

# Designing mandatory pension plans

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**Abstract:** Numerous countries have established mandatory pension savings schemes characterized by a predetermined contribution rate, investment strategy, and retirement payout policy. We set up a life-cycle model in which we can (i) evaluate the lifetime utility of a participating individual for any given mandatory savings scheme and (ii) determine the optimal design of the savings scheme. When the individual is rational and has access to the same investment opportunities as the pension fund, we show that typical pension scheme designs substantially reduce welfare but that an optimal pension design can marginally improve welfare through the more lenient return taxation on pension savings. However, we also find that a well-designed mandatory pension plan can substantially improve the welfare of individuals who either does not privately invest in stocks at all or only in an undiversified stock portfolio, or who procrastinate on retirement savings.

**Keywords:** Retirement savings, life cycle, consumption, investment, behavioral biases.

**JEL subject codes:** D91, G11, D14, E21, J32.

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# 1 Introduction

Numerous countries have established—or are moving towards—a mandatory retirement savings system in which workers are required to save a certain fraction (the contribution rate) of their labor income in a defined contribution pension plan.<sup>1</sup> Often the worker has virtually no influence on the contribution rate or on how the savings are invested, and is not allowed to make unscheduled withdrawals from the pension account at all or only do so subject to a significant penalty.<sup>2</sup> Such a setting raises a number of questions. How does the mandatory pension scheme affect the welfare, consumption, and private investment decisions of the worker? Given the existence of a mandatory pension scheme, which pension scheme design—in the form of contribution rate, fund investment strategy, and payout policy—would be optimal for the individual fund member? How sensitive is the optimal design to the characteristics of the fund member?

To answer the above questions, we set up a life-cycle utility maximization model that adds a mandatory illiquid pension scheme to common model ingredients such as consumption decisions, stochastic and unspanned labor income, stock investments, and portfolio constraints. We consider preferences of the Epstein-Zin class that allow for the separation of investor attitudes towards risk and intertemporal substitution of consumption. For a given design of the mandatory pension scheme, we solve this rich dynamic optimization problem for the optimal private consumption-investment decisions of the individual and derive the corresponding indirect utility. We do this under various assumptions about the abilities of the individual ranging from the fully rational, savvy investor to individuals procrastinating on savings and with limited investment skills. Of course, solving the individual’s optimization problem is already important for determining the extent to which an individual with a certain mandatory pension plan should build up additional liquid savings and how they should be invested. Then in a higher-level optimization, we solve for the optimal pension fund design, and we investigate how sensitive the optimal pension fund design is to the individual’s characteristics and skills.

Even a fully rational, savvy individual can benefit from saving in a retirement account due to the milder taxation of returns in such accounts compared to private investments. On the other hand, pension savings are predominantly illiquid, and a mandatory scheme with a given contribution rate and asset allocation may substantially distort the attainable life-cycle consumption plans away from the optimal plan. Hence, many mandatory savings

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<sup>1</sup>Defined contribution occupational pension plans play a significant role in the pension system in countries like Australia, Chile, Denmark, India, Indonesia, Israel, Latvia, Mexico, the Netherlands, Slovakia, Sweden, and the United Kingdom, cf. the OECD pension overview at <http://www.oecd.org/els/public-pensions/>.

<sup>2</sup>The low penalty rates in the United States are an exception, cf. the international comparison of Beshears, Choi, Hurwitz, Laibson, and Madrian (2015).

schemes lead to a significant welfare loss, but a carefully designed scheme may marginally improve the individual’s welfare. In our baseline parametrization of the model, we find that the best pension design involves a low (5-10%) contribution rate starting at age 25-35 and a fund stock weight following the ‘120 minus age’ rule, and the associated increase in the individual’s welfare corresponds to around 0.14% of initial wealth and lifetime labor income which in monetary terms is around \$1,800 in present value terms. For many fund designs near the optimal design, the individual can—through private savings and investments—more or less undo any undesired savings to the fund and asset allocation policies in the fund. However, for a rather common pension scheme (at least in Denmark) with a 17% contribution rate throughout working life and a 50/50 stock/bond allocation, the welfare loss of our baseline individual is 3.8% or roughly \$48,000; with the ‘120 minus age’ allocation, the loss is reduced to 3.5% or about \$44,000.

The behavioral household finance literature has documented that the consumption and investment decisions of many individuals deviate systematically from what standard theoretical models predict, cf., e.g., the surveys by [Campbell \(2006; 2016\)](#), [Guiso and Sodini \(2013\)](#), and [Beshears, Choi, Laibson, and Madrian \(2018\)](#). Some general findings are that many households invest a too small fraction of wealth in risky assets, are under-diversified, and obtain poor investment returns; cf., e.g., [Barber and Odean \(2000\)](#), [Calvet, Campbell, and Sodini \(2007\)](#), and [Campbell, Ramadorai, and Ranish \(2018\)](#). Furthermore, many individuals seem to procrastinate on retirement savings and refuse, or at least postpone, to set money aside for retirement; see [Bernheim, Skinner, and Weinberg \(2001\)](#), [Benartzi and Thaler \(2007\)](#), [Choi, Laibson, and Madrian \(2011\)](#), and [Gomes, Hoyem, Hu, and Ravina \(2018\)](#), among others. The behavioral biases translate into welfare losses that can have a moderate or large magnitude depending on the specific setting, cf. [Calvet et al. \(2007\)](#) and [Bhamra and Uppal \(2019\)](#). We show that a well-designed mandatory pension plan can mitigate these welfare losses by facilitating higher returns on investments, a better overall asset allocation, and ultimately consumption at a higher level and with a better life-cycle profile.

More specifically, with our baseline parameter values, we find that the welfare of an individual holding all private savings in the riskfree asset improves by up to around 7%—or \$88,800 in present value terms—when faced with mandatory savings in a pension fund with significant stock holdings. The best design with a constant contribution rate involves a 10% contribution rate from age 25 and fund investments according to the ‘120 minus age’ policy.<sup>3</sup> However, the maximum utility in this situation is still around 5% (or \$64,000) below the maximum utility when the individual is optimally investing her private wealth in both stocks and bonds. Hence, there are still significant potential welfare gains to

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<sup>3</sup>Some designs with a gradually increasing contribution rate are marginally better.

increased household participation in the stock market.

When the individual procrastinates on retirement savings, a mandatory plan can ensure that households build sufficient wealth for retirement and smooth consumption over the life cycle. To be specific, we assume that the individual evaluates consumption plans using a subjective discount factor of  $\beta = 0.96$ —a value often assumed in the life-cycle literature—but, due to lack of self-control, she applies the lower value  $\beta = 0.90$ —a higher impatience rate in line with the estimates of [Fagereng, Gottlieb, and Guiso \(2017\)](#) from Norwegian savings data—when making consumption and investment decisions. With our baseline parametrization, the individual’s welfare is reduced by around 20% due to procrastination in the absence of a mandatory savings program. We find that by requiring the individual to participate in a defined contribution retirement savings program, the welfare can be improved by as much as 12% for schemes with a constant contribution rate. The best design involves contributions of around 12% of income starting at age 35-40 together with the ‘120 minus age’ asset allocation policy.

The paper most closely related to our study is [Dahlquist, Setty, and Vestman \(2018\)](#). Motivated by the Swedish pension system, they determine the optimal default-choice stock-bond asset allocation strategy of a pension plan in a setting where the investor can—at a certain cost and only at age 25—actively switch to any asset allocation strategy.<sup>4</sup> The investor’s participation in the stock market outside the pension system is also an active decision that can be made at any age but only upon the payment of an entry cost. The authors allow for heterogeneous costs across investors to match the observed active decisions of Swedish investors. Four possible default fund strategies are considered: the popular ‘100 minus age’ rule, the strategy which is optimal in the absence of switching costs (depends on all state variables of the model), the average optimal age-based strategy, and a rule-of-thumb strategy developed by the authors. Throughout, the authors assume a given, fixed contribution rate and do not consider the optimal level of the rate (or whether it should be age dependent). They ignore taxes, although returns in many countries are differentially taxed whether they are made inside or outside retirement saving plans. In contrast, we incorporate taxes and search for the optimal contribution rate.

Our paper contributes to the literature on optimal consumption and investment decisions of an individual over the life cycle. This literature adds stochastic labor income, mortality risk, and other realistic features to the pioneering dynamic optimization models of [Samuelson \(1969\)](#) and [Merton \(1969; 1971\)](#). Prominent examples are [Viceira \(2001\)](#) and [Cocco, Gomes, and Maenhout \(2005\)](#) who, among other things, show that most individuals should invest all their savings in stocks early in life and then later in life gradually replace

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<sup>4</sup>The authors document that very few investors actively switch, which can justify our assumption of a fixed, pre-determined strategy with no opt-out possibility.

stocks by bonds, motivating the glidepath strategies of target date funds. While risky, the human capital of a typical individual is so little correlated with the stock market that it corresponds to an implicit position in bonds and, hence, financial investments should be tilted towards stocks. Over life, human capital declines so the optimal stock-bond allocation gradually approaches the Samuelson-Merton solution for the no-income case.<sup>5</sup>

Only few papers in the life-cycle literature explicitly model an illiquid pension account, although this is the predominant retirement savings vehicle for many individuals. As we do, [Campbell, Cocco, Gomes, and Maenhout \(2001\)](#) assume a predetermined, constant contribution rate and asset allocation strategy of the pension fund, and derive the individual's optimal consumption and private investments over the life cycle. They compare welfare and optimal individual decisions for two different fund allocation strategies, namely (i) 100% in the riskfree asset vs. (ii) 50% in stocks, 50% in the riskfree asset. We take the analysis a large step further by deriving the optimal combination of contribution rate and fund allocation strategy, and we also consider how the pension savings are optimally paid out in retirement.

