Terms-of-Trade Changes, Real GDP, and Real Value Added in the Open Economy: Reassessing Hong Kong’s Growth Performance

Ulrich Kohli

*Swiss National Bank

Abstract

Real GDP underestimates the increase in real domestic value added when the terms of trade improve. An improvement in the terms of trade is similar to a technological advance, but the national accounts treat these two phenomena very differently. Given its extreme openness and the massive terms-of-trade improvements it has enjoyed over the past forty years Hong Kong makes for an interesting case study. We find that average growth has been underestimated by real GDP by approximately 0.4% per annum between 1961 and 2003. This study also innovates in its measurement of the effect of real-exchange-rate changes on real income.

JEL Classification: O11, O41, C43, F11

Keywords: Terms of trade, real exchange rate, real GDP, real GDI, real value added

1. Introduction

Over the past four decades, Hong Kong’s terms of trade have improved massively. As shown in Figure 1, the terms of trade, measured by the price of exports relative to the price of imports, have increased by nearly 50% between 1961 and 2003. Much of the improvement occurred during the 1960s and 1970s. A bettering of the terms of trade amounts to a windfall gain for the country as a whole and it implies an increase in its
An improvement in the terms of trade essentially means that the country gets more for less. This phenomenon is similar to a technological progress. Contrary to a technological progress, however, a change in the terms of trade is treated by the national accounts as a price phenomenon, rather than as a real effect. Consequently, the beneficial effect of an improvement in the terms of trade is not taken into account by real gross domestic product (GDP). Real domestic value added growth will thus be underestimated in countries that experience an improvement in their terms of trade. Similarly, in international comparisons, the real income of countries that enjoy relatively favorable terms of trade will tend to be underestimated by real GDP. As shown in Figure 2, Hong Kong is an exceptionally open economy, with the GDP shares of imports and exports in excess of 1.6 in recent years. In view of this extreme openness, one can suspect that real GDP as it is conventionally measured has underestimated the growth in real domestic value added over the years, particularly so during the 1960s and 1970s. This paper examines this question in more detail. It essentially adopts the

Figure 1: Terms of trade and real exchange rate

![Figure 1: Terms of trade and real exchange rate](image)

Note: The real exchange rate is defined as the price of tradables relative to the price of nontradables; see expression (12). A fall in $e$ indicates a real appreciation of the HK dollar.

---

1 An improvement in the terms of trade is generally viewed as being welfare-enhancing, although some income redistribution might be needed for welfare to increase in the Pareto sense; see e.g. Krueger and Sonnenschein (1967) and Woodland (1982). Note, however, that the impact of an improvement in the terms of trade on welfare and future growth may be ambiguous in the presence of distortions such as trade and production taxes or subsidies, in case of departures from perfect competition in the products or factor markets, and under uncertainty; see e.g. Pattanaik (1970), Batra and Scully (1971), and Lahiri and Sheen (1990). By focusing on measurement issues, this analysis is one of comparative statics and it does not examine the growth implications of changes in the terms of trade and the real exchange rate.


approach proposed by Kohli (2004a), but improves upon it by adopting a definition of the real exchange rate that treats imports and exports symmetrically. Consequently the decomposition of the trading gains between the terms-of-trade effect and the real-exchange-rate effect is somewhat modified.

Figure 2: GDP shares of exports and imports

2. Nominal vs Real GDP

A country’s nominal GDP measures the total value of all final goods and services produced during a given period of time. Nominal GDP can also be interpreted as the country’s nominal gross domestic income (GDI) or its nominal domestic value added. Nominal GDP can be measured by looking at the expenditure side. Let it be assumed that gross output can be disaggregated into two components, a non-traded good (N) intended for domestic use (it is an aggregate of private household consumption, investment, and government purchases), and exports (X). Since some of these goods and services have foreign content, imports (M) must be subtracted. Nominal GDP ($\pi_t$) is thus equal to:

$$\pi_t = v_{N,t} + v_{X,t} - v_{M,t},$$

(1)

where the $v$’s denote nominal values and $t$ is for the time period. Nominal GDP amounted to about HK$1,220 billion in 2003. As a comparison, it was equal to HK$7 billion in 1961.

---

4 Throughout the paper the terms income and value added are used interchangeably.

5 In this paper (except for the simple illustration in Section 4) imports and exports are treated as middle products. Exports are thus conceptually different from goods intended for domestic use, since the former must still be processed by foreign producers before being ready to meet final demand.
For many purposes, it is real – rather than nominal – value added that is of interest. The nominal value of each GDP component can be interpreted as the product of a quantity and of an average price. Thus, for the $i^{\text{th}}$ GDP component:

$$v_{i,t} = p_{i,t} q_{i,t}, \quad i \in \{ N, X, M \},$$

where $p_{i}$ is the price of the $i^{\text{th}}$ component, and $q_{i}$ is the corresponding quantity. Prices may be normalized to unity for the base period (period 0). By dividing each component of nominal GDP by the corresponding price index and adding up the quantities thus obtained one gets real GDP ($q$) – also known as constant dollar GDP – such as it is measured in most countries, including in Hong Kong:

$$q_{t} = \frac{v_{N,t}}{p_{N,t}} + \frac{v_{X,t}}{p_{X,t}} - \frac{v_{M,t}}{p_{M,t}} = q_{N,t} + q_{X,t} - q_{M,t}.$$  

The real GDP index given by (3) is known in the literature as a Laspeyres quantity index. Taking again 2003 figures and using 2000 as the base period, one gets a value of real GDP of about HK$1,361 billion (at 2000 prices). The corresponding figure for 1961 is HK$87 billion. The path of real GDP between 1961 and 2003 is shown in Figure 3.

**Figure 3: Real value added and real GDP**

One can divide nominal GDP by real GDP to obtain a measure of the price of GDP, also known as the GDP deflator ($p$):

$$p_{t} = \frac{\pi_{t}}{q_{t}} = \frac{v_{N,t} + v_{X,t} - v_{M,t}}{s_{N,t} + s_{X,t} - s_{M,t}} = \frac{1}{P_{N,t}} + \frac{1}{P_{X,t}} - \frac{1}{P_{M,t}},$$

where $p_{i}$ is the price of the $i^{\text{th}}$ component, and $q_{i}$ is the corresponding quantity.
where \( s_N, s_X \) and \( s_M \) are the GDP shares of domestic goods, exports, and imports respectively. The GDP deflator is thus a harmonic mean of the prices of the various GDP components. It is known in the economic literature as a Paasche price index.

