over the period \([t, t + dt]\).\(^{12}\) Thus, the punishment intensity \( \xi \) can be viewed as a parameter governing the credibility of the punishment for deviating behavior. A higher \( \xi \) leads to a lower deviation incentive and thus sustains collusion better.

Formally, the set of incentive-compatible collusion agreements, denoted by \( \mathcal{C} \), consists of all continuous price-setting schemes \( P^C_i(\cdot) \equiv (P^C_{i1}(\cdot), P^C_{i2}(\cdot)) \), such that the incentive compatibility (IC) constraints are satisfied:

\[
V^D_{ij}(x) \leq V^C_{ij}(x), \quad \text{for all } x \in X \text{ and } j = 1, 2. \tag{2.19}
\]

Here, \( V^C_{ij,t} \equiv V^C_{ij}(x_{i,t}) \) is firm \( j \)'s value in the collusive equilibrium, pinned down recursively according to

\[
0 = \Lambda_t \Pi_{ij} (P^C_{ij,t}, P^C_{ij,t}) M_{ij,t} dt + \mathbb{E}_t \left[ d(\Lambda_t V^C_{ij}) \middle| P^C_{ij,t}, P^C_{ij,t} \right], \tag{2.20}
\]

where \( P^C_{ij,t} \equiv P^C_{ij}(x_{i,t}) \) with \( j = 1, 2 \) are the collusive prices. Further, \( V^D_{ij,t} \equiv V^D_{ij}(x_{i,t}) \) is firm

\(^{11}\)We adopt the non-collusive equilibrium as the incentive-compatible punishment for deviation, which follows the literature (see, e.g., Green and Porter, 1984; Rotemberg and Saloner, 1986). We can extend the setup to allow for finite-period punishment. The quantitative results are not altered significantly if the punishment lasts long enough.

\(^{12}\)Ex-post renegotiations can happen because the non-collusive equilibrium is not “immune to collective rethinking” or renegotiation-proof (see Farrell and Maskin, 1989). The strategy we consider is essentially a probabilistic punishment strategy.
\( j \)'s highest shareholder value if it deviates from the implicit collusion:

\[
0 = \max_{p_{ij,t}} \Lambda_i \Pi_{ij}(p_{ij,t}, p_{ij,t}^C) M_{ij,t} \, dt + \mathbb{E}_t \left[ d(\Lambda_i V_{ij,t}^D) \left| p_{ij,t}, p_{ij,t}^C \right. \right] + \Lambda_t \left( V_{ij,t}^N - V_{ij,t}^D \right) \, \zeta \, dt,
\]

if not punished  

if punished

In fact, there exist infinitely many elements in \( \mathcal{C} \) and hence infinitely many collusive equilibria. We focus on a subset of \( \mathcal{C} \), denoted by \( \mathcal{C} \), which consists of all price-setting schemes \( p^C_i(\cdot) \) such that the IC constraints (2.19) are binding state by state, i.e., \( V_{ij,t}^D(x) = V_{ij,t}^C(x) \) for all \( x \in \mathcal{X} \) and \( j = 1, 2 \).\(^{13}\) It is obvious that the subset \( \mathcal{C} \) is nonempty since it contains the non-collusive Nash equilibrium price-setting scheme. We further narrow our focus to the “Pareto-efficient frontier” of \( \mathcal{C} \), denoted by \( \mathcal{C}_p \), consisting of all pairs of \( p^C_i(\cdot) \) such that there does not exist another pair \( \tilde{p}^C_i(\cdot) \in \mathcal{C} \) with \( \tilde{p}_{ij}(x) \geq p_{ij}(x) \) for all \( x \in \mathcal{X} \) and \( j = 1, 2 \), and with strict inequality holding for some \( x \) and \( j \).\(^{14}\) Our numerical algorithm follows a method similar to those of Abreu, Pearce and Stacchetti (1990), Cronshaw and Luenberger (1994), Pakes and McGuire (1994), and Judd, Yeltekin and Conklin (2003).\(^{15}\) Deviation never occurs on the equilibrium path. Using the one-shot deviation principle (see Fudenberg and Tirole, 1991), it is clear that the collusive equilibrium characterized above is a subgame perfect Nash equilibrium.

### 2.3 Price Wars and Long-run Growth

In this subsection, we illustrate price war risks in collusive equilibria. We show that price war risks endogenously arise from long-run growth shocks and as a result, they amplify industries’ exposure to long-run growth shocks. All figures in Sections 2 - 4 are plotted based on the calibrated parameter values in Table 11.

**Key Mechanisms.** In our model, price war risks endogenously arise from long-run growth shocks. When the long-run growth rate \( \theta_t \) declines, profit margins follow suit because of endogenous declines in collusive prices \( p_{ij,t}^C \). Intuitively, the incentive to collude

\(^{13}\)Such equilibrium refinement in a GE framework is similar in spirit to Alvarez and Jermann (2000).

\(^{14}\)It can be shown that the “Pareto-efficient frontier” is not empty based on the fundamental theorem of the existence of Pareto-efficient allocations (see, e.g., Mas-Colell, Whinston and Green, 1995), as \( \mathcal{C} \) is non-empty and compact, and the order we are considering is complete, transitive, and continuous.

\(^{15}\)Proving the uniqueness of the equilibrium under our selection criterion is beyond the scope of the paper. We use different initial points in our numerical algorithm and find robust convergence to the same equilibrium.
on higher prices depends on how much firms value future revenues relative to their contemporaneous revenues. By deviating from collusive price-setting schemes, firms can obtain higher contemporaneous revenues and expand their customer base in the short run; however, firms run into the risk of losing future revenue because once the deviation is punished by the other firm, non-collusive equilibrium will be implemented. During periods with low long-run growth, firms expect lower future cash flows, so the future punishment for deviation becomes less costly. This situation makes firms more impatient for cash flows and gives them a stronger incentive to undercut their competitors’ prices.\textsuperscript{16} Therefore, a decline in long-run growth intensifies price competition, and reduces equilibrium product prices by weakening firms’ market power, leading to a price war (see, e.g., Rotemberg and Saloner, 1986; Lambson, 1987; Haltiwanger and Harrington, 1991).

Importantly, the decrease in profit margins after negative long-run growth shocks is caused by intensified competition and reduced market power rather than weakening aggregate demand. Although the two firms still maintain collusive price-setting schemes, the collusive prices are endogenously lower because of lower long-run growth rates. To elaborate on this point, in Panel A of Figure 2, we plot the supply and demand curves for firm \textit{j}’s product in collusive equilibrium. Fixing firm \textit{j}’s customer base \(M_{ij,t}\), the supply curve (blue solid line) is flat because firm \textit{j} agrees to sell its product at collusive price \(P_{ij,t}^C\), irrespective of the level of its contemporaneous demand. The demand curve (black dashed line) is downward-sloping, and represents equation (2.6). The initial equilibrium is given by point \(O_0\).

A negative shock to the long-run growth rate \(\theta_t\) reduces collusion incentives and weakens market power, shifting the supply curve downward to the blue dotted line. If the demand curve were unchanged, the new equilibrium would feature a much lower price and a much higher demand for firm \textit{j}’s goods (point \(O’\)). However, the demand curve also shifts downward to the black dash-dotted line, because the industry’s price index \(P_i\) endogenously declines dramatically due to the self-fulfilling price undercutting. The new equilibrium is given by point \(O_1\), featuring a price war with a much lower equilibrium price and a slightly higher equilibrium demand for firm \textit{j}’s goods. As illustrated in Panel D, the price war driven by negative long-run growth shocks is caused initially by the

\textsuperscript{16}The intuition is related to the Folk Theorem in game theory. The Folk Theorem asserts that provided players are sufficiently patient, repeated interaction can allow many subgame perfect outcomes, but more importantly subgame perfection can allow virtually any outcome in the sense of average payoffs. The effective discount rate is given approximately by \(r_t - \theta_t\). Thus, the periods with low \(\theta_t\) feature high discount rates and less patience.
A downward shift in firm j’s supply curve owing to the self-fulfilling decline in its market power. The shift in the supply curve reduces firm j’s price $P_{ij,t}$ and the industry’s price index $P_{i,t}$, which in turn increases firm j’s price elasticity of demand. When firm j’s price elasticity of demand is higher, forming a collusion is more difficult from competitor j’s perspective, because now firm j has more incentives to undercut prices. The diminished collusion incentive further induces a downward shift in firm j’s supply curve, which further reduces firm j’s price $P_{ij,t}$ and the industry’s price index $P_{i,t}$, increasing firm j’s price elasticity of demand. Such a feedback loop leads to self-fulfilling weakened market power and thus price wars.

![Diagram](image)

**Figure 2:** Impact of demand and supply shocks on equilibrium prices and quantities.

By contrast, Panels B and C highlight that short-run shocks cannot lead to self-fulfilling declines in firms’ market power. Panel B shows that a negative short-run demand shock (i.e., a decline in $C_t$) only generates a downward shift in the demand curve without affecting the supply curve. As a result, the change in equilibrium price and demand depends purely on the price elasticity of supply. Given a flat supply curve (infinite price elasticity of supply), firm j’s price in the new equilibrium (point $O_1$) is exactly the same.
as the initial equilibrium price (point $O_0$). Panel C shows that a negative short-run supply shock (i.e., an increase in $\omega$) only generates an upward shift in the supply curve without affecting the demand curve. As a result, the change in equilibrium demand and supply purely depends on the price elasticity of demand. As the demand curve is downward-sloping, the new equilibrium (point $O_1$) has a higher price and a lower demand for firm $j$’s goods.

Thus, the price war caused by negative long-run growth shocks involves shifts in both the demand and the supply curves owing to self-fulfilling intensified competition and weakened market power. Price war risks are generated by long-run growth shocks, not by short-run demand or supply shocks. In Online Appendix E.2, we show that price war risks subside when growth shocks become less persistent; specifically, price war risks become negligible if only short-run shocks are considered.

**Price War Risks.** We illustrate price war risks numerically in Figure 3. By exploiting the model’s homogeneity in $M_{i,t}C_t$, we can reduce the model to two state variables, $M_{i1,t}/M_{i,t}$ and $\theta_t$ (see Online Appendix H.1 for more discussion). We solve normalized firm values $\nu^C_{ij}(M_{i1}/M_{i,t}, \theta_t)$ and product prices $P^C_{ij,t}(M_{i1}/M_{i,t}, \theta_t)$ in a collusive equilibrium.

Panels A and B plot firm 1’s equilibrium product price for different long-run growth rates $\theta_t$. The blue solid line represents high long-run growth rates (i.e., $\theta_H$). As shown in Panel A, a decrease in the long-run growth rate (from $\theta_H$ to $\theta_L$) leads to a price war. Panel B shows dramatically narrowed profit margins.

Panel C illustrates the magnitude of price war risks by plotting the difference in profit margins between periods with high and low long-run growth rates. Price war risks display an inverted U shape, and are the largest when the two firms have comparable customer base shares (i.e., $M_{i1}/M_i = 0.5$). Intuitively, in an almost monopolistic industry, firms have weak collusion incentives because the difference between collusive and non-collusive profit margins is small. As a result, the variation in profit margins is small when long-run growth rates fluctuate. More discussion is presented in Online Appendix D.2.

The time-varying collusion incentive amplifies the effect of long-run growth shocks because during periods with low long-run growth rates, firms not only face low demand, but

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17If the marginal cost of production increases with output, the supply curve would be upward-sloping. Then, a negative short-run demand shock would result in a lower equilibrium price and a lower equilibrium demand for firm $j$’s good. This is the standard negative effect of demand shocks on equilibrium prices in models with decreasing-return-to-scale production technology. We intentionally assume a constant marginal cost of production $\omega$ to eliminate this effect and cleanly present the price war effect.
they also have low profit margins caused by weakened market power. To illustrate this amplification effect, we calculate the industry-level beta $\beta_{i,t}$ as the value-weighted firm-level beta $\beta_{ij,t}$:

$$\beta_{i,t} = \sum_{j \in F} w_{ij,t} \beta_{ij,t}, \text{ with } \beta_{ij,t} = \frac{v_{ij,t}(\theta_H)}{v_{ij,t}(\theta_L)} - 1 \text{ and } w_{ij,t} = \frac{v_{ij,t}(\theta_L)}{\sum_{k \in F} v_{ik,t}(\theta_L)}.$$

Panel D shows that the industry’s beta displays an inverted U shape (see the blue solid line) due to the price war risks exhibiting an inverted U shape. As a benchmark, the red dotted line plots the industry’s beta in the absence of price war risks (i.e., under the counterfactual that collusive prices do not change). When the two firms have comparable customer base shares, price war risks significantly amplify the industry’s exposure to long-run growth shocks owing to the large endogenous variation in profit margins.