[Blake, Wright, and Zhang \(2014\)](#) also investigate the optimal contribution rate and stock-bond allocation of the pension plan. However, they ignore the possibility of free savings outside the plan, which fixes consumption at a fraction of current income and thus prevents consumption smoothing. They also disregards bequests and taxes.

Most existing papers assume the individual can freely choose how much to contribute to the pension scheme and how to invest the balance of the pension account. Given the differential taxation of returns in pension accounts and in private accounts, a key focus is to allocate assets across these accounts in a tax-efficient way. As an example, [Dammon, Spatt, and Zhang \(2004\)](#) consider a life-cycle model in which the individual can save both in an illiquid, tax-deferred retirement account and in a liquid, taxable account. A predetermined fraction of non-financial income is allocated to the retirement account each year. While they take this contribution rate as given, we also discuss what the optimal rate is. In contrast to our setting, they let the individual freely choose how the balance of the retirement account is split between stocks and bonds. In their model, the asset allocation decisions are to a large extent driven by tax considerations. In our model, all returns (even unrealized gains) in both accounts are taxed proportionally. As an extension, they

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<sup>5</sup>The canonical life-cycle model has been extended in various dimensions such as labor supply flexibility ([Bodie, Merton, and Samuelson, 1992](#)), housing ([Cocco, 2005](#); [Fischer and Stamos, 2013](#); [Corradin, Fillat, and Vergara-Alert, 2014](#)), time-varying investment opportunities ([Kojen, Nijman, and Werker, 2010](#); [Munk and Sørensen, 2010](#); [Lynch and Tan, 2011](#)), unemployment risk ([Bremus and Kuzin, 2014](#)), income-stock market co-integration ([Benzoni, Collin-Dufresne, and Goldstein, 2007](#)), investments in annuity products ([Horneff, Maurer, and Rogalla, 2010](#); [Kojen, Nijman, and Werker, 2011](#)), habit formation in preferences ([Gomes and Michaelides, 2003](#); [Polkovnichenko, 2007](#); [Kraft, Munk, and Wagner, 2018](#)), and stock market entry/participation costs ([Fagereng et al., 2017](#)).

introduce the possibility to withdraw funds from the retirement account prematurely at a penalty. To simplify their numerical solution method, they make the highly unrealistic assumption that income is a given fraction of the investor’s contemporaneous financial wealth. [Amromin \(2003\)](#) illustrates in a simple setting how the risk of a significant drop in labor income can substantially change the optimal strategy away from the apparent tax-efficient strategy. Other papers focusing on tax-driven asset location and allocation decisions include [Gomes, Michaelides, and Polkovnichenko \(2009\)](#), [Zhou \(2009\)](#), and [Fischer and Gallmeyer \(2017\)](#).

The rest of the paper is organized as follows. Section 2 sets up the model and fixes the baseline parameter values. Section 3 illustrates how an individual optimally chooses private consumption and investment decisions for a fixed pension fund design. Section 4 determines and discusses the optimal pension fund design given the characteristics of the individual fund contributor, under the assumption that the individual acts rationally and has access to the same investment opportunities as the pension fund. Section 5 explores the implications on optimal pension fund design of behavioral biases in individual investment decisions. Finally, Section 6 summarizes and concludes.

## 2 The modeling framework

### 2.1 Description of the model

We use a discrete-time model with annual time steps. We model the decision problem of an individual who has just turned  $t_1$  years old, retires when she turns  $T_R$  years old, and may live on until the day before turning  $T_M + 1$  years old. Being alive at age  $t$ , there is a probability of  $p_t = \exp\{-\nu(t)\}$  of surviving another year, where  $\nu(t) > 0$  is the mortality intensity with  $p_{T_M} = 0$ .

At the beginning of year  $t$  (i.e. just after turning  $t$  years old), the individual has a private and perfectly liquid financial wealth of  $F_t$  and a pension account balance of  $A_t$ . Before retirement, she receives labor income of  $Y_t$  of which she pays the fraction  $\alpha_t \in [0, 1)$  into the pension fund and a proportional tax given by the rate  $\tau_Y$  on the remainder  $(1 - \alpha_t)Y_t$ , which leaves a total disposable wealth (aka. cash-on-hand) of

$$\tilde{F}_t = F_t + (1 - \tau_Y)(1 - \alpha_t)Y_t.$$

Of disposable wealth, she decides to consume a fraction  $\hat{c}_t \in (0, 1)$  and to invest the remainder  $(1 - \hat{c}_t)\tilde{F}_t$  in financial assets with a share of  $\pi_t$  in the stock market index and the rest in the riskfree asset. The pension fund invests  $A_t + \alpha_t Y_t$  over year  $t$  with a share of  $w_t$  in the stock market index and the rest in the riskfree asset.

We assume a constant log riskfree rate of  $r$  and assume that the log stock market return over any period  $dt$  is normally distributed with expectation  $(r + \mu_S - \frac{1}{2}\sigma_S^2) dt$  and standard deviation  $\sigma_S dt$ , and that returns are independent in the time dimension. The expected annual rate of return on the stock is thus  $\exp\{r + \mu_S\} - 1 \approx r + \mu_S$ . By assuming that the pension portfolio is continuously rebalanced through the year in order to maintain a constant stock weight of  $w_t$ , the log return on the portfolio over the year is normally distributed with expectation  $r + w_t\mu_S - \frac{1}{2}w_t^2\sigma_S^2$  and standard deviation  $w_t\sigma_S$ . All returns in the pension fund—realized or not—are taxed at year-end at a proportional rate of  $\tau_A$ , so that the after-tax gross return on the pension investment is

$$\begin{aligned} R_{At} &= 1 + (1 - \tau_A) \left[ \exp \left\{ r + w_t\mu_S - \frac{1}{2}w_t^2\sigma_S^2 + w_t\sigma_S\varepsilon_{St} \right\} - 1 \right] \\ &= \tau_A + (1 - \tau_A) \exp \left\{ r + w_t\mu_S - \frac{1}{2}w_t^2\sigma_S^2 + w_t\sigma_S\varepsilon_{St} \right\}, \end{aligned}$$

where  $\varepsilon_{St}$  is a standard normal shock to the stock price in year  $t$ . Hence, the dynamics of the pension account are

$$A_{t+1} = [A_t + \alpha_t Y_t] R_{At}, \quad t = t_1, t_1 + 1, \dots, T_R - 1. \quad (1)$$

Similarly, we assume the private portfolio is continuously rebalanced to the stock market weight  $\pi_t$  and is subject to year-end proportional taxation at the rate  $\tau_F$ . Therefore the dynamics of private financial wealth are

$$F_{t+1} = (1 - \hat{c}_t) \tilde{F}_t R_{Ft}, \quad (2)$$

where

$$R_{Ft} = \tau_F + (1 - \tau_F) \exp \left\{ r + \pi_t\mu_S - \frac{1}{2}\pi_t^2\sigma_S^2 + \pi_t\sigma_S\varepsilon_{St} \right\}$$

is the after-tax gross return on the private investment. We assume the individual is endowed with a liquid financial wealth of  $F_{t_1}$  and a pension balance of  $A_{t_1}$ .

The labor income before retirement, i.e. for  $t = t_1, t_1 + 1, \dots, T_R - 2$ , is assumed to evolve according to

$$Y_{t+1} = Y_t R_{Yt}, \quad R_{Yt} = \exp \left\{ \mu_Y(t) - \frac{1}{2}\sigma_Y(t)^2 + \sigma_Y(t)\varepsilon_{Yt} \right\}, \quad (3)$$

where  $\varepsilon_{Yt}$  is standard normally distributed and independent over time with contemporaneous correlation  $\rho_{YS}$  with  $\varepsilon_{St}$ . Hence,  $\rho_{YS}$  is the instantaneous correlation between log income growth and log stock returns,  $\sigma_Y(t)$  is the standard deviation of log income growth, and  $\mu_Y(t)$  the expected growth rate of income since  $E_t[Y_{t+1}/Y_t] = \exp\{\mu_Y(t)\}$ .

Here  $\mu_Y(t)$ —and maybe even  $\sigma_Y(t)$ —can be age-dependent to capture observed life-cycle patterns in labor income.

At retirement, labor income stops, but the individual starts receiving a pension from the mandatory fund. We assume that the pension received in the beginning of year  $t$  is  $A_t m_t$ , where

$$m_t = \frac{\tilde{r}}{1 - (1 + \tilde{r})^{-(T_M - t + 1)}},$$

i.e., the pension balance  $A_t$  is formally annuitized until the maximum age using an interest rate of  $\tilde{r} > 0$ . The remaining balance is still invested in a mix of stocks and bonds. The dynamics of the pension account thus become

$$A_{t+1} = A_t [1 - m_t] R_{At}, \quad t = T_R, T_R + 1, \dots, T_M - 1. \quad (4)$$

When the individual turns  $T_M$  years old, the fund pays out the remaining balance  $A_{T_M}$ , i.e.,  $m_{T_M} = 1$ . Intuitively, with  $\tilde{r}$  larger [smaller] than the expected after-tax pension account return, the pension payouts are expected to decrease [increase] through retirement.<sup>6</sup>

In retirement, disposable wealth is

$$\tilde{F}_t = F_t + (1 - \tau_Y) A_t m_t,$$

assuming that pension income is taxed at the same rate  $\tau_Y$  as pre-retirement labor income, and the dynamics of private financial wealth are still given by (2).<sup>7</sup>

The individual chooses  $\hat{c}_t$  and  $\pi_t$  for  $t = t_1, t_1 + 1, \dots, T_M$  to maximize lifetime utility. We let  $J_t$  denote the indirect utility at time  $t$ , conditionally on being alive, and this includes the utility of consumption in year  $t$  and subsequent dates, as well as any bequest utility. At the beginning of any year  $t$ , before receiving income and consuming in that year, the individual might die having a private wealth of  $F_t$  and a pension wealth of  $A_t$ . We assume this generates a bequest of  $B_t = F_t + A_t(1 - \tau_Y)$  so that ordinary income tax is deducted from the pension balance (as contributions were made out of pre-tax labor income). Leaving such an after-tax wealth is assumed to give the individual a bequest utility of  $\bar{U}_t = \xi^{\frac{1}{\psi-1}} B_t$ , where  $\xi \geq 0$  measures the strength of the bequest motive (see

<sup>6</sup>Conditional on survival, the expected pension payout next year relative to the payout received this year is  $E_t[m_{t+1}A_{t+1}]/(m_tA_t)$ , which can be shown to be smaller than 1 if and only if  $E_t[R_{At}] < m_{t+1}m_t/(1 - m_t) \approx 1 + \tilde{r}$ .