It is common practice to use real GDP data to compute growth rates over consecutive periods. Let \( Q_{t,t-1} \) be one plus the rate of growth of real GDP between period \( t-1 \) and period \( t \):

\[
Q_{t,t-1} = \frac{\sum_i \pm q_{i,t} s_i}{\sum_i \pm q_{i,t-1} s_i}, \quad i \in \{N, X, M\} ,
\]

where the sign is negative for imports and positive for the other two components, and where it has been taken into account that base-period (period \( 0 \)) prices are normalized to unity. It is important to note, however, that \( Q_{t,t-1} \) so defined is a ratio of two direct Laspeyres quantity indices, but it is not itself a Laspeyres quantity index, unless period \( t-1 \) happens to be the base period. Direct indices are defined relative to a base period, but they are not well suited to make comparisons over consecutive periods not involving the base period.\(^6\) Index \( Q_{t,t-1} \) can be contrasted with a true Laspeyres quantity index \( G_{t,t-1} \) of real GDP:

\[
G_{t,t-1} = \sum_i \pm s_{i,t-1} \frac{q_{i,t}}{q_{i,t-1}}, \quad i \in \{N, X, M\} .
\]

Time series of real GDP over longer periods of time can then be obtained as chained indices by compounding the individual elements. Several countries have recently switched from runs of direct Laspeyres quantity indices to chained Laspeyres quantity indices to measure real GDP in conformity with the recommendations by Eurostat in the context of the 1995 system of European Standardised Accounts (ESA95). In that case the implicit GDP price deflator has the form of a chained Paasche price index.

While the Laspeyres and Paasche indices are still today the ones most commonly used in practice, other, and better, formulas are available. One need only think of the Fisher and of the Törnqvist indices, two members of the family of superlative index numbers.\(^7\) The Törnqvist index is particularly relevant for what follows. Let \( P_{t,t-1} \) be the Törnqvist price index of GDP over consecutive periods. It is as follows:

\[
P_{t,t-1} = \exp \left[ \sum_i \pm \frac{s_{i,t} + s_{i,t-1}}{2} \ln \frac{p_{i,t}}{p_{i,t-1}} \right], \quad i \in \{N, X, M\} ,
\]

where the sign is negative for imports, and positive for the two other components. Let \( \Pi_{t,t-1} \) be one plus the rate of change in nominal GDP:

\(^6\) See Kohli (2007b).

\(^7\) See Diewert (1976).
Deflating the nominal GDP index by $P_{t-1}$ yields the implicit Törnqvist index of real GDP $(Y_{t-1})$, a superlative quantity index:

$$Y_{t-1} = \frac{\Pi_{t-1}}{P_{t-1}}.$$

### 3. Real Gross Domestic Income

Real GDI ($z$) can be defined as nominal GDI (or GDP) divided by the price of domestic expenditures ($p_N$). Thus:

$$z_t = \frac{\pi_t}{p_N} = \frac{p_{X,t}}{p_{N,t}} q_{N,t} - \frac{p_{M,t}}{p_{N,t}} q_{M,t} = q_{N,t} + e_t \frac{1}{2} q_{M,t} - e_t \frac{1}{2} q_{X,t},$$

where $h$ is the inverse of the terms of trade and $e$ is an index of the price of tradables relative to the price of non-tradables:

$$h_t = \frac{p_{M,t}}{p_{X,t}},$$

$$e_t = \frac{p_{X,t}}{p_{N,t}} \frac{\sqrt{q_{M,t}}}{\sqrt{q_{X,t}}}.$$

The numerator in (12) is a Cobb-Douglas price index of tradables, with imports and exports having equal weights. For given terms of trade, a change in $e$ can be interpreted as a change in the real exchange rate, an increase in $e$ being equivalent to a real depreciation of the home currency. The path of the real exchange rate so defined is shown in Figure 1 as well. It can be seen that the real exchange rate was fairly steady over the first half of the sample period. The fall in the price of tradables relative to the price in non-tradables during the 1980s and the early 1990s reveals a real appreciation of the Hong Kong dollar. Definition (12) is somewhat different from the one used in Kohli (2004a) where the real exchange rate was simply taken as the price of exports relative to the price of non-traded goods. The advantage of (12) is that both imports and exports

---

8 See Kohli (2004b).

9 Alternatively, the weights could be set to the average shares of imports and exports in total trade. It turns out, however, that this would make little difference in the case of Hong Kong, since trade was fairly balanced on average over the sample period.

10 See Salter (1959), Dornbusch (1980), Frenkel and Mussa (1984), Corden (1992), and the literature on what has become known as the “Australian model”. Note that the real exchange rate so defined does not coincide exactly with another common definition of the real exchange rate (sometimes called the PPP real exchange rate), namely the nominal exchange rate adjusted for inflation rate differentials; see Edwards (1989) for a review of competing definitions of the real exchange rate.
are treated symmetrically. Comparing (10) with (3), one sees that the crucial difference between real GDI and real GDP is that in the former the quantity of exports and imports are weighted by the terms of trade and the real exchange rate. A change in the terms of trade or in the real exchange rate will have a direct impact on real GDI, but not on real GDP. It will be shown in Sections 5 and 6 how the real GDI effects of changes in the terms of trade and the real exchange rate can be measured.

The change in real GDI over time is captured by index $Z_{t,t-1}$ defined as follows:

$$Z_{t,t-1} = \frac{z_t}{z_{t-1}} = \frac{\Pi_{x,t-1}}{P_{N,t,t-1}},$$ (13)

where $P_{N,t,t-1}$ is one plus the rate of inflation in terms of the domestic good:

$$P_{N,t,t-1} = \frac{P_{N,t}}{P_{N,t-1}}.$$ (14)

4. Preliminary Analysis

The conceptual difference between real GDP and real GDI can be illustrated with the help of a simple model. In what follows, imports will be treated as inputs to the production process, but the analysis is still valid in the context of consumer theory, as opposed to production theory. Let it be assumed that production involves one domestic factor of production, an aggregate of labor and capital, as well as imported products. Let it also be assumed for the time being that all outputs (domestic output and exports) can be aggregated into a composite good. The country’s technology can then be described by the following aggregate production function:

$$q_{Y,t} = f(q_{M,t}, v),$$ (15)

where $q_{Y}$ is the total quantity of output, and $v$ is the endowment of the domestic factor of production. It is assumed that this production function is increasing, linearly homogeneous and concave. Furthermore, perfect competition, given factor endowments, and exogenous terms of trade are assumed as it is standard in international trade theory.