### 3  The Extended Model with Full-blown Price Wars

In the baseline model, firms can costlessly monitor their competitors’ potential deviation from the collusive price-setting scheme, and as a result, the collusive price-setting scheme is always maintained in equilibrium. In that model, price wars are essentially endogenous declines in firms’ prices within the collusive regime. In this section, we introduce monitoring costs to generate endogenous full-blown price wars that involve shifts from the collusive regime to the non-collusive regime.
3.1 Imperfect Monitoring

Monitoring their competitors’ potential deviation over \([t, t + dt]\) requires firms’ shareholders to make an effort with intensity \(v_{i,t}\) per unit of customer base. The effort \(v_{i,t}\) can be viewed as an industry-specific non-pecuniary monitoring cost; it follows a two-state Markov process taking values in \(\mathcal{V} \equiv \{V_L, V_H\}\) with \(v_L = 0\) and \(v_H > 0\). The transition intensity from \(v_L\) to \(v_H\) is \(q^{V_L,V_H}\), and that from \(v_H\) to \(v_L\) is \(q^{V_H,V_L}\).

The fact that firms monitor each other is common knowledge. So if either firm chooses not to monitor the other, both firms would set non-collusive prices. The implication is that when deciding whether or not to implement collusive pricing, both firms must weigh the benefit of collusion against the disutility of making an effort to monitor the other’s price. If the benefit is lower than the cost for either firm, both firms will abandon collusion temporarily and enter into non-collusive price competition; hereafter, such a regime shift is referred to as a full-blown price war (see, e.g., Friedman, 1971; Green and Porter, 1984; Porter, 1985; Abreu, Pearce and Stacchetti, 1986).

Formally, we define the extended state vector as \(y_{i,t} \equiv (x_{i,t}, v_{i,t})\) and the extended state space as \(\mathcal{Y} \equiv \mathcal{X} \times \mathcal{V}\). The set of incentive-compatible collusion agreements, denoted by \(\mathcal{C}\), consists of all continuous price-setting schemes \(\mathcal{P}^C_i(\cdot) = (\mathcal{P}^C_{i_1}(\cdot), \mathcal{P}^C_{i_2}(\cdot))\), a pair of functions defined in state space \(\mathcal{Y}\), such that the following IC constraints are satisfied:

\[
V^C_{ij}(y) \leq V^D_{ij}(y), \quad \text{for all } y \in \mathcal{Y} \text{ and } j = 1, 2. \tag{3.1}
\]

Here \(V^D_{ij}(y_{i,t})\) is the highest shareholder value for firm \(j\) if it decides to deviate from the implicit collusion. Its theoretical characterization can be found in Appendix B. Further, \(V^C_{ij}(y_{i,t})\) is firm \(j\)’s value in the equilibrium where collusion and non-collusion are chosen optimally, pinned down by

\[
0 = \begin{cases} 
\Lambda_t \Pi_{ij}(P^C_{ij,t}, p^C_{ij,t})M_{ij,t}dt + \mathbb{E}_t \left[ d \left( \Lambda_t V^C_{ij,t} \right) \bigg| P^C_{ij,t}, P^C_{ij,t} \right], & \text{if } \Gamma_{ij,t} \geq v_{i,t} \text{ for all } j, \quad (I) \\
\max_{P_{ij,t}} \Lambda_t \Pi_{ij}(P_{ij,t}, p^N_{ij,t})M_{ij,t}dt + \mathbb{E}_t \left[ d \left( \Lambda_t V^C_{ij,t} \right) \bigg| P_{ij,t}, p^N_{ij,t} \right], & \text{otherwise}, \quad (II)
\end{cases}
\]

where \(P^C_{ij,t} \equiv P^C_{ij}(y_{i,t})\) with \(j = 1, 2\) in (I) are collusive prices and \(P^N_{ij,t} \equiv P^N_{ij}(y_{i,t})\) with \(j = 1, 2\) in (II) are non-collusive prices.

---

\(^{18}\)Intuitively, because monitoring is common knowledge, if firm \(j\) does not monitor, firm \(j\) will know and will rationally deviate from collusive pricing. Firm \(j\) knows this. As a result, firm \(j\) will also deviate conditional on firm \(j\)’s deviation, and so on. This scenario completely rules out any collusive equilibrium, making the non-collusive equilibrium a unique one.
where $\pi^C_{i,t} = (p^C_{i_1,t}, p^C_{i_2,t})$ and $\pi^N_{i,t} = (p^N_{i_1,t}, p^N_{i_2,t})$ are the collusive price and non-collusive price vectors, respectively. The intuition behind characterizations (I) and (II) of $\Pi^C_{ij}(y_i,t)$ is straightforward: firms of industry $i$ would choose to collude on the price-setting scheme $\pi^C_{i,t}$ over $[t, t + dt]$ if the benefit of collusion exceeds the non-pecuniary cost (i.e., $\Gamma_{ij,t} \geq \nu_{i,t}$ for both $j = 1, 2$). In this case, the relationship between $\Pi^C_{ij}(y_i,t)$ and $\Pi^C_{ij}(y_{i,t+dt})$ is characterized by (I). Alternatively, firms of industry $i$ would choose not to collude and would enter a full-blown price war over $[t, t + dt]$ if the benefit of collusion is smaller than the non-pecuniary cost (i.e., $\Gamma_{ij,t} < \nu_{i,t}$ for some $j$). In this case, the relationship between $\Pi^C_{ij}(y_i,t)$ and $\Pi^C_{ij}(y_{i,t+dt})$ is characterized by (II).

Because the benefit of collusion is monotonically increasing with the long-run growth rate $\theta_t$, the benefit of collusion can dominate the largest possible non-pecuniary cost $\nu_H$ when $\theta_t$ is high enough. In this case, full-blown price wars would never occur. When $\theta_t$ is low, the benefit of collusion can be dominated by $\nu_H$, and full-blown price wars would break out once $\nu_{i,t}$ switches from $\nu_L$ to $\nu_H$.

**Numerical Illustrations.** We illustrate full-blown price wars numerically in Figure 4. In Panel A, we consider a significant decrease in the long-run growth rate $\theta_t$ from $\theta_H$ to $\theta_L$. In the state of $\theta_L$, firms find the collusion benefit too small to justify monitoring cost $\nu_H$, so they enter a full-blown price war by setting non-collusive prices (black dashed line). By contrast, with $\nu_i = \nu_L$, full-blown price wars do not occur (gray dash-dotted line).

In Panel B, we consider a moderate decrease in $\theta$ from $\theta_H$ to $\theta_M < \theta_L$. In the state of a medium long-run growth rate $\theta_M$, whether the two firms collude with each other depends on their relative customer base shares because the benefit from collusion is smaller in a more concentrated industry (see our earlier discussion for Panel C of Figure 3). In particular, when the two firms’ customer base is not very different (i.e.,

\[ \Gamma_{ij,t} \equiv \Pi_{ij}(\pi^C_{i,t}) - \Pi_{ij}(\pi^N_{i,t}) + \frac{E_t[\Lambda_{t+dt} V^C_{ij,t+dt} | \pi^C_{i,t}] - E_t[\Lambda_{t+dt} V^C_{ij,t+dt} | \pi^N_{i,t}]}{\Lambda_t M_{ij,t} dt} \]

\[ \text{gain in profits} \]

\[ \text{gain in continuation values} \]
0.25 ≤ \( M_{i1} / M_i \leq 0.75 \), they will choose to maintain collusion, even though a lower long-run growth rate dampens the collusion incentive and results in lower collusive profit margins. However, when the industry is more concentrated (i.e., \( M_{i1} / M_i < 0.25 \) or \( M_{i1} / M_i > 0.75 \), the two firms will choose instead to enter a full-blown price war.

Importantly, although full-blown price wars occur only when the long-run growth rate is low, a decrease in the long-run growth rate increases the probability of entering a full-blown price war in the near future. Therefore, the occurrence of regime switching is endogenous in our model, but more importantly so is its time-varying probability. In Panel C, we plot the simulated probability that a full-blown price war will take place in the next five years. When the annualized long-run growth rate is above 5\%, the probability for a full-blown price war to occur is virtually zero. When the long-run growth rate decreases to −15\%, the probability increases to 37\%.

4 The Full Model with Creative Destruction

In this section, we further extend the model with full-blown price wars by allowing firms to snatch competitors’ customer bases through innovation. We emphasize the competition role of innovation activities for two reasons. First, product innovation is an important channel through which firms snatch competitors’ customer bases, besides strategic price undercutting. We show that the extent to which firms can collude crucially depends on their capacities for radical innovation, which determines the future market structure (i.e., the concentration of customer base in the industry). Second, introducing the competition...
role of innovation yields new important cross-sectional predictions, thereby expanding the scope of testing our asset pricing theory of price war risks. In particular, our model predicts that industries with a lower capacity for radical innovation are more exposed to price war risks and long-run growth shocks, and thus have higher expected stock returns.

4.1 Modeling the Competition Role of Innovation

Firms conduct innovation, with a constant success rate $\mu$ that is independent across firms. A successful innovation allows the innovating firm to snatch a $\tau_{ij,t}$ fraction of its competitor’s customer base, where $\tau_{ij,t}$ follows the Bernoulli distribution with two values:

$$
\tau_{ij,t} = \begin{cases} 
\tau_i, & \text{with probability } \lambda_{i,t}, \\
\tau_d, & \text{with probability } 1 - \lambda_{i,t}.
\end{cases}
$$

(4.1)

We assume $0 \approx \tau_i << \tau_d \approx 1$ to capture two different types of innovation. The event of snatching a small fraction $\tau_i$ reflects a successful non-radical innovation that cannot disrupt peer firms.\(^{20}\) One example of such type of innovation is Motorola Razr flip phone, which is similar to existing phones in the mobile phone market. The event of snatching a large fraction $\tau_d$ reflects a successful radical innovation (see Jaffe, 1986; Christensen, 1997; Manso, 2011; Kelly et al., 2018) that creates distinctive technologies and products to replace the existing ones, and disrupt peer firms. One example of radical innovation is the disruption of the mobile phone market by the first Apple iPhone. Variable $\lambda_{i,t}$ captures innovation similarity, which is the only industry characteristic that is ex-ante heterogeneous across industries. Intuitively, a lower $\lambda_{i,t}$ means that industry $i$ has a higher capacity for radical innovation at time $t$. Similarly, Acemoglu et al. (2018) also emphasize the importance of heterogeneous innovation capacities across firms and industries. In Section 5.1, we use patent data to construct an innovation similarity measure to capture $\lambda_{i,t}$.

In our model, the long-run growth of aggregate consumption $C_t$ is specified exogenously by equation (2.10). However, in principle, firms’ innovation can also drive the growth of aggregate consumption. In general, innovation can affect the economy through two channels: the technology diffusion channel and the product-market competition channel (see, e.g., Aghion et al., 2005; Bloom, Schankerman and Van Reenen, 2013; Acemoglu, Akcigit and Kerr, 2016). Our model focuses on the product-market competition channel.

\(^{20}\)The non-radical innovation is often referred to as incremental innovation in the literature.
and its asset pricing implications (see, e.g., Gârleanu, Kogan and Panageas, 2012; Kogan et al., 2017; Gârleanu and Panageas, 2018). We intentionally shut down the technology diffusion channel for endogenous growth to keep the model simple and transparent.

We assume that the value of $\lambda_{i,t}$ remains the same unless it is hit by an idiosyncratic Poisson shock with rate $\chi$. Conditional on receiving the Poisson shock, a new characteristic is drawn randomly from the set $\{\lambda_1, ..., \lambda_N\}$ each with equal probability, where $0 < \lambda_1 < ... < \lambda_N \leq 1$. With the competition role of innovation, the dynamics of the customer base (2.9) is modified as

$$\text{d}M_{ij,t} = -\delta M_{ij,t} \text{d}t + z(C_{ij,t}/C_t) \text{d}t + \tau_{ij,t} M_{ij,t} \text{d}I_{ij,t} - \tau_{ij,t} M_{ij,t} \text{d}I_{\bar{j}j,t} \quad (4.2)$$

where $I_{ij,t}$ and $I_{\bar{j}j,t}$ are independent Poisson processes capturing the success of firm $j$'s and $\bar{j}$'s innovation, respectively.

**Cross-Sectional Implications.** We now study the implication of innovation characteristics for an industry’s exposure to price war risks. To fix ideas, consider two industries different in innovation similarity $\lambda_{i,t}$. Panel A of Figure 5 plots the collusive equilibrium prices in the two industries with high and low long-run growth rates.

Two main implications are worth mentioning. First, profit margins are much lower in the industry with a higher capacity for radical innovation (i.e., lower $\lambda_{i,t}$) regardless of the long-run growth rate $\theta_t$. Second, profit margins drop more deeply in the industry with a lower capacity for radical innovation (i.e., higher $\lambda_{i,t}$) in response to a decline in $\theta_t$ from $\theta_H$ to $\theta_L$; in other words, the cash flows of firms in such industries are more exposed to long-run growth shocks. As discussed in Section 2.3, the incentive to collude exhibits an inverted U shape and becomes the lowest in monopoly industries. An industry with a higher capacity for radical innovation is more likely to be concentrated in the future because one firm may capture a large market share upon successful radical innovation. So even if the two firms currently have comparable customer base shares, the possibility of

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21Incorporating the asset pricing implications of endogenous long-run growth driven by innovations, like Gârleanu, Panageas and Yu (2012) and Corhay, Kung and Schmid (2017), would be an interesting extension of our model, but it is out of the scope of the current paper.

