Competition in Matching Markets: Evidence from College Admissions in China's Top Two Universities*

(This Version: November 2018)

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* We thank Lin Zhu, Tracy Xiao Liu, Qing Liu, Jian Huang, Ju Hu, Lixing Li, Naijia Guo, Xiaoyu Xia, Jeffrey Ely and participants in 2017 China Economy Conference (Beijing), PKU-NSD Microeconomics Workshop, 2018 China Meeting of the Econometric Society (Shanghai), 2018 CES China Conference (Hefei), 2018 International Conference on Economic Theory and Applications (Chengdu) for their helpful comments and suggestions. China Natural Science Foundation (No. 71173127) financially supports the paper.

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Abstract: In this paper, we introduce a model to study college competition in admissions quotas. Our simple model, by adapting matching model from Azevedo et al. (2014, 2016), delivers a few theoretical predictions: (1) When the quota of a college increases, the cutoffs of all colleges will (weakly) decrease. (2) Colleges tends to allocate their admission quotas across majors and regions so that their cutoffs are equalized. (3) Colleges will allocate more quotas to majors or regions more popular among students. We then provide empirical evidence by exploring data from the top two universities in China (i.e., Tsinghua and Peking University) to support our theoretical hypotheses.

Key words: college admissions; imperfect competition; matching markets

JEL classifications: C78; L13; I21

I. Introduction

Throughout the world colleges compete for high quality students. One important competitive strategy is by setting admission quotas not only for the whole college, but also among majors and student groups (e.g., by regions, tracks, ethnicities). How do changes of quotas affect student quality of each other? How do universities decide on their admission quotas? In this paper, we study strategic interactions between colleges in admitting students, adapting matching market models studied by Azevedo (2014) and Azevedo & Leshno (2016).

Our simple model delivers a few theoretical predictions on college behaviors: First, when the quota of a college increases, the cutoffs of all colleges will (weakly) decrease.

Second, colleges allocate their admission quotas across majors and regions so that major and regional cutoffs tend to be equalized. Third, a college will allocate more quotas to majors or regions becoming more popular among students. We then provide empirical evidence from top two universities in China, i.e., Tsinghua University and Peking University, to test our theoretical hypotheses.

I.1 From Traditional Markets to Matching Markets

It is easy to find similarities between college competition and competition in a traditional market, as colleges serve as a provider of higher education and students are customers demanding it. Yet colleges compete for students in a very different market – a matching market. In the traditional market (e.g. commodity market), traded goods are homogenous, and the equilibrium allocation (and social optimum) is determined as long as the price equalizing the quantity demanded and supplied are determined, no matter who trades with whom. In a matching market, agents in each side can be heterogeneous, and agents may value "matched" partners not only by the "price", i.e., money transfer, but also their other characteristics. For example, in college admissions, students not only care about the tuition fee paid, but also the quality or other characteristics (e.g., locations) of colleges; colleges also care about the student quality, not just (if any) the tuition revenue received.

Given heterogeneous "goods" in the market, matching markets are more similar to imperfectly competitive markets with product differentiation (Hotelling, 1929). However, in an imperfectly competitive market, heterogeneity occurs only on one side of the market: Consumers care about what kind of product they buy, but firms don't care about which consumers they serve. In a matching market, heterogeneity is usually two-sided. Furthermore, in a matching market, money transfers are often prohibited, and price used to clear the market is absent.

Matching theory (Roth and Sotomayor, 1993, etc.) has been successful to prove that in such a highly heterogeneous market without money, social desirable outcomes (e.g., *stable* outcomes, or core) often exist and can be achieved by some mechanisms/algorithms (e.g., Gale-Shapley mechanism). However, in matching model until now, agents (or matching objectives) from both sides are essentially *discrete*, therefore the model is not as tractable as an IO model.

A new literature has grown up to bridge the technical gap between matching theory and imperfect market (or IO) theory (Azevedo, 2014; Azevedo and Leshno, 2016). Their model (henceforth Azevedo-Leshno model) assumes a finite number of colleges, but a continuum mass of students. Priorities/preferences of colleges are based on some "indexes" of students, e.g., exam scores, and can be heterogeneous. *Demand* for a college is defined as, at any given *cutoffs*, i.e. minimum requirements of scores for admissions, the amount of students whose most preferred *affordable* college (i.e., college with a cutoff lower than their scores) is this college. Under quite general conditions, the stable matching is unique, under which a unique set of cutoffs equate the demand and supply (i.e., quotas) of every college. In other words, cutoffs play a role as market prices (Lemma 1 and Theorem 1, Azevedo and Leshno, 2016).

This set-up brings us a new way to study the strategic behavior of colleges or other "firms" in a matching market. Some of those behaviors have been studied in the "traditional" matching theory, such as quota or capacity control (Sonmez,1997; Kojima, 2007; Kesten, 2012), and quality improvement (Hatfield et al., 2016). Now they can be studied in a way that is very similar to study firm behavior in an imperfectly competitive market.

Our paper is an example of how to use such a IO-type method in a matching market to study college admissions problem. One unique feature of our model is that colleges have homogeneous priorities over students, based on their college entrance examination (or CEE) total scores. Although preference homogeneity is unnecessary for Azevedo-Leshno model, it would simplify analysis and deliver unique theoretical results. Our model is also closely related to quantity competition in imperfectly competitive market, first studied by Cournot (1838).

I.2 College Education as a Marketplace

There are still controversies whether market or IO analysis can be used to study college education in general. Two pioneer works have explored the behavior of colleges and found the similarity with firms in the marketplace. Rothschild and White (1993) argue that: "We believe that the market context of higher education – whether universities compete, how they compete, and the consequences of that competition for university input, production, pricing, and output decision – is interesting in its own right and important for understanding the cost and allocation issues that have concerned most researchers. …… The analysis of university behavior in a market context has been an under-researched area in economics."

Rothschild and White (1995) developed their ideas into a formal model to analyze "the pricing of higher education and other services in which the customers are inputs". Interestingly, they proved that prices that charge customers for what they get on net (output minus input) from the firm (or college) are competitive and support efficient allocations, and these price internalize the apparent external effects of customers on each other.

Our paper is different from theirs in two important ways. First, in their model, the objective function of a college (best understood as a private college) is to maximize profit, i.e., tuition revenue minus input resources, while in our model the objective function is to maximize the total quality of admitted students (as a total revenue) minus costs of admitting students. Second, in their model, colleges compete in a perfect market with given price parameters (i.e., tuitions), while in our model colleges competes in an imperfect market where there is no price.

I.3 Preview of the Paper

In Section 2, we build a simple model to study college competition in setting admission quotas. The model considers not only total quota a college may set, but also how to allocate total quota across different majors and regions. The model delivers three predictions on college behavior and its consequences, as we previously mentioned.

In Section 3, we describe institutional background, data and variables for empirical study. We collect data on college admissions competition between Tsinghua University and Peking University, the commonly recognized top two universities in China. They form a "duopoly market" for the most talented student group in China. Two key variables are cutoffs and quotas of two universities for different majors and regions. We also provide some stylized facts regarding quotas and cutoffs of those two colleges, to give us a first impression on how they compete.

In Section 4, we delineate our empirical method and report our empirical results. We explore time and regional variance of cutoffs and quotas to examine relationship between quotas and cutoffs and test our three theoretical hypotheses. All three theoretical hypotheses are supported by our empirical study. More concretely, when the quota of one university increases by 10%, the cutoff of the university and the rival university would decrease by 2.43 and 2.40% respectively. Second, two colleges set up their major and regional quotas so that all the majors and regions have roughly equalized cutoffs, measured by a small coefficient of variation (CV), compared to CV of quotas. Last, two colleges allocate more quotas to regions where the college becomes more popular among students.

Section 5 concludes the paper.

II. A Simple Model of College Admissions Competition with Homogeneous College Priority

We begin our theory from the simplest model with only two colleges without major divisions. Then we consider two colleges with two majors each, with vertical or lateral student preference over college-major bundles. We then provide a generalization of the model, where we have multiple college with multiple majors. At the end of this section we state our testable hypotheses. Colleges are assumed to have homogenous priorities over students (e.g, by their CEE total scores) throughout all the models.

II.1 Two-college model

Two colleges, Tsinghua University (TU) and Peking University (PU), compete for a continuum of students, with the amount normalized to 1. The quality of students, measured by their normalized CEE total scores, is uniformly distributed between 0 and 1. For any given quality, one half of the students prefer TU to PU, and the other half prefer PU to TU. All students prefer being matched to being unmatched. There are no tuition fees charged to students.

We consider the following duopoly game played by TU and PU:

1. Colleges simultaneously choose quotas $q_t, q_p \in [0,1]$, where q_t, q_p denote the quota of TU and PU, respectively.

2. After capacity choices, students are admitted according to the unique stable matching. Specifically, colleges will admit students by their CEE scores, from high to low, up to their committed quotas.

The objective function of colleges, i.e., profit, is defined as (Azevedo, 2014; Azevedo and Leshno, 2016):

Profit= total quality (or scores) of admitted students - total cost paid.

For each unit of students admitted, the college pays a cost of λ, $\lambda > 0$ but small enough. That is, the cost function is of the form: $C_i(q_i) = \lambda q_i$. [1]

Let's first distinguish two concepts of equilibrium. *Matching Equilibrium (ME)* is defined as the stable matching under any given quotas, i.e., the equilibrium in stage 2. It is also the matching market outcome where the cutoffs equate supply of and demand for two colleges. *Cournot Equilibrium (CE)* is defined as the Nash equilibrium for setting quotas: each college chooses a quota that is the optimal response for the quota chosen by the other college. The purpose of our analysis is to solve for CE, given that ME will always be achieved for any given quotas.

Without loss of generality, let $q_p \leq q_t$ at the Cournot equilibrium (CE). It is obvious that we cannot have: $q_t + q_p \geq 1$. Otherwise at least one college will admit students with quality (or marginal revenue) of 0, with a marginal cost of $\lambda > 0$. Therefore we have $q_p < 1/2$.

We first solve for ME, i.e., the equilibrium cutoffs given quotas. Let c_t, c_p be the cutoff scores of two colleges. Since $q_p \leq q_t$, we must have $c_p \geq c_t$. We call a college with a high (low) cutoff as *high(low)-positioning*. It is easy to derive the following relationship between the quotas and the cutoffs (see also Figure 1):

$$q_p = (1 - c_p)/2$$
$$q_t = \frac{1 - c_p}{2} + (c_p - c_t)$$

[1] Quotas are defined as the maximal number of students a college can admit. In our model, the quota must be met because: first, students always prefer being admitted to non-admitted, second, colleges will not set quotas so that the quota exceeds the number of students, since the marginal benefit would be zero (the ability of the least able student), while the marginal cost would be $\lambda > 0$.

Rearrangements lead to the equilibrium cutoffs as:

$$c_p = 1 - 2q_p$$
$$c_t = 1 - q_p - q_t$$

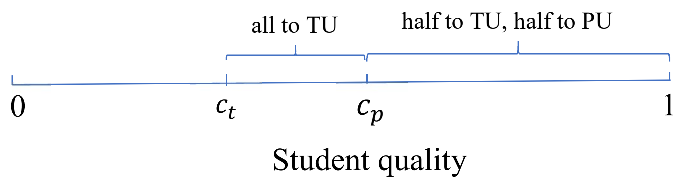

Figure 1 Two-college model

Claim 1. Under two-college model, (1) when a college raises its quota, it will ***weakly*** *lower the cutoffs of all colleges; (2) the marginal quota change of a* ***low-position*** *college will* ***NOT*** *affect the cutoff of the high-positioning college.*

Claim 1 part (1) replicates Lemma 2 in Azevedo (2014). It holds under a very general condition. Claim 1 part (2) is more restrictive, applying only for homogeneous college priorities. It says that there are only **one-way business-stealing externalities** from the low-positioning to the high-positioning.

We can solve for CE after solving for ME. First, the total student quality (or total revenue) of a college, denoted by R_t and R_p, can be expressed as:

$$R_p(q_t, q_p) = \frac{(1+c_p)}{2} * q_p = (1 - q_p)q_p,$$
$$R_t(q_t, q_p) = \frac{(1+c_p)}{2} * q_p + \frac{(c_p+c_t)}{2} * (q_t - q_p) = q_t - q_p q_t + \frac{1}{2}{q_p}^2 - \frac{1}{2}{q_t}^2.$$

It is easy to verify that the marginal revenues (MR):

$$MR_p \equiv \frac{\partial R_p(q_t, q_p)}{\partial q_p} = 1 - 2q_p = c_p,$$

$$MR_t \equiv \frac{\partial R_t(q_t, q_p)}{\partial q_t} = 1 - q_p - q_t = c_t.$$

Claim 2. MR=cutoff.

Claim 2 replicates Proposition 3 in Azevedo (2014). It applies for the same condition as Claim 1 part (2), i.e., if colleges have homogeneous priorities over students.

The profit $\pi_i = R_i - \lambda q_i, i = t, p$. The first order conditions (w.r.t q_i) for maximizing profit are:

$$MR_p = \lambda \Rightarrow 1 - 2{q_p}^* = {c_p}^* = \lambda$$

$$MR_t = \lambda \Rightarrow 1 - {q_p}^* - {q_t}^* = {c_t}^* = \lambda.$$

This leads to our solution for Cournot equilibrium(CE):

$${q_p}^* = {q_t}^* = (1 - \lambda)/2.$$

Monopoly Solution

It is interesting to look at the monopoly solution of this college admissions problem. That is, suppose two colleges jointly choose their quotas to maximize their total profit.

The monopoly solution can be solved as:

$$Min\{c_t^m, c_p^m\} = 1 - (q_t^m + q_p^m) = \lambda$$

That is, total profit is maximized as long as the total quota of two colleges is set such that MR, also the minimum cutoff among two colleges, equal to MC, no matter how students are allocated between two colleges.

We have the following conclusion:

Claim 3. CE maximizes monopoly profit.

The conclusion that CE implements monopoly outcome can be illustrated in Figure 2. Note that the MR curve of college i given q_{-i}, $MR_i(q_{-i})$, is kinked at $c_i = c_{-i}$ (or equivalently $q_i = q_{-i}$). In particular, given q_{-i}, when $c_i \leq c_{-i}$ (or $q_i \geq q_{-i}$), $MR_i(q_{-i})$ coincides with the MR curve for monopoly, $MR^m(q_{-i})$. Since MCs of both colleges are equal, then at Cournot equilibrium, their MRs, therefore cutoffs, are equal, i.e., $c_i = c_{-i}(=\lambda)$. This implies $MR^m(q_{-i}) = MR_i(q_{-i})$ at the CE. Therefore, CE implements monopoly outcome. Intuitively, when both colleges are equally positioning, an extreme case of low positioning, there is no “business-stealing” externalities from one college to the other. Then CE is consistent with monopoly solution.