The production function is shown in Figure 4, with gross output as a function of the quantity of imports, for a given endowment of the domestic factor.\textsuperscript{11} The slope of the production function can be interpreted as the marginal product of imports. Let the relative price of imports — the inverse of the terms of trade — be given by the slope of line $BC$. This slope is unity since all prices are typically normalized to one in the base period. Profit maximization by producers will lead to an equilibrium at point $C$ where the marginal product of imports is equal to their marginal cost. The volume of imports is the distance $OD$ and total output is equal to $OE$. If trade is balanced, exports are equal to $BE$. The distance $OB$ can be interpreted as real income, real value added, or real GDP:

$$q_i = q_{Y,t} - q_{M,t}.$$ (16)

\textsuperscript{11} See Kohli (1983, 2004a) for further details.
Assume now that the terms of trade improve, as the result, for instance, of a drop in import prices. The terms of trade are now given by the slope of $B'C'$, and equilibrium moves from point $C$ to point $C'$. The country imports more. The marginal product of imports admittedly falls, but their real price is now lower. Gross output is now equal to $OE'$. Real value added (real income) has clearly increased, going from $OB$ to $OB'$. Real GDP, on the other hand, falls from $OB$ to $OF$: point $F$ is at the intersection between the vertical axis and a unit-sloped line through $C'$. In accordance with the definition of real GDP, the distance $OF$ is equal to $OE'$ (gross output, $q_Y$) minus $OD'$ (imports, $q_M$). Thus, real GDP falls, even though real income and real value added unambiguously increase. This clearly illustrates the difficulties involved with this measure of a country’s activity.\(^\text{12}\)

**Figure 4: Imports as an input to the technology**

In the model of Figure 4, imports are treated as intermediate goods. This is consistent with the evidence that the bulk of world trade is in raw material and intermediate products. Moreover, even most so-called finished products are still subject to many domestic charges before they reach final demand, so that a significant proportion of their final price tag is generally accounted for by domestic value added. Nevertheless, the perverse effect of a change in the terms of trade on real GDP can also be illustrated with the conventional model of international trade theory, the Heckscher-Ohlin model, where all goods are treated as end products. In Figure 5, the initial terms of trade are given by the inverse of the slope of line $PC$. Production takes place at point $P$ on the production

\(^{12}\) Note that there are only two states (e.g. periods) in this example. Hence the demonstration is valid independently of whether one uses direct or chained indices.
possibilities frontier, whereas if trade is balanced, consumption takes place at point $C$. Next, let the terms of trade improve, so that the international price line is now given by $P'C'$. Production shifts towards the north-west, from $P$ to $P'$, whereas consumption moves from $C$ to $C'$. $C'$ is on a higher indifference curve, which clearly demonstrates the increase in real income. Yet real GDP falls, from $OA$ to $OA'$, $A'$ being at the intersection between the vertical axis and a line through $P'$, with the same slope as $PC$.

Figure 5: Trade in final goods

Thus, not only is the effect of an improvement in the terms of trade on real income underestimated by real GDP, but, even more seriously, the change in real GDP goes in the wrong direction. The intuitive explanation for this phenomenon is as follows. When import prices fall, the country can afford to import more. Yet, since real GDP is obtained by subtracting imports valued at their base period prices, i.e. without taking into account the lower price of imports, one ends up subtracting too much and one thus gets a real GDP figure that is too low.

Another way to look at the problem is to consider the effect of a change in the terms of trade on the GDP deflator. A drop in the price of imports leads to an increase in the deflator (since imports enter with a negative weight), even though no price has actually increased. Incidentally, this shows that the GDP deflator is a poor index of the general price level, since the drop in the price of imports has no inflationary effects — quite the
contrary. It is obvious, therefore, that if the GDP deflator overestimates the price level, real GDP will underestimate the quantity of real value added.\textsuperscript{13}

As an analogy, think of a farmer who grows wheat in his field, using his labor and fertilizer as only inputs (for simplicity, ignore the other inputs such as seeds and capital). Assume that the price of wheat is constant, but that for some reason the price of fertilizer falls. Everyone would agree that this is very good news for the farmer, whose income will increase even if he does not change his behavior. In fact, he will probably be tempted to increase his use of fertilizer, which has become cheaper, in order to increase his output of wheat somewhat, and thus to raise his income even more. True, using more fertilizer will increase the output of wheat by only a small amount since the marginal product of fertilizer is falling, but it would be absurd to simply subtract the quantity of fertilizer used from the quantity of wheat produced to come to the conclusion that the real value added by the farmer has fallen.

In a recent paper, Feenstra et al. (2004) draw a distinction between two measures of real GDP: an output side measure, which they denote by $GDP_o$, and an expenditure-side measure, which they denote by $GDP_e$. The former is obtained by deflating nominal GDP by a price index for output (the usual GDP deflator), while the latter is obtained by using instead a price index for expenditures (a domestic absorption price index). The distinction between $GDP_o$ and $GDP_e$ is thus essentially the same as between real GDP and real GDI.\textsuperscript{14} Besides terminology,\textsuperscript{15} the main differences between the two treatments are that (1) the measures in this paper are based on an explicit production model, (2) superlative indices are used rather than Paasche and Laspeyres indices, and (3) purchasing power differences are not corrected since, by focusing on a single economy (Hong Kong), there is no need to make international comparisons. The main purpose of Feenstra et al. (2004) is precisely to produce purchasing-power-parity adjusted expenditure price deflators in order to make it possible to compare international standards of living using the Penn World Tables.

5. Generalization

The model of the previous section was rather restrictive. Fortunately, it can easily be generalized to allow for technological change and to include many inputs and many outputs. In what follows, two domestic factors will be considered, labor ($L$) and capital

\textsuperscript{13} In case of a terms-of-trade deterioration, the opposite happens: the GDP price deflator underestimates the change in the price level.

\textsuperscript{14} Compare equations (1) and (2) in Feenstra et al. (2004) with equations (9) and (13) above, or equations (42) and (43) in Kohli (2004a).

\textsuperscript{15} Feenstra et al. (2004) reject the term GDI because “it is suggestive of the income-approach to measuring GDP (through adding up the earnings of factors) which we do not use”. This argument is not convincing, since nominal GDP is theoretically equivalent to nominal GDI, whereas nominal domestic expenditures will not equal nominal GDP if trade is not balanced. Furthermore, nominal GDP itself can be measured in two different ways, by adding up value added by industry, or, as is done here, by adding up expenditure components. Finally, the distinction output/expenditures suggests that trade is an activity that takes place once that production is completed, even though, as Feenstra et al. recognize, most trade is in intermediate goods and thus takes place during the production process.
(K), and again gross output will be split into exports (X) and domestic goods (N). The quantities of labor and capital services are denoted by \(x_L\) and \(x_K\), and their prices by \(w_L\) and \(w_K\). Let \(T_t\) be the production possibilities set at time \(t\). We assume that \(T_t\) is a convex cone. The aggregate technology can be described by a real GDI function defined as follows:

\[
\nu_t = \nu_0 + \left( q_N, q_M, q_K, q_L \right) \in \mathbb{R}^+ \ , \\
\nu_0 = \frac{w_L q_N + w_K q_M}{w_L + w_K}.
\]

Let \(T_t\) be the production possibilities set at time \(t\). We assume that \(T_t\) is a convex cone. The aggregate technology can be described by a real GDI function defined as follows:

\[
z_t = z(h_t, e_t, x_K, x_L, t) = \max_{q_N, q_M, q_K, q_L} \left\{ q_N, q_M, q_K, q_L \in T_t \right\}.
\]

It is shown in the Appendix that the real GDI function has the following slope properties:

\[
\frac{\partial z(\cdot)}{\partial h} = -\frac{1}{2} \frac{e_t}{h_t} \left( h_t^{-\frac{\nu}{3}} q_{X,t} + h_t^{\frac{\nu}{3}} q_{M,t} \right)
\]

\[
\frac{\partial z(\cdot)}{\partial e} = h_t^{\frac{\nu}{3}} q_{X,t} - h_t^{-\frac{\nu}{3}} q_{M,t}
\]

\[
\frac{\partial z(\cdot)}{\partial x_K} = \frac{w_{K,t}}{p_{N,t}}
\]

\[
\frac{\partial z(\cdot)}{\partial x_L} = \frac{w_{L,t}}{p_{N,t}}
\]

\[
\frac{\partial z(\cdot)}{\partial t} = \mu_t z_t,
\]

where \(\mu_t\) is the instantaneous rate of technological change.