22Moreover, when $M_{ij,t}/M_{ij,t} \to 0$, firm 1’s profit margin in both industries converges to the profit margin determined by the within-industry elasticity of substitution $\eta$, as we have shown in equation (2.7). When $M_{ij,t}/M_{ij,t} \to 1$, firm 1’s profit margin in both industries converges to the profit margin determined by the between-industry elasticity of substitution $\epsilon$. The limits of profit margins are almost the same in the two industries because all firms face exactly the same $\eta$ and $\epsilon$. 

26
Note: When plotting Panel B, we consider two firms having equal (revenue) market shares in each industry. We simulate the industry dynamics for 10 years and calculate the dispersion of market shares, measured by the standard deviation of the two firms’ market shares in year 10. Panel B plots the simulated probability density function of the dispersion of market shares across 10,000 simulations for each industry. The industry with a higher capacity for radical innovation will have a more right-skewed distribution (red bars) than the industry with a lower capacity for radical innovation (blue bars). This pattern also holds for the steady state.

Figure 5: Implications across industries with different capacities for radical innovation.

having a successful radical innovation in the future will largely dampen today’s collusion incentive, resulting in low collusive prices and low profit-margin sensitivities to long-run growth shocks. Our idea echoes and formalizes the important generic insight of Maskin and Tirole (1988a) and Fershtman and Pakes (2000): the tacit collusion among oligopolists arises in industries where each firm expects others to remain in the market for a long time; but if only one firm will end up dominating the market in the future, the incentive for collusive behavior becomes weaker.

Panel B confirms the above intuition by showing that the industry with a higher capacity for radical innovation has more dispersed market shares in the future. In Panel C, we compare the two industries’ exposure to long-run growth shocks for different levels of industry concentration, as reflected by firm 1’s customer base share. Conditional on the same level of concentration, firms in the industry with a higher capacity for radical innovation are less exposed to long-run growth shocks. The industry-level value-weighted beta exhibits an inverted U-shape in both industries. The difference in beta across the two industries is the largest when the two firms within the same industry have comparable customer base shares ($M_{i1}/M_i = 0.5$).

5 Empirical Analyses

In this section, we empirically test the main predictions of our model, and as a companion section, Section 6 reports the quantitative analyses of our model’s implications. We first
use patent data to construct an innovation similarity measure for industry characteristic $\lambda_{i,t}$ in our model. We then test the mechanism of our model by examining the time-series and cross-sectional property of profit margins. We find that average profit margins comove positively with long-run growth. In the cross section, we find that the profit margins of industries with higher innovation similarity are higher and more exposed to long-run growth shocks. We further exploit detailed product-price data and find these industries are more exposed to price war risks, and their product prices decrease to a greater extent in response to negative long-run growth shocks. In particular, these industries were more likely to engage in price wars after the Lehman crash in 2008. Finally, we test the asset pricing implications of our model. We find that the stock returns and dividend growth of industries with higher innovation similarity are more exposed to long-run growth shocks. These industries have higher average excess returns and risk-adjusted returns.

5.1 Data and the Innovation Similarity Measure

We first introduce the patent data and explain the construction of the innovation similarity measure. We then provide external validation tests for our measure and contrast it with the product similarity measure developed by Hoberg and Phillips (2016).

Patent Data and Our Merged Sample. We obtain the patent issuance data from PatentsView, a patent data visualization and analysis platform. PatentsView contains detailed and up-to-date information on granted patents from 1976 onward. Its coverage of recent patenting activities is more comprehensive than the National Bureau of Economic Research (NBER) patent data (see, Hall, Jaffe and Trajtenberg, 2001) and the patent data assembled by Kogan et al. (2017) combined. Patent assignees in PatentsView are disambiguated and their locations and patenting activities are tracked longitudinally. PatentsView categorizes patent assignees into different groups, such as corporations, individuals, and government agencies. The platform also provides detailed information of individual patents, including their grant dates and technology classifications.

We match patent assignees in PatentsView to US public firms in CRSP/Compustat,

\footnote{The PatentsView data cover all patents granted by the US Patent and Trademark Office (USPTO) from 1976 to 2017, while the NBER data and the data assembled by Kogan et al. (2017) only cover patents granted up to 2006 and 2010, respectively.}
and to US private firms and foreign firms in Capital IQ. Private firms are included in our sample because they play an important role in industry competition (see, e.g., Ali, Klasa and Yeung, 2008). We drop patents granted to individuals and government agencies. We use a fuzzy name-matching algorithm to obtain a pool of potential matches from CRSP/Compustat and Capital IQ for each patent assignee in PatentsView. We then manually screen these potential matches to identify the exact matches based on patent assignees’ names and addresses. In Online Appendix C.2, we detail our matching procedure. In total, we match 2,235,201 patents to 10,139 US public firms, 132,100 patents to 3,080 US private firms, 241,582 patents to 300 foreign public firms, and 35,597 patents to 285 foreign private firms. The merged sample covers 13,804 firms in 523 four-digit SIC industries from 1976 to 2017.

**Innovation Similarity Measure.** In light of previous studies (see, e.g., Jaffe, 1986; Bloom, Schankerman and Van Reenen, 2013), we construct our industry-level innovation similarity measure (called “innosimm”) based on the technology classifications of an industry’s patents. In Appendix C, we detail the construction method for innosimm. An industry in which firms have more similar patents has a higher innosimm.

The primary purpose of constructing the innosimm measure is to approximate industries’ capacity for radical innovation because radical innovation is necessarily distinctive to peers’. So industries with a lower innosimm tend to have a higher capacity for radical innovation. Our approach of approximating the capacity for radical innovation using innosimm is similar in spirit to those adopted by Bloom, Schankerman and Van Reenen (2013), Lin, Liu and Manso (2016), and Kelly et al. (2018).

Panel A of Figure 6 presents the time series of several industries’ innosimm measures. In the industry of “Search, Detection, Navigation, Guidance, Aeronautical, and Nautical Systems and Instruments”, innosimm is low throughout our sample period, suggesting that firms in this industry are able to consistently generate radical innovation. Innosimm keeps increasing in the “Drilling Oil and Gas Wells” industry, while it peaks in the early 2000s in the “Rubber and Plastics Footwear” industry.

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24 Capital IQ is one of the most comprehensive datasets covering private and foreign firms.

25 We use four-digit SIC codes in Compustat and Capital IQ to identify the industries of patent assignees. Both Compustat and Capital IQ are developed and maintained by S&P Global, and the SIC industry classifications in these two datasets are consistent with each other.
Figure 6: Examples of innosimm, consumption growth, and long-run growth shocks.

Validation of the Innosimm Measure. We perform two external validation tests for justifying the relationship between our innosimm measure and industries’ capacity for radical innovation. In the first validation test, we examine the relationship between innosimm and the brand perception of consumers. If a lower innosimm captures a higher capacity for radical innovation in an industry, we expect consumers to perceive firms’ brands to be more distinctive from their peers’ within industries of lower innosimm. We test this hypothesis by examining the relationship between innosimm and the relative change in brand distinctiveness over time, measured using the BAV consumer survey data. Column (1) of Table 1 shows that innosimm is negatively correlated with the two-year percentage change in industry-level brand distinctiveness, suggesting that industries with higher innosimm are associated with lower brand distinctiveness in the future.

We emphasize that our innosimm measure is conceptually different from the product similarity measure (called “prodsimm”) constructed by Hoberg and Phillips (2016). Innosimm captures to the extent to which firms in an industry can differentiate their products from peers’ products through innovation. It is a forward-looking measure

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26 The BAV database is regarded as the world’s most comprehensive database of consumers’ brand perception (see, e.g., Gerzema and Lebar, 2008; Keller, 2008; Mizik and Jacobson, 2008; Aaker, 2012; Lovett, Peres and Shachar, 2014; Tavassoli, Sorescu and Chandy, 2014; Dou et al., 2018). The BAV brand perception survey consists of more than 870,000 respondents in total, and it is constructed to represent the US population according to gender, ethnicity, age, income group, and geographic location.
that captures the (potential) similarity/distinctiveness of firms’ products in the future. Prodsimm, on the other hand, is derived from textual analyses based on firms’ current product descriptions (see Hoberg and Phillips, 2016). Therefore, it reflects the similarity of products produced by different firms as of today, rather than the potential similarity/distinctiveness of firms’ products in the future. In other words, prodsimm contains little information, if at all, about firms’ innovation activities, which are the necessary inputs for making products distinctive in the future. The conceptual difference between the two measures is confirmed formally by column (2) of Table 1, which shows that innosimm is unrelated to prodsimm.\textsuperscript{27} In Sections 5.3 and 5.4, we further show that, unlike innosimm, prodsimm is neither related to industries’ price war risks nor priced in the cross section.

Table 1: Validation of the innosimm measure (yearly analysis).

<table>
<thead>
<tr>
<th></th>
<th>(1) Industry-level percentage changes from year $t$ to $t+2$ (%)</th>
<th>(2) Brand distinctiveness Prodsimm</th>
<th>(3) Within-industry dispersion of market shares (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Innosimm$_t$</td>
<td>-0.69***</td>
<td>0.10</td>
<td>-1.26***</td>
</tr>
<tr>
<td></td>
<td>[-3.06]</td>
<td>[0.04]</td>
<td>[-2.60]</td>
</tr>
<tr>
<td>Year FE</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Observations</td>
<td>2466</td>
<td>5906</td>
<td>8967</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.298</td>
<td>0.002</td>
<td>0.008</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.033</td>
</tr>
</tbody>
</table>

Note: This table shows the relation of our innosimm measure with measures of brand distinctiveness, product similarity, and the dispersion of market shares at the four-digit SIC industry level. In column (1), the dependent variable is the two-year percentage change in industry-level brand distinctiveness. We compute industry-level brand distinctiveness as follows. First, from the BAV consumer survey we obtain brand-level brand distinctiveness, which is the fraction of consumers who consider a brand to be distinctive from others in the same industry. Next, we aggregate the brand-level distinctiveness measure to the firm level, and then further aggregate it to the four-digit SIC industry level. Prodsimm (i.e., product similarity measure) comes from Hoberg and Phillips (2016), and it is derived from textual analyses based on the business description in 10-K filings. We download the firm-level prodsimm from the Hoberg and Phillips Data Library, and aggregate it to the four-digit SIC industry level. The dispersion of market shares (in percent) is defined as the standard deviation of firms’ market shares (measured by sales) within the four-digit SIC industry. The sample in column (1) spans the period from 1993 to 2017, and the sample in column (2) spans the period from 1996 to 2015. The sample in columns (3) and (4) spans the period from 1988 to 2017. We include t-statistics in brackets. Standard errors are clustered by the four-digit SIC industry and year. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively.

In the second validation test, we examine the relationship between innosimm and dispersion of firms’ market shares. As explained in Figure 5, we expect that firms in industries with a lower capacity for radical innovation are associated with more evenly distributed market shares. If our innosimm measure is associated negatively with industries’ capacity to generate radical innovation, we expect industries with higher innosimm to have a lower dispersion of market shares. This is indeed reflected in the

\textsuperscript{27}The correlation between innosimm and prodsimm is low. Pearson correlation coefficient, Spearman’s rank correlation coefficient, and Kendall’s $\tau_A$ and $\tau_B$ coefficients between the two variables are 0.06, 0.02, 0.04, and 0.04, respectively.
data (see columns 3 and 4 of Table 1).

5.2 Sensitivity of Profit Margins to Long-run Growth Rates

We test our model’s predictions on profit margins in this subsection. First, consistent with the prediction of our baseline model in Section 2, the average profit margin of industries comoves positively with long-run growth. Second, consistent with the prediction of our full model in Section 4, profit margins in industries with higher innovation are higher and more exposed to long-run growth shocks.

Table 2: Sensitivity of average profit margins to long-run growth rates (yearly analysis).

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ln</td>
<td>Average industry profit margins</td>
<td>Average industry profit margins</td>
<td>ln</td>
</tr>
<tr>
<td>Long-run growth rates ($\theta_t$)</td>
<td>Filtered consumption growth rates</td>
<td>NBER-CES</td>
<td>Cumulative consumption growth rates</td>
<td>Compustat</td>
</tr>
<tr>
<td>$\theta_t$</td>
<td>0.53**</td>
<td>0.76*</td>
<td>0.35**</td>
<td>0.49*</td>
</tr>
<tr>
<td></td>
<td>[2.43]</td>
<td>[1.89]</td>
<td>[2.38]</td>
<td>[1.84]</td>
</tr>
<tr>
<td>Constant</td>
<td>0.00**</td>
<td>0.01***</td>
<td>0.00*</td>
<td>0.01**</td>
</tr>
<tr>
<td></td>
<td>[2.64]</td>
<td>[2.77]</td>
<td>[1.92]</td>
<td>[2.47]</td>
</tr>
<tr>
<td>Observations</td>
<td>51</td>
<td>47</td>
<td>53</td>
<td>47</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.115</td>
<td>0.064</td>
<td>0.080</td>
<td>0.042</td>
</tr>
</tbody>
</table>

Note: This table shows the sensitivity of average profit margins to long-run growth rates. The average profit margin in year $t$ is the simple average of the profit margins across all industries in year $t$. We compute industry-level profit margins based on Compustat and NBER-CES data as explained in Appendix A. Each dataset has its own advantage. Compustat covers public firms from all industries, while the NBER-CES database covers both public firms and private firms in the manufacturing sector. The sample of the Compustat data ends in 2017, while the sample of the NBER-CES data ends in 2011. Long-run growth rates ($\theta_t$) are measured by the annualized filtered consumption growth rates in the last quarter of year $t$ in columns (1)–(2), and by the cumulative consumption growth rates (demeaned and annualized) from year $t$ to year $t-1$ (8 quarter summation) in columns (3)–(4). The construction of the filtered consumption growth rates and cumulative consumption growth rates are explained in Appendix A. The sample in column (1) spans the period from 1965 to 2015, and that in column (3) the period from 1965 to 2017. In columns (2) and (4), the sample spans the period from 1965 to 2011. We include t-statistics in brackets. Standard errors are computed using the Newey-West estimator allowing for serial correlation in returns. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively.