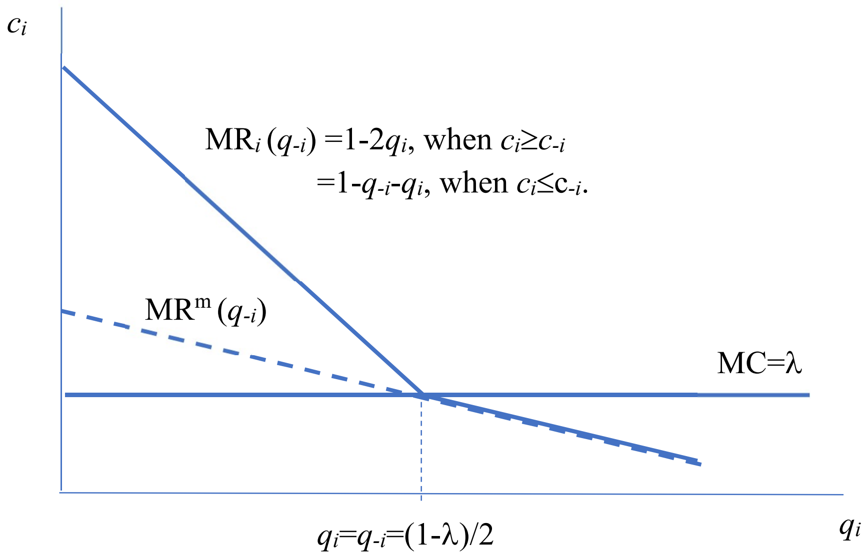

Figure 2 Cournot Equilibrium and Monopoly Solution

Extension 1: Heterogeneity in Costs

We consider two colleges have different marginal cost of admitting students, e.g., $\lambda_p > \lambda_t$. We hypothesize the college with lower marginal cost (here TU) would admit more students, i.e., ${q_t}^* > {q_p}^*$. Therefore, ${c_t}^* < {c_p}^*$. It is easy to find the Cournot equilibrium as:

$$
{c_p}^* = \lambda_p,\ {c_t}^* = \lambda_t
$$

$$
{q_p}^* = \frac{1-\lambda_p}{2},\ {q_t}^* = \frac{1+\lambda_p-2\lambda_t}{2}
$$

The monopoly solution which maximizes total profit for both colleges would require:

$$
q_t = 1 - \lambda_t, q_p = 0
$$

In this case, Cournot equilibrium *cannot* implement the monopoly profit. It turns out that equal MC is a critical condition for CE to implement monopoly solution.

Extension 2: Unbalanced Preferences

Suppose two colleges are not equally popular. For example, there are proportion α of all students prefer PU to TU, and $1-\alpha$ prefer TU to PU, with $\alpha \geq 1/2$.

The CE is:

$$
{c_p}^* = {c_t}^* = \lambda
$$

$$
{q_p}^* = \alpha(1-\lambda),\ \ {q_t}^* = (1-\alpha)(1-\lambda).
$$

More popular college would attract more students. However, their cutoffs are still equal.

It can be shown that the monopoly solution is still $q_t^m + q_p^m = 1 - \lambda$. Cournot equilibrium (CE) implements the monopoly profit. Equal popularity is not critical for CE to implement monopoly solution.

II.2 Two-college-two-major model

Suppose now each of two universities, TU and PU, has two majors: Science and Humanity. Student preferences are defined over 4 college-major bundles {TS, TH, PS, PH}. Here TS is Science major at TU, and so forth. Theoretically there are 4!=24 types of preference order for each student, and preference distribution (for any given student ability) should be defined over those types of preference order. To illustrate our basic idea, however, we simply consider two typical preference distribution: vertical or lateral student preference.

Vertical Student Preference

Suppose there are only two types of preference orders:

Type 1: TS>PH>PS>TH, with probability 1/2;

Type 2: PH>TS>TH>PS, with probability 1/2.

Student preference is vertical in the sense that they all prefer TS, PH to TH, PS. In addition, students who prefer Science major at TU to Humanity major at PU (slightly) favor Science major against Humanity major, so that they also prefer Science major at PU to Humanity major at TU. We assume for any given student quality (or score), each type of preference order is half populated.

Let $q_{ij}, i = t, p,\ and\ j = s, h$ be quotas for each major in each college. College-majors will admit students in a descending order of their college entrance exam scores, up to their committed quotas. The matching equilibrium (ME) is defined between college-majors and students on two sides. Let $c_{ij}, i = t, p,$ and $j = s, h$ be corresponding cutoffs. We first solve for ME, i.e., the equilibrium cutoffs.

It is obvious that $c_{ts}, c_{ph} \geq c_{th}, c_{ps}$. And without loss of generality, we assume $c_{ts} \geq c_{ph}$, and $c_{th} \geq c_{ps}$.Therefore we have $c_{ts} \geq c_{ph} \geq c_{th} \geq c_{ps}$.

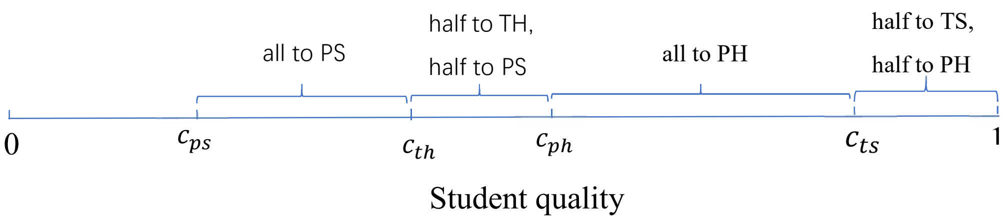

Figure 3 Two-college-two-major model (Vertical Preference)

It is easy to verify that (also referring to Figure 3):

$$q_{ts} = \frac{1}{2}(1 - c_{ts})$$
$$q_{ph} = \frac{1 - c_{ts}}{2} + (c_{ts} - c_{ph})$$
$$q_{th} = \frac{1}{2}(c_{ph} - c_{th})$$
$$q_{ps} = \frac{c_{ph} - c_{th}}{2} + (c_{th} - c_{ps})$$

Rearrangements lead to equilibrium cutoffs:

$$c_{ts} = 1 - 2q_{ts}$$
$$c_{ph} = 1 - q_{ts} - q_{ph}$$
$$c_{th} = 1 - q_{ts} - q_{ph} - 2q_{th}$$
$$c_{ps} = 1 - q_{ts} - q_{ph} - q_{th} - q_{ps}$$

We now solve for CE. We assume each college, NOT college-major, maximizes its total profit over two majors by choosing college-major quota, q_{ij}.[2] The Cournot equilibrium(CE) can be solved ass:

$$c_{ij}{}^{*} = \lambda, for \ \ i = t, p, and \ \ j = s, h.$$

$$q_{ts}{}^{*} = q_{ph}{}^{*} = \frac{1}{2} * (1 - \lambda),$$

$$q_{th}{}^{*} = q_{ps}{}^{*} = 0.$$

In the equilibrium, cutoffs are equalized among majors, which are also equal to MC. Therefore, colleges have no incentive to admit additional students into any major. The vertically less preferred major turns out to be shut down.

Lateral Student Preference

Suppose now student preference becomes more heterogeneous. In particular, except for Type-1 and 2, there are additional two types of students, Type-3 and 4:

Type 1: TS>PH>PS>TH, with probability $\alpha/2$,

Type 2: PH>TS>TH>PS, with probability $\alpha/2$,

Type 3: TH>PS>PH>TS, with probability $(1 - \alpha)/2$,

Type 4: PS>TH>TS>PH, with probability $(1 - \alpha)/2$.

Type 3 and 4 preference orders are just the reserve order of Type 1 and 2. Under such a type set, all four college-major bundles can be students' first choice. That is why we call it lateral preference. We assume $\alpha > 1/2$, i.e., type-1/2 are still more popular.

Let's hypothesize that: $c_{ts} \geq c_{ph} \geq c_{th} \geq c_{ps}$. ME is easy to solve as (by also referring to Figure 4):

[2] The result would be the same if each major in each college maximizes its own profit. A detailed discussion would be on Section II.4.

$$q_{ts} = \frac{1}{2}\alpha(1 - c_{ts})$$

$$q_{ph} = \frac{1}{2}\alpha(1 - c_{ts}) + \alpha(c_{ts} - c_{ph})$$

$$q_{th} = \frac{1-\alpha}{2}(1 - c_{ph}) + \frac{1}{2} * (c_{ph} - c_{th})$$

$$q_{ps} = \frac{1-\alpha}{2}(1 - c_{ph}) + \frac{1}{2} * (c_{ph} - c_{th}) + (c_{th} - c_{ps})$$

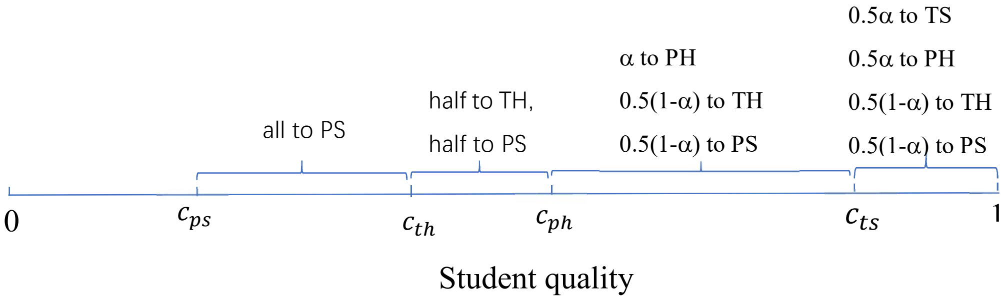

Figure 4 Two-college-two-major model (Lateral Preference)

Or equivalently:

$$c_{ts} = 1 - \frac{2}{\alpha} q_{ts}$$

$$c_{ph} = 1 - \frac{q_{ts}}{\alpha} - \frac{q_{ph}}{\alpha}$$

$$c_{th} = 1 - q_{ts} - q_{ph} - 2q_{th}$$

$$c_{ps} = 1 - q_{ts} - q_{ph} - q_{th} - q_{ps}$$

Under CE, we still consider the problem in which each college maximizes its total profit over two majors, by choosing q.

The Cournot equilibrium(CE) is:

$$c_{ij}^* = \lambda, for\ \ i = t, p, and\ \ j = s, h.$$

$$q_{ts}^* = q_{ph}^* = \frac{\alpha}{2}(1 - \lambda)$$

$$q_{th}{}^{*} = q_{ps}{}^{*} = \frac{(1-\alpha)}{2}(1-\lambda)$$

Under lateral student preference, all college-majors admit students, but more popular college-majors admit more students. Each major admit certain number of students such that their cutoffs are the same and equal to $\mathrm{MC} = \lambda$.

We summarize two-college-two-major model with either vertical and lateral student preference as the following:

Claim 4. Colleges equalize their (existing) major cutoffs.

Claim 5. Vertically less preferred majors will be eliminated by colleges; Laterally less popular majors will be allocated less quotas.

II.3 Multi-college-multi-major model: A Generalization

Let's generalize the model of two-college-two-major to arbitrary number of colleges and majors. We will restrict the model to the lateral student preference. It is not so restrictive, because, as we illustrated in Section II.2, colleges tend to eliminate their vertically less preferred majors so that any *existing* majors must be laterally preferred.

There are N colleges indexed by i, i.e., $N = \{i = 1, 2, \dots, N\}$. Each college has M_i majors, index $j(i)$, i.e. $M_i = \{j(i) = 1, 2 \dots, M_i\}$. The set of all college-major bundles are indexed by *k*, and denoted as $K = \{k = 1, \dots, K\} = \{(i, j(i)) | i \in N, j(i) \in M_i\}$.

Student quality, denoted by $\theta, \theta \in [0, 1]$, has a non-atomic distribution with a CDF as $F(\theta)$ and PDF as $f(\theta)$. F has a full support, i.e., the closure of $\{\theta | f(\theta) > 0\}$ is $[0, 1]$.

Student preference is independent of student quality. At each student quality $\theta,$ all the preference orders defined over the college-major bundle in set K are possible, i.e., with a positive probability. We further assume, at each student quality θ, there are proportions of $\alpha_{ij(i)}$ >0 who prefers college-major bundle $(i, j(i))$ *most*, with $\sum_{i,j(i)} \alpha_{ij(i)} = 1$.

The marginal cost of admitting students is constant among different majors within a college, but can be different across colleges. In particular, let λ_i be the MC for college i. $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_N$.

Lemma 1. *Given* q*, there exists a unique market clearing cutoff* c.

Lemma 1 shows the existence and uniqueness of matching equilibrium. It is noteworthy that Azevedo and Leshno (2016, Theorem 1) does not apply here. In their settings, colleges could have heterogeneous preferences over students, so the space of student's scores is $\Theta = [0,1]^K$ when there are K college-major bundles. Their theorem guarantees the existence and uniqueness of matching equilibrium when the distribution of student η has full support in Θ. But in our case, the CDF of student quality defined on Θ is $\eta(\theta_1, \theta_2, \dots, \theta_K) = F\left(\min_{1 \leq i \leq K} \theta_i\right)$ and it support $\operatorname{supp}(\eta) = \{(\theta_1, \theta_2, \dots, \theta_K) | \theta_1 = \theta_2 = \cdots = \theta_K\}$ (diagonal line only) is not Θ. [3]

Lemma 2. *When a college-major raises its quota, it will* ***weakly*** *lower the cutoffs of all college-majors.*

Lemma 2 replicates Lemma 2 in Azevedo (2014) and serves as an extension of Claim 1 part (1). As we said before, Lemma 2 holds in a very general condition. Basically, it only requires the matching equilibrium is unique.

We now strengthen Lemma 2 under the special case where colleges have homogeneous priorities over students.

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[3] Azevedo (2014) weakened the assumption in Azevedo and Leshno (2016). However, our settings do not satisfy the assumptions in Azevedo (2014) either.

Lemma 3. *Under matching equilibrium, marginal change of quota of any college-major* k*, i.e.,* q_k*, will not affect cutoff of any other college-major* $k' \neq k$, i.e., $c_{k'}$, if $c_k < c_{k'}$.

The proof is in Appendix 1. Lemma 3 extends Claim 1 part (2). After stating our lemmas, we are ready to state our main conclusion.