Following Diewert and Morrison (1986), and using (17) as a starting point, the following index can be defined to capture the contribution of changes in the terms of trade to real GDI:

\[
Z_{H,t \leftarrow \text{int}} = \ln \frac{\nu(x_{K,t \leftarrow \text{int}}, x_{L,t \leftarrow \text{int}}, t)}{\nu(x_{K,t \leftarrow \text{int}}, x_{L,t \leftarrow \text{int}}, t-1)}
\]

Index (23) can be interpreted as the geometric mean of Laspeyres-like and Paasche-like indices, and it thus has the Fisher form so to speak. Similarly, the contribution of changes in the real exchange rate can be identified as:

\[
Z_{E,t \leftarrow \text{int}} = \ln \frac{\nu(x_{K,t \leftarrow \text{int}}, x_{L,t \leftarrow \text{int}}, t)}{\nu(x_{K,t \leftarrow \text{int}}, x_{L,t \leftarrow \text{int}}, t-1)}
\]

---

\(\nu\) The real GDI function can be viewed as a modified nominal GDP function; see Kohli (2007a).
The contribution of changes in domestic factor endowments is obtained as:

\[
Z_{K,t,i} = \frac{z(h_{i,t}, e_{i,t}, x_{K,t}, x_{L,t}, t - 1)}{z(h_{i,t}, e_{i,t}, x_{K,t}, x_{L,t}, t)} \times \frac{z(h_{i,t}, e_{i,t}, x_{K,t}, x_{L,t}, t)}{z(h_{i,t}, e_{i,t}, x_{K,t}, x_{L,t}, t - 1)}
\]  

(25)

\[
Z_{L,t,i} = \frac{z(h_{i,t}, e_{i,t}, x_{K,t}, x_{L,t}, t - 1)}{z(h_{i,t}, e_{i,t}, x_{K,t}, x_{L,t}, t)} \times \frac{z(h_{i,t}, e_{i,t}, x_{K,t}, x_{L,t}, t)}{z(h_{i,t}, e_{i,t}, x_{K,t}, x_{L,t}, t - 1)}
\]

(26)

and, finally, the contribution of technological progress:

\[
Z_{T,t,i} = \frac{z(h_{i,t}, e_{i,t}, x_{K,t}, x_{L,t}, t)}{z(h_{i,t}, e_{i,t}, x_{K,t}, x_{L,t}, t - 1)} \times \frac{z(h_{i,t}, e_{i,t}, x_{K,t}, x_{L,t}, t)}{z(h_{i,t}, e_{i,t}, x_{K,t}, x_{L,t}, t)}
\]

(27)

6. Measurement

If the functional form of real GDI function (17) is known, it can be directly substituted into (23)–(27), and the various indices can thus easily be calculated provided that the necessary data are available. One form well suited for these purposes is the Translog function.\(^{17}\) It provides a second-order approximation in logarithms of an arbitrary real GDI function, and it is as follows:

\[
\ln z_t = \alpha_0 + \alpha_H \ln h_t + \alpha_E \ln e_t + \beta_K \ln x_{K,t} + (1 - \beta_K) \ln x_{L,t}
\]

\[+ \frac{1}{2} \gamma_{HH} (\ln h_t)^2 + \gamma_{HE} \ln h_t \ln e_t + \frac{1}{2} \gamma_{EE} (\ln e_t)^2
\]

\[+ \frac{1}{2} \phi_{KK} (\ln x_{K,t} - \ln x_{L,t})^2 + (\delta_{HK} \ln h_t + \delta_{EK} \ln e_t)(\ln x_{K,t} - \ln x_{L,t})
\]

\[+ (\delta_{HT} \ln h_t + \delta_{ET} \ln e_t) t + \phi_{KT} (\ln x_{K,t} - \ln x_{L,t}) t + \beta_T t + \frac{1}{2} \phi_T t^2
\]

(28)

In the case of (28), the first-order conditions (18)–(22) can be derived in share form through logarithmic differentiation:

\[
\frac{\partial \ln z_t}{\partial \ln h_t} = -s_{A,t} = \alpha_H + \gamma_{HH} \ln h_t + \gamma_{HE} \ln e_t + \delta_{HK} \frac{x_{K,t}}{x_{L,t}} + \delta_{HT} t
\]

(29)

\[
\frac{\partial \ln z_t}{\partial \ln e_t} = s_{B,t} = \alpha_E + \gamma_{HE} \ln h_t + \gamma_{EE} \ln e_t + \delta_{EK} \frac{x_{K,t}}{x_{L,t}} + \delta_{ET} t
\]

(30)

\[
\frac{\partial \ln z_t}{\partial \ln x_{K,t}} = s_{K,t} = \beta_K + \delta_{HK} \ln h_t + \delta_{EK} \ln e_t + \phi_{KE} \frac{x_{K,t}}{x_{L,t}} + \phi_{KT} t
\]

(31)

\[ \frac{\partial \ln z(\cdot)}{\partial \ln x_L} = s_L, t = 1 - \beta_K - \delta_{LK} \ln h_t - \delta_{LK} \ln e_t - \phi_{LK} \ln \frac{x_{K,t}}{x_{L,t}} - \phi_{LT} t \]  

(32)

\[ \frac{\partial \ln z(\cdot)}{\partial t} = \mu_t = \beta_T + \delta_{HT} \ln h_t + \delta_{ET} \ln e_t + \phi_{KT} \ln \frac{x_{K,t}}{x_{L,t}} + \phi_{LT} t , \]  

(33)

where \( s_A \) is the average share of foreign trade in GDP \( s_A = \frac{1}{2} s_X + \frac{1}{2} s_M \), \( s_B \) is the trade balance relative to GDP \( s_B = s_X - s_M \), and \( s_K \) and \( s_L \) are the GDP share of capital and labor, respectively.