Time-series Variation of Profit Margins. Table 2 shows that the average profit margin of industries comoves positively with the long-run growth rate. This pattern is robust to the choice of profit margin measures and long-run growth rates. Our finding is consistent with previous studies (see, e.g., Machin and Van Reenen, 1993; Hall, 2012; Anderson, Rebelo and Wong, 2018) showing that profit margins are strongly pro-cyclical.

Cross-Sectional Variation of Profit Margins. Table 3 shows that the one-year-ahead changes in industry-level profit margins are more positively correlated with long-run
growth in industries with higher innoimm (columns 1 and 5). One alternative explanation for our findings is that innoimm may be correlated with other industry characteristics such as the income elasticity of demand and the durability of a firm’s output. It is thus possible that these industry characteristics, not the channel of innoimm in Section 4, drive the heterogeneous sensitivity of profit margins to long-run growth. To mitigate this concern, we control for these industry characteristics and their interaction with long-run growth. We find that the coefficient of the interaction term between long-run growth and innoimm remains positive and statistically significant (columns 2–4 and columns 6–8), suggesting that our findings cannot be explained by other industry characteristics. In addition, Appendix Table D.1 shows that industry-level profit margins are associated positively with innoimm.

5.3 Sensitivity of Product Prices to Long-run Growth Rates

As shown in Section 4, the key mechanism of our model is that industries with higher innoimm collude to set higher product prices during periods with higher long-run growth rates, and their prices drop more deeply following negative long-run growth shocks due to the price wars that arise endogenously. We directly test this mechanism in this subsection. Specifically, we study the changes in product prices by exploiting a comprehensive product-level dataset, which allows us to track product prices over time and examine pricing behaviors across industries with different innoimm. We find that industries with higher innoimm are more exposed to price war risks.

5.3.1 The Nielsen Data for Product Prices

We use the Nielsen Retail Measurement Services scanner data to measure product price changes. The Nielsen data are used widely in the macroeconomics literature (see, e.g., Keller, 2008), which suggests that luxury goods producers tend to charge stable and high product prices to maintain their brand image and the perception of scarcity. The finding for durable industries is consistent with the fact that the Consumer Price Index (CPI) of durable goods is less volatile than that of non-durable goods (CPIs are available from St. Louis Fed’s website). Yogo (2006) shows that the consumption of durable goods is more pro-cyclical than non-durable goods, and Ait-Sahalia, Parker and Yogo (2004) show that luxury consumption is more pro-cyclical than basic consumption. Different from these two papers, we focus on the variation of profit margins instead of consumption demand.

Table 3 implies that the profit margins are less sensitive to long-run growth in luxury industries and durable industries. The finding for luxury industries is consistent with the marketing literature (see, e.g., Keller, 2008), which suggests that luxury goods producers tend to charge stable and high product prices to maintain their brand image and the perception of scarcity. The finding for durable industries is consistent with the fact that the Consumer Price Index (CPI) of durable goods is less volatile than that of non-durable goods (CPIs are available from St. Louis Fed’s website). Yogo (2006) shows that the consumption of durable goods is more pro-cyclical than non-durable goods, and Ait-Sahalia, Parker and Yogo (2004) show that luxury consumption is more pro-cyclical than basic consumption. Different from these two papers, we focus on the variation of profit margins instead of consumption demand.

The analyses are conducted by us based in part on data from The Nielsen Company (US), LLC and marketing databases provided through the Nielsen Datasets at the Kilts Center for Marketing Data Center.
Aguiar and Hurst, 2007; Broda and Weinstein, 2010; Hottman, Redding and Weinstein, 2016; Argente, Lee and Moreira, 2018; Jaravel, 2018). The Nielsen data record prices and quantities of every unique product that had any sales in the 42,928 stores of more than 90 retail chains in the US market from January 2006 to December 2016. In total, the Nielsen data cover more than 3.5 million unique products identified by Universal Product Codes (UPCs); they represent 53%, 55%, 32%, 2%, and 1% of all sales in grocery stores, drug stores, mass merchandisers, convenience stores, and liquor stores respectively (see, e.g., Argente, Lee and Moreira, 2018). We use the product-firm links provided by GS1, the official source of UPCs in the US, to match products in the Nielsen data to firms in CRSP/Compustat and Capital IQ. In Online Appendix C.3, we detail the matching procedure. Our merged data cover the product prices of 472 four-digit SIC industries.

5.3.2 Price War Risks across Industries with Different Innosimm

The Lehman crash is an event during which the US economy experienced a prominent negative long-run growth shock (see Panel C of Figure 6). In this subsection, we perform an event-type study to examine changes in product prices around the Lehman crash. To begin, we investigate the changes in media coverage about price wars around the Lehman crash. Because the Nielsen data mainly cover consumer goods sold by retailers and wholesalers, we focus on media coverage of the consumer goods sector and the retail/wholesale sector. Panel A of Figure 7 shows that, after the Lehman crash, the number of articles covering price wars increased dramatically. This pattern remains robust when we normalize the number of articles covering price wars using the total number of news articles (see Panel B of Figure 7).

Next, we examine the changes in product prices. We sort all industries into tertiles based on innosimm. Table 4 quantifies the changes in product prices of high-innosimm industries (top tertile) relative to low-innosimm industries (bottom tertile) around the Lehman crash. In particular, we restrict the sample to industries in the top and bottom tertiles, and create a top tertile indicator variable that equals one for the observations in the former group. We also create a post-Lehman indicator variable that equals one for observations in October 2008 and thereafter. We then regress the percentage change in product prices on the top tertile indicator variable, the post-Lehman-crash indicator at the University of Chicago Booth School of Business. The conclusions drawn from the Nielsen data are our own and do not reflect the views of Nielsen. Nielsen is not responsible for, had no role in, and was not involved in analyzing and preparing the results reported herein.
Table 3: Profit margin sensitivity of industries with different innosimm (yearly analysis).

<table>
<thead>
<tr>
<th>Long-run growth rates ( \theta_t )</th>
<th>Filtered consumption growth rates</th>
<th>Cumulative consumption growth rates</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta_t \times \text{innosimm}_t )</td>
<td>( \ln \left( \frac{\text{industry-level profit margins}<em>t+1}{\text{industry-level profit margins}</em>{t-1}} \right) )</td>
<td></td>
</tr>
<tr>
<td>( \theta_t \times \text{income elasticity of demand}_t )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Innosimm(_t)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>8848</td>
<td>6979</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.003</td>
<td>0.002</td>
</tr>
</tbody>
</table>

Note: This table shows the sensitivity of industry-level profit margins to long-run growth rates. Profit margins are computed based on Compustat data as explained in Appendix A. The construction of long-run growth rates \( \theta_t \) is explained in Appendix A. We estimate the income elasticity of demand based on the representative consumer’s income and expenditures (see Online Appendix C.4 for details). Luxury industries are the industries with an income elasticity of demand larger than one. The durability of firms’ output comes from Gomes, Kogan and Yogo (2009). The sample in columns (1)–(4) spans the period from 1988 to 2015, and that in columns (5)–(8) the period from 1988 to 2017. We include t-statistics in brackets. Standard errors are clustered by the four-digit SIC industry and year. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively.

variable, and an interaction term between the two. The coefficient of the interaction term is negative and statistically significant across different regression specifications, suggesting that product prices in high-innosimm industries decreased significantly relative to those in low-innosimm industries after the Lehman crash. The difference in product prices is economically significant. According to the regression without industry fixed effects (column 1 of Table 4), product prices decreased by 4.98% in high-innosimm industries after the Lehman crash, compared to those of low-innosimm industries.

Panel C of Figure 7 visualizes the difference in the annualized monthly percentage changes in product prices between low-innosimm and high-innosimm industries in the 36-month period centered at the Lehman crash. The plot clearly shows that product prices in high-innosimm industries reduced more than those in low-innosimm industries after the Lehman crash. Panel D of Figure 7 visualizes the monthly price-innosimm sensitivity.
Note: Panel A plots the number of articles (quarterly) that contain the term “price war” or “price wars” published in The Wall Street Journal, The New York Times, and Financial Times around the Lehman crash. We require that the articles cover the US region and cover either the consumer goods sector or the retail/wholesale sector. The gray vertical bar represents the Lehman crash. The black dashed and red solid lines represent the mean number of articles before and after the Lehman crash. Panel B plots the price war media coverage (in percent), which is the number of articles in Panel A normalized by the total number of articles published in the three journals. Panel C plots the difference in the annualized monthly percentage changes in product prices between high-innosimm (i.e., top tertile) and low-innosimm (i.e., bottom tertile) industries around the Lehman crash. The black circles and red triangles represent the differences in annualized monthly percentage price changes between high-innosimm and low-innosimm industries in the 18 months before and after the Lehman crash. The black dashed and red solid lines represent the mean values of the differences before and after the Lehman crash. Panel D shows the price-innosimm sensitivity around the Lehman crash. Panel E plots the difference in the percentage change in product prices between high-prodsimm (i.e., top tertile) and low-prodsimm (i.e., bottom tertile) industries. Panel F shows the price-prodsimm sensitivity. We estimate confidence intervals using the bootstrapping method. Specifically, for each panel, we construct 1 million time series by randomly drawing (with replacement) from a sample pool that contains observations both before and after the Lehman crash. We then estimate the 95% confidence interval (dotted lines) for the difference between the mean values before and after the Lehman crash. The differences between the mean values before and after the Lehman crash are statistically significant (insignificant) if the red solid lines are outside (within) the 95% confidence interval.

Figure 7: Price war media coverage and product prices around the Lehman crash.

(i.e., the sensitivity between the percentage change in product prices and industry-level innosimm estimated across all industries). We find that the price-innosimm sensitivity reduced significantly after the Lehman crash. We also use a regression approach to verify such a pattern in Appendix Table D.2. The above findings suggest that high-innosimm industries were more affected by the Lehman crash and that their product prices decreased to a greater extent than did product prices in low-innosimm industries, indicating that the former industries are more likely to engage in price wars following...
Table 4: Product prices around the Lehman crash (monthly analysis).

<table>
<thead>
<tr>
<th>Similarity measure</th>
<th>(1) Percentage change in industry-level product prices (monthly, annualized, %)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Innosimm</td>
<td><img src="image" alt="Table Content" /></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Prodsimm</td>
<td><img src="image" alt="Table Content" /></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: This table shows the changes in product prices around the Lehman crash. The dependent variable is the annualized monthly percentage change in product prices of four-digit SIC industries. To compute the monthly percentage change in product prices for four-digit SIC industries, we first compute the transaction-value-weighted price for each product across all stores in each month. We then calculate the monthly percentage change in prices for each product. Finally, we compute the transaction-value-weighted percentage change in product prices for each four-digit SIC industry across all products within the industry. In columns (1) and (2), the similarity measure is innosimm. In columns (3) and (4), the similarity measure is prodsimm. We consider the 36-month period centered at the Lehman crash. In Online Appendix F.1, we perform the analysis by considering the 24-month period centered at the Lehman crash and find similar results. We include t-statistics in brackets. Standard errors are clustered by the four-digit SIC industry and month. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively.

We also examine product prices around the Lehman crash for industries with different prodsimm (see Hoberg and Phillips, 2016). We find that product prices do not move differently for high-prodsimm and low-prodsimm industries (Panel E of Figure 7, columns 3 and 4 in Table 4). Moreover, we observe little change in price-prodsimm sensitivity following the Lehman crash (Panel F of Figure 7, columns 3 and 4 in Appendix Table D.2). These findings suggest that, unlike innosimm, prodsimm is not related to industries’ exposure to price war risks.

We now extend our analysis to the whole time period covered by the Nielsen data from 2006 to 2016. Specifically, we regress the percentage change in product prices on innosimm, long-run growth rates, and the interaction term between the two. Table 5 shows that the coefficient of the interaction term is positive and statistically significant. The result remains robust if we control for various industry characteristics, or if we use real product prices to adjust for inflation (see Online Appendix F.2). Our findings suggest that industries with higher innosimm have product prices that are more sensitive to long-run growth and hence are more exposed to price war risks.