Proposition 1. *The Cournot equilibrium is:*

$$c_{ij(i)} = \lambda_i, \forall j(i), i \neq N,$$
$$c_{Nj(N)} \leq \lambda_{N-1}, Min_{j(N)}\{c_{Nj(N)}\} = \lambda_N, \forall j(N)$$

The proof is in Appendix 1. Proposition 1 only characterize cutoffs under CE. Equilibrium quotas are difficult to characterized, unless we assume equal MC among colleges, i.e., $\lambda_i = \lambda$.[4]

Corollary 1. *If* $\lambda_i = \lambda, \forall i$*, then:*

$$c_{ij(i)} = \lambda, \forall i, j(i)$$
$$q_{ij(i)} = \alpha_{ij(i)}\big(1 - F(\lambda)\big), \forall i, j(i).$$

The proof is straightforward so we ignore. Proposition 1 and Corollary 1 extend Claim 4 and 5. In particular, major cutoffs are still equalized within colleges, except for the college with the solely lowest MC, and more popular majors are allocated to larger quota.

[4] When λ_i varies among different i, the equilibrium quotas would depend on a full characterization of student preference order over colleges. Define proportion α_k over all of those orders, with $k = 1, \dots, K = i!$. Then students admitted by the college with the highest cutoff, say λ_1,would be those with $\theta \geq \lambda_1$ whose first choice being a major in college 1. Students admitted by the college with the second highest cutoffs, say λ_2, would be those with $\theta \geq \lambda_2$whose first choice being a major in college 2, as well as those with $\lambda_1 > \theta \geq \lambda_2$ whose highest choice(s) is(are) major(s) in college 1, but the highest choice except those in college 1 is a major in college 2. And so on.

Suppose each college-major, instead of a college as a whole, maximizes its own profit independently. Let's call the solution *major-Cournot equilibrium (Major-CE)*. It is easy to derive that:

$$MR_{ij(i)} = c_{ij(i)}{}^{*} = \lambda_i, \forall i, j(i)$$

Equilibrium cutoffs are the same as under the Cournot equilibrium where colleges maximize their total profits for all its majors (i.e., *college-CE*), except for $i = N$. Equilibrium quotas under major-CE and college-CE for colleges other than *N* are also the same.

The idea of the (almost) equivalence between major-CE and college-CE is very similar to that of the equivalence between CE and monopoly under college competition without majors (as in Section II.1): when marginal cost is equal among majors, major cutoffs are equalized when majors are independent profit maximizers. But at such an optimum, *"stealing business" externalities* disappear so that it also realizes the "monopoly" profit of the whole college.

We state this conclusion as our second corollary.

Corollary 2. *The quotas and cutoffs under college-Cournot equilibrium are equal to those under major-Cournot equilibrium, except for college N.*

Regional quota allocation

For empirical relevance, we now consider quota allocation among regions, which is a simpler case than quota allocation among majors. The problem (ignoring major divisions) is a bit different from major quota allocation. In the major quota allocation, student preferences are defined over college-major bundles, implying that they can freely choose any college and major if available. In a regional quota allocation, students are often not allowed to move among regions. Therefore, their preferences are defined only over colleges, not regions.

There are N colleges indexed by i, i.e., $N = \{i = 1, 2, \ldots, N\}$, and J regions indexed as j. The total amount of students is normalized to unit, and β_j is the proportion of student in region j. Student quality in region j is denoted by $\theta_j, \theta_j \in [\underline{\theta}_j, \overline{\theta}_j]$, has a non-atomic distribution with a CDF as $F_j(\theta_j)$ and PDF as $f_j(\theta_j)$. F_j has a full support.

Student preference is defined over colleges, which are independent of student quality. For any region, all student preference orders over colleges are possible. We further assume, within region j, there are proportions of α_{ij}>0 who prefers college i *most*, with $\sum_i \alpha_{ij} = 1, \forall j$.

The marginal cost of admitting students is constant among different regions within a college, but can be different across colleges. In particular, let λ_i be the MC for college i. We assume $\underline{\theta}_j < \lambda_i < \overline{\theta}_j, \forall i, j$.

Each college i, by setting quotas for each region, i.e., q_{ij}, maximize its total profit among all regions:

$$\text{Max}_{\{q_{i1},\ldots,q_{iJ}\}} \sum\nolimits_j R_{ij}\,(q_j) - \lambda \sum\nolimits_j q_{ij}$$

where R_{ij} is the total quality of students (or total revenue) for college i from region j. Note that total revenue depend on quotas of all colleges at region j, i.e., q_j.

Lemma 4 *Under Cournot equilibrium, for any college i at any region j, marginal revenue is equal to cutoff, i.e.,* $MR_{ij} = c_{ij}, \forall i, j.$

Lemma 4 extends Claim 2 and replicates Proposition 3 in Azevedo (2014). It applies when colleges have homogeneous priorities over students, obviously true here.

Proposition 2. *The Cournot equilibrium (for regional quota allocation) is:*

$$c_{ij} = \lambda_i, \forall i, j$$

If $\lambda_i = \lambda, \forall i$, *then:*

$$q_{ij} = \beta_{ij}\alpha_{ij}\left(1 - F_j(\lambda)\right), \forall i, j.$$

Proposition 2 is derived from Lemma 4. The proof is straightforward so we ignore. The conclusions are essentially the same as for major quota allocation. That is, cutoffs are equalized among regions, and a larger quota is allocated to a region becoming more popular among students.[5]

II.4 Testable Hypotheses

The theory we developed in Section II.1-II.3 highlights three testable hypotheses.

Hypothesis 1: When the quota of a college (or college-major) increases, the cutoffs of all colleges (or college-majors) will (weakly) decrease.

Hypothesis 1 is predicted by Claim 1 part (1) and Lemma 2. We do not form hypothesis for asymmetric effect of quotas on cutoffs due to unequal positioning, stated in Claim 1 part (2) and Lemma 3. We will let our data tell us whether there are asymmetric effects of quotas on cutoffs between colleges.

Hypothesis 2: Colleges tends to allocate their quotas across their (existing) majors or regions so that cutoffs for each major or region are equalized.

[5] In Appendix 2, we consider another extension where colleges face constraints on total quotas they can allocate among different majors (or regions), but zero MC within those constraints. Proposition 2 (and 3) still holds: for colleges except the one with the largest quota, their cutoffs are equalized and more quotas are allocated to more popular majors (or regions).

Hypothesis 3: A college will allocate more quotas to more popular majors or regions becoming more popular.

Hypothesis 2 and 3 are predicted by Claim 4 and 5, Proposition 1 and Corollary 1, and Proposition 2. For Hypothesis 2, i.e., equalized cutoffs, equal MC for majors or regions are needed within colleges. For Hypothesis 3, to simplify our notations, in Corollary 1, we only give an expression for equilibrium quotas under equal college MCs. But Hypothesis 3 may not require equal MC among colleges.

III. Background and Data

III.1 Background

Tsinghua University and Peking University feature duopoly in China's college admissions. They are commonly recognized as the top two universities in China. In the QS World University Rankings of year 2015, the ending year of our sample, Tsinghua is placed at 47 and Peking at 57, representing the two highest-ranked universities in China (and their rankings are still rising!)[6]. In the "China Discipline Ranking" (2012), conducted by Ministry of Education, 16 disciplines offered at Peking University and 14 at Tsinghua are top-ranked in China – both of which far outpace any other institutions[7]. Their campuses' geographical proximity within Beijing only adds to the sense of similarity. News reports suggest that two universities indeed compete with each other for top-ranked students (*zhuangyuan*) in college admissions[8].

[6] The QS Rankings can be found at: https://www.topuniversities.com/

[7] Sina Education：http://edu.sina.com.cn/kaoyan/2013-01-29/1815370477.shtm

[8] "Guangdong CEE cutoffs have just been announced! Tsinghua and Peking University come to Huizhou for high-score students", News-163, 2017, http://news.163.com/17/0625/12/CNPBCH5O000187VE.html. "Tsinghua and

Someone may suspect that the two public universities have autonomy to change their quotas under government regulation. Although in each year universities need to report their total, regional and major quotas to the Ministry of Education, and publicize them before CEE, they still have discretions to vary their quotas to compete for high-quality students.

For example, Article 32 in Higher Education Law of the People's Republic of China (1999) states that: "Higher education institutions shall draw up enrolment plans in light of social needs, the conditions of the institutions, and the size of the student body verified by the State, and readjust on their own the proportions of enrolment for different faculties and subjects."

Article 27 in Regulations for Enrollment in Regular Colleges and Universities in 2017 states that: "Colleges and universities … may, within the scope of the national annual enrollment of regular higher education, draw up their own enrollment plan divided into provinces (regions, cities) and professionals, i.e., enrollment source plans, according to the relevant plan preparation work requirements and the principles and methods of enrollment planning defined in the enrollment regulations. The provincial education administrative departments, relevant departments (units), the Education Department (bureaus) and universities and colleges shall formulate, adjust, and execute the enrollment source plan within the scope of the annual national general authorized enrollment of higher education." Article 28 says: "Universities should strengthen the analysis and forecast of talent demand in light of the needs of China's economic and social development, and combine their own conditions for running schools, employment of graduates, and the conditions of student sources in each province (region, city), to make enrollment adjustment over professional structure, hierarchy, and regional

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Peking University fight for top students overnight", China Youth, 2016, http://d.youth.cn/sk/201606/t20160624_8183923.html "Tsinghua and Peking University enrollment chaos", News-163, 2015, http://news.163.com/15/0628/16/AT77IT810001124J.html.

structure, and arrange enrollment source plan independently, scientifically, and reasonably."

Our data also illustrate colleges can adjust admissions quotas quite flexibly: they can change their total admissions quota,[9] adjust quota allocations among different provinces,[10] and among different majors[11].

III.2 Data

The key variables in our empirical models are score cutoffs and admission quotas for Tsinghua University and Peking University. Our data for those variables are gathered from websites and brochures published by both universities. Data sources are listed in Table 1. The data represent the total university enrollment (cutoffs and quotas) in 30 provinces (excluding Tibet Autonomous Region and Hong Kong, Macao and Taiwan), as well as cutoffs and quotas for individual majors (mathematics, etc.), from 2011 to 2015, for both universities.

Table 1 Data source

Data	Source
Cutoffs of Tsinghua University in each province (2011-2015)	Tsinghua University Undergraduate Enrollment Website http://www.join-tsinghua.edu.cn/publish/bzw/9500/index.html
Cutoffs of majors in Tsinghua University in each province (2011-2015)	

[9] In 2011, Tsinghua planned to enroll 1200 students, and this number increased to 1400 in 2012, decreased to 1200 in 2015. Meanwhile, Peking University's quota is always around 1200. See also Figure 5.
[10] For instance, Tsinghua University enrolled 34 students through the science track in Anhui province in 2011, while in 2013, the number increased to 54 and in 2015 fell back to 40.
[11] For instance, the Environmental Engineering Major at Peking University enrolled 6 students through the science track in Beijing in 2012, while in 2014, the number decreased to 2 and in 2015 rose back to 4.

Cutoffs of Peking University in each province (2011-2015)	Peking University Undergraduate Enrollment Website http://www.gotopku.cn/programa/admitline/7.html
Cutoffs of majors in Peking University in each province (2011-2015)	Peking University 2012-2016 student recruitment brochure
Quota of Tsinghua University and its majors in each province (2011-2015)	Tsinghua University Undergraduate Enrollment Website http://www.join-tsinghua.edu.cn/publish/bzw/9501/index.html
Quota of Peking University and its majors in each province (2011-2015)	Peking University Undergraduate Enrollment Website http://www.gotopku.cn/programa/enrolstu/6.html
Major consolidation	College major catalogue issued by Ministry of Education (2012)

Table 2 provides a descriptive analysis of cutoffs and quotas. Table 3 details the consolidation of majors in these two universities, according to College Major Catalogue issued by Ministry of Education (2012). In total, there are 13 common or similar majors across the two universities.

It should be noted that admission quotas are released before the CEE is administered. Although it is defined as the maximal number of students to be admitted by a college in a major and region, in reality quotas are often fully occupied. Because Tsinghua and Peking Universities are so popular among students, to leave some pre-committed slots open can cause serious oppositions. Sometimes quotas are even broken, which lead to admission quotas be the *minimum* number of students that universities should admit. This is especially true for regional quotas. However, admission quotas are still very useful to consider the “supply” from colleges (and majors). When students fill in the application form, they have to refer to the admission quota, instead of the real admission number. Another reason for using admission quota is that the actual admission number is unavailable.

Table 2 Descriptive Statistic

（1）University Level					
Variable	Obs.	Mean	Std. Dev.	Min	Max
Cutoff	600	652.4	70.1	387	902
Quota	600	20.4	24.0	1	205
（2）Major Level					
Variable	Obs.	Mean	Std. Dev.	Min	Max
Cutoff	6,446	655.2	70.8	371	938
Quota	7,744	1.5	1.4	1	28

Table 3 Major category

Peking University			Tsinghua University			Consolidated	
Major name	Major code	Class code	Major name	Major code	Class code	Major Name	Major Code
Law	030101K	0301	Law	030101K	0301	Law	301
Japanese	050207	0502	Japanese	050207	0502	Foreign Language	502
English	050201	0502	English	050201	0502		
Journalism	0503	0503	Journalism	050301	0503	Journalism	503

Mathematics	0701	0701	Mathematics	070103T	0701	Mathematics	701
			Applied mathematics	070101	0701		
Physics	0702	0702	Physics	070201	0702	Physics	702
Chemistry	0703	0703	Chemistry	070301	0703	Chemistry	703
			Biochemistry	070303T	0703		
Biology	0710	0710	Biology	0710	0710	Biology	710
Electronic Information	0807	0807	Electronic Information	0807	0807	Electronic Information	807
Environmental Science and Engineering	0825	0825	Environmental Engineering	082502	0825	Environmental Science and Engineering	825
Urban-rural Planning	082802	0828	Architecture	082801	0828	Architecture	828
Information Management and Information System	120102	1201	Information Management and Information System	120102	1201	Information Science and Engineering	1201
Business Administration	1202	1202	Business Administration (Accounting)	120203K	1202	Economics, Finance and Accounting	1212
Economics	0201	0201	Economics and finance				
Note:							

① The following majors at Peking University cannot be consolidated: philosophy, international politics, sociology, China language literature, other languages (French, German, Spanish, Russian, Arabic, Korean, Thai, Filipino, Bahasa Indonesia, Urdu, and Sanskrit), archaeology, history, astronomy, geophysics, geology, psychology, public management, art theory, and experimental courses in engineering, science, and liberal arts.