If the true real GDI function is Translog, it turns out that indices (23) – (27) can be calculated on the basis of the data alone, that is without knowledge of the parameters of (28). Indeed, introducing (28) into (23) – (27) and making use of (29) – (33), one finds:

\[ Z_{H_t, t-1} = \exp \left[ -\frac{1}{2} (s_A, t + s_{A_{t-1}}) \ln \frac{h_t}{h_{t-1}} \right] \]  

(34)

\[ Z_{E_t, t-1} = \exp \left[ \frac{1}{2} (s_B, t + s_{B_{t-1}}) \ln \frac{e_t}{e_{t-1}} \right] \]  

(35)

\[ Z_{K_t, t-1} = \exp \left[ \frac{1}{2} (s_K, t + s_{K_{t-1}}) \ln \frac{x_{K,t}}{x_{K_{t-1}}} \right] \]  

(36)

\[ Z_{L_t, t-1} = \exp \left[ \frac{1}{2} (s_L, t + s_{L_{t-1}}) \ln \frac{x_{L,t}}{x_{L_{t-1}}} \right] \]  

(37)

\[ Z_{T_t, t-1} = \frac{Z_{t, t-1}}{X_{t, t-1}} . \]  

(38)

In this last expression \( X_{t, t-1} \) is a Törnqvist index of domestic input quantities:

\[ X_{t, t-1} = \exp \left[ \frac{s_{K_t} + s_{L_t}}{2} \ln \frac{x_{K, t}}{x_{K, t-1}} + \frac{s_{L_t} + s_{L_{t-1}}}{2} \ln \frac{x_{L, t}}{x_{L, t-1}} \right] . \]  

(39)

It can easily be seen that – still assuming that the true real GDI function is indeed given by (28) – indices (34) – (38) together give a complete decomposition of the growth in real GDI:\(^\text{18}\)

\[ Z_{t, t+1} = Z_{H_t, t-1} \times Z_{E_t, t-1} \times Z_{K_t, t-1} \times Z_{L_t, t-1} \times Z_{T_t, t-1} . \]  

(40)

\(^\text{18}\) See Kohli (1990, 2007a).
That is, the actual change in real GDI can be decomposed into five real factors: the terms-of-trade effect, the real exchange-rate effect, the capital endowment effect, the labor endowment effects, and the technological change (also known as total factor productivity) effect.

From (7), (14), (34) and (35), and recalling that \( s_N + s_X - s_M = 1 \), one finds:

\[
P_{t,t-1} = P_{N,t,t-1} \times Z_{H,t,t-1} \times Z_{E,t,t-1}.
\]

In other words, the Törnqvist GDP price deflator incorporates the terms-of-trade and the real-exchange-rate effects, two effects that should be viewed as real – as opposed to price – effects. Conversely, in view of (9) and (13), (41) implies:

\[
Z_{t,t-1} = Y_{t,t-1} \times Z_{H,t,t-1} \times Z_{E,t,t-1}.
\]

That is, real GDI \( (Z_{t,t-1}) \) is equal to real GDP \( (Y_{t,t-1}) \) augmented by the terms-of-trade and real-exchange-rate effects. Put differently, as indicated by (40), real GDP, by excluding \( Z_{H,t,t-1} \) and \( Z_{E,t,t-1} \), takes account of only three out of five real forces.

7. Estimates for Hong Kong

The discussion now turns to the empirical application. The data are for Hong Kong, they are annual, and they cover the period 1961–2003. They consist of current and constant price values of the major components of GDP: consumption, investment, government purchases, exports and imports. Consumption, investment and government purchases were aggregated into a composite, domestic good. The corresponding price \( (P_{N,t,t-1}) \) was computed as a Törnqvist price index.

One characteristic of the Hong Kong economy, as mentioned in the introduction, is its extreme openness, with the GDP shares of imports and exports well in excess of 100% in recent years. One reason for this situation has to do with what is sometimes called “entrepot trade”. Many products destined for overseas markets are imported into Hong Kong from China and elsewhere, and then re-exported shortly afterwards. Another reason has to do with what is referred to as “processing trade”. Thus, there are many instances of production taking place in stages, with production stations located some in Hong Kong and some in China, so that the goods involved cross the border a number of times during the entire process. The question then arises how to treat this type of trade, and in particular how to handle re-exports, i.e. products that merely transit through Hong Kong.

One might think that one way to deal with this situation would be to entirely exclude entrepot trade from the analysis. The current and constant dollar values of total exports and imports would decrease accordingly, with little or no impact on GDP. This would only be admissible if it were indeed true that re-exports contain zero domestic value added. Note that while data on re-exports are available, data on imported products destined to be re-exported are not. However, still assuming that no domestic transformation occurs, the price and the quantity of imports intended to be re-exported
could be taken to be identical to the price and quantity of re-exports. Domestic imports and exports would thus be reduced by the same amounts.

The hypothesis that no domestic intervention takes place is not tenable, however. After all, there must be a reason why these goods transit through Hong Kong, rather than being shipped directly to their final destination. In all likelihood re-exports involve an array of domestic activities, such as unloading and reloading, storing, repackaging, trading, wholesaling, financing, marketing, insuring, etc., so that re-exports will contain some domestic value added after all.\(^{19}\) In fact, there is evidence that the margins added by Hong Kong are quite substantial, with Chinese goods being much more expensive when they leave Hong Kong than when they enter.\(^{20}\) In that sense, re-exports are not fundamentally different from other exports (the so-called “domestic exports”), which too typically have domestic as well as foreign content. This approach that treats all imports and exports as middle products – which enter and exit the production sector – is therefore well suited to handle re-exports without the need for any adjustment.\(^{21}\)

On the contrary, it would be incorrect to exclude re-exports from the analysis. It is noteworthy that the foreign content of domestic exports may vary a great deal, from close to zero to a proportion close to one. It is therefore difficult to know where to draw the line, and if one were to disaggregate exports into two categories, it is not obvious that the distinction between domestic exports and re-exports would be the most fruitful, particularly so since this would provide no guidance as how to disaggregate imports. We therefore proceed without attempting to disaggregate imports and exports, although it should be noted that our model can easily handle a larger number of inputs and outputs if the data are available.

Shown in Table 1 are estimates of \(Q_{t-1}, G_{t-1}, Y_{t-1}, Z_{t-1}, Z_{d,t-1}\), and \(Z_{e,t-1}\), based on (5), (6), (9), (13), (34) and (35), respectively.\(^{22}\) Geometric means for each decade and for the entire period are reported at the bottom of the table. The path of real GDI (the cumulated value of \(Z_{t-1}\)) can be seen in Figure 3. The yearly growth factors of real GDP (\(Q_{t-1}\)) and real GDI (\(Z_{t-1}\)) are shown in Figure 6.

\(^{19}\) Note that many of these activities are non-traded activities. This shows that re-exports (and exports, for that matter) may contain traded as well as non-traded components, just like non-traded goods incorporate traded and non-traded intermediate goods and services; see Burstein, Eichenbaum, and Rebelo (2005a, 2005b), for instance. Domestic, non-traded intermediate activities need not be modeled explicitly since they net out in the aggregate; see Woodland (1982).