37
Table 5: Price war risks across industries with different innosimm (quarterly analysis).

<table>
<thead>
<tr>
<th>Long-run growth rates ($\theta_t$)</th>
<th>Filtered consumption growth rates</th>
<th>Cumulative consumption growth rates</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_t \times \text{innosimm}_t$</td>
<td>1.17** 1.13* 1.17** 1.16** 0.80** 0.75* 0.81** 0.79**</td>
<td>[2.37] [2.24] [2.34] [2.35] [2.56] [2.15] [2.33] [2.28]</td>
</tr>
<tr>
<td>$\theta_t \times \text{income elasticity of demand}_t$</td>
<td>-0.36 -0.77***</td>
<td>[-0.75] [-5.69]</td>
</tr>
<tr>
<td>$\theta_t \times \text{luxury industries}_t$</td>
<td>-0.52 -0.91***</td>
<td>[-1.56] [-3.92]</td>
</tr>
<tr>
<td>$\theta_t \times \text{durable industries}_t$</td>
<td>-1.27*</td>
<td>[-2.14] [0.51]</td>
</tr>
<tr>
<td>Income elasticity of demand</td>
<td>0.01 0.00</td>
<td>[0.52] [-0.68]</td>
</tr>
<tr>
<td>Luxury industries</td>
<td>-0.00 -0.01</td>
<td>[-0.22] [-0.94]</td>
</tr>
<tr>
<td>Durable industries</td>
<td>-0.10***</td>
<td>[-5.29] [-2.71]</td>
</tr>
<tr>
<td>$\theta_t$</td>
<td>-0.20 0.26 0.07 -0.02 0.11 1.07** 0.58 0.07</td>
<td>[-0.13] [0.17] [-0.02] [0.23] [3.07] [1.20] [0.15]</td>
</tr>
<tr>
<td>Innosimm</td>
<td>0.00 0.00 0.00 0.00 -0.00 -0.00 -0.00 -0.00</td>
<td>[0.17] [0.18] [0.17] [0.17] [-0.24] [-0.21] [-0.13] [-0.25]</td>
</tr>
<tr>
<td>Observations</td>
<td>7338 7338 7338 7338 8208 8208 8208 8208</td>
<td>0.002 0.003 0.002 0.006 0.004 0.006 0.005 0.010</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.002 0.003 0.002 0.006 0.004 0.006 0.005 0.010</td>
<td></td>
</tr>
</tbody>
</table>

Note: This table shows the sensitivity of percentage changes in product prices to long-run growth rates across four-digit SIC industries with different innosimm. The dependent variable is the one-year-ahead percentage change in industry-level product prices. Long-run growth rates are measured by annualized filtered consumption growth rates in quarter $t$ in columns (1)–(4), and by the cumulative consumption growth rates from quarter $t - 7$ to $t$ (annualized) in columns (5)–(8). Income elasticity of demand, luxury industries, and durable industries are defined in Table 3. The sample spans the period from 2006 to 2016. We include t-statistics in brackets. Standard errors are clustered by the four-digit SIC industry and year. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively.

### 5.4 Asset Pricing Tests

We now test the asset pricing implications of our model. Our model shows that long-run growth shocks are priced because they affect firms’ cash flows. In the times-series data, we find that the average real profits of industries comove positively with long-run growth rates, indicating that long-run growth shocks affect a firm’s cash flows significantly. In the cross section, we find that the stock returns and dividend growth of industries with higher innosimm are more exposed to long-run growth shocks. Moreover, we find that industries with higher innosimm have higher average excess returns and risk-adjusted returns. The spreads between high-innosimm industries and low-innosimm industries (called innosimm spreads) are persistent and robust after controlling for various related measures. These findings are consistent with the prediction of our full model in Section 4. Finally, we find that the innosimm spreads become much weaker in the group of
Table 6: Sensitivity of average industry profits to long-run growth rates (yearly analysis).

<table>
<thead>
<tr>
<th>Long-run growth rates ($\theta_t$)</th>
<th>Filtered consumption growth rates</th>
<th>Cumulative consumption growth rates</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Gross profits</td>
<td>Net profits</td>
</tr>
<tr>
<td>$\theta_t$</td>
<td>2.62***</td>
<td>1.96***</td>
</tr>
<tr>
<td></td>
<td>[5.72]</td>
<td>[3.91]</td>
</tr>
<tr>
<td>Constant</td>
<td>0.04***</td>
<td>0.04***</td>
</tr>
<tr>
<td></td>
<td>[5.76]</td>
<td>[4.41]</td>
</tr>
<tr>
<td>Observations</td>
<td>51</td>
<td>51</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.281</td>
<td>0.099</td>
</tr>
<tr>
<td></td>
<td>0.343</td>
<td>0.155</td>
</tr>
</tbody>
</table>

Note: This table shows the sensitivity of average real profits of industries to long-run growth rates. The profit in year $t$ is the simple average of the real profits across all industries in year $t$. We compute industry-level gross and net profits based on Compustat data as explained in Appendix A. The construction of long-run growth rates ($\theta_t$) is explained in Appendix A. The sample in columns (1) and (2) spans the period from 1965 to 2015, and that in columns (3) and (4) spans the period from 1965 to 2017. We include t-statistics in brackets. Standard errors are computed using the Newey-West estimator allowing for serial correlation in returns. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively.

Industries that have experienced antitrust enforcement in recent years, suggesting that the observed spreads are related to firms’ collusion incentive, as predicted by our model (see Online Appendix E.3 for detailed discussion).

**Comovement between Average Profits and Long-run Growth Rates.** Table 6 shows that both the growth rates of average real gross profits and net profits of industries comove positively with long-run growth rates. This relationship is economically significant. For 1% increase in the filtered consumption growth rates, the growth rates of average gross profits and net profits increase by 2.62% and 1.96%, respectively. Our findings suggest that long-run growth shocks have a substantial impact on firms’ cash flows.

**Exposure of Stock Returns and Dividend Growth to Long-run Growth Shocks.** In the cross section, we first examine the exposure of stock returns to long-run growth shocks across industries with different innosimm. We sort all industries into quintile portfolios based on innosimm and regress the cumulative returns of each portfolio on long-run growth. Table 7 tabulates the betas to long-run growth shocks for each portfolio. The difference in betas between Q1 and Q5 is positive and statistically significant, suggesting that the stock returns of industries with higher innosimm are more exposed to long-run growth shocks.

Next, we examine the exposure of real dividend growth to long-run growth shocks for the long-short portfolio sorted on innosimm. We construct the real dividend growth...
Table 7: Exposure of stock returns to long-run growth shocks (quarterly analysis).

<table>
<thead>
<tr>
<th>Industry portfolios sorted on innosimm</th>
<th>1 (Low)</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5 (High)</th>
<th>5 – 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Betas to long-run growth shocks</td>
<td>-0.05</td>
<td>4.45***</td>
<td>-0.62</td>
<td>3.57**</td>
<td>4.69**</td>
<td>4.75**</td>
</tr>
<tr>
<td>Filtered consumption growth rates</td>
<td>[-0.05]</td>
<td>[3.16]</td>
<td>[-0.52]</td>
<td>[2.57]</td>
<td>[2.41]</td>
<td>[2.61]</td>
</tr>
<tr>
<td>Betas to long-run growth shocks</td>
<td>1.78</td>
<td>6.55***</td>
<td>3.48***</td>
<td>5.51***</td>
<td>5.24***</td>
<td>3.46**</td>
</tr>
<tr>
<td>Cumulative consumption growth rates</td>
<td>[1.58]</td>
<td>[5.09]</td>
<td>[3.12]</td>
<td>[3.73]</td>
<td>[4.03]</td>
<td>[2.09]</td>
</tr>
</tbody>
</table>

Note: This table shows the exposure to long-run growth shocks for industry portfolios sorted on innosimm. In June of year \( t \), we sort industries into five quintiles based on innosimm in year \( t – 1 \). Once the portfolios are formed, their monthly returns are tracked from July of year \( t \) to June of year \( t + 1 \). We estimate the betas to long-run growth shocks by regressing the eight-quarter cumulative portfolio returns on the eight-quarter cumulative filtered consumption growth rates: \( \sum_{i=0}^{7} R_{i, T-j} = \alpha_i + \beta_i \sum_{j=0}^{7} \hat{\eta}_{j-\tau} + \varepsilon_{i, T} \), where \( \hat{\eta}_{j} \) is the demeaned quarterly consumption growth rate at quarter \( \tau \). Consumption and stock returns are deflated to real terms using the personal consumption expenditure deflator from the US Bureau of Economic Analysis (BEA). The sample spans the period from 1988 to 2015 because our data on the filtered consumption growth rates end in 2015. Following Dittmar and Lundblad (2017), we also estimate the betas to long-run growth shocks by regressing the eight-quarter cumulative portfolio returns on the eight-quarter cumulative consumption growth rates: \( \sum_{i=0}^{7} R_{i, T-j} = \alpha_i + \beta_i \sum_{j=0}^{7} \hat{\xi}_{j-\tau} + \varepsilon_{i, T} \), where \( \hat{\xi}_{j} \) is the demeaned quarterly consumption growth rate at quarter \( \tau \). Consumption and stock returns are deflated to real terms using the Newey-West estimator allowing for serial correlation in returns. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively.

Table 8: Exposure of dividend growth to long-run growth shocks (quarterly analysis).

<table>
<thead>
<tr>
<th>Panel A: Dividend growth spreads (Q5 – Q1, annualized, %)</th>
<th>Panel B: Exposure to long-run growth shocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Filtered consumption growth rates</td>
<td>Cumulative consumption growth rates</td>
</tr>
<tr>
<td>2.09</td>
<td>6.23***</td>
</tr>
<tr>
<td>[0.48]</td>
<td>[4.72]</td>
</tr>
<tr>
<td></td>
<td>7.68***</td>
</tr>
<tr>
<td></td>
<td>[4.75]</td>
</tr>
</tbody>
</table>

Note: Panel A shows the annualized spreads of dividend growth for the long-shot industry portfolio sorted on innosimm. In Panel B, we regress the four-quarter cumulative dividend growth of the long-shot industry portfolio sorted on innosimm on the annualized quarterly filtered consumption growth rates (\( \sum_{j=1}^{4} (D_{Q5,j+i} - D_{Q1,j+i}) \)) and the annualized eight-quarter lagged cumulative consumption growth rates (\( \sum_{j=1}^{4} (D_{Q5,j+i} - D_{Q1,j+i}) = \alpha + \beta \sum_{j=0}^{3} \hat{\eta}_{j-\tau} + \varepsilon_{i, T} \)). We exclude financial firms and utility firms from the analysis. We include t-statistics in brackets. Standard errors are computed using the Newey-West estimator allowing for serial correlation in returns. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively.

rate following previous studies (see, e.g., Campbell and Shiller, 1988; Bansal, Dittmar and Lundblad, 2005; Hansen, Heaton and Li, 2005, 2008; Bansal, Kiku and Yaron, 2016). Importantly, we account for stock entries and exits when computing a portfolio’s dividend growth rate (see Online Appendix C.5 for details). Table 8 shows that the dividend growth of industries with higher innosimm is also more exposed to long-run growth shocks.

**Innosimm Spreads across Industries.** We now examine whether innosimm is priced in the cross section. Panel A of Table 9 presents the value-weighted average excess returns and alphas for the industry portfolios sorted on innosimm. The panel shows that the portfolio consisting of high-innosimm industries (i.e., Q5) exhibits significantly higher average excess returns and alphas. The annualized spread in average excess returns
between Q1 and Q5 is 3.41% and the annualized spreads in alphas are 5.22% and 4.75% for the Fama-French three-factor model and the Carhart four-factor model, respectively. We also perform the same analysis for prodsimm, and we find that prodsimm is not priced in the cross section. The return difference between the high-prodsimm portfolio and the low-prodsimm portfolio is statistically insignificant (see Panel B of Table 9).

Table 9: Average excess returns and alphas of portfolios sorted on innosimm and prodsimm (monthly analysis).