② The following majors at Tsinghua University cannot be consolidated: engineering mechanics, mechanical engineering, vehicle engineering, measurement and control technology and instrumentation, materials science and engineering, polymer materials and engineering, energy and power engineering, electrical engineering and automation, automation, computer science and technology, software engineering, civil engineering, building environment and energy engineering, water conservancy and hydropower engineering, aerospace engineering, nuclear engineering and technology, engineering physics, biomedical engineering, medical physics experimental class, clinical medicine, pharmacy, industrial engineering, social science and humanities experimental classes, chemical engineering, and industrial biological engineering.

III.3 Stylized Facts

Before we conduct formal regressions, we first look at some stylized facts of college admissions of these two universities. Figure 5 shows how the cutoffs (normalized to 100 points) of two universities vary across years. Cutoffs are grouped by CEE tracks, i.e., Humanity or Science, averaged across provinces. Cutoffs for first-batch colleges (including roughly 100 elite universities) are also shown. For either track, two universities have roughly the same cutoffs.

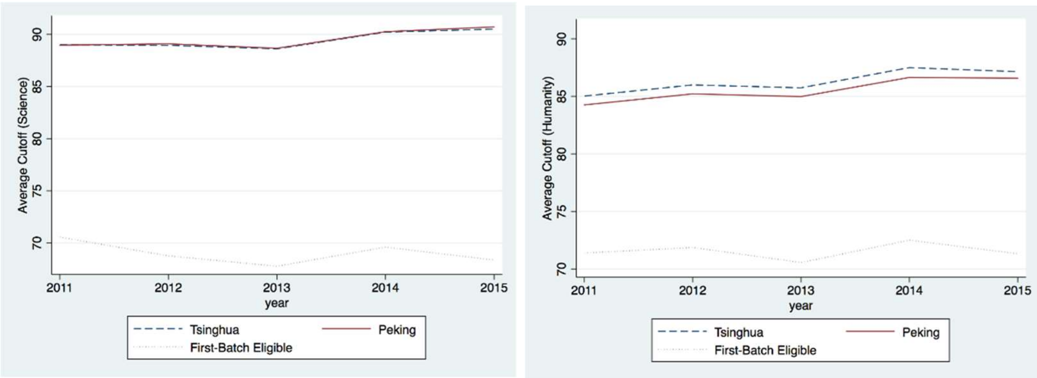


Figure 5 Cutoffs at University Level through Years (Provincial Average)
Science(Left), Humanity(Right), Normalized to 100pts

Figure 6 shows the total quotas (summed across provinces and majors) of two universities across years and by tracks. Tsinghua university has a larger total quota than Peking University. As for tracks, Peking university has a larger quota for Humanity but smaller quota for Science. Quotas also vary across years, with a range of 1,100 to 1,450 for Tsinghua university, and 1,100 to 1,200 for Peking university.

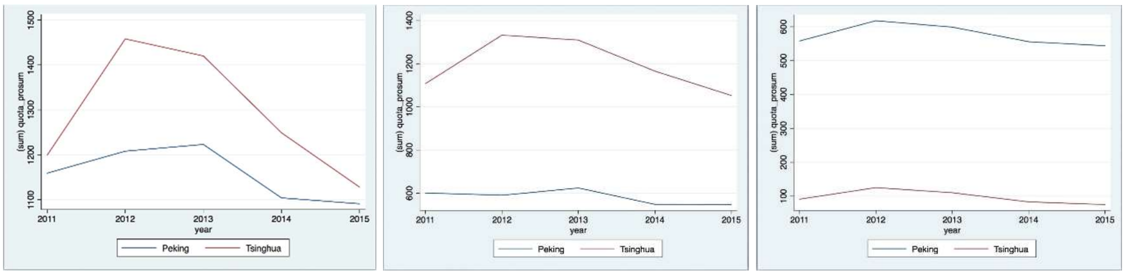
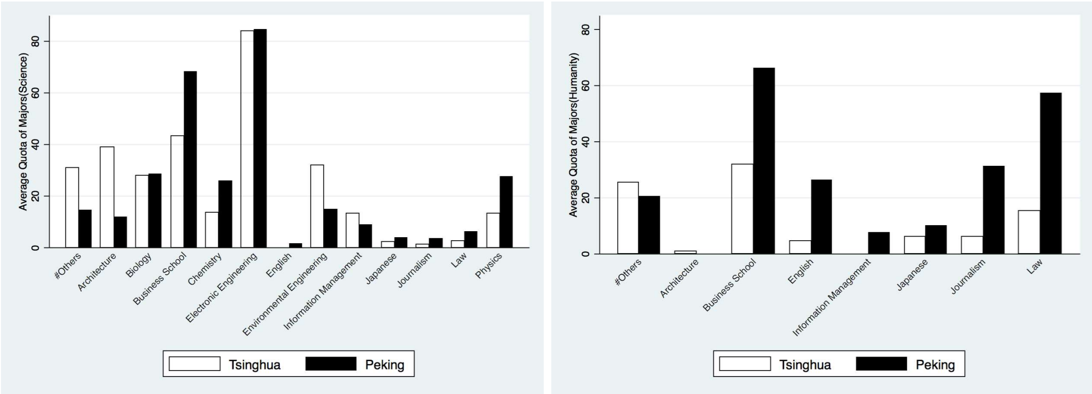


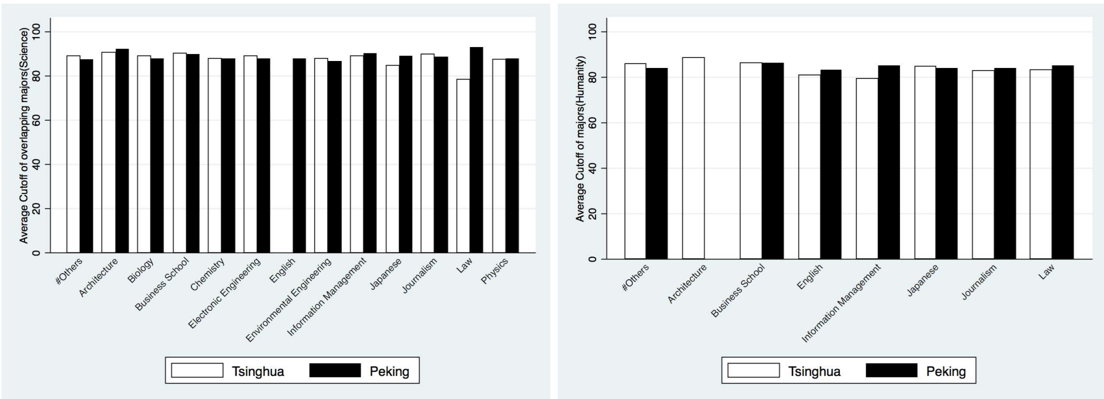
Figure 6 Quotas at University Level through Years
Total (Left), Science(Middle), Humanity (Right)

Figure 7 and 8 show quotas and cutoffs for each overlapping (or consolidated) major of two universities, averaged across years. Admission quotas vary a lot across majors, yet cutoffs are surprisingly equalized. The pattern is consistent with our Hypothesis 2 that colleges allocate quotas among majors to equalize cutoffs. Even among those non-overlapping majors, the same pattern stands out: quotas vary a lot while cutoffs tend to be equal.



#: Others are the average number of quota of all the non-overlapping majors from two universities.

Figure 7 Major Quotas (Yearly Average)
Science(Left), Humanity (Right)



#: Others are the average cutoffs of all the non-overlapping majors from two universities.

Figure 8 Major Cutoffs (Yearly and Provincial Average)
Science(Left), Humanity (Right)

Figure 9 and 10 show the provincial quotas and cutoffs by track. They reveal the same pattern as major quotas and cutoffs: quotas vary much yet cutoffs are equalized. The equalization of cutoffs is even more surprising here, because CEE contents are often different among regions and their scores are in general not fully comparable.

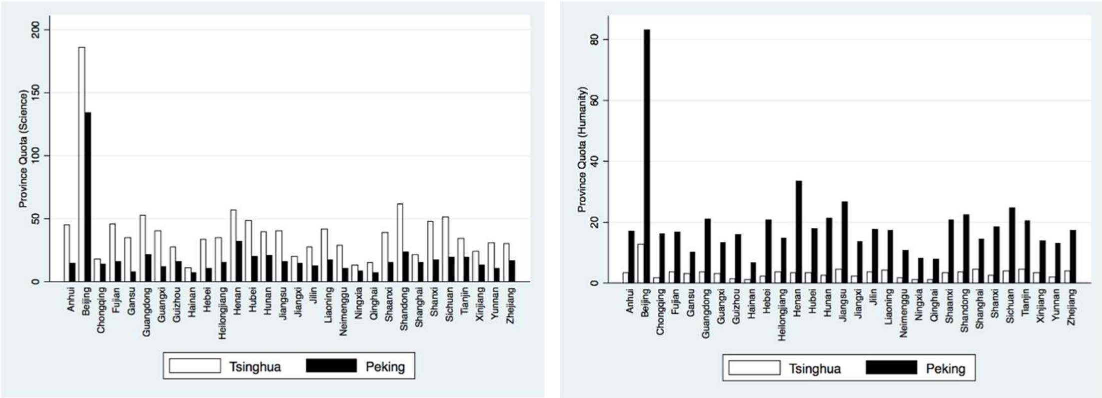


Figure 9 Quotas at Provincial Level (Yearly Average)
Science(Left), Humanity (Right)

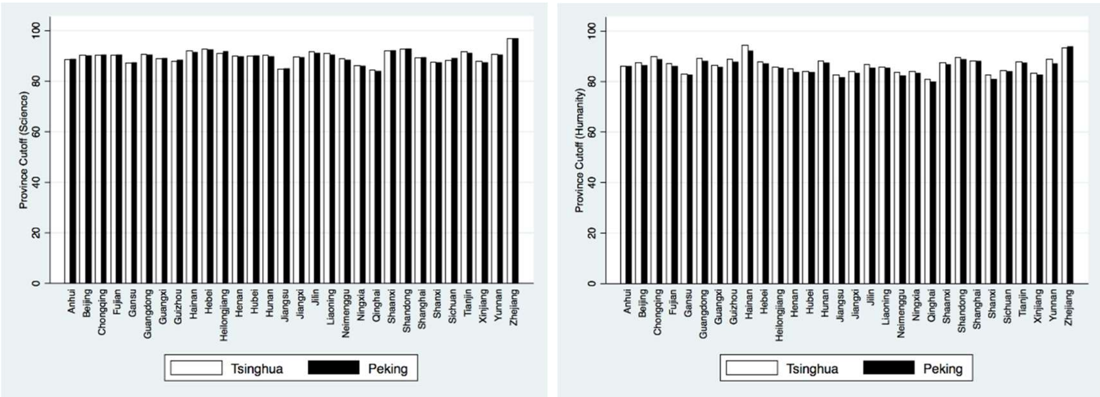


Figure 10 Cutoffs at Provincial Level (Yearly Average, Normalized)
Science(Left), Humanity (Right)

IV. Empirical Tests

We now provide our empirical tests for our three Hypothesis. The relationships between those three hypotheses can be shown in a conceptual framework in Figure 11. Colleges play a Cournot Competition at the first stage, while at the second stage, the matching market determines the equilibrium cutoffs. Hypothesis 1 is about how the cutoffs are determined, especially how they depend on quotas. Note that when quotas are determined (by Cournot competition), all the supply-side factors are fully captured by them. In other words, quotas are exogenous when testing Hypothesis 1 which concerns only the matching equilibrium. However, student characteristics are factors affecting both matching equilibrium and Cournot equilibrium. Therefore, we have to include them into our testing for Hypothesis 1, as well as Hypothesis 3 concerning Cournot equilibrium. We cannot observe college characteristics such as marginal cost, or even colleges' objective function - we just assume them. Finally, Hypothesis 2 is just about statistical characteristics of cutoffs.

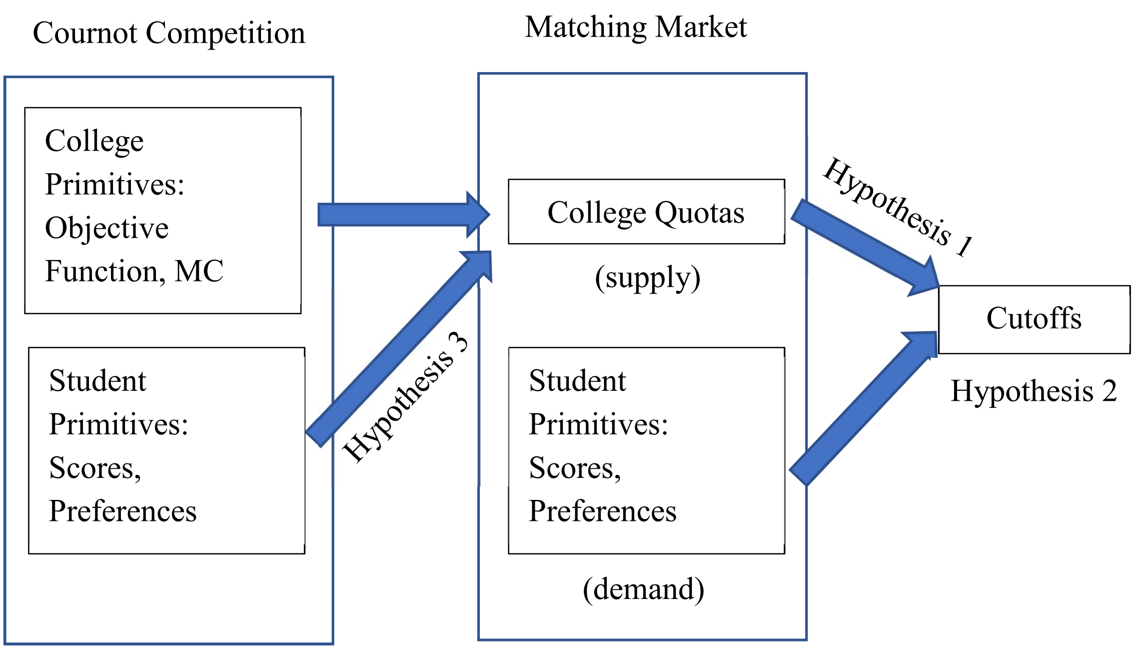


Figure 11 Conceptual Framework

IV.1 Testing Hypothesis 1: How Quotas Affect Cutoffs

We explore time and regional variants of quotas to test its causal effects on cutoffs. The benchmark regression model is as followed:

$$
\begin{aligned}
Cutoff(i,j,k,t) = \alpha &+ \beta_1 * Quota(i,j,k,t) + \beta_2 * Quota(-i,j,k,t) \\
&+\beta_3 * Cutoff(i,j,k,t-1) + \beta_4 * Cutoff(-i,j,k,t-1) \\
&+Province_{dummy} + University_{dummy} + Track_{dummy} \\
&+Year_{dummy} + u(i,j,k,t) \qquad (1)
\end{aligned}
$$

Here i represents a generic university (Tsinghua University or Peking University), $-i$ represents the rival university, j represents province, k represents track (humanities or science), t represents year. $Cutoff(i,j,k,t)$ represents cutoffs of university i in province j through track k in year t. $Quota(i,j,k,t)$ represents the corresponding admission quotas. $Quota(-i,j,k,t)$ represents the quota of the rival university. Cutoffs and quotas are expressed in logarithmic form. $u(i,j,k,t)$ is the residual term.