\(^{20}\) See Hanson and Feenstra (2004).

\(^{21}\) The term “middle product” was coined by Sanyal and Jones (1982).

\(^{22}\) If data on the prices and quantities of labor and capital services are available, estimates of (36)–(38) could readily be computed.
Table 1
Real GDP, real GDI, terms-of-trade, and real exchange-rate effects

<table>
<thead>
<tr>
<th>Year</th>
<th>$Q_{t-1}$</th>
<th>$G_{t-1}$</th>
<th>$Y_{t-1}$</th>
<th>$Z_{t-1}$</th>
<th>$Z_{Al,t-1}$</th>
<th>$Z_{E,t-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1962</td>
<td>1.1415</td>
<td>1.0920</td>
<td>1.0937</td>
<td>1.1479</td>
<td>1.0476</td>
<td>1.0018</td>
</tr>
<tr>
<td>1963</td>
<td>1.1570</td>
<td>1.1394</td>
<td>1.1392</td>
<td>1.1768</td>
<td>1.0341</td>
<td>0.9989</td>
</tr>
<tr>
<td>1964</td>
<td>1.0857</td>
<td>1.0844</td>
<td>1.0847</td>
<td>1.0915</td>
<td>1.0061</td>
<td>1.0002</td>
</tr>
<tr>
<td>1965</td>
<td>1.1448</td>
<td>1.1498</td>
<td>1.1491</td>
<td>1.1674</td>
<td>1.0163</td>
<td>0.9996</td>
</tr>
<tr>
<td>1966</td>
<td>1.0172</td>
<td>1.0124</td>
<td>1.0157</td>
<td>1.0373</td>
<td>1.0229</td>
<td>0.9993</td>
</tr>
<tr>
<td>1967</td>
<td>1.0169</td>
<td>1.0303</td>
<td>1.0326</td>
<td>1.0563</td>
<td>1.0231</td>
<td>1.0000</td>
</tr>
<tr>
<td>1968</td>
<td>1.0333</td>
<td>1.0316</td>
<td>1.0331</td>
<td>1.0658</td>
<td>1.0293</td>
<td>1.0023</td>
</tr>
<tr>
<td>1969</td>
<td>1.1129</td>
<td>1.1123</td>
<td>1.1125</td>
<td>1.1188</td>
<td>1.0041</td>
<td>1.0016</td>
</tr>
<tr>
<td>1970</td>
<td>1.0918</td>
<td>1.0962</td>
<td>1.0968</td>
<td>1.0947</td>
<td>1.0007</td>
<td>0.9974</td>
</tr>
<tr>
<td>1971</td>
<td>1.0708</td>
<td>1.0521</td>
<td>1.0532</td>
<td>1.1027</td>
<td>1.0468</td>
<td>1.0001</td>
</tr>
<tr>
<td>1972</td>
<td>1.1033</td>
<td>1.1111</td>
<td>1.1103</td>
<td>1.1202</td>
<td>1.0111</td>
<td>0.9979</td>
</tr>
<tr>
<td>1973</td>
<td>1.1236</td>
<td>1.1500</td>
<td>1.1148</td>
<td>1.1124</td>
<td>0.9951</td>
<td>1.0028</td>
</tr>
<tr>
<td>1974</td>
<td>1.0233</td>
<td>1.0255</td>
<td>1.0286</td>
<td>0.9782</td>
<td>0.9480</td>
<td>1.0032</td>
</tr>
<tr>
<td>1975</td>
<td>1.0033</td>
<td>0.9992</td>
<td>1.0000</td>
<td>0.9304</td>
<td>1.0325</td>
<td>0.9980</td>
</tr>
<tr>
<td>1976</td>
<td>1.1623</td>
<td>1.1655</td>
<td>1.1663</td>
<td>1.2111</td>
<td>1.0371</td>
<td>1.0012</td>
</tr>
<tr>
<td>1977</td>
<td>1.1173</td>
<td>1.1199</td>
<td>1.1200</td>
<td>1.1138</td>
<td>0.9942</td>
<td>1.0002</td>
</tr>
<tr>
<td>1978</td>
<td>1.0850</td>
<td>1.0893</td>
<td>1.0892</td>
<td>1.0913</td>
<td>1.0021</td>
<td>0.9999</td>
</tr>
<tr>
<td>1979</td>
<td>1.1152</td>
<td>1.0847</td>
<td>1.0856</td>
<td>1.1030</td>
<td>1.0161</td>
<td>0.9999</td>
</tr>
<tr>
<td>1980</td>
<td>1.1021</td>
<td>1.0752</td>
<td>1.0773</td>
<td>1.1106</td>
<td>1.0308</td>
<td>1.0000</td>
</tr>
<tr>
<td>1981</td>
<td>1.0924</td>
<td>1.0937</td>
<td>1.0935</td>
<td>1.0765</td>
<td>0.9842</td>
<td>1.0003</td>
</tr>
<tr>
<td>1982</td>
<td>1.0279</td>
<td>1.0211</td>
<td>1.0213</td>
<td>1.0339</td>
<td>1.0121</td>
<td>1.0003</td>
</tr>
<tr>
<td>1983</td>
<td>1.0580</td>
<td>1.0452</td>
<td>1.0463</td>
<td>1.0450</td>
<td>0.9985</td>
<td>1.0002</td>
</tr>
<tr>
<td>1984</td>
<td>1.0999</td>
<td>1.0959</td>
<td>1.0977</td>
<td>1.1292</td>
<td>1.0267</td>
<td>1.0019</td>
</tr>
<tr>
<td>1985</td>
<td>1.0051</td>
<td>1.0185</td>
<td>1.0195</td>
<td>1.0432</td>
<td>1.0251</td>
<td>0.9981</td>
</tr>
<tr>
<td>1986</td>
<td>1.1078</td>
<td>1.1053</td>
<td>1.1039</td>
<td>1.0589</td>
<td>0.9637</td>
<td>0.9953</td>
</tr>
<tr>
<td>1987</td>
<td>1.1302</td>
<td>1.1160</td>
<td>1.1178</td>
<td>1.1317</td>
<td>1.0155</td>
<td>0.9970</td>
</tr>
<tr>
<td>1988</td>
<td>1.0803</td>
<td>1.0660</td>
<td>1.0667</td>
<td>1.0662</td>
<td>1.0047</td>
<td>0.9949</td>
</tr>
<tr>
<td>1989</td>
<td>1.0265</td>
<td>1.0190</td>
<td>1.0195</td>
<td>1.0415</td>
<td>1.0263</td>
<td>0.9953</td>
</tr>
<tr>
<td>1990</td>
<td>1.0370</td>
<td>1.0334</td>
<td>1.0341</td>
<td>1.0368</td>
<td>1.0062</td>
<td>0.9964</td>
</tr>
<tr>
<td>1991</td>
<td>1.0564</td>
<td>1.0491</td>
<td>1.0509</td>
<td>1.0641</td>
<td>1.0162</td>
<td>0.9965</td>
</tr>
<tr>
<td>1992</td>
<td>1.0659</td>
<td>1.0718</td>
<td>1.0735</td>
<td>1.0859</td>
<td>1.0146</td>
<td>0.9969</td>
</tr>
<tr>
<td>1993</td>
<td>1.0635</td>
<td>1.0536</td>
<td>1.0549</td>
<td>1.0626</td>
<td>1.0114</td>
<td>0.9959</td>
</tr>
<tr>
<td>1994</td>
<td>1.0548</td>
<td>1.0667</td>
<td>1.0667</td>
<td>1.0526</td>
<td>0.9882</td>
<td>0.9985</td>
</tr>
<tr>
<td>1995</td>
<td>1.0389</td>
<td>1.0437</td>
<td>1.0412</td>
<td>1.0099</td>
<td>0.9697</td>
<td>1.0003</td>
</tr>
<tr>
<td>1996</td>
<td>1.0431</td>
<td>1.0431</td>
<td>1.0427</td>
<td>1.0547</td>
<td>1.0093</td>
<td>1.0021</td>
</tr>
<tr>
<td>1997</td>
<td>1.0507</td>
<td>1.0504</td>
<td>1.0514</td>
<td>1.0565</td>
<td>1.0027</td>
<td>1.0022</td>
</tr>
<tr>
<td>1998</td>
<td>0.9503</td>
<td>0.9438</td>
<td>0.9441</td>
<td>0.9542</td>
<td>1.0100</td>
<td>1.0008</td>
</tr>
<tr>
<td>1999</td>
<td>1.0342</td>
<td>1.0270</td>
<td>1.0268</td>
<td>1.0124</td>
<td>0.9857</td>
<td>1.0002</td>
</tr>
<tr>
<td>2000</td>
<td>1.1016</td>
<td>1.0942</td>
<td>1.0938</td>
<td>1.0816</td>
<td>0.9869</td>
<td>1.0019</td>
</tr>
<tr>
<td>2001</td>
<td>1.0046</td>
<td>1.0046</td>
<td>1.0049</td>
<td>1.0112</td>
<td>1.0065</td>
<td>0.9998</td>
</tr>
<tr>
<td>2002</td>
<td>1.0189</td>
<td>1.0193</td>
<td>1.0201</td>
<td>1.0353</td>
<td>1.0139</td>
<td>1.0011</td>
</tr>
<tr>
<td>2003</td>
<td>1.0322</td>
<td>1.0342</td>
<td>1.0337</td>
<td>1.0148</td>
<td>0.9798</td>
<td>1.0020</td>
</tr>
</tbody>
</table>

1962-1970 | 1.0708 | 1.0728 | 1.0740 | 1.0895 | 1.0145 | 1.0001
1971-1980 | 1.0897 | 1.0828 | 1.0836 | 1.0959 | 1.0110 | 1.0003
1981-1990 | 1.0658 | 1.0608 | 1.0614 | 1.0657 | 1.0061 | 0.9980
1991-2003 | 1.0390 | 1.0381 | 1.0382 | 1.0375 | 0.9995 | 0.9999
From 1962 to 2003, the official measure of real GDP suggests that real growth has reached 6.8% on average, a truly remarkable performance. The chained Laspeyres quantity index indicates a somewhat lower value, namely 6.3%, whereas the implicit Törnqvist measure yields a value of 6.4%. The fact that the implicit Törnqvist shows a higher value than the chained Laspeyres quantity index is consistent with the fact that in the supply context the latter is biased downwards.

The fourth column of Table 1 shows estimates of real GDI. It averaged 7.2% over the sample period, substantially more than either measure of real GDP. This difference can be attributed to the trading gains, i.e. the combined effect of the terms-of-trade and the real-exchange-rate effects. These two effects are shown in the last two columns of the table. The terms-of-trade effect is found to be particularly large: it has added about 0.82% of real growth per year. The real-exchange-rate effect, on the other hand, was very small. Nonetheless, it cannot be ignored if one wants the decomposition (38) to hold exactly. While the real-exchange-rate effect is small on average, it can be quite large for particular observations. Thus, it reduced real value added by as much as 0.5% in 1986, 1988, and 1989, and it added about 0.3% in 1973 and 1974. The terms-of-trade effect too was far from steady. Thus, there are six years where it added more than 3% in growth (1962, 1963, 1971, 1975, 1976, 1980) and three years (1974, 1986, 1995) when it reduced growth by more than 3%. The time profile of \( Z_{\text{H}, t-1} \) and \( Z_{\text{E}, t-1} \) can also be depicted graphically. This is done in Figure 7. The dominating role of the terms-of-trade effect is clearly apparent.
Looking at the various sub-periods, it is found that the terms-of-trade effect was particularly strong in the 1960s and 1970s. This of course is when the improvement in the terms of trade was largest (Figure 1). However, this is also when the trade shares were lowest (Figure 2). The GDP shares of imports and exports increased substantially during the following two decades, and thus the terms-of-trade effect was still sizable during the 1980s, averaging 0.6% per year, even though the improvement in the terms of trade had become less pronounced. The terms-of-trade effect turned out to be slightly negative during the 1991−2003 period. The real-exchange-rate effect was slightly negative during the 1980s, and essentially nil during the other sub-periods.

It is interesting to note that the recession of 1998 was somewhat less pronounced than suggested by the real GDP figures. Real GDI fell by approximately 4.6%, as opposed to a drop of 5.6% as indicated by the implicit Törnqvist index of real GDP. What helped in this particular year were the positive terms-of-trade and real-exchange-rate effects (1.0% and 0.1%, respectively). Over the course of the following two years, however, the terms-of-trade effect turned strongly negative, penalizing real GDI by a total of 2.7%.

8. Conclusion

To sum up, it seems that Hong Kong’s growth performance over the past four decades has been even more stellar than the data suggest. Real growth may have been underestimated by real GDP by about 0.43% per year. Much of the discrepancy originates in the 1960s, 1970s, and 1980s. Moreover, it appears that this average figure masks large positive or negative deviations in particular years. The discrepancy in the growth rates of real GDP and real domestic income consists of four elements: over the sample period, the direct Laspeyres index of real GDP overestimated growth by about
0.40% compared to a Laspeyres chained index, and by an additional 0.04% by ignoring the real-exchange-rate effect. On the other hand, it underestimated growth by about 0.06% by providing a linear, rather than a superlative measure, and by an additional 0.82% by neglecting the terms-of-trade effect. By compounding these deviations, one arrives at the figure of 0.43% mentioned above. A discrepancy of 0.43% might not seem like much, but compounded over a period of 42 years, it adds up to nearly 20% of GDP.

It is not suggested that real GDP should be abandoned as a statistical concept. Nonetheless, it is instructive to ask oneself what it is that real GDP is supposed to measure. Is it output, is it real value added, is it real income? If international trade were taking place in final goods exclusively (the case depicted by Figure 5), one would be justified in arguing that real GDP is a measure of domestic output and real value added. The drop in real GDP following an improvement in the terms of trade that was flagged in Section 4 is due to the specific index number formula that was postulated (the Laspeyres quantity index): if one used instead an index that is exact for the true transformation function, one would find that real GDP is untouched by variations in the terms of trade. Real GDP would simply mirror the position of the production possibilities frontier, and it would be affected by changes in domestic factor endowments and the technology only.

As argued earlier, however, the view all international trade is in final products does not match reality. On the contrary, nearly all trade is in middle products. Almost all of it is carried out by firms, not by households. International trade is an intimate part of production, rather than an afterthought. Thus, it is really the model underlying Figure 4 that is relevant. So again the question is asked: what is real GDP meant to measure? Is it output, is it real value added, is it real income? Referring to the analysis of Section 4, the correct answer appears to be “none of the above”. The change in the terms of trade depicted in Figure 4 leads to an increase in gross and net output, in real value added, and in real income, and yet real GDP declines. Admittedly, if instead of the Laspeyres one used an index of real GDP that is exact for the true production function, real GDP would be found to remain unchanged, but this would still be far off the mark in comparison to output, income, and value added. In our view, real GDP is essentially an indicator of gross output possibilities, but because it leaves out an essential element of the aggregate transformation technology (the international trade element), it cannot be expected to provide a full picture of the country’s performance. Put in the context of Figure 4, real GDP reflects the position of the production function, but it disregards the terms of trade line that is key for the determination of actual gross output and for the projection of real value added, net output and real income onto the vertical axis. Real GDP does increase as domestic factor endowments grow and as the technology improves, but it fails to register the (gross and net) output effect of an enhancement in the transformation possibilities offered by international trade. This is why we believe that real GDI is a more comprehensive and more relevant measure of real value added than real GDP.

Our measure of real value added is somewhat similar to the Command-Basis GNP indicator that the US Bureau of Economic Analysis has been publishing for the past 20

---

23 Feenstra et al. (2004) argue that countries whose GDP\(e\) are larger than their GDP\(o\) as a result of relatively unfavourable terms of trade are really more productive than their real income suggests. A cynical view would be that these countries are mostly good at producing things people do not want.
years. This indicator attempts to take trading gains changes into account in a rather crude way, by deflating nominal exports by the price of imports – rather than by the price of exports – in (3) above. Even if this procedure goes in the right direction, it is rather ad hoc and it rests on little economic analysis. As a measure of income it also suffers from the drawback that it depends on the position of the trade balance, that is, ultimately, on a savings/consumption decision. Our analysis on the contrary suggests that a preferable approach is to deflate nominal GDP by a domestic price index. While we have a preference for a Törnqvist index, a Fisher index could also be envisaged; the results it would produce would be numerically very close.

Finally, it should be emphasized that this discussion should in no way be interpreted as a criticism of the statistical unit that computes Hong Kong national accounts data, or of any other agency for that matter. Computing real GDP as a Laspeyres quantity index is standard practice almost everywhere in the world. It just so happens that given the huge terms-of-trade improvements that Hong Kong has enjoyed over the years and in view of its exceptional openness, Hong Kong makes for an interesting case study.

Appendix

The purpose of this appendix is to derive expressions (18) – (22) of the main text. Let \( \pi(p_M, p_X, p_N, x_K, x_L, t) \) be the country’s nominal GDP function. It is defined as follows (see Kohli, 1978, 1991; the time subscripts are omitted for simplicity):

\[
\pi(p_M, p_X, p_N, x_K, x_L) = \max_{q_N, q_X, q_M \in T} \left\{ p_N q_N + p_X q_X - p_M q_M : (q_N, q_X, q_M, x_K, x_L) \in T \right\}.
\]

(A1)

From (17) it can easily be seen that:

\[
\pi(p_M, p_X, p_N, x_K, x_L, t) = p_N z(h, e, x_K, x_L, t).
\]

(A2)

The slope properties of \( \pi(p_M, p_X, p_N, x_K, x_L, t) \) are well known (Kohli, 1978, 1991):

\[
\frac{\partial \pi(\cdot)}{\partial p_M} = -q_M
\]

(A3)

\[
\frac{\partial \pi(\cdot)}{\partial p_X} = q_X
\]

(A4)

\[
\frac{\partial \pi(\cdot)}{\partial p_N} = q_N
\]

(A5)

\[
\frac{\partial \pi(\cdot)}{\partial x_K} = w_K
\]

(A6)

\[24\] See Denison (1981).
Making use of (A2) these conditions can be rewritten as:

\[
\frac{\partial \pi}{\partial p_M} = p_N \frac{dz}{dp_M} = p_N \left( \frac{z_h}{p_X} + \frac{1}{2} z_e p_N^{-1/2} p_X^{1/2} \right) = \frac{p_N}{p_M} \left( hz_h + \frac{1}{2} ez_e \right) = -q_M \tag{A9}
\]

\[
\frac{\partial \pi}{\partial p_X} = p_N \frac{dz}{dp_X} = p_N \left( -z_h \frac{p_M}{p_X} + \frac{1}{2} z_e p_M^{1/2} p_X^{-1/2} \right) = \frac{p_N}{p_X} \left( -hz_h + \frac{1}{2} ez_e \right) = q_X \tag{A10}
\]

\[
\frac{\partial \pi}{\partial p_N} = z + p_N \frac{dz}{dp_N} = z - z_e p_M^{1/2} p_X^{-1/2} = z - ez_e = q_N \tag{A11}
\]

\[
\frac{\partial \pi}{\partial x_K} = p_N \frac{dz}{dx_K} = p_N z_K = w_K \tag{A12}
\]

\[
\frac{\partial \pi}{\partial x_L} = p_N \frac{dz}{dx_L} = p_N z_L = w_L \tag{A13}
\]

\[
\frac{\partial \pi}{\partial t} = p_N \frac{dz}{dt} = p_N z_t = \mu p_N z , \tag{A14}
\]

where \( z_h = \frac{\partial \pi}{\partial h} , z_e = \frac{\partial \pi}{\partial e} \), and so on. Jointly solving (A9) and (A10) for \( z_h \) and \( z_e \) we find:

\[
z_h = -\frac{p_X q_X - p_M q_M}{2hp_N} = \frac{1}{2} \frac{e}{h} \left( h^{1/2} q_X + h^{-1/2} q_M \right) \tag{A15}
\]

\[
z_e = \frac{p_X q_X - p_M q_M}{ep_N} = h^{1/2} q_X - h^{-1/2} q_M , \tag{A16}
\]

i.e. expressions (18) and (19). Expressions (20) – (22) follow immediately from (A12) – (A14).

References