<table>
<thead>
<tr>
<th></th>
<th>Panel A: Industry portfolios sorted on innosimm</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Average excess returns</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \mathbb{E}[R] - r_f ) (%)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 (Low)</td>
<td>6.13***</td>
<td>8.37***</td>
<td>7.35***</td>
<td>8.62***</td>
<td>9.54***</td>
<td>3.41***</td>
</tr>
<tr>
<td></td>
<td>[2.74]</td>
<td>[3.73]</td>
<td>[3.01]</td>
<td>[4.42]</td>
<td>[3.17]</td>
<td>[2.71]</td>
</tr>
<tr>
<td></td>
<td>Fama-French three-factor model (see Fama and French, 1993)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a (%)</td>
<td>(-2.51^{**})</td>
<td>(0.07)</td>
<td>(-1.34)</td>
<td>(1.08)</td>
<td>(2.71^{**})</td>
<td>(5.22^{***})</td>
</tr>
<tr>
<td></td>
<td>[(-2.48)]</td>
<td>[0.10]</td>
<td>[(-0.69)]</td>
<td>[1.21]</td>
<td>[2.49]</td>
<td>[3.54]</td>
</tr>
<tr>
<td></td>
<td>Carhart four-factor model (see Carhart, 1997)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a (%)</td>
<td>(-2.47^{***})</td>
<td>(0.99)</td>
<td>(-1.25)</td>
<td>(1.43)</td>
<td>(2.28^{***})</td>
<td>(4.75^{***})</td>
</tr>
<tr>
<td></td>
<td>[(-2.70)]</td>
<td>[0.18]</td>
<td>[(-0.78)]</td>
<td>[1.50]</td>
<td>[2.63]</td>
<td>[4.01]</td>
</tr>
</tbody>
</table>

|          | Panel B: Industry portfolios sorted on prodsimm |          |          |          |          |          |
|          | Average excess returns                          |          |          |          |          |          |
|          | \( \mathbb{E}[R] - r_f \) (%)                 |          |          |          |          |          |
| 1 (Low)  | 4.94**                                         | 6.44**   | 8.07**   | 6.25*    | 6.19*    | 1.25     |
|          | [2.29]                                          | [2.05]   | [2.59]   | [1.77]   | [1.91]   | [0.42]   |
|          | Fama-French three-factor model (see Fama and French, 1993) |
| a (%)    | \(-0.89\)                                     | \(0.03\) | \(2.21^{**}\) | \(0.88\) | \(0.82\) | \(1.70\) |
|          | [\(-0.48\)]                                    | [0.01]   | [2.53]   | [0.06]   | [0.86]   | [0.64]   |
|          | Carhart four-factor model (see Carhart, 1997)   |
| a (%)    | \(-0.57\)                                     | \(0.42\) | \(2.19^{**}\) | \(0.69\) | \(0.70\) | \(1.26\) |
|          | [\(-0.36\)]                                    | [0.20]   | [2.43]   | [0.68]   | [0.75]   | [0.53]   |

Note: This table shows the value-weighted average excess returns and alphas for the four-digit SIC industry portfolios sorted on innosimm. In June of year \( t \), we sort the four-digit SIC industries into five quintiles based on their innosimm in year \( t - 1 \). Once the portfolios are formed, their monthly returns are tracked from July of year \( t \) to June of year \( t + 1 \). The sample period is from July 1988 to June 2018. We exclude financial firms and utility firms from the analysis. We include t-statistics in brackets. Standard errors are computed using the Newey-West estimator allowing for serial correlation in returns. We annualize average excess returns and alphas by multiplying them by 12. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively.

We further examine the persistence of the return spreads and the betas to long-run growth shocks around the portfolio sorting period. Panels A and B of Figure 8 show that the positive relation between return spreads and innosimm exists three years before and continues to exist three years after portfolio formation. This result reinforces the findings in Table 9 because it indicates that innosimm is a persistent firm characteristic priced in the cross section. Panel C of Figure 8 shows that the positive relation between portfolio betas and innosimm remains stable five years after portfolio formation. The above findings

30The correlation in innosimm is 0.99 between years \( t \) and \( t - 1 \) and 0.86 between years \( t \) and \( t - 5 \).
of persistent innosimm spreads and betas support our theory of heterogeneous persistent risk exposure due to persistent industry characteristics, rather than time-varying betas (see Daniel and Moskowitz, 2016).

![Graphs A, B, and C showing excess returns, Fama-French three-factor alphas, and pre- and post-sorting betas.](image)

Note: Panels A and B plot the annualized excess returns and alphas, averaged across different portfolio-formation months, associated with the portfolios sorted on innosim three years before and three years after portfolio formation. Specifically, we conduct event studies for different portfolio-formation months \( t \), spanning the period from 1988 to 2018. In each portfolio formation month \( t \), we sort industries into quintiles based on innosim to construct portfolios. Both industry allocations and weights in each portfolio are fixed at their values in portfolio-formation month \( t \). We then compute the value-weighted returns for each of the portfolios sorted on innosim across time. Next, for each month \( t' \in [t - 36, t + 36] \), we estimate the parameters of the Fama-French three-factor models based on portfolio returns during \([t' - 36, t']\). Using the estimated parameters and the portfolio returns in month \( t' \), we estimate the Fama-French three-factor alphas in month \( t' \). Finally, we compute the average alpha for each month across all portfolio formation-months \( t \), and obtain annualized alphas by multiplying the monthly alphas by 12. Panel C plots the pre- and post-sorting betas to long-run growth shocks, which are estimated using the same approach as the one in Panel B of Table 7.

Figure 8: Before- and after-sorting return spreads and betas to long-run growth shocks.

Finally, we perform a series of double-sort analyses. As shown by Appendix Table D.3, the innosimm spreads are robust after controlling for various related industry characteristics including the measures of profit margins, prodsimm, innovation originality, asset growth rates, income elasticity of demand, and durability of firms’ output.

**The Impact of Antitrust Enforcement.** To test whether the observed innosimm spreads are related to firms’ collusion incentive, we exploit the variation in antitrust enforcement, which punishes collusive behavior and thus dampens firms’ incentive to collude.

Specifically, we split all industries into two groups in each year based on whether they have experienced any antitrust enforcement in the past 10 years.\(^{31}\) As shown in Table 10, the innosimm spreads are much smaller in the industries that have recently experienced

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\(^{31}\) The antitrust enforcement cases are hand collected from the websites of the US Department of Justice (DOJ) and the Federal Trade Commission (FTC). DOJ provides four-digit SIC codes for the firms in some cases. For the remaining DOJ cases and all FTC cases, we match the firms involved in antitrust enforcement to CRSP/Compustat and Capital IQ, from which we collect the four-digit SIC codes of these firms.
Table 10: Antitrust enforcement and innoSimm spreads (monthly analysis).

<table>
<thead>
<tr>
<th></th>
<th>Excess returns (%)</th>
<th>Fama-French three-factor alpha (%)</th>
<th>Carhart four-factor alpha (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel A: Industries with antitrust enforcement in the past 10 years</td>
<td>−0.81, [−0.33]</td>
<td>0.59, [0.24]</td>
<td>−0.44, [−0.21]</td>
</tr>
<tr>
<td>Panel B: Industries without antitrust enforcement in the past 10 years</td>
<td>3.27**, [2.01]</td>
<td>5.44**, [2.91]</td>
<td>5.54***, [3.00]</td>
</tr>
</tbody>
</table>

Note: This table presents the average excess returns and alphas (both in percent) of the value-weighted long-short four-digit SIC industry portfolio sorted on innoSimm in the industries with (Panel A) and without (Panel B) antitrust enforcement in the past 10 years. We exclude financial firms and utility firms from the analysis. We include t-statistics in brackets. Standard errors are computed using the Newey-West estimator allowing for serial correlation in returns. We annualize the average excess returns and the alphas by multiplying them by 12. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively.

antitrust enforcement, suggesting that innoSimm spreads are driven by heterogeneous collusion incentives across industries with different innoSimm as illustrated by our model.

6 Quantitative Analyses

In this section, we conduct quantitative analyses. We solve the model numerically (see Online Appendix H). The model’s parameters are calibrated based on both existing estimates and micro data without referring to asset pricing information. Then we examine whether the calibrated model can quantitatively explain the observed asset pricing patterns in both the time series and the cross section.

6.1 Calibration

We calibrate the model monthly. Some parameters are determined using external information without simulating the model (see Panel A of Table 11). The remaining parameters are calibrated internally from moment matching (see Panel B of Table 11).

**Externally Determined Parameters.** Following standard practice, we set the risk aversion parameter at $\gamma = 9$ and the elasticity of intertemporal substitution at $\psi = 1.5$. We set $\varphi_\theta = 0.044$ following Bansal and Yaron (2004). We set the within-industry elasticity of substitution at $\eta = 15$ and the between-industry elasticity of substitution at $\epsilon = 2$, which are broadly consistent with the values of Atkeson and Burstein (2008). We choose a low customer base depreciation rate ($\delta = 0.002$) and accumulation rate ($z = 0.004$) to capture a sticky customer base (see, e.g., Gourio and Rudanko, 2014; Gilchrist et al., 2017). We set
Table 11: Calibration and parameter choice.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel A: Externally Determined Parameters</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Risk aversion</td>
<td>$\gamma$</td>
<td>9</td>
<td>Elasticity of intertemporal substitution</td>
<td>$\psi$</td>
<td>1.5</td>
</tr>
<tr>
<td>Predictable variation in consumption growth</td>
<td>$\phi_\psi$</td>
<td>0.044</td>
<td>Customer base depreciation</td>
<td>$\delta$</td>
<td>0.002</td>
</tr>
<tr>
<td>Between-industry elasticity of substitution</td>
<td>$\epsilon$</td>
<td>2</td>
<td>Within-industry elasticity of substitution</td>
<td>$\eta$</td>
<td>15</td>
</tr>
<tr>
<td>Transition rate from $v_L$ to $v_H$</td>
<td>$q^{v_L, v_H}$</td>
<td>0.008</td>
<td>Transition rate from $v_H$ to $v_L$</td>
<td>$q^{v_H, v_L}$</td>
<td>0.33</td>
</tr>
<tr>
<td>Customer base accumulation</td>
<td>$z$</td>
<td>0.004</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Panel B: Internally Calibrated Parameters</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Persistence of expected growth rate</td>
<td>$\kappa$</td>
<td>0.01</td>
<td>Average long-run growth rate</td>
<td>$\bar{\theta}$</td>
<td>0.0015</td>
</tr>
<tr>
<td>Volatility of consumption growth</td>
<td>$\sigma_c$</td>
<td>0.0078</td>
<td>Subjective discount factor</td>
<td>$\beta$</td>
<td>0.0018</td>
</tr>
<tr>
<td>Marginal cost of production</td>
<td>$\omega$</td>
<td>0.16</td>
<td>Punishment rate</td>
<td>$\zeta$</td>
<td>0.015</td>
</tr>
<tr>
<td>Price monitoring effort</td>
<td>$v_H$</td>
<td>0.037</td>
<td>Lowest innovation similarity</td>
<td>$\lambda$</td>
<td>0.75</td>
</tr>
<tr>
<td>Innovation success rate</td>
<td>$\mu$</td>
<td>0.047</td>
<td>Customer base stealing (non-radical)</td>
<td>$\tau_i$</td>
<td>0.015</td>
</tr>
<tr>
<td>Persistence of innovation similarity</td>
<td>$\chi$</td>
<td>0.0002</td>
<td>Customer base stealing (radical)</td>
<td>$\tau_d$</td>
<td>0.90</td>
</tr>
</tbody>
</table>

the transition intensity between states of high and low monitoring efforts at $q^{v_L, v_H} = 0.008$ and $q^{v_H, v_L} = 0.33$. Thus, on average, the state of high monitoring effort lasts for one quarter and the state of low monitoring effort lasts for 10 years.

Internally Calibrated Parameters. The other parameters are determined by matching moments in Table 12. We calibrate the persistence of expected growth rate $\kappa$ to match the auto-correlation of annual consumption growth rates. We set $\bar{\theta}$ and $\sigma_c$ to match the average annual consumption growth rate and its standard deviation. We set the subjective discount factor $\beta$ to match the risk-free rate.

The marginal cost of production $\omega$ is determined to match the asset-to-sales ratio. The punishment rate $\zeta$ determines collusion incentive and profit margins, and the price monitoring effort $v_H$ determines the threshold of regime switching or the full-blown price war. We calibrate the values of $\zeta$ and $v_H$ by matching the average net profit margin and the volatility of the growth rates of net profits.

The innovation success rate $\mu$ is calibrated to match the fraction of industries with patent issuance in a year. Parameters $\tau_i$ and $\tau_d$ determine the fraction of customer base.

---

32In our model, we can think of firms using rental capital $K_{ij,t}$ to produce goods, and the parameter $\omega$ captures the rental price per unit of $K_{ij,t}$. Thus, asset to sales $s_{ij,t} = K_{ij,t} / (P_{ij,t} Y_{ij,t}) = 1 / P_{ij,t}$. The net profit margin is calculated as follows: net profit margin $\text{margin}_{ij,t} = (1 - \text{corporate tax rate}) \times (\text{gross profit margin}_{ij,t} - \text{asset to sales}_{ij,t} / \text{financial leverage} \times \text{corporate bond yield})$, where gross profit margin $\text{margin}_{ij,t} = (P_{ij,t} - \omega) / P_{ij,t}$. We use a corporate tax rate of 30% and a corporate bond yield of 5%. The financial leverage, or asset to debt ratio, is set at 5/3 (see Papanikolaou, 2011).
Table 12: Targeted moments in the data and model.