We include year, province, track, and university dummies to control for the corresponding demand/student-side fixed effects. Year dummies control the overall trend of CEE scores due to factors like national policy changes or cohort effects of student quality. Track dummies control the time independent student difference between the humanities and science tracks in CEE. Province dummies control the time persistent difference in student quality, CEE difficulty, and full score (scale) among different provinces. Universities dummies control time-fixed student preferences for Tsinghua and Peking University. In alternative specifications, we also consider the interaction of some of these dummy variables, to incorporate more complex fix effects.

We include cutoffs of previous year to control for non-fixed demand changes. The cutoffs of the previous year are usually used by students as a reference when applying for universities. A rise in cutoffs of last year will signal to applicants that the university's popularity has improved. Note that universities may also adjust their admissions quotas according to historical cutoffs. So if those cutoffs are not controlled, it may cause an endogenous problem of omitted variables.[12]

One concern of the model is that the dependent variable (cutoffs) and the main independent variable (quotas) are first order integrated i.e., I(1), processes. Although first-order differencing the model may well eliminate the concern, it also has the problem of "eliminating" cross-sectional variations important for our identifications, noting that our sample has 120 panels but only 5 years. To check whether those two variables are indeed I(1) process, we use the Harris-Tzavalis unit-root test for panel data, and find that, for both variables, the null hypothesis that panels contain unit roots are strongly rejected. We also included first difference outcome in our regression tables.[13]

The above model is to test causality between quotas and cutoffs at university level. The regression model at major-level is almost the same as the one for university-level, except that major dummies are added to control for student preferences toward different majors. We focus on the same or similar majors of two universities (defined in Table 3), because they are presumably close substitutes to each other.

Empirical Results

The results for testing effects of quotas on cutoffs at university-level are shown in Table 4. Column 1 sticks to the benchmark model (eq.1). Other regressions add more controls. Column 2 controls interactions between provinces and tracks, or equivalently, the matching system dummies. Column 3 further controls the interactions between the matching systems and universities, which can reflect the time-invariant student preferences for each university varying among different provinces and tracks. Column 4, based on Column 2, further controls the interaction between the matching systems and years, to capture the possible time-variant differences among different matching

[12] Using lagged dependent variables as proxies for unobserved explanatory variable is a common practice in cross-sectional regression analysis; see for example Wooldridge (2003), Section 9.2, pp. 300-302.

[13] A few tests for unit root under panel data are available. Harris and Tzavalis (1999)'s test is suitable when the number of periods are relatively small while the number of panels are large.

systems. Column 5, based on Column 4, further adds last year's quotas of both universities. It is possible that students take into account not only last year's cutoffs but also admissions quotas when they apply for universities. In each specification, we cluster residuals by tracks and provinces, but allow residuals to be correlated among years and universities.[14] Column 6 uses first-order difference. The result is insignificant while the sign of the coefficients is still consistent with other specifications.

Table 4 How Quotas Affect Cutoffs: University-level

Explanatory variables	Explained variable：Log_Cutoff(i,j,k,t)					Explained variable：ΔLog_Cutoff(i,j,k,t)	
	(1)	(2)	(3)	(4)	(5)		(6)
Log_Quota(i, j,k,t)	-0.009**	-0.017***	-0.017***	-0.243***	-0.366***	ΔLog_Quota(i, j,k,t)	-0.002
	(0.004)	(0.005)	(0.005)	0.000	(0.001)		(0.005)
Log_Quota(-i,j,k,t)	-0.006	-0.014**	-0.014**	-0.240***	-0.363***	ΔLog_Quota(-i,j,k,t)	0.003
	(0.004)	(0.005)	(0.006)	0.000	(0.001)		(0.005)
Other control variables							
Log_Cutoff (i,j,k,t-1)	0.245***	0.103**	-0.085*	2.319***	2.702***		
	(0.049)	(0.043)	(0.043)	(0.058)	(0.058)		
Log_Cutoff(-i,j,k,t-1)	0.028	-0.114*	0.0752	2.102***	2.488***		
	(0.055)	(0.066)	(0.068)	(0.058)	(0.058)		
Log_Quota(i,j,k, t-1)					0.021***		
					-0.001		
Log_Quota (-i,j,k,t-1)					0.021***		
					-0.001		
Province	Y	N	N	N	N		N
Track	Y	N	N	N	N		N
University	Y	Y	N	Y	Y		N
Year	Y	Y	Y	N	N		Y
Track*Province	N	Y	N	N	N		N
Track*Province* University	N	N	Y	N	N		N
Track*Province*Year	N	N	N	Y	Y		N
Observations	480	480	480	480	480		480

[14] As different provinces, tracks and years belong to different CEE systems, CEE scores are less likely to be related across provinces, tracks and years. However, as we have introduced time lagged variable, CEE scores within the same province and track could be related over time. In addition, we are considering the competitive relationship between Tsinghua University and Peking University, so residuals could be related across universities.

R^2	0.978	0.984	0.985	0.999	0.999		0.1

Notice: The clustered robust standard deviation of track and province are in the brackets. *** p<0.01, ** p<0.05, * p<0.1.

The results from all specifications (Column 1-5 in Table 4) are consistent with Hypothesis 1. In fact, any increase in admissions quotas in one university will significantly reduce its own cutoffs, as well as the other university's cutoffs. Using Column 4, which we believe represent the best specification, if a university's quota increases 10%, the other university's cutoff will decrease by 2.40%, while its own cutoff will also fall by 2.43%.

It is also interesting to see whether quotas have asymmetric cross-effects on cutoffs, as illustrated by Lemma 3. The results are shown in Table 5. The quota changes in Peking University have significantly negative effects on Tsinghua University's cutoffs, while those effects do not show up in the opposite direction. According to Lemma 3, it seems that Peking University is a bit higher positioning than Tsinghua University. Note that Tsinghua University has a larger total quota than Peking University (Figure 2), consistent with their relative positioning.

Table 5 How Quotas affect Cutoffs: University Level (University Subsample)

Explanatory variables	Explained variable：Log_Cutoff(i,j,k,t)					
	TU Subsample			PU Subsample		
	(1)	(2)	(3)	(4)	(5)	(6)
Log_Quota(i, j,k,t)	-0.00803	-0.0140*	-0.00681	-0.0190***	-0.0280**	-0.0263***
	(0.00538)	(0.00751)	(0.00593)	(0.00655)	(0.0120)	(0.00937)
Log_Quota(-i,j,k,t)	-0.0157**	-0.0231*	-0.0218**	-0.000472	-0.00801	-0.00186
	(0.00702)	(0.0128)	(0.0103)	(0.00500)	(0.00804)	(0.00589)
Other control variables						
Log_Cutoff (i,j,k,t-1)	0.0247	-0.188	0.00555	0.241	0.0265	0.225
	(0.196)	(0.170)	(0.207)	(0.164)	(0.175)	(0.176)
Log_Cutoff(-i,j,k,t-1)	0.249	0.187	0.268	0.0267	-0.0379	0.0383

	(0.177)	(0.192)	(0.189)	(0.179)	(0.145)	(0.187)
Log_Quota(i,j,k, t-1)			-0.00217			0.00963
			(0.00502)			(0.00808)
Log_Quota (-i,j,k,t-1)			0.00913			0.00246
			(0.00875)			(0.00539)
Province	Y	N	Y	Y	N	Y
Track	Y	N	Y	Y	N	Y
Year	Y	Y	Y	Y	Y	Y
Track*Province	N	Y	N	N	Y	N
Observations	240	240	240	240	240	240
R^2	0.978	0.984	0.978	0.979	0.986	0.979

Notice: The clustered robust standard deviation of track and province are in the brackets. *** $p<0.01$, ** $p<0.05$, * $p<0.1$.

The result for testing the effects of quotas on cutoffs at major level is shown in Table A1 in appendix. In general, we do not find strong evidence supporting negative effects of quotas on cutoffs between the same or similar major within or across two universities. We can think of two reasons for insignificant results.

First, the effects of major quotas to major cutoffs may be diffused because of the large number of majors, esp. when students don’t have strong major preferences. Students regard two universities as *substitutes*, but not any specific major between them. As we assume in Section II.2 and II.3, students may regard a Science major at Tsinghua University as a close substitute to a Humanity major (instead of a Science major) at Peking University.

Second, major quota setting is in general very flexible: colleges sometimes negotiate with top students on major provisions during the admission procedure, which may lead to major quota reallocation among regions. Due to those unobservable changes, major quotas may not be exogenous.

IV.2 Testing Hypothesis 2: Equalization of Major and Regional Cutoffs

The way we test the equalization of major (or regional) cutoffs is to compare the coefficient of variation (CV) of major cutoffs to the CV of major quotas. According to Hypothesis 2, because colleges vary their quotas among majors to equalized cutoffs, the CV of major quotas should be higher than that of major cutoffs.

For equalization of major cutoffs, since CEE scores are in general not comparable across years, provinces and tracks, and our theory does not require major cutoffs to be equalized across colleges, we compare CV of cutoffs to quotas within each university, original provinces, track, and year. The results are shown in Table 6. No matter we consider all majors or overlapping majors, the CV for major cutoffs is only 0.014-0.015, while the CV for quotas is 0.44-0.50, roughly 30 times larger than CV of major cutoffs; the difference is very significant.

Table 6 Equalization of Major/Provincial Cutoffs

	CV of cutoffs	**CV of quotas**	**Diff. (=q-c)**	**T Statistics**	**p-value**	**# of obs.**
All Majors	0.0146	0.4452	-0.4306	-45.1934	0.0000	353
Overlapping Majors	0.0139	0.4923	-0.4784	-42.7620	0.0000	309
Provinces	0.1067	0.8354	-0.7287	-14.8825	0.0000	20

Someone may suspect that CV of cutoffs are underestimated. Although CEE scores can go from zero to full scores, for those elite students, their possible scores cannot be too low. We then conduct a robust check by adjust CEE scores by subtracting first-batch cutoffs from the original CEE scores. The adjustment increases the CV of cutoffs a lot, but the test still delivers the same results: CV of cutoffs are still significantly lower than CV of quotas (Table A2).

For equalization of provincial cutoffs, since different provinces may use different CEE test contents, the scores are often incomparable across provinces. Yet we still calculate CV of provincial cutoffs, because those incomparability seems more likely to cause upward bias of the CV. The results are shown in the third line of Table 7. Under the very limited sample size (20), the CV of provincial cutoffs are proved to be significantly lower than that of provincial quotas. As a whole, Hypothesis 2 is justified by our data.

IV.3 Testing Hypothesis 3: How Popularity Affect Quotas

We now examine how the two universities compete with each other through setting their admission quotas. We consider quota setting at two levels. The first is total quota (including all majors) allocation among regions; the second is major quota allocation among regions. Unfortunately, our empirical method does not allow us to identify the determinants of total quota including all regions and majors.

Out major interest is how popularity of the university or its majors would affect its quota setting. We use the cutoffs in previous year in each region or major as the proxy of popularity in current year. An increases in last year's cutoffs indicates an increase in popularity. Assuming there are time-continuous changes in popularity, the rise of cutoffs for a certain province or major in previous year suggests that its popularity in this province/major will still increase this year. Therefore, the university will allocate more quotas to this province/major to compete for high-quality student, according to Hypothesis 3.

The empirical model is as follows:

$$Quota(i,j,k,t) = \alpha + \beta_1 * Cutoff(i,j,k,t-1) + \beta_2 * Cutoff(-i,j,k,t-1)$$
$$+\beta_3 * Quota(i,j,k,t-1) + \beta_4 * Quota(-i,j,k,t-1)$$
$$+Province_{dummy} + University_{dummy} + Track_{dummy}$$
$$+Year_{dummy} + u(i,j,k,t) \quad (2)$$

$Cutoff(i,j,k,t-1)$ and $Cutoff(-i,j,k,t-1)$ are our main regressors. All the control variables have been described in eq. (1), except $Quota(i,j,k,t-1)$ and $Quota(-i,j,k,t-1)$, which represent previous year's quota for two universities. We add them as control variables because quota adjustment is usually based on quotas of previous year. We also consider the first difference model as in Table 4.

The regression results are shown in Tables 7 and 8. All the dummy variables are the same as in Column 1-4, Table 4 for testing Hypothesis 1. Table 7 illustrates the regression results for quota setting at regional level. Last year's cutoffs have a positive effect on the admissions quotas for the current year, significant at the 5% level at Column 4. Furthermore, last year's cutoff of the *competitor* has a *negative* effect on the admissions quotas for the current year, significant at 10% level in Column 1-2 and at 5% level in Column 5. This is also consistent with Hypothesis 3, since given the relatively stable student group demanding either Tsinghua or Peking University, the popularity increase of the competitor implies the popularity decrease of your own. Column 6 shows the fist-order difference results, which is insignificant but consistent with other specifications.