<sup>7</sup>To facilitate the reduction in the number of state variables and thus numerical complexity, our model ignores any basic state pension paid to all citizens independently of their wealth and other income as well as any pre-retirement minimum income ensured by the state in the form of various welfare benefits. To the extent that the state-sponsored minimum income and basic pension coincide with a subsistence minimum consumption level that gives zero utility, our model would still apply with model income representing the income above the subsistence level and assuming that it is only a fraction of this income component which is contributed to the pension fund.



Appendix A). Should the individual reach the maximum age, the pension account has already been paid out, so the bequest is then  $B_{T_M+1} = F_{T_M+1}$ .

We assume Epstein-Zin utility so that  $J_t$  satisfies the recursive relation

$$J_t = \max_{\hat{c}_t, \pi_t} \left\{ \left( \hat{c}_t \tilde{F}_t \right)^{1-\frac{1}{\psi}} + \beta \text{CE}_t^{1-\frac{1}{\psi}} \right\}^{\frac{1}{1-\frac{1}{\psi}}}, \quad (5)$$

where

$$\text{CE}_t = \left( p_t \text{E}_t \left[ J_{t+1}^{1-\gamma} \right] + (1-p_t) \text{E}_t \left[ \bar{U}_{t+1}^{1-\gamma} \right] \right)^{\frac{1}{1-\gamma}} \quad (6)$$

is the certainty equivalent of next period's utility which is given by  $J_{t+1}$  if the individual stays alive and  $\bar{U}_{t+1}$  otherwise. Here,  $\gamma > 0$  is the relative risk aversion,  $\psi > 0$  is the elasticity of intertemporal substitution, and  $\beta > 0$  is the subjective time preference factor.<sup>8</sup> An auxiliary parameter is  $\theta = (1-\gamma)/(1-\frac{1}{\psi})$ . The case  $\psi = 1/\gamma$  (and thus  $\theta = 1$ ) corresponds to time-additive power utility.

Given our set-up, the indirect utility will be a function  $J_t = J_t(F_t, A_t, Y_t)$  in the working phase and  $J_t = J_t(F_t, A_t)$  in retirement. The initial indirect utility is  $J_{t_1}(F_{t_1}, A_{t_1}, Y_{t_1})$ . We show in Appendix A that the dimension of the state space can be reduced by one by exploiting a homogeneity property. More precisely, in retirement

$$J_t = ([1-\tau_Y]A_t + F_t) G_t(a_t), \quad a_t = \frac{[1-\tau_Y]A_t}{[1-\tau_Y]A_t + F_t} \quad (7)$$

where  $G_t$  is linked to  $G_{t+1}$  through a recursive equation involving an expectation over the distribution of the shock  $\varepsilon_{S_t}$  to stock prices. Similarly, before retirement,

$$J_t = ([1-\tau_Y]A_t + F_t) G_t(a_t, y_t), \quad y_t = \frac{[1-\tau_Y]Y_t}{[1-\tau_Y]A_t + F_t}, \quad (8)$$

and  $G_t$  is linked to  $G_{t+1}$  through a recursion involving an expectation over the distribution of the shock  $\varepsilon_{S_t}$  to stock prices and the shock  $\varepsilon_{Y_t}$  to labor income. Since payouts from the pension fund are taxed at the rate  $\tau_Y$ , whether paid out as a pension flow to the saver or as a bequest, we can interpret  $[1-\tau_Y]A_t + F_t$  as the total after-tax financial wealth at time  $t$ . The state variables of the reduced problem are  $a_t$ , the pension share of total financial wealth, and  $y_t$ , the ratio of after-tax income to total financial wealth. The problem is solved by backwards dynamic programming on a grid of points  $(a_i, y_j)$ . The expectations are approximated by Gauss-Hermite quadrature. This leads to the indirect utility and the optimal decisions each year in all grid points. To obtain life-cycle patterns, we simulate many possible paths forward drawing random shocks to stock prices and labor

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<sup>8</sup>We assume  $\gamma \neq 1$  and  $\psi \neq 1$ , but cases with  $\gamma = 1$  or  $\psi = 1$  or both can be studied separately with appropriate adjustments of (5) and (6).

income, using interpolation and extrapolation when simulated values of  $a$  and  $y$  are off the grid. We report averages and selected percentiles at each age to indicate an expected life-cycle pattern as well as a confidence interval.

## 2.2 Baseline parameter values

In subsequent numerical examples we use the baseline parameter values listed in Table 1. The financial asset parameter values are standard with a (real) riskfree rate of 1%, an equity premium of 4%, and an equity volatility of 15.7%. The assumed preference parameter values are frequently used in the theoretical life-cycle literature. With  $\psi = 1/\gamma$ , we take the classical time-additive power utility as the baseline setting, and assume  $\gamma = 4$  which is in the range generally considered realistic based on introspection and empirical studies (Meyer and Meyer, 2005). The subjective discount factor of 0.96 is a standard choice (Cocco et al., 2005, e.g.), and the bequest coefficient also seems consistent with the empirical literature (see Kværner (2018) and the discussion therein) and values typically considered in related papers. To better match observed savings and investment decisions, several papers consider other preference parameters. For example, Fagereng et al. (2017) and Dahlquist et al. (2018) calibrate life-cycle models to observed consumption and investment patterns and find a risk aversion in the range 11-15 (to produce low stock weights) and a subjective discount factor in the range 0.75-0.93 (to produce low savings).

The tax rates, retirement age, and mortality rates vary somewhat across countries and, to be specific, we take values relevant in a Danish context. The income tax rate is set to 34%, the average income tax rate in Denmark in 2016 as estimated by the Danish tax authorities. Pension fund returns, realized or not, are taxed at 15.3%, whereas we set the tax rate on returns on private financial investments to 27%.<sup>9</sup> The retirement age is fixed at 70, which in the model is where labor income ends and the pension payout period begins.<sup>10</sup> We use the mortality rates for women published by the Danish Financial Supervisory Authority in 2017.<sup>11</sup>

<sup>9</sup>See <http://taxsummaries.pwc.com/ID/Denmark-Individual-Taxes-on-personal-income> for an English summary of Danish tax laws and rates. The tax is 27% on stock returns up to approximately \$8,000 and 42% on additional stock returns. Only realized returns are taxed and losses can be carried forward. The labor income tax system is progressive and involves various allowances, tax brackets, and tax rates, which is not tractable in a model like ours, where non-proportional taxes would eliminate the homogeneity needed to reduce the number of state variables.

<sup>10</sup>The official retirement age in Denmark is currently 68 years but, following a broad political agreement, is expected to increase as life expectancy increases, and for a person currently 25 years old the expected retirement age is 72. The official retirement age is the earliest age at which an individual has the right to receive a state pension, but the individual can retire up to a few years earlier and start receiving payouts from her labor market pension fund. Our baseline model does not include state pensions, so the retirement age only determines when labor income stops and payouts from the individual's pension fund start.

<sup>11</sup>The data are at <https://www.finanstilsynet.dk/Tal-og-Fakta/Oplysninger-for-virksomheder/Oplysningstal-om-forsikring-og-pension/Levetidsmodel>.

Parameter	Description	Value
<i>Horizon, preferences, and initial wealth</i>		
$t_1$	Initial age in years	25
$T_R$	Retirement age in years	70
$T_M$	Maximum age in years	110
$\gamma$	Relative risk aversion	4
$\psi$	Elasticity of intertemporal substitution	0.25
$\beta$	Subjective discount factor	0.96
$\xi$	Bequest strength parameter	4
$F_{t_1}$	Initial financial wealth	5
$A_{t_1}$	Initial pension wealth	0
<i>Financial assets</i>		
$r$	Interest rate	0.01
$\mu$	Expected excess stock return	0.04
$\sigma_S$	Stock volatility	0.157
<i>Income</i>		
$Y_{t_1}$	Initial annual income	40
$\sigma_Y$	Income volatility	0.1
$\rho_{YS}$	Income-stock correlation	0
<i>Tax rates</i>		
$\tau_Y$	Income tax rate	0.34
$\tau_F$	Tax rate on private returns	0.27
$\tau_A$	Tax rate on pension returns	0.153

Table 1: **Baseline parameter values.**

Finally, we assume that the 25-year old has a financial wealth of \$5,000, yet no pension savings, and a current pre-tax labor income of \$40,000 per year. As Cocco et al. (2005) and others, we assume that the expected labor income growth can be described by a third-order polynomial, and we determine the coefficients of the polynomial so that labor income is expected to peak at age 55 at a value 50% above initial income and is subsequently expected to drop by 10% until retirement.

### 2.3 Welfare metric

In our setting, the pension fund design is characterized by a sequence of contribution rates  $\alpha_{t_1}, \alpha_{t_1+1}, \dots, \alpha_{T_R-1}$  and stock weights  $w_{t_1}, w_{t_1+1}, \dots, w_{T_M-1}$  (as well as the payout parameter  $\tilde{r}$  which we fix for now). For simplicity, we represent such a design by  $(\alpha, w)$ .

A key element of our analysis is to compare the utility or welfare that an individual can generate under different designs of the mandatory pension plan but under the same assumptions about income, initial wealth, etc. Suppose we want to compare two pension designs,  $(\alpha, w)$  and  $(\hat{\alpha}, \hat{w})$ . The individual obtains an initial indirect utility of  $J = J_{t_1}(F, A, Y; \alpha, w)$  with the first pension design and  $\hat{J} = J_{t_1}(F, A, Y; \hat{\alpha}, \hat{w})$  with the second design. If  $J < \hat{J}$ , the individual obviously prefers  $(\hat{\alpha}, \hat{w})$  to  $(\alpha, w)$ .

We can quantify how much better off the individual is with  $(\hat{\alpha}, \hat{w})$  than with  $(\alpha, w)$  by the fraction  $\lambda$  of both the labor income stream and the initial wealth (both free and illiquid) that the individual would give up to replace  $(\alpha, w)$  by  $(\hat{\alpha}, \hat{w})$ . With reduced income and wealth, the indirect utility with the design  $(\hat{\alpha}, \hat{w})$  is

$$J_{t_1}([1 - \lambda]F, [1 - \lambda]A, [1 - \lambda]Y; \hat{\alpha}, \hat{w}) = (1 - \lambda)J_{t_1}(F, A, Y; \hat{\alpha}, \hat{w}). \quad (9)$$

Equating this with  $J_{t_1}(F, A, Y; \alpha, w)$ , we find

$$\lambda = 1 - \frac{J_{t_1}(F, A, Y; \alpha, w)}{J_{t_1}(F, A, Y; \hat{\alpha}, \hat{w})}. \quad (10)$$

We can transform the relative loss  $\lambda$  into an initial dollar amount if we multiply  $\lambda$  by the sum of the initial financial wealth and the present value of after-tax labor income until retirement. Suppose we calculate this present value by discounting expected after-tax labor income by the riskfree rate of 1%. Given the assumptions about labor income and relevant parameter values, the present value of income turns out to be \$1,279,347 (based on 10,000 simulated income paths), which is substantially larger than the assumed initial financial wealth of \$5,000.<sup>12</sup> In this case, a  $\lambda$  of 1% corresponds roughly to \$12,840.

<sup>12</sup>Since the income is not fully spanned by traded assets, there is no unique way to calculate the present value of future income.

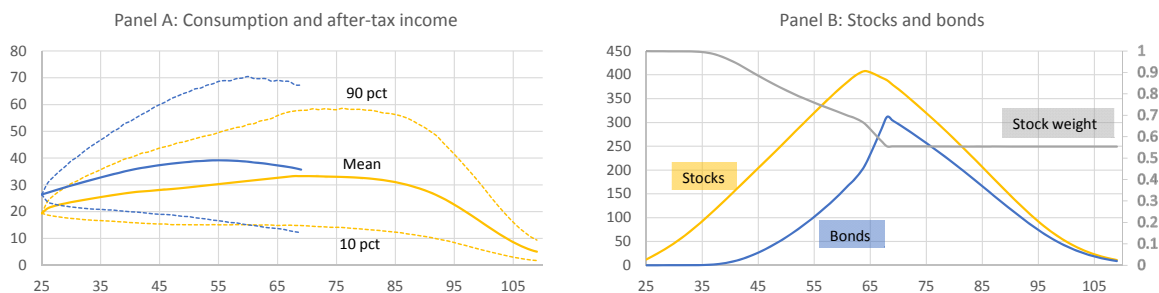


Figure 1: **The no pension case.** Panels A shows the annual consumption (yellow lines) and after-tax labor income (blue lines). The solid lines represent the mean across 10,000 simulated paths, whereas the dotted lines indicate the 10th and 90th percentile at each age. Panel B illustrates the mean investments over the life cycle divided into stocks (yellow line) and bonds (blue line). The grey line indicates the mean stock weight and is to be read off the right-hand axis.

### 3 Individual decisions with a given pension fund design

As an important benchmark, we first consider the case without a pension plan so that all retirement savings are done through private, liquid asset holdings, and the individual is free from behavioral biases. This is the canonical life-cycle model (with taxes on income and returns), and the output illustrated in Figure 1 comply with the well-known results in this literature. Panel A shows that consumption is hump-shaped over the life cycle as observed in the data (Thurow, 1969; Gourinchas and Parker, 2002). In the early adult years, the individual consumes a large share of disposable wealth and thus accumulates wealth rather slowly. Panel B documents that the optimal portfolio weight of the stock is 100% until around age 35, after which the weight is gradually reduced to around 55% at retirement and maintained at that level in the remaining life span. The initial indirect utility is 6.976, which is used for comparisons below.

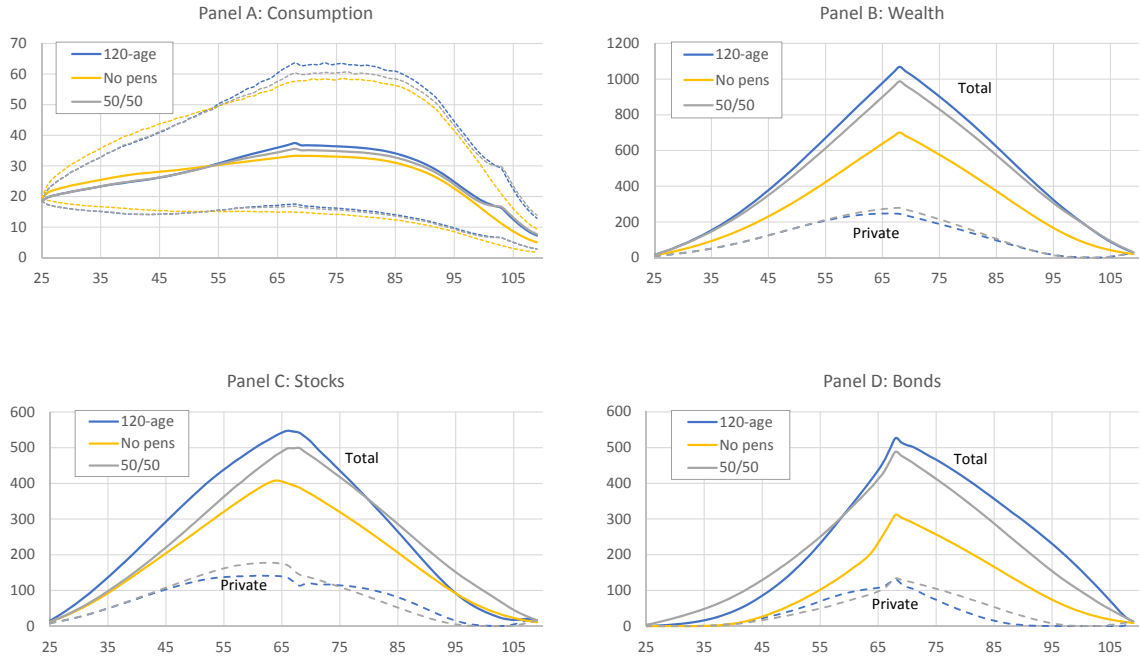
Next, we illustrate the distorting effects a typical mandatory savings scheme can have on consumption, investments, and welfare relative to the no-pension case from above. We assume the plan stipulates a 17% contribution rate from age 25 and until retirement. The balance of the fund is invested with 50% in stocks and 50% in bonds at any time, both before and in retirement. For brevity we refer to this plan as a 25y/17pct/50-50 pension plan. This plan resembles many mandatory pension plans in Denmark. Payouts from the fund are defined through a variable annuity rate of  $\tilde{r} = 3\%$ ; as this is slightly higher than the after-tax expected return of the pension fund (2.58%), the expected pension payouts are slowly decreasing through retirement. The indirect utility of the individual is 6.713 with the mandatory 25y/17pct/50-50 plan. In the metric defined by (10), this is about 3.8% worse than in the no-pension case, which means that the individual would give up

3.8% of her current wealth and lifetime income—around \$48,800 using the present value of income calculated earlier—to avoid the mandatory 25y/17pct/50-50 plan. The fact that investment returns are taxed more leniently within the pension fund cannot make up for the distortions in asset holdings and ultimately in consumption that this mandatory pension plan induces. With a ‘120 minus age’ investment strategy—meaning that the stock share is  $120 - t$  percent (and thus the bond share  $t - 20$  percent) when the individual has age  $t$ —the maximal lifetime utility with a 17% contribution rate is 6.734, a welfare loss of 3.5%, and thus only slightly better compared to the 50/50 allocation.

Figure 2 illustrates consumption and investments over the life cycle without a pension plan (yellow lines), the plan with 17% contributions and 50/50 asset allocation (grey lines), and the plan with 17% contributions and the ‘120 minus age’ strategy (blue lines). All quantities are in thousands of USD. Panel A shows that, until age 55, consumption is lower with the mandatory plans than without it, with the reduction being up to 9%. After age 55, consumption is larger with the mandatory pension plans, especially for very old individuals. Due to the compulsory savings, the life-cycle consumption path is less smooth than the individual would prefer.

Panel B of Figure 2 shows that the total savings (disposable wealth plus any pension balance) are substantially larger with the mandatory pension plan than without. Hence, the individual’s private savings in the presence of the plan exceeds the difference between the optimal savings without the plan and the mandatory savings in the plan. In part, the larger total savings are due to the lower tax rate on pension returns than private returns. In addition, such excess savings can be explained by the desire to hold more stocks than what the pension fund savings entail. As Panels C and D illustrate, all the private investments early in life are in stocks. Still, the total stock holdings in and outside the pension fund lack somewhat behind the optimal stock holdings in the absence of the mandatory plan, at least until age 55 or so. Without the mandatory plan, the individual prefers not to invest in bonds until around age 55. With the plan, the fund invests 50% of the mandatory savings in bonds. The mandatory plan thus causes excessive bond investments that are particularly large early in life but remain significant throughout life.

Figure 3 compares the outcomes of the no-pension case and a very different mandatory pension scheme, the 40y/14pct/120-age scheme. With this scheme, contributions are assumed to be 14% of income but are not beginning until age 40, and the pension fund determines the stock share by following the ‘120 minus age’ rule. Obviously, the dynamics of the pension fund balance is now very different than for the 25y/17pct/50-50 plan considered above. However, the individual adjusts her own savings and asset allocation decision to get closer to the preferred overall positions. When pension savings are not initiated until age 40, the individual builds up larger private savings in the early years



**Figure 2: Consumption and asset holdings: No pension v. 25y/17pct pension plans.** Panel A shows consumption in thousand USD over life with the solid lines representing the mean and the dotted lines the 10th and 90th percentiles across the 10,000 simulated paths. Orange lines are for the no-pension case, grey [blue] lines for the pension plan with a 17% contribution rate and a stock share equal to 50% [resp.,  $(120 - \text{age})\%$ ]. Panels B, C, and D show the means across the 10,000 paths of wealth, stock holdings, and bond holdings over life, all in thousands USD. In each of these panels, the dashed grey and blue lines represent the privately held amounts, and the solid grey and blue lines the total holdings.

than with the 25y/17pct/50-50 plan. At age 39, for example, the mean private wealth is now 139.9 thousand dollars—close to the 140.3 thousand in the no-pension case—whereas with the previous plan it is 76.1 thousand on top of the pension balance of 142.0 thousand.

With the new plan, early-life total wealth as well as the separate positions in stocks and bonds are closer to the optimum in the no-pension case. Later in life, total stock and bond holdings are larger with any of the two pension plans than in the no-pension scenario. Again, Panel A reveals that the resulting life-cycle consumption profile is remarkably insensitive to the introduction of the mandatory pension scheme. Compared to the no-pension case, mean consumption with the new plan is slightly larger (up to 1%) until around age 45, after which it is slightly lower until around age 100 after which the still substantial payouts from the pension fund lead to higher consumption (up to 9%) than desired in the absence of a pension scheme. The indirect utility with this alternative plan is 6.984, which is marginally larger than in the no-pension case (6.976) and much larger than with the 25y/17pct/50-50 plan (6.713).

## 4 Pension design for “rational” individuals

In the previous section, we explained how to derive an individual’s optimal lifetime utility for a given pension design  $(\alpha, w)$ , that is  $J_{t_1}(F_{t_1}, A_{t_1}, Y_{t_1}; \alpha, w)$ , and illustrated how the pension design can affect welfare, consumption, and investments over the life cycle. In this section, we intend to find the *optimal* pension design, at least in some tractable family of possible designs, i.e., we want to find

$$\operatorname{argmax}_{(\alpha, w)} J_{t_1}(F_{t_1}, A_{t_1}, Y_{t_1}; \alpha, w).$$

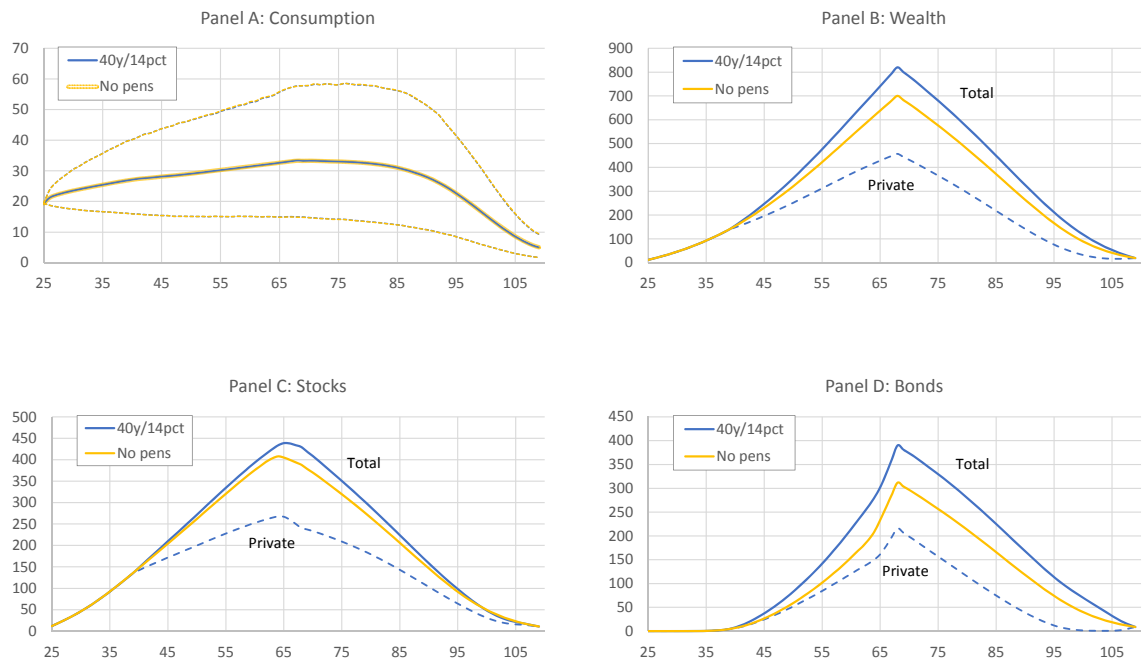
With individualized pension accounts, the contribution rates and stock weights could in principle depend on the individual’s current account balance and maybe even her labor income and private wealth. In practical implementations, both  $\alpha$  and  $w$  will depend at most on the age of the individual and, if they do so, only in rather simple ways that are easily communicated to fund members.<sup>13</sup> Hence, we restrict ourselves to age-dependent contribution rates and fund allocation.

The following two subsections consider two different types of contribution policies  $(\alpha_t)$ . The first type has a constant contribution rate and contributions beginning either at the initial age of 25 years or at some later date. In the second type, the contribution rate is zero until a certain start age, then flat at a positive level until a predetermined age from which it gradually increasing until another predetermined age, and then constant until retirement.

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<sup>13</sup>With time-varying investment opportunities,  $\alpha$  and  $w$  could potentially depend on return predictors.





**Figure 3: Consumption and asset holdings: No pension v. 40y/14pct/120-age plan.** Panel A shows consumption in thousand USD over life with the solid lines representing the mean and the dashed lines the 10th and 90th percentiles across the 10,000 simulated paths. Orange lines are for the no-pension case, blue lines for the pension plan with contributions start at age 40 and are 14% of income and with the stock share following the '120 minus age' rule. Panels B, C, and D show the means across the 10,000 paths of wealth, stock holdings, and bond holdings over life, all in thousands USD. In each of these panels, the dashed blue line represents the privately held amounts, and the solid blue line the total holdings.

Both types of contribution policies are mixed with two fund investment strategies ( $w_t$ ) in which the stock share is either constant over time at 50% or 75% or follows the ‘120 minus age’ rule, i.e. starts at 95% at age 25 and is reduced by one percentage point per year throughout life. Throughout this section we assume that the individual is a rational utility maximizer with access to the same investment opportunities as the pension fund.

We assume for now that the individual starts saving in the pension fund at a certain age and then pays a constant fraction of his labor income into the fund until retirement. We assume the fund investment strategy is either to maintain a constant stock share of 50% or to follow the ‘120 minus age’ rule. The fund payout rule is characterized by the variable annuity rate  $\tilde{r}$ , which we fix at 3% for now. For each contribution start age and investment strategy, we then determine the optimal constant contribution rate and the associated lifetime utility.

Table 2 summarizes our findings. When contributions are required from age 25 or 30 and are invested with the 50-50 strategy, no positive contribution rate would allow the individual to generate a higher lifetime utility than in the case without mandatory pension savings. Hence, the optimal contribution rate is listed as 0%. For the other considered combinations of the contribution initiation age and the fund asset allocation policy, a higher lifetime utility is attainable with non-zero contributions. Across all the considered designs, the largest utility is obtained when contributions start at age 25 with a contribution rate of 5% and investments according to the ‘120 minus age’ strategy. We find that lifetime utility tends to be quite flat around the optimal contribution rate, which can be explained by the fact that the individual can adjust her private savings and asset allocation decisions to almost fully compensate for a slightly suboptimal contribution rate. Note that the maximum welfare gain is only 0.14% (corresponding to around \$1,800) relative to the case where the individual makes optimal decisions in the absence of a mandatory pension plan. For a carefully designed pension plan, the return taxation benefits of fund investments only just exceed the distorting impact on the individual’s consumption and asset allocation over the life cycle.

Many occupational pension schemes (at least in Denmark) impose a constant contribution rate of 10-17% beginning immediately when the individual takes up a job covered by that scheme, which is often around age 25. Moreover, the pensions savings are in many cases invested without consideration of the individual’s age, corresponding to a constant stock share of around 50%. Table 2 indicates that young, rational, and savvy individuals generally dislike such a pension design. Under the assumptions of mainstream life-cycle models, they prefer 100% in stocks and would also prefer first saving outside the pension fund to build a wealth buffer they can tap into in case of poor labor market outcomes. As explained in Section 3 a mandatory pension scheme with a 17% contribution rate from

Contributions start	50% stocks		(120-age)% stocks	
	Contrib	Utility gain	Contrib	Utility gain
Age 25	0%	0.00%	5%	0.14%
Age 30	0%	0.00%	7%	0.14%
Age 35	5%	0.01%	10%	0.14%
Age 40	10%	0.03%	14%	0.13%
Age 45	16%	0.06%	18%	0.12%
Age 50	23%	0.08%	24%	0.11%

Table 2: **Constant contribution rates: optimal rates and utility gains.** The utility gain is the relative difference to the no-pension case.

age 25 and a 50-50 fund asset allocation strategy leads to a welfare loss of 3.8%.

We emphasize that differences in utility both across the savings start date and fund investment strategies are modest, because the individual to a large extent can make up for the distortions created by the mandatory savings through private investment decisions. Obviously, this outcome depends heavily on the assumption that the individual (i) has access to the same investment opportunities as the fund and (ii) is a rational utility maximizer. We relax these assumptions in Section 5.

The assumptions considered above imply that pension payouts are expected to slowly decrease through retirement since the variable annuity rate  $\tilde{r}$  of 3% exceeds the after-tax expected return on pension savings, which is 2.58% with the 50/50 asset allocation and lower (in retirement) with the ‘120 minus age’ policy as the stock weight is then below 50%. Typical real-life pension schemes feature a constant or slowly decreasing payout stream. However, due to the rapidly increasing mortality rate late in life, the individual is planning to have a more steeply decreasing consumption pattern in that life phase, as can be seen from Panel A of Figure 2. Therefore, the individual might prefer a larger value of  $\tilde{r}$ . Indeed, the best scheme found above with 5% contributions from age 25 and the ‘120 minus age’ rule can be further improved by changing  $\tilde{r}$  from 3% to 5%. The increases in the utility gain is slim, though, from 0.14% to 0.17%.

Small improvements are also possible by allowing the contribution rate to be piecewise linear.

## 5 Pension design for “irrational” individuals

A large body of research has documented that many individual investors fail to follow the theoretically optimal consumption and investment strategy. Many young individuals invest too little or nothing at all in stocks. A large share of the individuals who are participating in the stock market own very few stocks and thus hold a rather undiversi-

fied portfolio. For example, based on the 2007 Survey of Consumer Finance (SCF) in the United States, Favilukis (2013) reports that 24.2% of households held stocks outside retirement accounts, a number increasing to 40.6% if stocks in retirement accounts are included. According to Polkovnichenko (2005), the median number of directly owned stocks was 3 among households in the 2001 SCF. Other evidence from the US is reported by Blume and Friend (1975) and Goetzmann and Kumar (2008), among others, and Campbell et al. (2018) present similar findings from India, Fagereng et al. (2017) from Norway, and Florentsen, Nielsson, Raahauge, and Rangvid (2019) from Denmark. Also, many households maintain large deposits in commercial banks at low interest rates. This section investigates the optimal design of a mandatory pension fund when the fund member—by choice or mistake—faces different investment opportunities than the pension fund.

## 5.1 No private stock investments

In this subsection, we assume that the individual is investing all of her private wealth in the riskfree asset and is thus not participating in the stock market. With our baseline parameters, the initial indirect utility is then only 6.212 in the absence of a pension saving scheme, which is 11% below the case where the individual invests optimally in both stocks and bonds. Under these circumstances a mandatory pension plan is potentially quite attractive as it provides access to higher average returns. Still, it is not clear how that plan should be designed.

First, consider the case with a constant contribution rate. Suppose that the fund invests 50% in stocks and 50% in the riskfree asset. If contributions are required to start at age 30 and continue until retirement at a constant rate, the optimal contribution rate turns out to be 15% with an associated lifetime utility 6.2% higher than without any pension plan; again, this can roughly be interpreted as an increase of 6.2% in lifetime wealth and income. On the other hand, the utility is still 5.4% below the maximum utility with optimal participation in the stock market. As shown in the left part of Table 3, this exceeds the maximum utility attainable when contributions begin at age 25, 35, or 40. The right part of the table present similar results when fund investments follow the ‘120 minus age’ rule. Here, utility is maximized when contributions begin already at age 25, and the optimal contribution rate is now 10% with an associated utility 6.9% larger than without a pension plan.

We have experimented with plans involving a non-constant contribution rate. We find that small additional welfare gains are possible. In one such plan the contribution rate is 5% at age 25-30, then increasing linearly up to 15% at age 40, and fixed there until retirement. Coupled with the ‘120 – age’ asset allocation strategy, this plan generates a welfare gain of 7.0% relative to the no-pension case without private stock investments.

Start	50% stocks			(120-age)% stocks		
	Contrib	No stocks	Stocks	Contrib	No stocks	Stocks
Age 25	11%	6.1%	-5.5%	10%	6.9%	-4.9%
Age 30	15%	6.2%	-5.4%	14%	6.8%	-4.9%
Age 35	20%	6.2%	-5.5%	20%	6.6%	-5.0%
Age 40	27%	5.8%	-5.8%	27%	6.1%	-5.5%

Table 3: **No private stocks: optimal constant contribution rates and utilities.** The numbers in the column ‘No stocks’ represent the utility gain relative to no-pension case with no private stocks. The numbers in the column ‘Stocks’ are the utility loss relative to no-pension case with private stocks.

## 5.2 Undiversified private stock investments

In this subsection, we consider the case in which the individual can invest private funds in stocks but not in the well-diversified stock index. For concreteness and tractability, we assume that the individual can only invest in a stock (portfolio) that has the same expected return as the index, is perfectly correlated with it, but has a standard deviation which is twice as high. Without a mandatory pension system, the maximum lifetime utility of the individual is reduced from 6.976 to 6.466 when she invests in the less diversified stock portfolio instead of the overall stock index. This is a welfare reduction of 7.3%.

Table 4 summarizes the results with constant contribution rates. The individual prefers to postpone fund contributions until age 30, then contribute 14% of income until retirement, and that the fund follows the ‘120 minus age’ rule. The associated lifetime utility with the best design is 3.9% larger than in the case where the individual does not have a mandatory pension plan and invests in stocks only through an underdiversified portfolio. The optimal pension design is thus worth roughly \$50,000 to the individual. However, the utility is still 3.7% below what the individual could have obtained without a pension plan if she would invest (optimally) in the stock market index instead of the underdiversified portfolio.

Also in this case can welfare be improved somewhat by allowing age-dependent contribution rates. For example, suppose that contributions start at age 30 at 10%, stays at that level until age 35, and then increases linearly to 20% at age 45 and stays there. Assuming that the fund applies the ‘120 minus age’ rule, this generates a maximal utility which is 4.0% above the case without a pension plan.

## 5.3 Procrastinating on retirement savings

We assume that the utility the individual derives from any consumption plan is associated with the baseline subjective discount factor  $\beta = 0.96$ . However, due to lack of self-

Start	50% stocks			(120-age)% stocks		
	Contrib	Undivers	Divers	Contrib	Undivers	Divers
Age 25	10%	3.1%	-4.4%	10%	3.7%	-3.9%
Age 30	14%	3.4%	-4.2%	14%	3.9%	-3.7%
Age 35	18%	3.4%	-4.1%	18%	3.8%	-3.8%
Age 40	24%	3.3%	-4.3%	24%	3.5%	-4.1%

Table 4: **Undiversified private stocks and constant contribution rates: optimal rates and utility gains and losses.** The numbers in the columns ‘Undivers’ show the utility gain relative to the no-pension case where the individual privately invests in an underdiversified stock portfolio. The numbers in the columns ‘Divers’ show the utility loss relative to the no-pension case where the individual privately invests in the entire stock market index.

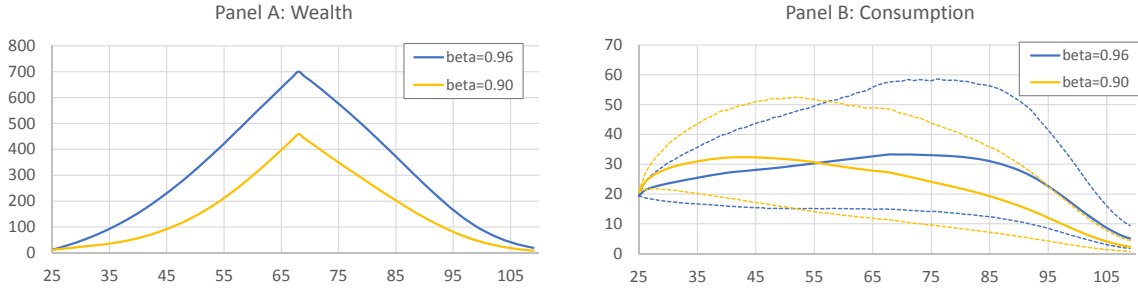


Figure 4: **Consumption and wealth: Procrastination without pension plan.** Panel A shows the mean accumulation of wealth over the life cycle. In Panel B the solid lines show the mean annual consumption and the dotted lines the 10th and 90th percentiles. Blue lines are for a subjective discount factor of 0.96 and yellow lines for a subjective discount factor of 0.90. Results are derived from 10,000 simulated paths. Both wealth and consumption are measured in thousands of USD.

control, the individual applies the lower value  $\beta = 0.90$  (i.e., a higher impatience rate) when making consumption decisions. Hence, the individual will save too little and thus potentially benefit significantly from a mandatory pension plan. In the absence of a mandatory pension scheme, Figure 4 compares the outcome of the optimal decisions for the two different  $\beta$ -values. Panel A confirms that the less patient individual accumulates much less wealth. Panel B shows that the less patient individual consumes significantly more than the patient individual early in life but has a lower consumption from age 55 and on, due to the lower savings to draw on.

For the individual with  $\beta = 0.96$ , the lifetime utility of the consumption plan derived with  $\beta = 0.90$  is 5.595 and thus 19.8% lower than the utility generated by the truly optimal consumption plan for  $\beta = 0.96$ . In this sense, the procrastination of savings has sizeable welfare costs. To be clear, we assume in this calculation that  $\hat{c}_t(a_t, y_t)$  and  $\pi_t(a_t, y_t)$  are derived with  $\beta = 0.90$  and then evaluated with  $\beta = 0.96$ . The evaluation of

Start	50/50			120 minus age		
	Contrib	No pens	No procrast	Contrib	No pens	No procrast
Age 25	10%	8.8%	-12.7%	10%	9.5%	-12.2%
Age 30	15%	10.2%	-11.6%	14%	10.7%	-11.2%
Age 35	21%	11.6%	-10.4%	21%	12.1%	-10.1%
Age 40	29%	12.4%	-9.8%	29%	12.7%	-9.6%
Age 45	39%	12.3%	-10.0%	39%	12.5%	-9.8%

Table 5: **Procrastination: optimal contribution rates and utility gains and losses.** The numbers in the columns ‘No pens’ show the utility gain relative to no-pension case with procrastination. The numbers in the columns ‘No procrast’ show the utility loss relative to the no-pension case without procrastination.

a given strategy  $(\hat{c}_t, \pi_t)$  follows the same backwards iterative approach as when deriving the optimal strategy, except that no maximization is performed.

Table 5 lists the optimal constant contribution rates for different saving initiation ages and the two asset allocation strategies. The best pension design involves a 29% contribution rate from age 40 paired with the ‘120 minus age’ glidepath strategy. The welfare gain is as large as 12.7% compared to the case where the procrastinating individual has no mandatory savings. However, the individual’s welfare is still far below what she could obtain without both procrastination and mandatory savings. If we increase the annuity rate  $\tilde{r}$  determining the profile of pension payouts from 3% to 4%, the welfare gain is marginally increased, but still 12.7% rounded to one decimal point. As in previous cases, a non-constant contribution rate can lead to slightly higher utility. For example, the welfare gain is 12.8% if the contributions start at 5% at age 30 and then gradually increases from age 35 to age 45 by two percentage points per year.

Table 6 combines procrastination on savings and non-participation in the stock market. Without a compulsory retirement savings scheme, the individual’s lifetime utility is only 5.118. This utility level is 26.6% below the no-pension case in without procrastination and with optimal private investments in the stock market, and it is thus also considerably below the above case where the procrastinating individual is able to make optimal stock investment decisions. The welfare improvement of a mandatory pension scheme is thus also larger now with gains up to 16.1%, obtained with 28% contributions beginning at age 40 and invested according to the ‘120 minus age’ rule.

Not surprisingly, the degree of procrastination has a significant effect on the optimal contribution rate. As an illustration, suppose that the individual is even more impatient than assumed above and makes decision applying a subjective discount factor of  $\beta = 0.80$ , which is in line with the estimates of Fagereng et al. (2017) based on Norwegian data. The individual still evaluates consumption streams with  $\beta = 0.96$ , and we assume the

Start	50/50			120 minus age		
	Contrib	No pens	No biases	Contrib	No pens	No biases
25	14%	12.5%	-17.5%	14%	13.3%	-16.8%
30	15%	13.2%	-17.0%	15%	13.8%	-16.5%
35	21%	14.7%	-15.9%	20%	15.1%	-15.5%
40	28%	15.8%	-15.1%	28%	16.1%	-14.8%
45	37%	15.5%	-15.3%	37%	15.7%	-15.1%

Table 6: **Procrastination and no private stocks: optimal contribution rates and utility gains and losses.** The numbers in the columns ‘No pens’ show the utility gain relative to no-pension case featuring procrastination and no private stocks. The numbers in the columns ‘No biases’ show the utility loss relative to the no-pension case without procrastination.

individual does not invest in stocks out of private savings. In this case, lifetime utility without a pension scheme is only 2.304. The best pension plan involves 25% contributions from age 25 invested confirming to the ‘120 minus age’ principle, and with this plan the biased individual can obtain a lifetime utility of 4.516, which is a stunning 96% welfare improvement.

## 6 Conclusion

Our analysis shows that the optimal design and welfare implications of a mandatory defined contribution pension system depends heavily on how individuals handle “free” wealth. Assuming individuals are rational and have access to the same investment opportunities as the pension fund, many pension designs significantly reduce the welfare of the individual compared to the case without a mandatory pension system. However, for carefully designed schemes, the milder taxation of returns inside the fund can outweigh the distortions of the life-cycle consumption plan that the mandatory savings may cause. In our baseline setting, this occurs with a low (5-10%) contribution rate starting at age 25-35 and the fund investing according to the ‘120 minus age’ rule, but the welfare gains are only around 0.14% and thus marginal.

If the individual privately either does not participate in the stock market or only invests in a poorly diversified stock portfolio, a mandatory pension scheme can provide larger welfare gains. The optimal design involves a medium (10-14%) contribution rate starting at age 25-35, again coupled with the ‘120 minus age’ investment rule. With such a pension system, the welfare of the individual increases by up to 7% but is still significantly lower than in the case where the individual also invests private savings in the stock market index.

We find in our baseline parametrization that procrastination on savings reduces indi-



vidual welfare by as much as 20% in the absence of a compulsory pension savings program. With an appropriately designed mandatory pension plan, the welfare can be improved by as much as 12% for schemes with a constant contribution rate. This is obtained with contributions of around 12% of income starting at age 35-40. Again, the ‘120 minus age’ asset allocation policy is preferred to the even stock-bond allocation.

A number of relevant extensions easily come to mind. First, while our model of labor income dynamics is standard in the literature, it ignores unemployment risk and recent empirical studies have identified various other inadequacies of the model ([Guvenen, Karahan, Ozkan, and Song, 2019](#)). Second, if we include public pensions, the optimal individual decisions and the optimal design of the defined contribution plan might change significantly. Third, we ignored housing which is a key concern of households who can invest in residential real estate, enjoy the benefits from living in the home, and—if home prices increase or mortgage debt is paid down (or both)—accumulate wealth in terms of home equity. Note, however, that the optimal pension design might then be substantially different for homeowners than for homerenters. Fourth, we assume constant interest rates, but an extensive literature has shown that the optimal stock-bond allocation is considerably twisted once time-varying interest rates are introduced ([Campbell and Viceira, 2001](#); [Munk and Sørensen, 2010](#)), which may have repercussions for the optimal pension design. Other interesting extensions are to introduce an option to access the pension savings at some penalty rate or allow for non-proportional taxation of returns and income. Note, however, that any of these extensions necessitate an additional state variable that further complicates the numerical solution approach.

## A Solving the utility maximization problem

Through a normalization by total after-tax wealth  $(1 - \tau_Y)A_t + F_t$ , we can reduce the number of state variables by one. After normalization, the state variables are

$$a_t = \frac{(1 - \tau_Y)A_t}{(1 - \tau_Y)A_t + F_t}, \quad y_t = \frac{(1 - \tau_Y)Y_t}{(1 - \tau_Y)A_t + F_t}.$$

Final year. In this case, death is certain at the end of the period ( $p_{T_M} = 0$ ) and since  $m_{T_M} = 1$  the bequest is

$$B_{T_M+1} = F_{T_M+1} = (1 - \hat{c}_{T_M})\tilde{F}_{T_M}R_{F,T_M} = (1 - \hat{c}_{T_M})(F_{T_M} + (1 - \tau_Y)A_{T_M})R_{F,T_M}.$$

The certainty equivalent is

$$\begin{aligned} \text{CE}_{T_M} &= \left( \mathbb{E}_{T_M} \left[ \xi^{\frac{1-\gamma}{\psi-1}} B_{T_M+1}^{1-\gamma} \right] \right)^{\frac{1}{1-\gamma}} \\ &= \xi^{\frac{1}{\psi-1}} (F_{T_M} + (1 - \tau_Y)A_{T_M}) (1 - \hat{c}_{T_M}) \left( \mathbb{E}_{T_M} \left[ R_{F,T_M}^{1-\gamma} \right] \right)^{\frac{1}{1-\gamma}}. \end{aligned}$$

The recursive utility maximization problem simplifies to

$$\begin{aligned} J_{T_M} &= \max_{\hat{c}_{T_M}, \pi_{T_M}} \left\{ \left( \hat{c}_{T_M} \tilde{F}_{T_M} \right)^{1-\frac{1}{\psi}} + \beta \text{CE}_{T_M}^{1-\frac{1}{\psi}} \right\}^{\frac{1}{1-\frac{1}{\psi}}} \\ &= (F_{T_M} + (1 - \tau_Y)A_{T_M}) \max_{\hat{c}_{T_M}, \pi_{T_M}} \left\{ \hat{c}_{T_M}^{1-\frac{1}{\psi}} + \beta \xi^{\frac{1}{\psi}} (1 - \hat{c}_{T_M})^{1-\frac{1}{\psi}} \left( \mathbb{E}_{T_M} \left[ R_{F,T_M}^{1-\gamma} \right] \right)^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right\}^{\frac{1}{1-\frac{1}{\psi}}} \end{aligned}$$

Here, the optimal stock weight  $\pi_{T_M}^*$  is determined by maximizing  $\left( \mathbb{E}_{T_M} \left[ R_{F,T_M}^{1-\gamma} \right] \right)^{1/(1-\gamma)}$ , and then optimal consumption and the value function are given by

$$\hat{c}_{T_M}^* = \frac{1}{1 + \xi \beta^\psi \left( \mathbb{E}_{T_M} \left[ R_{F,T_M}^{1-\gamma} \right] \right)^{\frac{\psi-1}{1-\gamma}}}, \quad (11)$$

$$J_{T_M} = (F_{T_M} + (1 - \tau_Y)A_{T_M})G_{T_M}, \quad (12)$$

where  $\pi_{T_M}^*$  is applied for generating the return and

$$G_{T_M} = \left\{ \left( \hat{c}_{T_M}^* \right)^{1-\frac{1}{\psi}} + \beta \xi^{\frac{1}{\psi}} (1 - \hat{c}_{T_M}^*)^{1-\frac{1}{\psi}} \left( \mathbb{E}_{T_M} \left[ R_{F,T_M}^{1-\gamma} \right] \right)^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right\}^{\frac{1}{1-\frac{1}{\psi}}}. \quad (13)$$

Note that for large values of  $\xi$ , only a small fraction of wealth is consumed in the final year so that the end-of-year bequest is large. As  $\xi$  approaches zero, more of the wealth is consumed and less left for bequest. These observations confirm that  $\xi$  measures the strength of the bequest motive.

In retirement, i.e. for  $t = T_R, T_R + 1, \dots, T_M - 1$ . For an induction argument, we assume that

$J_{t+1} = ((1 - \tau_Y)A_{t+1} + F_{t+1})G_{t+1}(a_{t+1})$  which implies that the certainty equivalent is

$$\begin{aligned} \text{CE}_t &= \left( p_t \text{E}_t \left[ ((1 - \tau_Y)A_{t+1} + F_{t+1})^{1-\gamma} G_{t+1}(a_{t+1})^{1-\gamma} \right] + (1 - p_t) \text{E}_t \left[ \xi^{\frac{1-\gamma}{\psi-1}} ((1 - \tau_Y)A_{t+1} + F_{t+1})^{1-\gamma} \right] \right)^{\frac{1}{1-\gamma}} \\ &= ((1 - \tau_Y)A_t + F_t) \left( \text{E}_t \left[ \left( \frac{(1-\tau_Y)A_{t+1} + F_{t+1}}{(1-\tau_Y)A_t + F_t} \right)^{1-\gamma} \left\{ p_t G_{t+1}(a_{t+1})^{1-\gamma} + (1 - p_t) \xi^{\frac{1-\gamma}{\psi-1}} \right\} \right] \right)^{\frac{1}{1-\gamma}}. \end{aligned}$$

Note that

$$\begin{aligned} (1 - \tau_Y)A_{t+1} + F_{t+1} &= (1 - \tau_Y)A_t(1 - m_t)R_{At} + (1 - \hat{c}_t)(F_t + (1 - \tau_Y)A_t m_t)R_{Ft} \\ &= (1 - \tau_Y)A_t(1 - m_t)R_{At} + (1 - \hat{c}_t)(F_t + (1 - \tau_Y)A_t - (1 - m_t)(1 - \tau_Y)A_t)R_{Ft} \end{aligned}$$

and, thus,

$$\frac{(1 - \tau_Y)A_{t+1} + F_{t+1}}{(1 - \tau_Y)A_t + F_t} = a_t(1 - m_t)R_{At} + (1 - \hat{c}_t)(1 - (1 - m_t)a_t)R_{Ft}.$$

Furthermore,

$$\begin{aligned} a_{t+1} &= \frac{(1 - \tau_Y)A_{t+1}}{(1 - \tau_Y)A_{t+1} + F_{t+1}} \\ &= \frac{(1 - \tau_Y)A_t(1 - m_t)R_{At}}{(1 - \tau_Y)A_t(1 - m_t)R_{At} + (1 - \hat{c}_t)(F_t + (1 - \tau_Y)A_t - (1 - m_t)(1 - \tau_Y)A_t)R_{Ft}} \\ &= \frac{a_t(1 - m_t)R_{At}}{a_t(1 - m_t)R_{At} + (1 - \hat{c}_t)(1 - (1 - m_t)a_t)R_{Ft}}. \end{aligned}$$

Hence, we can write

$$\text{CE}_t = ((1 - \tau_Y)A_t + F_t) \mathcal{C}_t(a_t),$$

where

$$\begin{aligned} \mathcal{C}_t(a_t)^{1-\gamma} &= \text{E}_t \left[ \left\{ a_t(1 - m_t)R_{At} + (1 - \hat{c}_t)(1 - (1 - m_t)a_t)R_{Ft} \right\}^{1-\gamma} \right. \\ &\quad \times \left. \left\{ p_t G_{t+1} \left( \frac{a_t(1 - m_t)R_{At}}{a_t(1 - m_t)R_{At} + (1 - \hat{c}_t)(1 - (1 - m_t)a_t)R_{Ft}} \right)^{1-\gamma} + (1 - p_t) \xi^{\frac{1-\gamma}{\psi-1}} \right\} \right]. \end{aligned}$$

The utility recursion (5) implies that

$$\begin{aligned} J_t &= \max_{\hat{c}_t, \pi_t} \left\{ \left( \hat{c}_t \tilde{F}_t \right)^{1-\frac{1}{\psi}} + \beta ((1 - \tau_Y)A_t + F_t)^{1-\frac{1}{\psi}} \mathcal{C}_t(a_t)^{1-\frac{1}{\psi}} \right\}^{\frac{1}{1-\frac{1}{\psi}}} \\ &= \max_{\hat{c}_t, \pi_t} \left\{ \hat{c}_t^{1-\frac{1}{\psi}} (F_t + (1 - \tau_Y)A_t - (1 - m_t)(1 - \tau_Y)A_t)^{1-\frac{1}{\psi}} + \beta ((1 - \tau_Y)A_t + F_t)^{1-\frac{1}{\psi}} \mathcal{C}_t(a_t)^{1-\frac{1}{\psi}} \right\}^{\frac{1}{1-\frac{1}{\psi}}} \\ &= ((1 - \tau_Y)A_t + F_t) \max_{\hat{c}_t, \pi_t} \left\{ (\hat{c}_t[1 - (1 - m_t)a_t])^{1-\frac{1}{\psi}} + \beta \mathcal{C}_t(a_t)^{1-\frac{1}{\psi}} \right\}^{\frac{1}{1-\frac{1}{\psi}}} \\ &\equiv ((1 - \tau_Y)A_t + F_t) G_t(a_t). \end{aligned}$$

Since the expectation in  $\mathcal{C}_t$  involves  $G_{t+1}(\cdot)$ , we get a recursion for  $G$ . We solve this backwards

starting with  $t = T_M - 1$  in which case  $G_{T_M}$  is the constant known from (13).

Before retirement, i.e. for  $t = t_1, \dots, T_R - 1$ . For an induction argument, we assume that  $J_{t+1} = ((1 - \tau_Y)A_{t+1} + F_{t+1})G_{t+1}(a_{t+1}, y_{t+1})$  which implies that the certainty equivalent is

$$CE_t = ((1 - \tau_Y)A_t + F_t) \left( E_t \left[ \left( \frac{(1 - \tau_Y)A_{t+1} + F_{t+1}}{(1 - \tau_Y)A_t + F_t} \right)^{1-\gamma} \left\{ p_t G_{t+1}(a_{t+1}, y_{t+1})^{1-\gamma} + (1 - p_t) \xi^{\frac{1-\gamma}{\psi-1}} \right\} \right] \right)^{\frac{1}{1-\gamma}}.$$

Now

$$(1 - \tau_Y)A_{t+1} + F_{t+1} = (1 - \tau_Y)(A_t + \alpha_t Y_t)R_{At} + (1 - \hat{c}_t)(F_t + (1 - \tau_Y)(1 - \alpha_t)Y_t)R_{Ft}$$

and, thus,

$$\frac{(1 - \tau_Y)A_{t+1} + F_{t+1}}{(1 - \tau_Y)A_t + F_t} = (a_t + \alpha_t y_t)R_{At} + (1 - \hat{c}_t)(1 - a_t + (1 - \alpha_t)y_t)R_{Ft}.$$

Furthermore,

$$\begin{aligned} a_{t+1} &= \frac{(1 - \tau_Y)A_{t+1}}{(1 - \tau_Y)A_{t+1} + F_{t+1}} = \frac{(1 - \tau_Y)(A_t + \alpha_t Y_t)R_{At}}{(1 - \tau_Y)(A_t + \alpha_t Y_t)R_{At} + (1 - \hat{c}_t)(F_t + (1 - \tau_Y)(1 - \alpha_t)Y_t)R_{Ft}} \\ &= \frac{(a_t + \alpha_t y_t)R_{At}}{(a_t + \alpha_t y_t)R_{At} + (1 - \hat{c}_t)(1 - a_t + (1 - \alpha_t)y_t)R_{Ft}} \end{aligned}$$

and

$$\begin{aligned} y_{t+1} &= \frac{(1 - \tau_Y)Y_{t+1}}{(1 - \tau_Y)A_{t+1} + F_{t+1}} = \frac{(1 - \tau_Y)Y_t R_{Yt}}{(1 - \tau_Y)(A_t + \alpha_t Y_t)R_{At} + (1 - \hat{c}_t)(F_t + (1 - \tau_Y)(1 - \alpha_t)Y_t)R_{Ft}} \\ &= \frac{y_t R_{Yt}}{(a_t + \alpha_t y_t)R_{At} + (1 - \hat{c}_t)(1 - a_t + (1 - \alpha_t)y_t)R_{Ft}}. \end{aligned}$$

Hence, we can write

$$CE_t = ((1 - \tau_Y)A_t + F_t) \mathcal{C}_t(a_t, y_t),$$

and the utility recursion (5) implies that

$$\begin{aligned} J_t &= \max_{\hat{c}_t, \pi_t} \left\{ \left( \hat{c}_t \tilde{F}_t \right)^{1-\frac{1}{\psi}} + \beta ((1 - \tau_Y)A_t + F_t)^{1-\frac{1}{\psi}} \mathcal{C}_t(a_t, y_t)^{1-\frac{1}{\psi}} \right\}^{\frac{1}{1-\frac{1}{\psi}}} \\ &= \max_{\hat{c}_t, \pi_t} \left\{ (\hat{c}_t)^{1-\frac{1}{\psi}} (F_t + (1 - \tau_Y)(1 - \alpha_t)Y_t)^{1-\frac{1}{\psi}} + \beta ((1 - \tau_Y)A_t + F_t)^{1-\frac{1}{\psi}} \mathcal{C}_t(a_t, y_t)^{1-\frac{1}{\psi}} \right\}^{\frac{1}{1-\frac{1}{\psi}}} \\ &= ((1 - \tau_Y)A_t + F_t) \max_{\hat{c}_t, \pi_t} \left\{ (\hat{c}_t [1 - a_t + (1 - \alpha_t)y_t])^{1-\frac{1}{\psi}} + \beta \mathcal{C}_t(a_t, y_t)^{1-\frac{1}{\psi}} \right\}^{\frac{1}{1-\frac{1}{\psi}}} \\ &\equiv ((1 - \tau_Y)A_t + F_t) G_t(a_t, y_t). \end{aligned}$$

Since the expectation in  $\mathcal{C}_t$  involves  $G_{t+1}(\cdot, \cdot)$ , we get a recursion for  $G$ . For  $t = T_R - 1$ , note that  $G_{t+1} = G_{T_R}$  only depends on  $a_{t+1}$ .

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