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average consumption growth rate (%)</td>
<td>1.79</td>
<td>1.85</td>
<td>2.65</td>
<td>3.22</td>
</tr>
<tr>
<td>AR(1) of consumption growth</td>
<td>0.49</td>
<td>0.55</td>
<td>8.48</td>
<td>8.43</td>
</tr>
<tr>
<td>Average dispersion of market shares (%)</td>
<td>18.9</td>
<td>20.7</td>
<td>8.95</td>
<td>8.71</td>
</tr>
<tr>
<td>Frac. of industries with patent issuance (%)</td>
<td>67.6</td>
<td>67.3</td>
<td>2.74</td>
<td>3.36</td>
</tr>
<tr>
<td>Volatility of net profits' growth rates (%)</td>
<td>4.95</td>
<td>6.54</td>
<td>0.991</td>
<td>0.992</td>
</tr>
<tr>
<td>Real risk-free interest rate (%)</td>
<td>1.39</td>
<td>1.36</td>
<td>4.40</td>
<td>4.44</td>
</tr>
<tr>
<td>Patent value/market cap. (%)</td>
<td>67.6</td>
<td>67.3</td>
<td>2.74</td>
<td>3.36</td>
</tr>
<tr>
<td>Asset-to-sales ratio (%)</td>
<td>1.39</td>
<td>1.36</td>
<td>4.40</td>
<td>4.44</td>
</tr>
</tbody>
</table>

Note: Following Bansal and Yaron (2004), the real consumption data are constructed based on BEA data and cover the period from 1929 to 2018. Industry-level profit margins are computed based on Compustat data as explained in Appendix A. Within-industry dispersion of market shares is computed based on Compustat data as explained in Table 1. Volatility of net profits' growth rates is the volatility of the growth rates of the real average industry net profits. Asset-to-sales ratio is computed based on Compustat at the firm level. We construct the above Compustat-based moments using the data from 1988 to 2017, which is the sample period of our innosimm measure. Fraction of industries with patent issuance is computed based on PatentsView data. Patent value/market cap. is the total value of patents granted to a firm in a given year normalized by the firm’s lagged market cap. The patent value is measured in dollars based on stock market reaction to the patent issuance (see Kogan et al., 2017). Real risk-free interest rate is the average of the difference between the annual returns of one-month treasury bills (from CRSP) and the rate of change in CPI from 1988 to 2018. We bootstrap the data moments with 1000 replications and report the 5th and 95th percentiles of the bootstrapped distribution (in brackets). When constructing the model moments, we simulate a sample of 500 industries for 70 years with a 20-year burn-in period. We then compute the model-implied moments similar to the data. For each moment, the table reports the average value of 1,000 simulations and the 5th and 95th estimated percentiles of the simulated distribution (in brackets).

We set their value to match the dispersion of market shares at the industry level, and the value of the patents (obtained from Kogan et al. (2017)) granted to a firm in a year as a percentage of the firm’s market capitalization. We assume that the industry-level innovation similarity \( \lambda_{i,t} \) is bounded between \( \lambda \) and \( \bar{\lambda} \). We discretize \([\lambda, \bar{\lambda}]\) into \( N = 11 \) grids with equal spacing, so that \( \lambda_1 = \lambda \) and \( \lambda_{N} = \bar{\lambda} \). The parameter \( \chi \) determines the persistence of innovation similarity \( \lambda_{i,t} \). We calibrate its value so that the yearly autocorrelation of \( \lambda_{i,t} \) in our model is 0.99, which is consistent with the yearly correlation of our innosimm measure. The parameter \( \bar{\lambda} \) is normalized to one so that the most extreme industry is the one with no capacity for radical innovation. The parameter \( \Delta \) is determined by matching the median net profit margin.

6.2 Quantitative Results

We first investigate the model-implied sensitivity of average net profits to long-run growth rates. The model-implied regression coefficient is 1.05 with \( R^2 = 0.218 \), comparable to
the data (see Table 6 and Panel A of Table 13). This implies that the low $R^2$ in the data is caused by estimating regressions at the yearly frequency, not by the poor explanatory power of long-run growth shocks.

Table 13: Sensitivity of net profits and asset pricing implications in the data and model.

Panel A: Sensitivity of average net profits to long-run growth rates

<table>
<thead>
<tr>
<th></th>
<th>$\theta_t$</th>
<th>Constant</th>
<th>R-squared</th>
<th>Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>1.96</td>
<td>0.04</td>
<td>0.099</td>
<td>51</td>
</tr>
<tr>
<td></td>
<td>[0.86, 3.04]</td>
<td>[0.02, 0.05]</td>
<td>[0.017, 0.227]</td>
<td></td>
</tr>
<tr>
<td>Model</td>
<td>1.05</td>
<td>0.04</td>
<td>0.218</td>
<td>51</td>
</tr>
<tr>
<td></td>
<td>[0.57, 1.56]</td>
<td>[−0.17, 0.27]</td>
<td>[0.041, 0.428]</td>
<td></td>
</tr>
</tbody>
</table>

Panel B: Asset pricing implications

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Full model</th>
<th>Model-based counterfactuals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean of real risk-free rates (%)</td>
<td>1.39</td>
<td>1.38</td>
<td>1.38</td>
</tr>
<tr>
<td></td>
<td>[0.55, 2.19]</td>
<td>[−0.25, 2.99]</td>
<td>[−0.25, 2.99]</td>
</tr>
<tr>
<td>Volatility of real risk-free rates (%)</td>
<td>2.79</td>
<td>1.51</td>
<td>1.51</td>
</tr>
<tr>
<td></td>
<td>[2.30, 3.09]</td>
<td>[0.97, 2.27]</td>
<td>[0.97, 2.27]</td>
</tr>
<tr>
<td>Volatility of market excess returns (%)</td>
<td>13.09</td>
<td>11.32</td>
<td>10.67</td>
</tr>
<tr>
<td></td>
<td>[11.60, 14.76]</td>
<td>[9.45, 13.25]</td>
<td>[8.94, 12.45]</td>
</tr>
<tr>
<td>Mean of market excess returns (%)</td>
<td>7.54</td>
<td>9.04</td>
<td>8.15</td>
</tr>
<tr>
<td></td>
<td>[2.44, 12.32]</td>
<td>[6.27, 11.73]</td>
<td>[5.73, 10.42]</td>
</tr>
<tr>
<td>Innosimm Q1 (average excess return, %)</td>
<td>6.13</td>
<td>8.21</td>
<td>7.93</td>
</tr>
<tr>
<td></td>
<td>[4.96, 7.42]</td>
<td>[4.79, 11.59]</td>
<td>[5.51, 10.33]</td>
</tr>
<tr>
<td>Innosimm Q5 (average excess return, %)</td>
<td>9.54</td>
<td>11.45</td>
<td>8.19</td>
</tr>
<tr>
<td></td>
<td>[8.34, 10.74]</td>
<td>[8.17, 14.69]</td>
<td>[5.80, 10.42]</td>
</tr>
<tr>
<td>Innosimm Q5 – Q1 (average spread, %)</td>
<td>3.41</td>
<td>3.24</td>
<td>0.26</td>
</tr>
<tr>
<td></td>
<td>[1.59, 5.10]</td>
<td>[1.12, 5.35]</td>
<td>[−1.84, 2.31]</td>
</tr>
</tbody>
</table>

Note: In Panel A, the data moments come from column (2) of Table 6. We bootstrap the regression with 1000 replications and report the 5th and 95th percentiles of the bootstrapped distribution (in brackets). In the model, we simulate a sample of 500 industries for 70 years. The first 20 years are dropped as burn-in. We calculate the average net profits across all industries in each year. We then estimate the sensitivity by regressing the year-$t$ growth rate of average net profits on $\theta_t$. The 5th and 95th estimated percentiles of the simulated distribution of regression coefficients are reported in brackets. In Panel B, real average risk-free interest rate is the average of the difference between the annual returns of one-month treasury bills (from CRSP) and the rate of change in CPI from 1988 to 2018. In each year we compute the annualized volatility based on the 12 monthly market excess returns. We then average the annualized volatility from 1988 to 2018 to obtain the volatility of market excess returns. Mean of market excess returns is the average of the value-weighted market excess returns from 1988 to 2018. We bootstrap the data moments with 1000 replications and report the 5th and 95th percentiles of the bootstrapped distribution (in brackets). The last three rows compare the value-weighted average excess returns of portfolios sorted on innosimm between model and data (returns in the data column are from Panel A of Table 9). In the model, in each year $t$, we sort the simulated firms into five quintiles based on their $\lambda_i$ at the beginning of the year. We then compute the value-weighted average excess returns of each quintile. Stock returns are adjusted for financial leverage.

Next, we check whether our model can quantitatively replicate the main asset pricing findings in Panel B of Table 13. The model-implied average risk-free rate, volatility of real risk-free rates, volatility of market excess returns, and average market excess returns are roughly in line with the data (columns 1 and 2). The last three rows of Table 13 show that the model-implied spread in annualized average excess returns between Q1 and Q5 is
about 3.24\% (last row of column 2), which is also roughly consistent with the data.

To understand the importance of price war risks in generating the cross-sectional implications, we use the model to conduct two counterfactual analyses. In the first counterfactual (column 3), we shut down the price war risks by not allowing any firms to collude with each other. That is, the two firms in the same industry adopt non-collusive pricing, and both set prices taking the other’s price as given. The equilibrium in each industry is the non-collusive Nash equilibrium. We find that the average market excess return decreases from 9.04\% (full model) to 8.15\% by about 10\%; the change in the average excess market return of Q5 is especially large (about 30\%). The volatility of market excess returns decreases from 11.32\% (full model) to 10.67\% by about 5.7\%. The model-implied Q5-Q1 spread in average excess returns is largely reduced from 3.24\% (full model) to 0.26\%. The spread is still a positive number because even in non-collusive equilibrium, profit margins can vary with customer base shares (see the red dotted line in Panel B of Figure 3) because of the difference between within- and between-industry competition, captured by elasticities \( \eta \) and \( \epsilon \). Industries with a higher capacity for radical innovation are less exposed to long-run growth shocks and have lower expected stock returns, because their endogenously more concentrated market structure dampens the within-industry competition.

In the second counterfactual (column 4), we set \( \epsilon = \eta > 0 \) and \( z = 0 \).\(^{33}\) As we discuss in Online Appendix E.1, given the same elasticity and no customer base accumulation (\( z = 0 \)), the two firms will always set the same prices regardless of their current customer base shares. The collusive equilibrium is exactly the same as the non-collusive equilibrium because within-industry competition is identical to between-industry competition. In fact, the economy is isomorphic to the one with a continuum of firms having monopolistic competition. Thus, we can analytically derive the model’s asset pricing implications. In this counterfactual, the cross-industry difference in innovation activities has no effect on industries’ exposure to price war risks.\(^{34}\) Thus, the Q5-Q1 spread is about zero. The mean and volatility of market excess returns are slightly smaller than those in the first counterfactual.

Overall, by comparing the implications of our full model with those from the two

\(^{33}\)Although the levels of profit margins and firm values depend on the levels of \( \epsilon \) and \( \eta \), the asset pricing implications are exactly the same when \( \epsilon = \eta \).

\(^{34}\)The firm-level returns are different depending on the firm’s customer base share and the industry’s capacity for radical innovation. We show in Online Appendix Figure OA.11 that the firm’s conditional expected stock returns decrease with its customer base share due to the lower expected growth rate.
counterfactuals, we have shown that price war risks significantly contribute to the equity premium and stock return volatility; price war risks are also the key to explaining our cross-sectional asset pricing patterns in the data.

7 Conclusion

In this paper, we investigate the origin of systematic price war risks and explore their implications. We develop a general-equilibrium asset pricing model incorporating dynamic games of price competition among firms. In our model, price wars can arise endogenously from declines in long-run growth, because firms become effectively more impatient for cash flows and their incentives to undercut prices become stronger. The exposure to price war risks reflects predictable and persistent heterogeneous industry characteristics. Firms in industries with a higher capacity for radical innovation are more immune to price war risks because of the higher likelihood of market disruption in the future. Exploring detailed patent, brand, profit margin, product price, and media and analyst coverage data, we find evidence for the existence of price war risks, as well as the evidence that price war risks are priced in the cross section of industries’ capacity for radical innovation. Our theoretical and empirical studies shed new light on the relationship between competition and stock returns – we emphasize that what matters for understanding asset prices is forward-looking competition for future market dominance, rather than current concentration or product similarity.

References


He, Zhiguo, and Arvind Krishnamurthy. 2013. “Intermediary asset pricing.” American Economic Review,


Appendix

A Construction of Variables in Figure 1.

Long-run Growth Rates. Long-run growth rates of consumption in year $t$ are measured by: (1) the consumption growth rates filtered by a Bayesian mixed-frequency approach as in Schorfheide, Song and Yaron (2018), and (2) the cumulative demeaned consumption growth rates from year $t$ to year $t - 1$ (eight-quarter summation), as in Bansal, Dittmar and Lundblad (2005) and Dittmar and Lundblad (2017). Filtered consumption growth rates come from Schorfheide, Song and Yaron (2018) and the data end in 2015.35 Following Bansal, Dittmar and Lundblad (2005) and Dittmar and Lundblad (2017), we measure the demeaned consumption growth rate at quarter $t$ as the difference in log consumption growth rate at quarter $t$ and the unconditional mean of log consumption growth rate over the post-war period (from 1947 to 2018). Consumption is measured as per-capita real personal consumption expenditures on non-durable goods and services, and is deflated to real terms using the personal consumption expenditure deflator. The two measures of long-run growth rates are highly correlated (see Panel B of Figure 6).

35We are grateful to Amir Yaron for sharing data on the filtered consumption growth rates.
**Profit Margins.** We construct two measures of gross profit margins based on the NBER-CES Manufacturing Industry Database and Compustat, and two measures of net profit margins based on the BEA data and Compustat. Following Domowitz, Hubbard and Petersen (1986) and Allayannis and Ihrig (2001), we construct the NBER-CES-based profit margin for industry $i$ at year $t$ as (Value of shipments$_{ij,t}$ + ΔInventory$_{ij,t}$ – Payroll$_{ij,t}$ – Cost of material$_{ij,t}$) / (Value of shipments$_{ij,t}$ + ΔInventory$_{ij,t}$). Following Anderson, Rebello and Wong (2018), we construct the Compustat-based profit margin for industry $i$ at year $t$ as (Sales$_{ij,t}$ – COGS$_{ij,t}$) / Sales$_{ij,t}$. We measure the BEA-based aggregate net profit margin as the profits after tax for the nonfinancial corporate business scaled by the GDP in the nonfinancial sector. We construct the Compustat-based net profit margin for industry $i$ at year $t$ as (Sales$_{ij,t}$ – COGS$_{ij,t}$ – SG&A$_{ij,t}$ – Interest$_{ij,t}$ – Tax$_{ij,t}$) / Sales$_{ij,t}$. We remove R&D expenditures from SG&A following Peters and Taylor (2017).