Table 7 How Popularity Affects Quotas: Region-level

Explanatory variables	Explained variable: Log_Quota (i,j,k,t)					Explained variable: ΔLog_Cutoff(i,j,k,t)
	(1)	(2)	(3)	(4)	(5)	(6)

Log_Cutoff (i,j,k,t-1)	1.285	1.197	0.414	3.251**	1.212	ΔLog_Quota(i, j,k,t)	0.796
	(1.161)	(1.218)	(1.641)	(1.569)	(0.971)		(0.934)
Log_Cutoff (-i,j,k,t-1)	-2.102*	-2.190*	-1.408	-0.136	-2.205**	ΔLog_Quota(-i,j,k,t)	-0.983
	(1.183)	(1.219)	(1.735)	(1.569)	(1.023)		(0.934)
Other control variables							
Log_Quota (i,j,k,t-1)	0.712***	0.594***	0.158**	0.582***	0.597***		
	(0.030)	(0.041)	(0.067)	(0.008)	(0.049)		
Log_Quota (-i,j,k,t-1)	-0.285***	-0.403***	0.0326	-0.415***	-0.406***		
	(0.027)	(0.039)	(0.050)	(0.008)	(0.049)		
Province	Y	N	N	N	N		N
Track	Y	N	N	N	N		N
University	Y	Y	N	Y	N		N
Year	Y	Y	Y	N	N		Y
Track*Province	N	Y	N	N	Y		N
Track*Province*University	N	N	Y	N	N		N
Track*Province*Year	N	N	N	Y	N		N
University*Year	N	N	N	N	Y		N
Observations	480	480	480	480	480		360
R^2	0.966	0.969	0.979	0.978	0.974		0.089

Notice: The clustered robust standard deviation of track and province are in the brackets. *** p<0.01, ** p<0.05, * p<0.1。

Notice: The clustered robust standard deviation of track and province are in the brackets. *** p<0.01, ** p<0.05, * p<0.1

Table 8 explains how major quotas are determined. Unfortunately, we find no significant effect of previous year's cutoffs (from one's own university or the rival university) on this year's quotas. One possible reason is that major quotas for any given year are so flexible that colleges need not to fix their quotas by considering cutoffs of one year ago. Universities sometimes negotiate with students (often the top-guys) for their major choice and change major quotas according to negotiation results.

Table 8 How Popularity Affects Quotas: Major-level

Explanatory variable	Explained variable：Log_Quota (i,j,k,l,t)				
	(1)	(2)	(3)	(4)	(5)
Log_Cutoff (i,j,k,l,t-1)	-0.116	-0.0727	0.00196	-0.539	-0.0323
	(0.404)	(0.421)	(0.658)	(0.760)	(0.450)
Log_Cutoff (-i,j,k,l,t-1)	0.0969	0.131	-0.539	-0.299	0.144
	(0.439)	(0.466)	(0.678)	(0.811)	(0.448)
Other control variables					
Log_Quota (i,j,k,l,t-1)	0.609***	0.601***	0.0649	0.622***	0.638***
	(0.0348)	(0.0360)	(0.0638)	(0.0375)	(0.0354)
Log_Quota (-i,j,k,l,t-1)	0.0594**	0.0521*	0.0322	0.0724**	0.0566*
	(0.0284)	(0.0280)	(0.0460)	(0.0294)	(0.0294)
Province	Y	N	N	N	N
Track	Y	N	N	N	N
University*Major	Y	Y	N	Y	N
Year	Y	Y	Y	N	N
Track*Province	N	Y	N	N	Y
Track*Province*University*Major	N	N	Y	N	N
Track*Province*Year	N	N	N	Y	N
University*Major*Year	N	N	N	N	Y
Observations	1,088	1,088	1,088	1,088	1,088
R2	0.783	0.787	0.903	0.816	0.823

Notice: The clustered robust standard deviation of track and province are in the brackets. *** $p<0.01$, ** $p<0.05$, * $p<0.1$, l represents major.

As a whole, the results are consistent with Hypothesis 3 that universities try to expand admission quotas where its popularity increases.

V. Model Extensions

In this section we consider some further extensions of our theoretical model.

V.1 2-Dimensional Abilities

Until now our theoretical model assumes colleges have homogeneous preferences (or priorities) over students; it solely depends on their quality, e.g., total scores in CEE. In other words, student ability is one-dimensional. In reality, universities may develop their own admissions rules, at least deviate from the single index widely-accepted.

Assume student i's ability $(\theta^i, \gamma^i) \sim U[0,1] \times [-\bar{\gamma},\ \bar{\gamma}]$. θ is vertical ability and γ is horizontal ability, both being observable. There are two colleges (without major division). College 1 prefers student with higher $e_1 = \theta + \gamma$, while college 2 prefers student with higher $e_2 = \theta - \gamma$.

We can have the following results:

Proposition 3. *Under matching equilibrium:*

(1) $q(c)$ *and* $c(q)$ *are bijections.*

(2) When $|c_1 - c_2| < 2\bar{\gamma}$, $q_i = \frac{1}{2}(1 - c_i) + \frac{(c_j - c_i + 2\bar{\gamma})^2}{16\bar{\gamma}}$ *(Otherwise the expression is as in Section II.1).*

(3) $\left.\frac{\partial c}{\partial q}\right|_{q_1 = q_2} = C_0 \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$*, where* $C_0 < 0$ *is a constant.*

Proof is in Appendix 1. Figure 12 illustrates the matching equilibrium. Panel (a) and (b) show 2-dimensional student ability distributions, corresponding to students who prefer college 1 to 2 and college 2 to 1 respectively. For given cutoffs c_1, c_2 (corresponding to some level of e_1, e_2), students eligible for college 1 and 2 are shown by areas above the 45 degree lines crossing c_1, c_2 in both panels. In panel (a), students eligible for college 1 would be admitted by college 1, while students eligible *only* for college 2 would be admitted by college 2. And vice versa in panel (b).

Note that when college 1 lowers its cutoff, it decreases the amount of students admitted by college 2 (in panel (a)). The same thing happens when college 2 lowers its cutoff (as in panel(b)). The business-stealing negative externality is always there, regardless of relative positioning of two colleges.

Proposition 4. *(1) The Cournot equilibrium is characterized as below: Cutoffs are* $c_i^* = \lambda + \frac{\bar{\gamma}}{8}$, *profits are* $\Pi_i^* = \frac{11}{64}\bar{\gamma}^2 + \left(\frac{\lambda\bar{\gamma}}{16} + \frac{\lambda\bar{\gamma}^2}{2}\right) + \frac{1}{4} - \frac{1}{2}\lambda + \frac{1}{4}\lambda^2$.*(2) The monopoly solution is: cutoff* $c_i^m = \lambda + \frac{\bar{\gamma}}{2}$, *profit* $\Pi_i^m = \frac{15}{16}\bar{\gamma}^2 + \left(\frac{\lambda\bar{\gamma}}{4} + \frac{\lambda\bar{\gamma}^2}{2}\right) + \frac{1}{4} - \frac{1}{2}\lambda + \frac{1}{4}\lambda^2$.

Proof is in Appendix 1. Note that Cournot equilibrium is not equal to monopoly solution, because of the business-stealing externality. It sets a lower cutoffs and admit more students than monopoly. Proposition 4 also explain colleges often develop diverged admission criteria if they value different student abilities: Cournot equilibrium profit would be lower when "horizontal ability" (γ) is NOT taken into consideration (Let $\bar{\gamma} = 0$ in the profit equation).

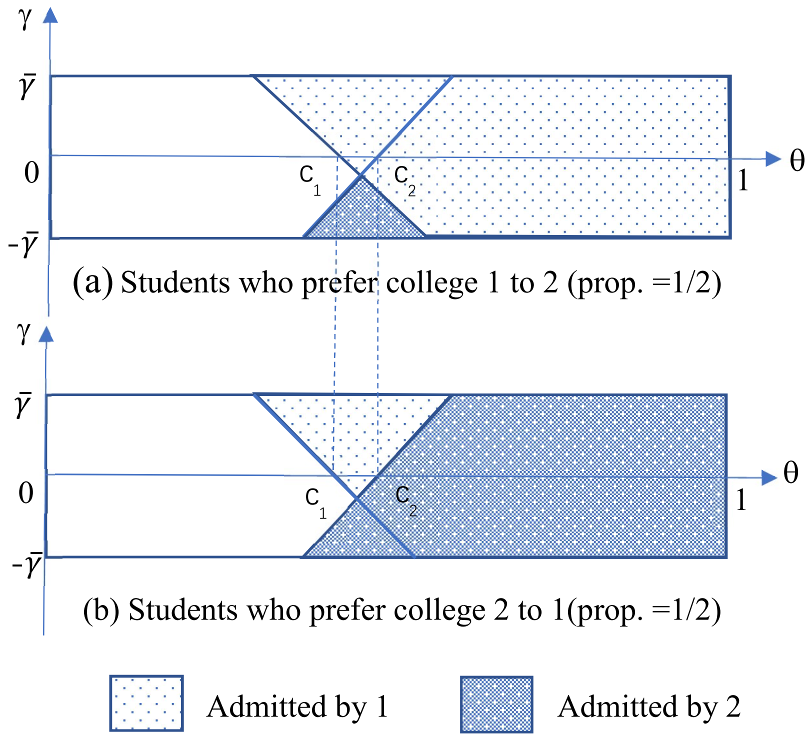


Figure 12 2-Dimensional Abilities

V.2 Cutoff-dependent Student Preference

Students may prefer colleges/majors with higher cutoffs. This can happen in several cases. First, students may have insufficient information to form an "independent" preferences over colleges, but interpret a high cutoff as a "signal" of high-quality of a college. Second, students may prefer a college with high-quality peers, inducing a preference order favoring high cutoffs. In either case students have some form of "social" preferences.

We introduce a cutoff-dependent student preference in this way: Assume that there are students of mass $\frac{c_1}{c_1+c_2}$ prefer college 1 to 2, and students of mass $\frac{c_2}{c_1+c_2}$ prefer 2 to 1. That is, a higher cutoff of a college makes it more popular.

In all our previous models, cutoff and quota are one-to-one mappings (i.e., bijections) under matching equilibrium. Under a cutoff-dependent student preference, it is no longer the case.

Proposition 5. *Under matching equilibrium (or market clearing conditions), q is a function of c, but c is not a function of q.*

A formal proof is omitted. We use an example to illustrate the second part (the first part is obvious): when $q_1 = q_2 = \frac{1}{2}$, $c = (\frac{1}{2}, 0)$ and $(0, \frac{1}{2})$ are two matching equilibria. In the former equilibrium, all students prefer 1 to 2, and those with $\theta \in (\frac{1}{2}, 1]$ go to college 1 and those with $\theta \in (0, \frac{1}{2}]$ go to college 2. The latter one is similarly derived.

Proposition 6. *(1) Under monopolistic setting, cutoff* $c^m = (\lambda, \lambda)$. *(2) Under Cournot equilibrium, there are **3** equilibria* c^{*1}, c^{*2}, c^{*3}. $c^{*1} = (\lambda, \lambda)$ *is symmetric, while* $c^{*2} = (a,\, b)$ *and* $c^{*3} = (b,\, a)$ *are asymmetric, where* $\lambda < a < b$. *In addition,* $\Pi_i(c^{*1}) > Max\{\Pi_i(c^{*2}), \Pi_i(c^{*3})\}, \forall i.$

Proof is in Appendix 1. Note that in this case Cournot competition *can* lead to cutoff non-equalization, as well as profit loss.

VI. Conclusions

The paper investigates college competition in admission quotas. Our simple theory generated three hypotheses: First, when the quota of one college increases, the cutoffs of all colleges will (weakly) decrease. Second, colleges tend to allocate their admission quotas across majors or regions so that cutoffs across majors or regions are equalized. Third, a college will allocate more quotas to more popular majors, or regions where it becomes more popular. Empirical evidence from the admission competition between Tsinghua University and Peking University, the two best-known universities in China, supports our theoretical hypotheses.

Although quantity (or Cournot) competition is a well-studied issue in the IO theory, such an issue has almost never been studied in a "higher education marketplace". The paper is the first (to our knowledge) to bring this issue into theoretical and empirical investigation. Our research is also connected to matching theory, in particular college admissions/school choice literature concerning strategic behaviors of colleges/schools. The key connecting point between IO and matching theory is admission cutoffs, which play a similar role of market price, in the matching market without monetary transfers.

One future research direction is to apply the framework to study empirically competition among multiple universities. For this purpose, a more advanced econometric model should be developed to examine interactive effects among multiple pairs of universities. Another research direction is to explore exogenous events to study strategic behavior of universities and their consequences. For example, starting from the end of last century, China experienced a large expansion in college education mainly driven by government initiates. How universities responded to this policy change and how those responses affect higher education marketplace remain an open question.

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Appendix

Appendix 1: Proofs

Proof of Lemma 1

Proof. We give a constructive proof using mathematical induction.

When there are $K = 1$ college-major pair, we have $c_1 = \max\{1 - q_1, 0\}$. If the lemma holds for $K \leq m - 1$, we now prove that the lemma holds for $K = m$. The set of colleges is also denoted as K.

If $\min_{k \in K} \frac{q_k}{\alpha_k} \geq 1$, i.e., for each college-major bundle k, the mass of the students who prefer k most is no less than k's quota, then $c = 0$ is the only stable matching. Now we turn to the case when $\min_{k \in K} \frac{q_k}{\alpha_k} < 1$.

The remaining proof has 2 steps: we first solve for the highest cutoff, and then define a sub-problem based on this cutoff.

Step 1: Let $r = \min_{k \in K} \frac{q_k}{\alpha_k}$ *and set* $S = \arg\min_{k \in K} \frac{q_k}{\alpha_k}$*. Then if stable matching exists,* c *is the cutoff in any one of these matchings, and set* $T = \arg\max_{k \in K} c_k$*, then we have* $S = T$ *and* $\max_{k \in K} c_k = F^{-1}(1 - r)$.

Suppose $S \neq T$. There are two possibilities.

(i) $T \backslash S \neq \emptyset$. Then for any $k \in S \cap T$ and $k' \in T \backslash S$, $c_k = c_{k'}$ and the students matched to college-major k are those who prefer k most and $\theta \geq c_k$, and it is similar for college-major k'. Then $q_k = \alpha_k\big(1 - F(c_k)\big), q_{k'} = \alpha_{k'}\big(1 - F(c_{k'})\big), \frac{q_k}{\alpha_k} = \frac{q_{k'}}{\alpha_{k'}}$, so $k' \in S$. Contradiction.

(ii) $T \backslash S = \emptyset$. Then $T \subseteq S$ and $T \neq S$. For any $k \in S \backslash T$ and $k' \in T$, we have $c_k < c_{k'}$. The students matched to college-major k' are those who prefer k' most and $\theta \geq c_{k'}$ and $q_{k'} = \alpha_{k'}\big(1 - F(c_{k'})\big)$. For college-major k, the students matched to it include those who prefer k most and $\theta \geq c_k$, and a part of those whose $c_k \leq \theta < c_{k'}$, so $q_k \geq \alpha_k\big(1 - F(c_k)\big)$. Then $\frac{q_k}{\alpha_k} \geq 1 - F(c_k) > 1 - F(c_{k'}) = \frac{q_{k'}}{\alpha_{k'}}$. Contradiction.

Note that $r = \min_{k \in K} \frac{q_k}{\alpha_k} \in (0,1)$. In stable matching $\max_{k \in K} c_k = F^{-1}(1-r)$ directly follows $S = T$. The existence of F^{-1} is guaranteed by that F has full support.

Step 2: The lemma holds when $K = m$.

Let $C = \max_{1 \leq k \leq m} c_k > 0$. Those students whose $\theta \geq C$ will be matched to the college-major bundle they prefer most. If set $K = S$, all students are matched, and the existence and uniqueness are proved. Otherwise, for the student whose $\theta < C$, we can ignore colleges in set S, and define a sub-problem of the original matching problem as below.

In this sub-problem, there are $K - |S|$ college-major bundles, the set of which is $K \backslash S$. Student quality $\phi = \frac{\theta}{C} \in [0,1]$, where θ is the quality in the original matching problem, and its distribution function $G(\phi) = \frac{F(C\phi)}{F(C)}$ has full support. At each student quality ϕ, there are proportions of γ_k who prefers college-major bundle k most, $\sum_{k \in K \backslash S} \gamma_k = 1$. The quotas for college-major bundle k is $q_k' = \frac{q_k - \alpha_k(1 - F(C))}{F(C)} > 0$.

Since the lemma holds for $K \leq m - 1$, there exists a unique cutoff c^s in this sub-economy. Then there exists a unique cutoff in the original economy:

$$c_k = \begin{cases} C, & k \in S \\ C \cdot c_k^s, & k \in K \backslash S \end{cases}.$$

Proof of Lemma 3

Proof. Consider the construction of the unique cutoff in the proof of Lemma 1. We first find the set of college-major bundles $S_1 = \arg\min_{1 \leq k \leq K} \frac{q_k}{\alpha_k}$. Their cutoffs are $C_1 =$

$F^{-1}\left(1-\min\limits_{1\le k\le K}\frac{q_k}{\alpha_k}\right)$. After all those students whose $\theta \ge C_1$ are matched to the college-major bundle they prefer most, we ignore the college-major bundles in S_1 and determine the cutoffs for the remaining college-major bundles. Note that college-major bundles in S_1 are in higher position than the remaining ones. If $c_{k_1} < c_{k_2}$ and $k_2 \in S$, then with marginal changes in q_{k_1}, $\min\limits_{1\le k\le K}\frac{q_k}{\alpha_k}$ will not change, neither will S, so c_{k_2} is not affected.

The same logic applies in any sub-problems: Suppose the recursive process of determining cutoffs ends in the t-th recursion. We have $t \le K$. Let S_i be the set of college-major bundles whose cutoffs are determined in the i-th recursion, $1 \le i \le t$. The cutoffs of college-major bundles in S_i is the same, denoted as C_i. From the construction process, $C_1 > C_2 > \cdots > C_t$. Then if $c_{k_1} < c_{k_2}$, we have $k_1 \in S_i$ and $k_2 \in S_j$ where $i > j$. For marginal changes in q_{k_1}, $S_j = \min\limits_{\{1,\ldots,K\}\backslash(S_1 \cup S_2 \cup \ldots \cup S_{j-1})}\frac{q_k}{\alpha_k}$ will not be affected.

Proof of Proposition 1

Proof. We first prove for all i, $j(i)$, $Min_{j(i)}c_{ij(i)} = \lambda_i$. Suppose not. That is, for some $i, Min_{j(i)}c_{ij(i)} = c_{ij'} > \lambda_i$. Then if college *i* increases the quota of college-major (i,j') at the margin, it can admit into college-major (i,j') a positive amount of students with ability higher than λ_i=MC, at least those who prefer (i,j') most but with abilities slightly lower than c_{ij} , without affecting any other college-major cutoffs within this college (by Lemma 3). This contradicts that college *i* maximizes its profit.

Now consider $i = N-1$. We prove $c_{N-1,j(N-1)} = \lambda_{N-1}, \forall j(N-1) \in M_{N-1}$. Suppose not. That is, there exists some major $j \in M_{N-1}$ such that $c_{N-1,j} \equiv Min_{j(N-1)}\{c_{N-1,j(N-1)} | c_{N-1,j(N-1)} \neq \lambda_{N-1}\} > \lambda_{N-1}$. That is, $c_{N-1,j}$ is the second lowest cutoff in college $N-1$.

Suppose we increase $q_{N-1,j}$ by an arbitrarily small amount δ. This δ amount of quota can be fulfilled by "stealing business" *only* from *any* college-majors with lower original cutoffs (by Lemma 3, and our assumption that all preference orders are possible given student ability). Part of it is from outside college $N-1$, at least from major $j(N)$ in college N such that $c_{N,j(N)} = \lambda_N \leq \lambda_{N-1} < c_{N-1,j}$. Part of it is also fulfilled by "stealing business" from within college $N-1$, in particular, college-major $(N-1, j')$ such that $c_{N-1,j'} = \lambda_{N-1}$, which then admits students to fully compensate its quantity loss. Suppose among newly admitted students in college-major $(N-1, j)$, the amount of $\delta_1 > 0$ comes from outside the college $N-1$, while $\delta_2 = \delta - \delta_1$ comes from college-major $(N-1, j')$. Then the total quality change of college $N-1$ at the margin would be $\delta_1 c_{N-1,j} + \delta_2 \lambda_{N-1} > \delta \lambda_{N-1}$. Therefore, marginal increase in $q_{N-1,j}$ would increase college $N-1$'s profit, contradicting its profit maximization. So $c_{N-1,j(N-1)} = \lambda_{N-1}$ for all $j(N-1) \in M_{N-1}$.

The reasoning for $i = N-1$ goes back to $i = N-2, \ldots 1$.

Finally, we prove for $i = N$, $c_{Nj(N)} \leq \lambda_{N-1}$. If not, i.e., $c_{N,j} > \lambda_{N-1}$, for some $j \in M_N$, then college *N* can profitably steal business from any major in college $N-1$, contradicting college *N*'s profit maximization. □

Proof of Proposition 3

Proof.

Part 1: According to the definition of demand, q is a function of c. Notice that $\mathrm{q_i}$ is strictly decreasing with $\mathrm{c_i}$, and weakly increasing with $\mathrm{c_j}, \mathrm{j} \neq \mathrm{i}$. If c and c' satisfy $\mathrm{q(c) = q(c'), c \neq c'}$, there is a contradiction.

Part 2: The proportion of students admitted by college 1 is composed of light shaded areas in panel (a) and (b), Figure 12, normalized by total (or rectangular) area.

Therefore, $q_1 = \frac{1}{2} * \frac{(1-c_1)2\bar{\gamma}}{2\bar{\gamma}} + \frac{1}{2} * \frac{\left[\left(\frac{c_2-c_1}{2}+\bar{\gamma}\right)^2\right]}{2\bar{\gamma}} = \frac{1}{2}(1-c_1) + \frac{(c_2-c_1+2\bar{\gamma})^2}{16^{-}}$, where the term $2\bar{\gamma}$ is used to normalize the area. The formula for q_2 can also be derived similarly.

Part 3: From part 2, $\frac{\partial q}{\partial c} = Q_0 \begin{pmatrix} 3+\frac{c_2-c_1}{2\bar{\gamma}} & -1+\frac{c_1-c_2}{2\bar{\gamma}} \\ -1+\frac{c_2-c_1}{2\bar{\gamma}} & 3+\frac{c_1-c_2}{2\bar{\gamma}} \end{pmatrix}$ and thus $\left.\frac{\partial q}{\partial c}\right|_{c_1=c_2} = Q_0 \begin{pmatrix} 3 & -1 \\ -1 & 3 \end{pmatrix}$, where $Q_0 < 0$ is a constant. Implicit function theorem implies part 3.

Proof of Proposition 4

If $|c_1{}^* - c_2{}^*| < 2\bar{\gamma}$, according to the proof of Proposition 3 we have $\frac{\partial q}{\partial c} = Q_0 \begin{pmatrix} 3+\frac{c_2-c_1}{2\bar{\gamma}} & -1+\frac{c_1-c_2}{2\bar{\gamma}} \\ -1+\frac{c_2-c_1}{2\bar{\gamma}} & 3+\frac{c_1-c_2}{2\bar{\gamma}} \end{pmatrix}$ and thus $\frac{\partial c}{\partial q} = C_0 \begin{pmatrix} 3+\frac{c_1-c_2}{2\bar{\gamma}} & 1+\frac{c_2-c_1}{2\bar{\gamma}} \\ 1+\frac{c_1-c_2}{2\bar{\gamma}} & 3+\frac{c_2-c_1}{2\bar{\gamma}} \end{pmatrix}$, where $Q_0, C_0 < 0$ are constants.

Suppose college 1 increases its quota q_1 by one unit at the margin. Then the cutoff of college 1, c_1, would decrease by $3+\frac{c_1-c_2}{2\bar{\gamma}}$ units (by a scale of C_0), while the cutoff of college 2, c_2 , would decrease by $1+\frac{c_1-c_2}{2\bar{\gamma}}$ units. In other words, as in Figure A1, the line B_1E_1, B_2E_2 would shift left by $3+\frac{c_1-c_2}{2\bar{\gamma}}$ units, while the line C_1D_1, C_2D_2 would shift left by $1+\frac{c_1-c_2}{2\bar{\gamma}}$ units. Then the total quality change of college 1 (i.e., MR_1) is composed of three parts: (1) the left shift of line B_1E_1 by $3+\frac{c_1-c_2}{2\bar{\gamma}}$ units; (2) the left shift of line B_2A_2 by $3+\frac{c_1-c_2}{2\bar{\gamma}}$ units; (3) the left shift of A_2D_2 by $1+\frac{c_1-c_2}{2\bar{\gamma}}$ units. The first two changes are positive, while the third is negative. To calculate the area of these parallelograms, note that the heights of B_1E_1, B_2A_2 and A_2D_2 are $2\bar{\gamma}$, $\bar{\gamma}+\frac{c_2-c_1}{2}$ and $\bar{\gamma}+\frac{c_2-c_1}{2}$ respectively. In addition, the average student quality for college 1 at the margin along the line B_1E_1 and B_2A_2 is c_1, while the average quality along the line A_2D_2 is $\mathrm{c}_1 + \bar{\gamma} + \frac{\mathrm{c}_2 - \mathrm{c}_1}{2}$.

Let $\mathrm{h}(x)$ be the height of x, then the total quality change is:

$$\left(3 + \frac{\mathrm{c}_1 - \mathrm{c}_2}{2\bar{\gamma}}\right) * \left(\mathrm{h}(B_1E_1) + \mathrm{h}(B_2A_2)\right) * c_1 - \left(1 + \frac{\mathrm{c}_1 - \mathrm{c}_2}{2\bar{\gamma}}\right) * \mathrm{h}(A_2D_2) * \left(c_1 + \bar{\gamma} + \frac{\mathrm{c}_2 - \mathrm{c}_1}{2}\right)$$
$$= 8 * \bar{\gamma} * \left(c_1 - \frac{(2\bar{\gamma} + \mathrm{c}_2 - \mathrm{c}_1)^2(2\bar{\gamma} + \mathrm{c}_1 - \mathrm{c}_2)}{64\bar{\gamma}^2}\right)$$

and the corresponding total cost change is $\left(\left(3 + \frac{\mathrm{c}_1 - \mathrm{c}_2}{2\bar{\gamma}}\right) * \left(\mathrm{h}(B_1E_1) + \mathrm{h}(B_2A_2)\right) - \left(1 + \frac{\mathrm{c}_1 - \mathrm{c}_2}{2\bar{\gamma}}\right) * \mathrm{h}(A_2D_2)\right) * \lambda = 8 * \bar{\gamma} * \lambda$. Then *MR*=*MC* leads to

$$\lambda = c_1 - \frac{(2\bar{\gamma} + \mathrm{c}_2 - \mathrm{c}_1)^2(2\bar{\gamma} + \mathrm{c}_1 - \mathrm{c}_2)}{64\bar{\gamma}^2}.$$

Similarly,

$$\lambda = c_2 - \frac{(2\bar{\gamma} + \mathrm{c}_1 - \mathrm{c}_2)^2(2\bar{\gamma} + \mathrm{c}_2 - \mathrm{c}_1)}{64\bar{\gamma}^2}.$$

Subtracting the two equation above we have

$$(c_1 - c_2)\left(1 + \frac{(2\bar{\gamma} + \mathrm{c}_2 - \mathrm{c}_1)(2\bar{\gamma} + \mathrm{c}_1 - \mathrm{c}_2)}{32\bar{\gamma}^2}\right) = 0.$$

Since $|{c_1}^* - {c_2}^*| < 2\bar{\gamma}$ and $\frac{(2\bar{\gamma} + \mathrm{c}_2^* - \mathrm{c}_1^*)(2\bar{\gamma} + \mathrm{c}_1^* - \mathrm{c}_2^*)}{32\bar{\gamma}^2} > 0$, we have ${c_1}^* = {c_2}^*$, i.e. the Cournot equilibrium must be symmetric. Then

$$c_1{}^* = c_2{}^* = \lambda + \frac{\bar{\gamma}}{8}.$$

If $|c_1{}^* - c_2{}^*| \geq 2\bar{\gamma}$, the expression of c_1, c_2 is as in Section II.1, and $c_1{}^* = c_2{}^* = \lambda$ in equilibrium, which contradicts $|c_1{}^* - c_2{}^*| \geq 2\bar{\gamma}$.

The Cournot equilibrium profit can be calculated according to the cutoffs.

Consider monopoly solution when $|c_1{}^m - c_2{}^m| < 2\bar{\gamma}$. When changing q_1, the total quality change for *college 2* is:

$$\left(1 + \frac{c_1 - c_2}{2\bar{\gamma}}\right) * \left(\mathrm{h}(A_1C_1) + \mathrm{h}(C_2D_2)\right) * c_2 - \left(3 + \frac{c_1 - c_2}{2\bar{\gamma}}\right) * \mathrm{h}(A_1E_1) * (c_2 + \bar{\gamma} + \frac{c_1 - c_2}{2})$$
$$= -\left(3 + \frac{c_1 - c_2}{2\bar{\gamma}}\right) * \left(\bar{\gamma} + \frac{c_1 - c_2}{2}\right) * \left(\bar{\gamma} + \frac{c_1 - c_2}{2}\right)$$

The total quality change for both colleges a whole is:

$$8 * \bar{\gamma} * \left(c_1 - \frac{(2\bar{\gamma} + c_2 - c_1)^2(2\bar{\gamma} + c_1 - c_2)}{64\bar{\gamma}^2}\right) - \left(3 + \frac{c_1 - c_2}{2\bar{\gamma}}\right) * \left(\bar{\gamma} + \frac{c_1 - c_2}{2}\right)^2$$
$$= 8 * \bar{\gamma} * \left(c_1 - \frac{(2\bar{\gamma}+c_1-c_2)\left(8\bar{\gamma}^2+2\bar{\gamma}(c_1-c_2)+(c_1-c_2)^2\right)}{32^{-2}}\right).$$

The total cost change is $8 * \bar{\gamma} * \lambda$. And the monopoly solution is by equalizing total quality change and total cost change:

$$\lambda = c_1 - \frac{(2\bar{\gamma} + c_1 - c_2)(8\bar{\gamma}^2 + 2\bar{\gamma}(c_1 - c_2) + (c_1 - c_2)^2)}{32\bar{\gamma}^2}$$

Similarly,

$$\lambda = c_2 - \frac{(2\bar{\gamma} + \mathrm{c}_2 - \mathrm{c}_1)(8\bar{\gamma}^2 + 2\bar{\gamma}(\mathrm{c}_2 - \mathrm{c}_1) + (\mathrm{c}_2 - \mathrm{c}_1)^2)}{32\bar{\gamma}^2}$$

Subtracting the two equation above we have

$$(c_1 - c_2)\left(\frac{(2\bar{\gamma} + \mathrm{c}_2 - \mathrm{c}_1)(2\bar{\gamma} + \mathrm{c}_1 - \mathrm{c}_2)}{16\bar{\gamma}^2}\right) = 0.$$

Since $|c_1{}^m - c_2{}^m| < 2\bar{\gamma}$ and $\frac{(2\bar{\gamma}+\mathrm{c}_2^{\mathrm{m}}-c_1^m)(2\bar{\gamma}+\mathrm{c}_1^{\mathrm{m}}-\mathrm{c}_2^{\mathrm{m}})}{16\bar{\gamma}^2} > 0$, we have $c_1{}^m = c_2{}^m$, i.e. the monopoly equilibrium must be symmetric. Then

$$c_1{}^m = c_2{}^m = \lambda + \frac{\bar{\gamma}}{2}.$$

The sum of profits is $\Pi^m = \Pi_1^m + \Pi_2^m = \frac{5}{8}\bar{\gamma}^2 + \left(\frac{\lambda\bar{\gamma}}{2} + \lambda\bar{\gamma}^2\right) + \frac{1}{2} - \lambda + \frac{1}{2}\lambda^2$.

If $|c_1{}^m - c_2{}^m| \geq 2\bar{\gamma}$, without loss of generality, assume $c_1{}^m \geq c_2{}^m$. Then the total quality is $\left(\left(\frac{c_1+(1-\bar{\gamma})}{2} - \lambda\right) \cdot \frac{(1-\bar{\gamma})-c_1}{2} + \left(\frac{2(1-\bar{\gamma})+(1+\bar{\gamma})}{3} - \lambda\right) \cdot \frac{\bar{\gamma}}{2}\right) + \left(\left(\frac{c_2+(c_1-2\bar{\gamma})}{2} - \lambda\right) \cdot (c_1 - c_2 - 2\bar{\gamma}) + \left(\frac{2(c_1-2\bar{\gamma})+(c_1+2\bar{\gamma})}{3} - \lambda\right) \cdot (2\bar{\gamma}) + \left(\frac{(c_1-2\bar{\gamma})+2(c_1+2\bar{\gamma})}{3} - \lambda\right) \cdot (\bar{\gamma}) + \left(\frac{(c_1+2\bar{\gamma})+(1-\bar{\gamma})}{2} - \lambda\right) \cdot \frac{(1-\bar{\gamma})-(c_1+2\bar{\gamma})}{2} + \left(\frac{2(1-\bar{\gamma})+(1+\bar{\gamma})}{3} - \lambda\right) \cdot \frac{\bar{\gamma}}{2}\right) = -\frac{c_2^2}{2} + \frac{\bar{\gamma}^2}{2} + \frac{1}{2} - \lambda(1 - c_2)$. When $c_2{}^m = \lambda$, total quality reaches its maximum, $\frac{1}{2}\bar{\gamma}^2 + \frac{1}{2} - \lambda + \frac{1}{2}\lambda^2 < \Pi^m$.

Therefore, the monopolist will set $c_1{}^m = c_2{}^m = \lambda + \frac{\bar{\gamma}}{2}$ and the solution is unique.

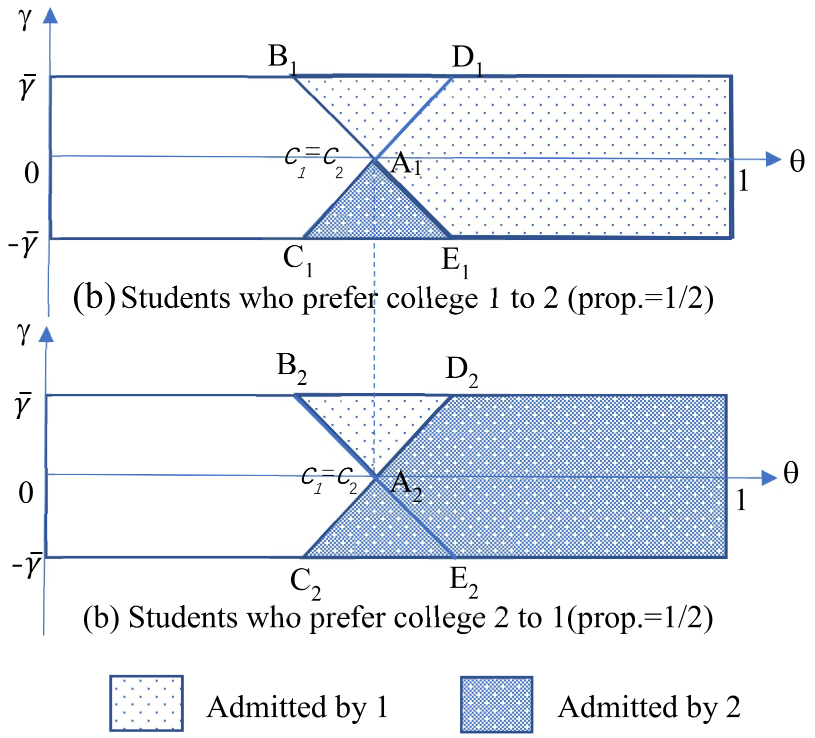

Figure A1 Symmetric Solution for 2-Dimensional Abilities

Proof of Proposition 6

Under Cournot equilibrium, without loss of generality, suppose $c_1 \geq c_2$. Then:

$$\begin{cases} \Pi_1 = \frac{c_1}{c_1+c_2}(1-c_1)\left(\frac{1+c_1}{2}-\lambda\right) \\ \Pi_2 = \frac{c_2}{c_1+c_2}(1-c_1)\left(\frac{1+c_1}{2}-\lambda\right)+(c_1-c_2)\left(\frac{c_1+c_2}{2}-\lambda\right) \end{cases},$$

numerically solving the first order conditions yields the results.

Under monopoly, two colleges jointly maximize:

$$\Pi_1+\Pi_2 = (1-c_1)\left(\frac{1+c_1}{2}-\lambda\right)+(c_1-c_2)\left(\frac{c_1+c_2}{2}-\lambda\right),$$

and the unique maximum is reached when $c_1 = c_2 = \lambda$.

Appendix 2: Alternative Model: Total Quota Constraints

We propose an alternative modelling where we suppose each college faces a total quota constraint, but without any cost of recruiting students (or only fixed cost irrelevant to student number). We restrict our discussion within two-college-two-major model.

A1.1 Vertical Student Preference

Student preferences are the same as in Part 1, Section II.2

The problem for TU is:

$$Max_{q_{ts},\ q_{th}}\ R_{ts} + R_{th}$$

$$\text{s.t}\ q_{ts} + q_{th} \leq q_t$$

The problem for PU is:

$$Max_{q_{ps},\ q_{ph}}\ R_{ps} + R_{ph}$$

$$\text{s.t}\ q_{ps} + q_{ph} \leq q_p$$

where $q_i, i = p, t$ are total quota of college i, which is exogenously given. We assume $q_t \leq q_p$. And we also assume $q_t + q_p \leq 1$.

The solution would satisfy:

$$1 - 2{q_{ts}}^* - {q_{th}}^* = l_t$$

$$1 - {q_{ts}}^* - {q_{ph}}^* - 2{q_{th}}^* = l_t$$

$$1 - {q_{ts}}^* - {q_{ph}}^* - {q_{ps}}^* = l_p$$

$$1 - q_{ts}{}^{*} - q_{ph}{}^{*} - q_{th}{}^{*} - q_{ps}{}^{*} = l_p$$

together with the two total quota constraints with equality, where $l_t, l_p \geq 0$ are Lagrange multipliers.

Solving it leads to:

$$q_{ph}{}^{*} = q_{ts}{}^{*} = q_t$$
$$q_{th}{}^{*} = 0$$
$$q_{ps}{}^{*} = q_p - q_t$$
$$l_t = 1 - 2q_t$$
$$l_p = 1 - q_t - q_p$$

For equilibrium cutoffs, we have:

$$c_{ts}{}^{*} = c_{ph}{}^{*} = c_{th}{}^{*} = 1 - 2q_t$$
$$c_{ps}{}^{*} = 1 - q_t - q_p$$

The cutoff of TH (the less favorable major of the smaller college) is set so that the quota being zero. The smaller college uses all its capacity for its more favorable majors. The larger college will match the quota of the smaller one for its favorable majors. Therefore, cutoffs for both majors are equal. This conclusion is the same as the main model with positive MC. The larger college can then use its excess capacity to admit students into its less favorable majors, up to its total quota.

However, the larger college can also use its extra quota to admit students into its favorable major (because its rival has reach its capacity), or even split arbitrarily those excess quotas between two majors.

A1.2 Lateral Student Preference

Now student preference is lateral (as in part 2. Section II.2). The solution would satisfy:

$$1-\frac{3+\alpha}{2\alpha}q_{ts}{}^{*}-q_{th}{}^{*}+\frac{1-\alpha}{2\alpha}q_{ph}{}^{*}=l_t$$
$$1-q_{ts}{}^{*}-q_{ph}{}^{*}-2q_{th}{}^{*}=l_t$$
$$1-\frac{\alpha+1}{2\alpha}q_{ts}{}^{*}-\frac{\alpha+1}{2\alpha}q_{ph}{}^{*}-q_{ps}{}^{*}=l_p$$
$$1-q_{ts}{}^{*}-q_{ph}{}^{*}-q_{th}{}^{*}-q_{ps}{}^{*}=l_p$$

and the two total quota constraints with equality, where $l_t, l_p \geq 0$ are Lagrange multipliers. Solving it leads to:

$$q_{ph}{}^{*}=q_{ts}{}^{*}=\alpha q_t$$
$$q_{th}{}^{*}=(1-\alpha)q_t$$
$$q_{ps}{}^{*}=q_p-\alpha q_t$$
$$l_t=1-2q_t$$
$$l_p=1-q_t-q_p$$

For equilibrium cutoffs, we have:

$$c_{ts}{}^{*}=c_{ph}{}^{*}=c_{th}{}^{*}=1-2q_t$$
$$c_{ps}{}^{*}=1-q_t-q_p$$

The quotas of TU (the smaller college) is set so that cutoffs are equalized among two majors (just as in the main model). The larger college will match quota of the smaller college for its favorable major. Therefore, cutoffs for both majors in smaller college and for favorable major in large college is equal. Yet the larger college will set a lower cutoff for its less favorable major to exhaust its quota.

However, the solution is not the only one. It is easy to see that the larger college call still arbitrarily divide its excess capacity among two majors, leading to lower cutoffs of both majors.

The general lesson is that when one college has a larger total quota than the other, it can allocate its excess quota in an arbitrary way, without considering competition from its rival college. Note that our model only includes two colleges. When there are many colleges, only the college with the *largest* quota can use its excess quota in an arbitrary way, without fearing competitions from other colleges. Therefore, this “excess capacity effect” can be ignored for almost all colleges.

Table and Figure Appendix

Table A1 How Quotas Affect Cutoffs: Major-level

Explanatory variables	Explained variable：Log_Cutoff (i,j,k,l,t)				
	（1）	（2）	（3）	（4）	（5）
Log_Quota (i,j,k,l,t)	-0.002	-0.008**	-0.012*	-0.008	-0.006
	(0.002)	(0.004)	(0.006)	(0.005)	(0.004)
Log_Quota(-i,j,k.l,t)	-0.007	-0.005	0.007	-0.005	-0.001
	(0.006)	(0.003)	(0.017)	(0.005)	(0.003)
Other control variables					
Log_Cutoff (i,j,k,l,t-1)	0.218***	0.020	-0.240**	0.085	0.086
	(0.047)	(0.061)	(0.103)	(0.090)	(0.100)
Log_Cutoff (-i,j,k,l,t-1)	-0.135	-0.135**	0.094	-0.058	-0.082
	(0.086)	(0.053)	(0.065)	(0.087)	(0.104)
Log_Quota (i,j,k,l,t-1)					-0.002
					(0.004)
Log_Quota (-i,j,k,l,t-1)					-0.009
					(0.010)
Province	Y	N	N	N	N
Track	Y	N	N	N	N
University*Major	Y	Y	N	Y	Y
Year	Y	Y	Y	N	N
Track*Province	N	Y	N	N	N
Track*Province *University*Major	N	N	Y	N	N
Track*Province*Year	N	N	N	Y	Y
Observations	1,059	1,059	1,059	1,059	930
R^2	0.716	0.722	0.785	0.757	0.727

Notice: The clustered robust standard deviation of track and province are in the brackets. *** p<0.01, ** p<0.05, * p<0.1。l represents major.

Table A2 Equalization of Major Cutoffs
((Major cutoffs adjusted by subtracting provincial first-batch cutoffs)

	CV of cutoffs	**CV of quotas**	**Diff. (=q-c)**	**T Statistics**	**p-value**	**# of obs.**
All Majors	0.0776	0.4452	-0.3676	-36.8544	0	350
Overlapping Majors	0.0677	0.4315	-0.3638	-35.0535	0	277
Provinces	0.2670	0.7375	-0.4705	-6.2416	0	20