**Media and Analyst Coverage of Price Wars.** We use textual analysis to measure the media and analyst coverage of price wars. Following Baker, Bloom and Davis (2016), we quantify the prevalence of price wars using the targeted-phrase search approach, which is “one of the simplest but at the same time the most powerful approaches” in textual analysis (see Loughran and McDonald, 2016). The price war media coverage is the number of articles that contain the term “price war” or “price wars” normalized by the number of articles published in The Wall Street Journal, The New York Times, and Financial Times. We consider articles covering the US region obtained from the Dow Jones Factiva. The price war analyst coverage is the number of analyst reports that contain the term “price war” or “price wars” normalized by the number of articles published in The Wall Street Journal, The New York Times, and Financial Times. We consider analyst reports covering the US region obtained from Thomson ONE Investext. Following Huang, Zang and Zheng (2014), we plot the price war analyst coverage after 1996, because the data coverage for the full text of analyst reports is limited before 1996.

**B Highest Deviation Value with Full-blown Price Wars**

In equation (3.1), firm $j$’s deviation value $V^{D}_{ij,t} = V^{D}_{ij}(y_{i,t})$ is given by the following HJB equations:

$$0 = \begin{cases} \max_{P_{ij}} \Lambda_t \Pi_{ij}(P_{ij,t}, P^C_{ij,t})M_{ij,t}dt + \mathbb{E}t \left[ d \left( \Lambda_t V^{D}_{ij,t} \right) \right] |_{P_{ij,t}, P^C_{ij,t}} + \Lambda_t \left( V^{N}_{ij,t} - V^{D}_{ij,t} \right) \xi dt, & \text{if } \Gamma_{ij,t} \geq v_{i,t} \text{ for all } j, \quad (III) \\ \max_{P_{ij}} \Lambda_t \Pi_{ij}(P_{ij,t}, P^N_{ij,t})M_{ij,t}dt + \mathbb{E}t \left[ d \left( \Lambda_t V^{D}_{ij,t} \right) \right] |_{P_{ij,t}, P^N_{ij,t}}, & \text{if } \Gamma_{ij,t} \geq v_{i,t} \text{ for some } j, \quad (IV) \end{cases}$$

where $P^N_{ij,t} = P^N_{ij}(y_{i,t})$ with $j = 1, 2$ are the non-collusive prices that solve the maximization problems in (IV), and $V^{N}_{ij,t} = V^{N}_{ij}(y_{i,t})$ with $j = 1, 2$ are firm values in the non-collusive equilibrium.

**C Innovation Similarity Measure**

We define the cosine similarity between two patents, $a$ and $b$, as:
similarity \((a, b) = \frac{A \cdot B}{\|A\| \|B\|}\) (C.1)

where \(A\) and \(B\) are the technology vectors of patent \(a\) and patent \(b\).\(^{36}\) If the two patents share exactly the same technology classifications, the cosine similarity attains the maximum value 1. If the two patents are mutually exclusive in their technology classifications, the cosine similarity reaches the minimum value 0. Because patent technology classifications are assigned according to the technical features of patents, the cosine similarity measure captures how similar the patents are in terms of their technological positions. Based on the pairwise cosine similarity of patents, we take the following steps to construct innosimm.

First, we construct the patent-level similarity measure to capture the extent to which a patent is differentiated from other patents recently developed by peer firms. In particular, for a patent granted to firm \(i\) in year \(t\), the patent-level similarity measure is the average of the pairwise cosine similarity (defined by equation C.1) between this patent and the other patents granted to firm \(i\)'s peer firms in the same four-digit SIC industry from year \(t - 5\) to year \(t - 1\).

Next, we aggregate patent-level similarity measures to obtain industry-level similarity measures. For example, a four-digit SIC industry’s similarity measure in year \(t\) is the average of patent-level similarity measures associated with all the patents granted to firms in the industry in year \(t\). Because not all industries are granted patents every year, we further average the industry-level similarity measures over time to filter out noise and better capture firms’ ability to generate differentiated innovation. In particular, our innosimm measure in industry \(i\) and year \(t\) (i.e., \(\text{innosimm}_{it}\)) is constructed as the time-series average of industry \(i\)'s similarity measures from year \(t - 9\) to year \(t\). In the regression analyses of our paper, we standardize innosimm using its unconditional mean and the standard deviation of all industries’ innosimm across the entire period from 1976 to 2017 to ease the interpretation of the regression coefficients.

### D Supplementary Empirical Results

**Profit Margins Across Industries with Different Innosimm.** In Table D.1, we show that industry-level profit margins are positively associated with innosimm. This relationship is robust to the measures of profit margins constructed from both the Compustat and NBER-CES data. The coefficient of innosimm is economically significant. According to the regressions with year fixed effects (columns 2, 4, and 6), a one-standard-deviation increase in innosimm is associated with a 2.34-percentage-point increase in the Compustat-based gross profit margins, a 3.19-percentage-point increase in the NBER-CES-based gross profit margins, and a 1.44-percentage-point increase in the Compustat-based net profit margins.

[^36]: PatentsView provides both the Cooperative Patent Classification (CPC) and the US Patent Classification (USPC), the two major classification systems for US patents. As in Kelly et al. (2018), we use CPC for our analyses because USPC is no longer available after 2015. Our results are robust to the classification based on USPC for data prior to 2015. There are 653 unique CPC classes (four-digit level) in PatentsView. The technology classification vector for a patent consists of 653 indicator variables that represent the patent’s CPC classes.
Table D.1: Profit margins across industries with different innosimm (yearly analysis).

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Industry-level profit margins (%)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gross profit margins, Compustat</td>
<td>Gross profit margins, NBER-CES</td>
<td>Net profit margins, Compustat</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Innosimm</td>
<td>2.45 ***</td>
<td>2.34 ***</td>
<td>3.29 ***</td>
<td>3.19 ***</td>
<td>1.58 ***</td>
<td>1.44 ***</td>
</tr>
<tr>
<td></td>
<td>[4.13]</td>
<td>[3.87]</td>
<td>[3.65]</td>
<td>[3.51]</td>
<td>[4.19]</td>
<td>[3.76]</td>
</tr>
<tr>
<td>Year FE</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Observations</td>
<td>9179</td>
<td>9179</td>
<td>2787</td>
<td>2787</td>
<td>9179</td>
<td>9179</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.002</td>
<td>0.006</td>
<td>0.063</td>
<td>0.072</td>
<td>0.018</td>
<td>0.028</td>
</tr>
</tbody>
</table>

Note: This table shows the relation between innosimm and industry-level profit margins. Profit margins are computed as in Appendix A. The sample in columns (1)–(2) and columns (5)–(6) spans the period from 1988 to 2017, and that in columns (3)–(4) the period from 1988 to 2011. We include t-statistics in brackets. Standard errors are clustered by the four-digit SIC industry and year. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively.

Price-similarity Sensitivity around the Lehman Crash. Columns (1) and (2) of Table D.2 show that the price-innosimm sensitivity reduced significantly following the Lehman crash, indicating that industries with higher innosimm were more affected by the Lehman crash and that their product prices decreased to a greater extent. We do not observe any significant change in the price-prodsimm sensitivity (columns 3 and 4).

Table D.2: Price-similarity sensitivity around the Lehman crash (monthly analysis).

<table>
<thead>
<tr>
<th>Similarity measure</th>
<th>(1) Percentage change in industry-level product prices (monthly, annualized, %)</th>
<th>(2) Innosimm</th>
<th>(3) Prodsimm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Similarity_{t-1} × post Lehman crash_{t}</td>
<td>−3.04 *** [−3.19]</td>
<td>−2.84 *** [−2.79]</td>
<td>−0.07 [−0.20]</td>
</tr>
<tr>
<td>Similarity_{t-1}</td>
<td>−1.00 [−1.32]</td>
<td>−2.05 [−1.45]</td>
<td>−0.01 [−0.01]</td>
</tr>
<tr>
<td>Post Lehman crash_{t}</td>
<td>−1.61 [−1.44]</td>
<td>−1.64 [−1.47]</td>
<td>−2.25* [−1.80]</td>
</tr>
<tr>
<td>Industry FE</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Observations</td>
<td>7641</td>
<td>7641</td>
<td>7192</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.004</td>
<td>0.040</td>
<td>0.001</td>
</tr>
</tbody>
</table>

Note: This table shows the changes in price-similarity sensitivity around the Lehman crash. The dependent variable is the annualized monthly percentage change in product prices of four-digit SIC industries. We consider the 36-month period centered at the Lehman crash. In Online Appendix F.1, we perform the analysis by considering the 24-month period centered at the Lehman crash and find similar results. We include t-statistics in brackets. Standard errors are clustered by the four-digit SIC industry and month. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively.

Double-sort Analyses. We have shown that the profit margins of industries with higher innosimm are more exposed to long-run growth shocks. According to our model, innosimm is priced in the cross section because it captures the sensitivity of profit margins to long-run growth shocks. One alternative explanation is that innosimm may be priced via its positive correlation with the level of profit margins instead of its relation with the sensitivity of profit margins to long-run growth shocks. This is a valid concern because previous studies have shown that profitability is strongly related to asset returns (see, e.g.,
Novy-Marx, 2013; Fama and French, 2015; Hou, Xue and Zhang, 2015). We present two sets of evidence against this alternative explanation. First, the innoimm spreads documented in Table 9 are robust when we control for the profitability factor using the Fama-French five-factor model (Fama and French, 2015) and the Hou-Xue-Zhang \( q \) factor model (Hou, Xue and Zhang, 2015). The annualized spreads in alphas between the high-innoimm industries (Q5) and the low-innoimm industries (Q1) are 9.24% and 8.88%, while the t-statistics are 4.11 and 6.38, in these two models. Second, the innoimm spreads remain robust after we double sort on the profit margins (see Table D.3). These findings suggest that innovimm and profitability are likely priced through different underlying economic mechanisms, which is perhaps not surprising given that the level of profitability is affected by many other factors besides its sensitivity to long-run growth shocks.

Besides the level of profitability, we also conduct a number of double-sort analyses for other related variables. We find that innoimm spreads are robust after controlling for various related variables including prodsimm, innovation originality, asset growth rate, income elasticity of demand, and the durability of firms’ outputs (see Table D.3).

### Table D.3: Double-sort analyses (monthly analysis).

<table>
<thead>
<tr>
<th>Double-sort variable</th>
<th>Excess returns (%)</th>
<th>Fama-French three-factor alpha (%)</th>
<th>Carhart four-factor alpha (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profit margins</td>
<td>2.53*** [2.80]</td>
<td>4.43** [5.00]</td>
<td>4.14*** [4.42]</td>
</tr>
<tr>
<td>Prodsimm</td>
<td>2.15* [1.92]</td>
<td>3.81*** [4.27]</td>
<td>3.72*** [4.81]</td>
</tr>
<tr>
<td>Durability of firms’ outputs</td>
<td>3.66** [2.27]</td>
<td>3.96** [2.07]</td>
<td>3.69** [2.48]</td>
</tr>
</tbody>
</table>

Note: This table shows the average excess returns and alphas from double-sort analyses. In the double-sort analyses, we first sort the four-digit SIC industries into three groups based on profit margins, prodsimm, innovation originality, asset growth rate, or income elasticity of demand in June of year \( t \). We then sort firms within each group into five quintiles based on innoimm in year \( t - 1 \). Once the portfolios are formed, their monthly returns are tracked from July of year \( t \) to June of year \( t + 1 \). Industry-level profit margins are computed based on Compustat data as explained in Figure 1. Prodsimm is the product similarity measure as in Hoberg and Phillips (2016), which is derived from textual analysis based on the business description in 10-K filings. Innovation originality is constructed following Hirshleifer, Hsu and Li (2017) to capture the patents’ originality. In particular, we count the number of unique technology classes contained in a patent’s citation list. We then obtain the industry-level innovation originality measure by averaging the number of unique technology classes across all patents in a four-digit SIC industry every year. Asset growth rate is the growth rate of the total asset. We obtain the industry-level asset growth rate by averaging the firm-level asset growth rate in a four-digit SIC industry every year. We estimate the industry-level income elasticity of demand based on the representative consumer’s income and expenditures on different products (see Section C.4 for details). We also perform the double-sort analysis in which we first sort industries into categories based on the durability of firms’ output. The durability of firms’ output comes from Gomes, Kogan and Yogo (2009), who classify each SIC industry into six categories (durables, non-durables, services, private domestic investment, government, and net exports) based on its contributions to final demand. We exclude financial firms and utility firms from our analyses. We include t-statistics in brackets. Standard errors are computed using the Newey-West estimator allowing for serial correlation in returns. We annualize average excess returns and alphas by multiplying them by 12. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively.