Estate Taxation With Both Accidental and Planned Bequests

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Abstract

Actual inheritances are a hybrid of canonical types of bequests and, in particular, of accidental bequests and altruistic bequests. In this paper, a bequeathed estate consists of two components: an amount intended by altruistic parents and an amount which results from the “premature” death of parents. Altruistic parents can also invest in their children’s education. Taxing those two types of bequests separately is known to have different implications. The purpose of this paper is to see the distributive incidence of estate taxation when those two components are indistinguishable. The substitutability between education and intended bequests plays a key role in the tax design.

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1. Introduction

Nobody likes paying taxes, especially after death, and this seems to be truer today than ever before. An increasing number of countries do not have an inheritance or estate tax and some, including the United States, are contemplating phasing it out in the near future. This is surprising for a tax long thought to be one of the most efficient and the most equitable of taxes. For a number of social philosophers and classical economists estate or inheritance tax is the ideal tax: it is highly progressive and it has few disincentive effects, since it is only payable at death, and it is fair since it concerns unearned resources. Yet opponents of the “death tax” (as they have dubbed it) claim that it is unfair and immoral. It penalizes those frugal and loving parents who pass wealth on to their children, reducing the incentive to save and invest.

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Why so much controversy? One of the reasons is that there are different types of bequests, more precisely different reasons for leaving bequests, and for each of them the social desirability of a tax may vary. For example, the advocates of estate taxation have often in mind accidental bequests, the taxation of which is supposed to be harmless. Opponents of the “death tax” focus on altruistic bequests and the disincentive effects of taxing them. They also claim that it prevents small businesses from passing on from generation to generation.

The purpose of this paper is to assess the desirability of estate taxation when bequests result from lifetime uncertainty (i.e. when the date of the death is unknown) and from a mere joy of giving.¹ In the absence of a private annuity market, uncertainty about the length of life leads to some unexpected bequests. At the same time, parents may very well draw joy from bequeathing some of their wealth (human and physical) to their children. These two types of bequests – accidental bequests and bequests based on the joy of giving – are known to have different implications and particularly to react to taxation in contrasting ways.² If they can be distinguished, they should be taxed differently. Unfortunately they cannot easily be distinguished, and this makes the problem of estate taxation rather difficult. Not surprisingly, its incidence is highly sensitive to the relative importance of the two bequest motives.

To study this issue we use a two-period overlapping generations (OLG) growth model cast in a closed economy. There is some idiosyncratic uncertainty on the length of life in the second-period and there is no annuity market. This leads to accidental bequests and to a certain heterogeneity among individuals. If there were no joy of giving and individuals had the same labor productivity, the standard result is that a 100% tax on accidental bequests has no adverse effect on efficiency but can contribute to more equity. There is another source of bequest. Parents can leave part of their savings to their children out of some joy of giving. This type of bequest can take two forms: education spending and financial bequest. As shown by Becker and Tomes (1979) investment in education has the priority as long as its marginal return exceeds the rate of interest. Education spending is not directly taxed, unlike intended financial bequests. This taxation is likely to have some effect on the level of capital accumulation (positive or negative depending on the intertemporal elasticity of substitution). In this paper we use one type of planned or intentional bequest that is based on the joy of giving or also on paternalistic altruism. This is to be distinguished from another type of planned bequest based on pure altruism, according to which the parents’ utility function has the welfare of his descendants as argument.

In this paper we are concerned with the effect of estate taxation on the coefficient of variation of lifetime income and on average income.³ In other words, we are not concerned with the optimal taxation issue but rather by the marginal effect of estate taxation on what is considered a reasonable index of inequality. The reason for this choice (coefficient of variation rather than social welfare function, and tax reform

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¹ This paper is an expansion of an earlier paper by Michel and Pestieau (2002). Philippe Michel passed away in summer 2004.
² For an overview see Cremer and Pestieau (2005), Kaplow (2001).
³ The coefficient of variation is equal to the standard deviation divided by the mean. In the Appendix we use the square of the coefficient of variation, namely the variance over the squared mean.
rather than optimal taxation) is one of analytical simplicity. Even within this simple specification the problem remains complex.

Another source of heterogeneity is productivity. Individuals have different productivities which are not necessarily correlated across generations, but which are statistically independent of lifetime uncertainty. We will see that the desirability of an estate tax increase depends on the relative importance of accidental and intended bequests, the balance between education investment and intended financial bequests, and the extent of intergenerational mobility.

Michel and Pestieau (2002) consider a much simpler version of this model. In their paper the only source of heterogeneity is lifetime uncertainty. Individuals have the same productivity. Preferences and technology are homothetic and strictly concave. There is no transmission of human capital. If estate taxes could be distinguished according to the bequest motive, the tax on accidental bequest would always be desirable (i.e. it would lower the coefficient of variation without depressing average output). The tax on intended bequests is only desirable when the intertemporal elasticity of substitution is less than or equal to 1. When it is higher than 1, the reduction in capital accumulation can more than outweigh the reduction in inequality. When the two taxes are merged, there is a value of the elasticity of substitution higher than 1, above which the tax is undesirable.

In this paper we introduce productive heterogeneity and intergenerational mobility. We also look at the impact of alternative taxes on the coefficient of variation and on the mean of income. There is a price to pay for this generalization: we can only use log-linear utilities and Cobb-Douglas production functions.

The rest of the paper is organized as follows. In section 2 the basic OLG model is introduced with the steady-state values of capital accumulation, aggregate production and human capital. We also calculate the coefficient of variation of lifetime income. Section 3 analyzes the effects of alternative tax tools, particularly estate taxes, on the steady-state values of this coefficient and of output. The final section concludes this paper.

2. The model

2.1. Consumers

To deal with the problem at hand, we adopt a standard OLG model with lifetime uncertainty. Individuals belonging to generation $t$ and having productivity $i$ live for at most two periods. They work and earn $w_i h_i$ in the first period with $w_i$ being the standard wage rate and $h_i$ an index of human capital. They also inherit $b_i$ at the beginning of this period. They then devote their resources, $w_i h_i + b_i$, to present consumption, $c_i$, education

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4 The conventional wisdom that accidental bequests if they could be taxed separately should be subject to a 100% tax has been recently challenged by Blumkin and Sadka (2004) who show that in an optimal income tax setting à la Mirrlees leaving some accidental bequests untaxed can be desirable as it relaxes the self-selection constraints. In our model, there is no optimal taxation and intended bequests come from the joy of giving and not from pure altruism.
investment, \( e'_{t+1} \), and savings, \( s'_{t+1} \); \( e'_{t+1} \) serves to enhance the productivity of the next generation’s worker. Savings are then devoted to consumption, \( d'_{t+1} \), in their retirement period and to some intentional bequests, \( x'_{t+1} \). We assume zero population growth, which implies that each parent has only one child.

Uncertainty in the length of lifetime is captured by assuming that each individual lives with certainty for the entire first period but that they either live for the entire second period with probability \((1 - \pi)\) or die prematurely at the beginning of the second period with probability \(\pi\). Probability \(\pi\) is the same for all generations; its value is common knowledge.

Individual type is defined by an ability parameter \(a'_i\) which, combined with some education investment, \(e'_i\), supplied by altruistic parents, generates the level of human capital, \(h'_i\), with:

\[
h'_i = h(a'_i, e'_i).
\]

As already mentioned, we use a Cobb-Douglas function and then:

\[
h'_i = (a'_i)^\mu (e'_i)^{1-\mu} \quad 0 < \mu \leq 1.
\]

The distribution of \(a'_i\) is time invariant with unitary mean \(\bar{a}=1\) and variance \(\sigma^2_a\). If there is perfect correlation between the parent’s and child’s ability, both have the same type; otherwise, a child of type \(i\) does not necessarily inherit from a parent with the same productivity. We will denote this intergenerational correlation by \(\rho\).

Individuals preferences are represented by a log-linear utility function with three arguments: \(c'_i, d'_{t+1}, l'_{t+1}\), namely, first period consumption, second period consumption and total intended transfers to children. We write:

\[
U_i = \log c'_i + (1-\pi) \bar{\beta} \log d'_{t+1} + \gamma \log l'_{t+1}
\]

where \(\bar{\beta}\) and \(\gamma\) are parameters reflecting time preference and altruism respectively. For simplicity reasons, we use the notation \(\beta = \bar{\beta}(1-\pi)\), the product of time preference and survival probability.

In this setting financial bequests consist of an unintended part, the second period consumption of a parent who prematurely died, and an intended part, \(x_{t+1}\). There are two ways of bequeathing voluntarily: by investing in education, \(e'_{t+1}\), or by leaving a financial bequest, \(x_{t+1}\). Note that the argument of the utility function is \(x_{t+1}\), that is, after tax bequest, as we show below. An individual of type \(i\) and belonging to generation \(t\) receives from his parent \(e'_i\), which implies an effective wage of \(h'_i w_i\). He also receives \(b'_i = x'_i\) if his parent lives through the second period or \(b'_i = x'_i + d'_i\) if his parent dies. From now on, we will use a second superscript, \(j = 1, 2\), for this. Individuals are thus characterized by their ability, \(i\), their generation, \(t\), and whether or not they benefit from accidental bequest (\(j = 1\) or 2). The same individual intentionally leaves \(x'_{t+1}\) and \(e'_{t+1}\) taking into account the effects of these two transfers on the expected income of his child:

\[
l'_{t+1} = x'_{t+1} + \theta w_{t+1} h(e'_{t+1}, a'_{t+1})
\]
where \( \theta \) denotes the (subjective) weight given to human capital relative to physical capital. As a benchmark, \( \theta = 1 \), but we allow for the possibility that education receives more or less weight than physical bequest. In our formulation, individuals derive some joy of giving from intentional transfers, but not from an accidental bequest, if any. This is the consequence of our specification of paternalistic altruism.

The budget constraints for individuals belonging to generation \( t \) and type \( ji \) are simply:

\[
\omega_{it} = w_i h_{it} + b_{it} = c_{it} + s_{it} + e_{i+1}^{ji}
\]

and:

\[
R_{i+1} s_{it} = d_{i+1}^{ji} + x_{i+1}^{ji}
\]

where \( \omega_{it} \) is the lifetime income, and \( s_{it} \), savings. The subscript \( j = 1, 2 \) denotes whether or not there is an unexpected bequest. \( R_{i+1} \) is one plus the rate of interest. Both \( R \) and \( w \) are to be determined by the productive side of the model.

Although formally modeled as a two-period model, we have in fact three overlapping generations. In period \( t \) we have the working generation, \( t \), the surviving retired generation, \( t - 1 \), and the generation, \( t + 1 \), of children, who have a passive role and receive an amount, \( e_{i+1} \), of education spending from their parents.

**Figure 1: Intergenerational transfers**

<table>
<thead>
<tr>
<th>Period</th>
<th>t - 1</th>
<th>t</th>
<th>t +1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Generation</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>t - 1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>t</td>
<td>( x_t )</td>
<td>( d_t )</td>
<td>( s_{t+1} )</td>
</tr>
<tr>
<td>t +1</td>
<td>( e_{i+1} )</td>
<td>( d_{i+1} )</td>
<td></td>
</tr>
</tbody>
</table>

Figure 1 depicts the intergenerational flows \( x_t, e_t \) and \( d_t \) (with probability \( \pi \)). Education \( e_{i+1} \) is transferred to \( t + 1 \) generation at period \( t \), and bequest \( x_{i+1} \) is given at period \( t + 1 \).

**2.2. Taxes and Transfers**

Let us now introduce alternative taxes. First we have a wage tax, \( \tau_w \), and a capital income tax, \( \tau_c \). Then we have an estate tax that is denoted \( \tau_e \), but for the sake of presentation we also distinguish a tax on intended bequest \( \tau_e \) and a tax on unintended bequest, \( \tau_u \). The government also makes a uniform lump-sum transfer \( T \) to the young generation. There is no public debt: tax revenue finances this uniform transfer. We posit
time-invariant tax rates; only $T_t$ depends on time to satisfy the revenue constraint. We thus write:

\[ T_t = r_t \bar{b}_t^r \pi (\bar{b}_t^r + \bar{b}_t^s) + r_u w_t \bar{h}_t + r_t R_t K_t, \]  

(3)

where the upperbar denotes average values, and $K_t$, the stock of capital. With these tax parameters we rewrite the above individual budget constraints as:

\[
\omega_t^{ji} = b_t^{ji} + w_t (1 - r_t) h_t^{ji} + T_t = c_t^{ji} + x_t^{ji} + e_t^{ji} \\
R_{t+1} (1 - r_t) s_t^{ji} = d_t^{ji} + (1 - r_t) x_t^{ji} + e_t^{ji},
\]

where:

\[
b_t^{ji} = x_t^{ji} + \frac{d_t^{ji}}{1 - r_u} \quad \text{and} \quad b_t^2 = x_t^{ji}.
\]

Combining these two constraints, we obtain:

\[
\omega_t^{ji} = c_t^{ji} + c_t^{ji} + \frac{d_t^{ji}}{R_{t+1} (1 + r_t)} + x_t^{ji} (1 + r_t) \\
\omega_t^{ji} = c_t^{ji} + c_t^{ji} + x_t^{ji} + e_t^{ji}
\]

(4)

With these taxes, (2) becomes

\[
I_t^{ji} = x_t^{ji} + \theta w_{t+1} (1 - r_w) h_t^{ji} + e_t^{ji}
\]

(2A)

As we can see, the joy of giving relies on net-of-tax planned bequests and net-of-tax earnings.

We now consider the choice of an individual belonging to generation $t$, of type $i$, and either having or not having received an accidental bequest. It amounts to maximize (1), subject to (2A) and (3), with respect to $c_t^{ji}, d_t^{ji}, x_t^{ji},$ and $e_t^{ji}$. Assuming interior solutions for these four variables, we obtain the following demand and supply functions:

\[
c_t^{ji} = \frac{1}{1 + \beta + \gamma} \left[ \omega_t^{ji} + \theta \mu \frac{w_{t+1} q h_t^{ji}}{R_{t+1}} \right]
\]

(5)

\[
d_t^{ji} = \frac{\beta R_{t+1} (1 - r_t)}{1 + \beta + \gamma} \left[ \omega_t^{ji} + \theta \mu \frac{w_{t+1} q h_t^{ji}}{R_{t+1}} \right]
\]

\[
x_t^{ji} = \frac{\gamma R_{t+1}}{1 + \beta + \gamma} \left[ \omega_t^{ji} + \theta \mu \frac{w_{t+1} q h_t^{ji}}{R_{t+1}} \right] - z \theta w_{t+1} q h_t^{ji}
\]

(6)

\[
e_t^{ji} = \theta q w_{t+1} h_t^{ji} \frac{1 - \mu}{R_{t+1}} = d_t^{ji} \left( \theta q (1 - \mu) \frac{w_{t+1}}{R_{t+1}} \right)^{1/\mu}
\]

(7)

where $z = \frac{1 - r_x}{1 + r_x}$ and $q = \frac{1 - r_w}{\hat{r}}$. 

\[
\theta q w_{t+1} h_t^{ji} \frac{1 - \mu}{R_{t+1}} = d_t^{ji} \left( \theta q (1 - \mu) \frac{w_{t+1}}{R_{t+1}} \right)^{1/\mu}
\]
These two price parameters $z$ and $q$ can easily be interpreted. They incorporate the tax wedge that distorts the choice of intended bequests relative to consumption and the choice of education relative to intended bequest. The consumer’s price of intended bequest is $\frac{1 + \tau_z}{(1 - \tau_z) R} = \frac{1}{zR}$. Another way to express this is to say that the effective tax on intended bequests is $1 - z = \frac{\tau_z}{1 + \tau_z}$. In choosing between education and intended bequests (assumed to be positive), the parent equates their respective rate of return: $R_z$ and $w \frac{\partial h}{\partial e} (1 - \tau_w)$. The ratio of these rates of return is simply:

$$\frac{w \frac{\partial h}{\partial e} (1 - \tau_w)}{R_z} = q \frac{w \frac{\partial h}{\partial e}}{R}$$

With the log-linear utilities and Cobb-Douglas education function, $c$, $d$ and $e$ are necessarily positive. As to $x$, it could be negative; this is why we generally assume non-negative bequests. Here, to keep the problem simple, we assume that $x$ is positive. Later we provide the necessary condition for this to hold.

2.3. Production

The production sector is summarized by a profit maximizing firm with a Cobb-Douglas production function:

$$Y_t = AK_t^\alpha \overline{h}_t^{1-\alpha}$$

where $Y_t$ is aggregate output, $K_t$, the capital stock, and $\overline{h}_t$, aggregate human capital. Population $N$ is constant and normalized to 1. Consequently aggregate output and per capita output are equal. More generally, with this normalization the average and the aggregate values of variables such as saving, output, consumption, bequests and education can be used interchangeably.

We assume total depreciation after one period. Profit maximization implies:

$$R_t = \alpha Y_t / K_t \text{ and } w_t = (1 - \alpha) Y_t / \overline{h}_t$$

where $w_t$ is the wage rate per efficiency unit. For further use, we write:

$$k_t = K_t / \overline{h}_t$$

Capital accumulation with total depreciation is equal to aggregate saving:

$$K_{t+1} = \overline{s}_t$$
Both saving and human capital can be obtained from individual choices. We can show that saving is motivated by two objectives: second period consumption ($\beta$) and intended bequest ($\gamma$). Thus, using (5) and (6), we write:

$$s''_i = \frac{\beta + \gamma}{1 + \beta + \gamma} \left( \omega_i'' + \theta \mu \frac{W_{t+1}}{R_{t+1}} q h''_i + \theta w_{t+1} q h''_{t+1} \right) - \theta \frac{W_{t+1}}{R_{t+1}} q h''_{t+1}$$

Summing up over all individuals, $j_i$, we obtain:

$$K_{t+1} = \left( \frac{\beta + \gamma}{1 + \beta + \gamma} \right) \left( \omega_i'' + \theta \mu \frac{W_{t+1}}{R_{t+1}} q h''_i + \theta w_{t+1} q h''_{t+1} \right) - \theta \frac{W_{t+1}}{R_{t+1}} q h''_{t+1}$$

In the same way, we aggregate education and then the resulting human capital. We use (7) to obtain:

$$\bar{e}_{t+1} = \left( \theta q (1-\mu) \frac{1-\alpha}{\alpha} k_{t+1} \right)^{1/\mu}$$ (8)

and hence:

$$\bar{h}_{t+1} = \left( \frac{1-\alpha}{\alpha} (1-\mu) \right)^{1/\mu} \left( \frac{1-\alpha}{\alpha} (1-\mu) \right)^{1/\mu}$$ (9)

It is important to understand the dynamics of this model. At the start of period $t$, an individual of productivity $i$ inherits either $b^{1i}$ or $b^{2i}$ depending on whether or not his parent belonging to generation $t-1$ and being of type $i$, dies prematurely. In other words, it is important to distinguish $(i, t-1)$ from $(i, t)$.

### 2.4. Bequests and Lifetime Income

Now we introduce the two types of bequests. In the case of the early death of his parents of ability $i$, a child inherits:

$$b^{1i}_{t+1} = x_{t+1} + \frac{d_{t+1}}{1 + r_u}$$

$$= z R_{t+1} \gamma + \beta \varphi \left( \omega_i'' + \theta \mu \frac{W_{t+1}}{R_{t+1}} q h''_i + \theta w_{t+1} q h''_{t+1} \right) - z \theta q w_{t+1} h''_{t+1}$$ (10.1)

In the case of late death, inheritance is exclusively intended:

$$b^{2i}_{t+1} = x_{t+1}$$

$$= z R_{t+1} \gamma + \beta \varphi \left( \omega_i'' + \theta \mu \frac{W_{t+1}}{R_{t+1}} q h''_i + \theta w_{t+1} q h''_{t+1} \right) - z \theta q w_{t+1} h''_{t+1}$$ (10.2)

where $\varphi = \frac{1 + r_u}{1 + r_u} = 1$ when the two types of bequests are undistinguished ($r_u = r_h$).
As already mentioned, we assume that $x_{i,t} > 0$. For further reference, we now show the average levels of bequests. Making use of (7), they can be expressed as:

$$\overline{b}_{t+1} = zR_{t+1} \frac{\gamma + \beta \varphi}{\gamma + \beta} \left[ K_{t+1} + \theta \frac{w_{t+1}}{R_{t+1}} q \overline{h}_{t+1} \right] - z\theta q w_{t+1} \overline{h}_{t+1}$$

and:

$$\overline{b}_{t+1} = \overline{b}_{t+1} - zR_{t+1} \frac{\beta \varphi}{\gamma + \beta} \left[ K_{t+1} + \theta \frac{w_{t+1}}{R_{t+1}} q \overline{h}_{t+1} \right]$$

From the Cobb-Douglas assumption, $RK = \alpha Y$ and $w\overline{h} = (1-\alpha)Y$.

Thus:

$$\overline{b}_{t+1} = \alpha z \frac{\beta \varphi + \gamma}{\beta + \gamma} Y_{t+1} - \frac{\beta (1-\varphi)}{\beta + \gamma} (1-\tau_w) (1-\alpha) Y_{t+1} \theta$$

and:

$$\overline{b}_{t+1} = \alpha z \frac{\gamma}{\beta + \gamma} Y_{t+1} - \frac{\beta (1-\tau_w) (1-\alpha) Y_{t+1} \theta}{\beta + \gamma}$$

As we assume that $\overline{b}_{t+1} = \overline{x}_{t+1} > 0$, we have: $\frac{\alpha \gamma}{1-\alpha} > \beta q \theta$.

Depending on the death of his parent, a child of ability $i$ will have an income $\omega_{i,t}^1$ or $\omega_{i,t}^2$. More precisely, making use of (10.1) and (10.2):

$$\omega_{i,t}^1 = zR_i \frac{\gamma + \beta \varphi}{1+\beta + \gamma} \left[ \omega_{i,t-1} + \theta \mu \frac{w_i}{R_i} q h_i \right] + (1-\theta) z q w_i h_i + T_i$$

and:

$$\omega_{i,t}^2 = zR_i \frac{\gamma}{1+\beta + \gamma} \left[ \omega_{i,t-1} + \theta \mu \frac{w_i}{R_i} q h_i \right] + (1-\theta) z q w_i h_i + T_i$$

In aggregate terms, we write:

$$\overline{\omega}_t = \pi \overline{\omega}_t^1 + (1-\pi) \overline{\omega}_t^2 = \overline{b}_t^1 + \pi (\overline{b}_t^1 - \overline{b}_t^2) + (1-\tau_w) w_i \overline{h}_i + T_i$$

$$= (1-\tau_w) \overline{b}_t^1 + \pi (1-\tau_w) (\overline{b}_t^1 - \overline{b}_t^2) + w_i \overline{h}_i + \tau R_i K_i$$

$$= Y_i \left[ 1 - \frac{\beta}{\beta + \gamma} (1-\tau_w) (1-\tau_c) (\alpha + (1-\alpha) q \theta) \right]$$

In the second equation we use (3). It is interesting to observe that average lifetime income and average output do not coincide whether with or without taxation. Without tax, and with $\theta = 1$, we have:

$$\overline{\omega}_t = Y_i \left( \frac{\gamma + \beta \varphi}{\gamma + \beta} \right) < Y_i$$
Another way of presenting this difference is to write:

\[
\bar{\omega}_t = Y_t - (1-\pi)\bar{d}_t < Y_t
\]

where \( \bar{d}_t \) is the average consumption of the older generation at period \( t \). It should be noted that in the present model, lifetime income is income accruing to the younger generation.

2.5. Capital Accumulation

With full depreciation, saving is equal to the capital stock used in the next period.

\[
s_t = K_{t+1} = \frac{\beta + \gamma}{1+\beta + \gamma} \bar{\omega}_t - \left(1 - \frac{\beta + \gamma}{1+\beta + \gamma} \right) \theta q \frac{1-\alpha}{\alpha} K_{t+1}
\]

Substituting (13), we establish the following:

\[
K_{t+1} \left[1 + \frac{1-\alpha}{\alpha} \theta q \left(1 - \frac{\beta + \gamma}{1+\beta + \gamma} \right)\right] = \frac{Y_t}{1+\beta + \gamma} \left[\beta + \gamma - \beta(1-\pi)(1-\tau_r)\left(\alpha+(1-\alpha)q\theta\right)\right]
\]

(14)

2.6. Steady State

We now turn to the steady-state solutions to which the economy converges. Dropping the time index \( t \), we rewrite (9) and (14):

\[
\bar{h} = \left[\frac{1-\alpha}{\alpha} (1-\mu)\right]^{1-\mu} \frac{1-\mu}{\theta q} \frac{1-\mu}{\mu} k^{1-\mu}
\]

(15)

\[
k^{1-\alpha} = \frac{A}{1+\beta + \gamma} \frac{\beta + \gamma - \beta(1-\pi)(1-\tau_r)\left(\alpha+(1-\alpha)q\theta\right)}{1 + \frac{1-\alpha}{\alpha} \theta q \left(1 - \frac{\beta + \gamma}{1+\beta + \gamma} \right)}
\]

(16)

From (16) we have \( k=k(q, \tau_r) \) with \( \frac{\partial k}{\partial \tau_r} > 0 \) and \( \frac{\partial k}{\partial q} < 0 \). This in turn implies that \( \frac{\partial R}{\partial \tau_r} < 0 \) and \( \frac{\partial R}{\partial q} > 0 \).

We then obtain output in the steady-state:

\[
Y = A k^\alpha \bar{h} = A \left[\frac{1-\alpha}{\alpha} (1-\mu)\theta q\right]^{\frac{1-\mu}{\mu}} \frac{1-\mu}{\mu} + \alpha
\]

(17)
For further use, let us differentiate \( Y = \text{constant} + \frac{1-\mu}{\mu} \log q + \left( \alpha + \frac{1-\mu}{\mu} \right) \log k \) with respect to \( q \). This yields:

\[
\frac{d \log Y}{dq} = \frac{1-\mu}{\mu q} - \left( \frac{1}{\mu} \right) \left\{ \frac{\beta(1-\pi)(1-\tau) \theta}{\beta + \gamma - \beta(1-\pi)(1-\tau)(\alpha + (1-\alpha) \theta q)} \right. \\
+ \left. \frac{\theta \left( 1 - \frac{\beta + \gamma}{1 + \beta + \gamma} \right)}{\alpha + (1-\alpha) \theta \left( 1 - \frac{\beta + \gamma}{1 + \beta + \gamma} \right) q} \right\}
\]

Assuming that \( \log Y \) is strictly concave, \( Y \) is a single-peaked function of \( q \) with maximum at \( q^* \). We thus have:

\[
Y = Y(q, \tau, \ldots)
\]

The intuition is straightforward. For \( q < q^* \), enhancing human capital relative to physical capital is desirable; it is growth-promoting to raise \( q \), by lowering \( \tau_w \) relative to \( \tau_x \) and \( \tau_r \). The opposite occurs once we reach \( q > q^* \). The reason why capital tax \( \tau_r \) raises \( Y \) is that in the present model we suppose that the collected tax revenue is transferred to the savings of the younger generation.

2.7. Coefficient of variation

We now turn to the coefficient of variation (CV) of the lifetime income which is going to be our measure of inequality. It is derived in the Appendix.\(^5\) We can establish that the CV unambiguously depends on our five tax parameters as follows:

\[
CV(\omega) = CV(q, \tau, \phi, z)
\]

where \( z = \frac{1 - \tau_r}{1 + \tau_r} \), \( q = \frac{1 - \tau_w}{z} \) and \( \phi = \frac{1 + \tau_r}{1 + \tau_w} \).

Recall that \( z \) represents the net-of-tax price of planned bequests. \( q \) denotes the relative net-of-tax price of earnings relative to planned bequests including the tax rates. When these rates cannot be distinguished, \( \phi = 1 \). Increasing these price terms \( q, \phi, z \) exacerbates lifetime income inequality.

---

\(^5\) For reasons of analytical simplicity and without loss of generality, we use the square of the coefficient of variation in the Appendix.
3. The Incidence of Taxes on Welfare

To assess the effect of these tax parameters on welfare and not just on inequality, we need to know their impact on per capita income. We have seen that $Y = Y(q, \tau_r)$. The effect of $q$ on average income depends on whether $q > q^*$ (see 2.6). For tractability reasons we did not use a social welfare function. We can, however, talk of an unambiguous increase in welfare if we have an increase in $Y$ combined with a decrease in $CV$.

Table 1: Welfare effect of price parameters

<table>
<thead>
<tr>
<th>Effect of an increase of</th>
<th>$q$</th>
<th>$\tau_r$</th>
<th>$\varphi$</th>
<th>$z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>on $CV$</td>
<td>(+)</td>
<td>(-)</td>
<td>(+)</td>
<td>(+)</td>
</tr>
<tr>
<td>on $Y$</td>
<td>(+/-)</td>
<td>(+)</td>
<td>(0)</td>
<td>(0)</td>
</tr>
<tr>
<td>on social welfare</td>
<td>(?/-)</td>
<td>(+)</td>
<td>(-)</td>
<td>(-)</td>
</tr>
</tbody>
</table>

Table 1 gives the direct effects of price parameters. For example, we observe that the direct effect of an interest income tax increase improves welfare, but there are indirect effects on $z$ and $q$ that can change this conclusion. What is clear is that a relative increase in the tax on unplanned bequests ($\varphi$ going down) improves welfare. Our approach differs from the standard optimal taxation approach in two respects. First, as in tax reform work we are concerned by marginal and local results and not by global ones. Second, we assess the desirability of these local moves by their effect not on social welfare but on both the coefficient of variation and the mean of lifetime income.

Unambiguity with respect to $q$, $\tau_r$, $\varphi$ and $z$ does not mean unambiguity towards the tax rates themselves as illustrated in Table 2. The effect of a tax on unintended bequests is not surprising. That of a wage tax is due to the absence of labor supply distortion. As to the two other taxes, their ambiguous incidence can be explained by the fact that they intervene at different levels. Finally, we consider the case where $\varphi = 1$ or $\tau_u = \tau_r = \tau_b$. In that case, we have:

$$\frac{\partial CV}{\partial \tau_b} = \left[ \frac{\partial CV}{\partial q} \frac{1 - \hat{\tau}}{z^2} - \frac{\partial CV}{\partial \varphi} \right] \frac{1 - \tau_r}{(1 + \tau_b)^2} > ou < 0$$

The effect of such a tax is still ambiguous.

Note that if we assume away human capital formation, all these taxes would have no effect on the capital stock, and thus on $Y$ (see Michel and Pestieau (2002)). Introducing human capital formation, it is clear that a tax on earnings discourages education spending and a tax on both capital income and unintended bequests induces a substitution in favor of education. Table 2 summarizes these findings.
Table 2: Welfare effect of alternative taxes

<table>
<thead>
<tr>
<th>Effect of an increase of</th>
<th>$\tau_w$</th>
<th>$\tau_r$</th>
<th>$\tau_u$</th>
<th>$\tau_x$</th>
<th>$\tau_b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>on $CV$</td>
<td>(-)</td>
<td>?</td>
<td>(-)</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>on $Y$</td>
<td>$q &lt; q^*$</td>
<td>(-)</td>
<td>(+)</td>
<td>0</td>
<td>(+)</td>
</tr>
<tr>
<td></td>
<td>$q &gt; q^*$</td>
<td>(+)</td>
<td>(+)</td>
<td>0</td>
<td>(-)</td>
</tr>
<tr>
<td>on $SW$</td>
<td>$q &lt; q^*$</td>
<td>?</td>
<td>?</td>
<td>(+)</td>
<td>?</td>
</tr>
<tr>
<td></td>
<td>$q &gt; q^*$</td>
<td>(+)</td>
<td>?</td>
<td>(+)</td>
<td>?</td>
</tr>
</tbody>
</table>

We observe a number of ambiguous cases. The positive effect of $\tau_u$ on welfare is not surprising. All the others are ambiguous. Yet from Table 1 we know that welfare can always be improved by using a combination of instruments. For example, assume $q < q^*$. Then raising $q$ is growth-enhancing. This, however, raises $CV$. So we need to lower $z$ to achieve $dCV < 0$. Increasing $q$ and decreasing $z$ is possible by raising $\tau_x$ and adjusting $\tau_w = 1 - zq$ so that $d\tau_w = -qdz - zdq$ given $\tau_r$.

We can write the following proposition:

**Proposition 1.** Let $\tau_x = \tau_u = \tau_b$. We can make tax reforms so as to increase $Y$ but decrease $CV$ by manipulating $q$ and $z$, and therefore by changing $\tau_b$ and $\tau_w$ as follows:

(a) $Y_q(q, \tau_r) \, dq > 0$ with $dq \geq 0 \iff q \geq q^*$

(b) $CV_q(q, z, \tau_r, \varphi) \, dq + CV_z \, dz < 0$ or $dz < -\frac{CV_q}{CV_z} dq$,

where $d\tau_w = -zdq - zdq$ and $d\tau_b = -\frac{1 - \tau_r}{z^2}$, from the definition of $q$ and $z$: $q = \frac{1 - \tau_w}{z}$ and $z = \frac{1 - \tau_r}{1 + \tau_r}$.

To further our interpretation of tax incidence we now consider two simple cases.

We first observe the following:
- $\tau = 0$ or 1 means that there is no uncertainty on longevity and thus no accidental bequest.
- $\theta = 0$ means that transferring human capital does not generate any joy of giving. This assumption is equivalent to $\mu = 1$ (education has no effect on human capital).
- $\varphi = 0$ means that there is no intergenerational correlation of ability.
- $\sigma_a = 0$ means that everyone has the same capacity towards the human capital technology.

**Case 1:** $\theta = 0$ and $\mu = 1$.

In that case $k$ and $y$ do not depend on $q$ but only on $\tau_r$. We assume that $\varphi = 1$, and thus we have $\frac{\partial CV}{\partial \tau_b} < 0$, $\frac{\partial CV}{\partial \tau_w} < 0$, $\frac{\partial CV}{\partial \tau_r} < 0$. These signs can be obtained from (A.4) in the Appendix.
Henceforth, those three taxes have a positive effect on equality. The positive role of \( \tau_w \) depends on \( \sigma_a^2 > 0 \) (and on \( \theta \)). The positive effect of either \( \tau_r \) or \( \tau_b \) is independent of \( \sigma_a^2 \) and \( \theta \). When \( \sigma_a^2 = 0 \), the second and third terms of the RHS of (A.4) vanish. This is the case studied by Michel and Pestieau (2002).

**Case 2:** \( \pi = 0, \xi = 0 \)

There are no accidental bequests and the source of inequality is \( \sigma_a^2 \).

If we assume that \( \frac{\partial (zR)}{\partial z} < 0 \) which is possible,\(^6\) then \( \frac{\partial CV}{\partial \tau_r} > 0 \) and \( \frac{\partial CV}{\partial \tau_b} > 0 \) as shown in (A.4) in the Appendix. If, furthermore, \( q > q^* \), we have the paradoxical case of a tax on bequests that increases income inequality and decreases average income.

### 4. Conclusion

We have studied the incidence of alternative taxes on the steady-state coefficient of variation of lifetime income and on average production in an overlapping generations model with two types of bequests, accidental and planned, and two types of planned transfers, physical and human capital.

In spite of our very simple setting (Cobb-Douglas production function and logarithmic utilities), we only get unambiguous results for the wage tax and for an estate tax restricted to accidental bequests. A tax on interest income and a tax on planned bequests have an ambiguous incidence on the coefficient of variation. Ambiguity results from the tax-induced substitution between the education and intended bequest.

Finally our model rests on two key assumptions. The first is the welfare criterion used, namely the minimization of the coefficient of variation. Even though in a static framework there is a close relation between maximizing a utilitarian social welfare function and minimizing the coefficient of variation, this is not clear in a dynamic framework. We also look for the conditions under which average income is increasing and inequality is decreasing. This approach is surely more acceptable, but it is also highly demanding.

The second assumption is that of logarithmic preferences implying identical substitution between \( c \) and \( d \) on the one hand and between \( d \) and \( x \) on the other hand. Empirically it seems that the substitutability between \( c \) and \( d \) is much lower than that between \( d \) (or \( c \)) and \( x \). We plan in future work to adopt a truly normative approach and to use a more general utility function.

Going back to the observed trend towards relying less and less on inheritance taxation, our paper shows that one can most often find a combination of inheritance taxes and other taxes that decreases inequality and even increases welfare. In other words there is no good economic reason for depriving one-self from such policy tools.

\(^6\) To obtain this result, we need

\[
\frac{\mu (\beta + \gamma)}{1 + \beta + \gamma} (\alpha + \beta (1 - \pi) (1 - \tau_r) (1 - \alpha) \theta q) (\alpha + \beta (1 - \pi) (1 - \tau_r) (1 - \alpha) \theta q) < \beta (1 - \pi) (1 - \tau_r) (\alpha + (1 - \alpha) \theta q) (1 - \beta (1 - \pi) (1 - \tau_r) (1 - \alpha) \theta q)
\]
Appendix

Determination of the coefficient of variation

From (10.1), (10.2) and (12.1), (12.2), we write:

\[ \bar{w}_{t+1} - \bar{w}_{t+1}^2 = \bar{b}_{t+1} - \bar{b}_{t+1}^2 = \frac{\beta \varphi}{\beta + \gamma} (az + (1 - \tau_w)(1 - \alpha) \theta) T_{t+1} \]

We also compute the deviations from the mean:

\[ \omega_{t+1}^1 - \bar{w}_{t+1}^1 = z R_{t+1} \left( \frac{\gamma + \beta \varphi}{1 + \beta + \gamma} \right) \left( \omega_{t+1}^1 - \bar{w}_{t+1}^1 \right) + w_{t+1} \left( 1 - \theta + \theta \mu \frac{\gamma + \beta \varphi}{1 + \beta + \gamma} \right) \left( \omega_{t+1}^1 - \bar{w}_{t+1}^1 \right) \bar{h}_{t+1} \]

\[ \omega_{t+1}^2 - \bar{w}_{t+1}^2 = z R_{t+1} \left( \frac{\gamma + \beta \varphi}{1 + \beta + \gamma} \right) \left( \omega_{t+1}^2 - \bar{w}_{t+1}^2 \right) + w_{t+1} \left( 1 - \theta + \theta \mu \frac{\gamma + \beta \varphi}{1 + \beta + \gamma} \right) \left( \omega_{t+1}^2 - \bar{w}_{t+1}^2 \right) \bar{h}_{t+1} \]

From these deviations, we calculate the variance of \( \omega_{t+1}^1 \):

\[ \text{Var} (\omega_{t+1}^1) = \pi E (\omega_{t+1}^1 - \bar{w}_{t+1}^1)^2 + (1 - \pi) E (\omega_{t+1}^2 - \bar{w}_{t+1}^2)^2 + \left[ \pi (1 - \pi)^2 + (1 - \pi) \pi^2 \right] (\bar{w}_{t+1} - \bar{w}_{t+1}^2)^2 \]

Using the above expressions, we obtain:

\[ \text{Var} (\omega_{t+1}^1) = \text{Var} (\omega_{t+1}^1) \left( \frac{z R_{t+1}^2}{1 + \beta + \gamma} \right) \left( \pi (\gamma + \beta \varphi)^2 + (1 - \pi) \gamma^2 \right) + \sigma^2 \left( 1 - \alpha \right)^2 (1 - \tau_w)^2 Y_{t+1} \]

\[ + \pi \left( 1 - \theta + \theta \mu \frac{\gamma + \beta \varphi}{1 + \beta + \gamma} \right) \left( 1 - \theta - \theta \mu \frac{\gamma + \beta \varphi}{1 + \beta + \gamma} \right)^2 \]

\[ + 2z R_{t+1} (1 - \tau_w) (1 - \alpha) Y_{t+1} \text{ cov} (\omega_{t+1}^1, \omega_{t+1}^2) \left( \frac{\theta}{1 + \beta + \gamma} \right) \]

\[ + \pi (1 - \pi) \left( \frac{\beta \varphi}{\beta + \gamma} \right)^2 (az + (1 - \tau_w)(1 - \alpha) \theta) Y_{t+1} \]

In this expression we used the following results:

\[ \bar{w}_{t+1} = Y_{t+1} \left[ 1 - \frac{\beta}{\beta + \gamma} (1 - \pi)(1 - \tau_w)(\alpha + (1 - \alpha)q \theta) \right] \]

and:

\[ \omega_{t+1}^1 = \frac{z R_{t+1}^2}{1 + \beta + \gamma} \left( \pi \gamma^2 + (1 - \pi) \gamma^2 \right) + \sigma^2 \left( 1 - \alpha \right)^2 (1 - \tau_w)^2 Y_{t+1} \]

\[ + \pi \left( 1 - \theta + \theta \mu \frac{\gamma + \beta \varphi}{1 + \beta + \gamma} \right) \left( 1 - \theta - \theta \mu \frac{\gamma + \beta \varphi}{1 + \beta + \gamma} \right)^2 \]

\[ + 2z R_{t+1} (1 - \tau_w) (1 - \alpha) Y_{t+1} \text{ cov} (\omega_{t+1}^1, \omega_{t+1}^2) \left( \frac{\theta}{1 + \beta + \gamma} \right) \]

\[ + \pi (1 - \pi) \left( \frac{\beta \varphi}{\beta + \gamma} \right)^2 (az + (1 - \tau_w)(1 - \alpha) \theta) Y_{t+1} \]
$\text{cov}(\omega_{t+1}, \omega_{t+1}) = zR_{t+1} \frac{\gamma + \pi \beta \varphi}{1 + \beta + \gamma} \rho \text{cov}(\omega_t, \omega_t)$

\begin{equation}
+ (1 - \tau_u)(1 - \alpha) \sigma_a^2 Y_{t+1} \left(1 - \theta + \theta \mu \frac{\gamma + \pi \beta \varphi}{1 + \beta + \gamma}\right)
\end{equation}

where $\rho$ is the correlation between $\omega_t$ and $\omega_{t+1}$, and $\sigma_a^2$ is the variance of $\omega_t$, which is time invariant.

In the steady-state, we can write:

$$
\text{cov}(\omega_t, \omega_t) = \frac{1 - \theta + \theta \mu \frac{\gamma + \pi \beta \varphi}{1 + \beta + \gamma}}{1 - \rho \frac{\gamma + \pi \beta \varphi}{1 + \beta + \gamma} zR} (1 - \tau_u)(1 - \alpha) \sigma_a^2 Y
$$

Hence we have:

$$
\frac{\text{Var}(\omega)}{Y^2} \left[1 - \left(\frac{zR}{1 + \beta + \gamma}\right) (\pi (\gamma + \beta \varphi)^2)\right] =
$$

$$
\sigma_a^2 (1 - \alpha)^2 (1 - \tau_u)^2 \left[\pi \left(1 - \theta + \theta \mu \frac{\gamma + \beta \varphi}{1 + \beta + \gamma}\right)^2 + (1 - \pi) \left(1 - \theta + \theta \mu \frac{\gamma}{1 + \beta + \gamma}\right)^2\right]
$$

$$
+ \pi(1 - \pi) \left(\frac{\beta \gamma}{1 + \beta + \gamma}\right) (z \alpha + (1 - \tau_u)(1 - \alpha) \theta)^2 + 2 \frac{\rho zR}{1 + \beta + \gamma} (1 - \tau_u)^2 (1 - \alpha)^2 \sigma_a^2
$$

$$
\frac{1 - \theta + \theta \mu \frac{\gamma + \pi \beta \varphi}{1 + \beta + \gamma}}{1 - \rho \frac{\gamma + \pi \beta \varphi}{1 + \beta + \gamma} zR} \left[\pi (\gamma + \beta \varphi) \left(1 - \theta + \theta \mu \frac{\gamma + \beta \varphi}{1 + \beta + \gamma}\right) + (1 + \pi) \gamma \left(1 - \theta + \theta \mu \frac{\gamma}{1 + \beta + \gamma}\right)\right]
$$

(A.3)

To obtain the square of the coefficient of variation, we substitute (A.1) in (A.3):

$$
\text{CV}(\omega) \left[1 - \frac{\beta}{\beta + \gamma} (1 - \pi)(1 - \tau) (\alpha + (1 - \alpha) q \theta)^2\right]
$$

$$
\left[1 - \left(\frac{zR}{1 + \beta + \gamma}\right)^2 (\pi (\gamma + \beta \varphi)^2 + (1 - \pi) \gamma^2)\right] =
$$

$$
\sigma_a^2 (1 - \alpha)^2 (1 - \tau_u)^2 \left[\pi \left(1 - \theta + \theta \mu \frac{\gamma + \beta \varphi}{1 + \beta + \gamma}\right)^2 + (1 - \pi) \left(1 - \theta + \theta \mu \frac{\gamma}{1 + \beta + \gamma}\right)\right]
$$
After some manipulations we obtain an expression for the coefficient of variation of $\omega$ as a function of policy variables and $R$, itself a function of the policy variables $(q, \tau_r)$. We denote the RHS of (A.4) by $\psi$ and the LHS after $CV(\omega)$ by $\Delta$. Then we have:

$$CV(\omega) = \Delta(q, \tau_r, \varphi, z)$$

We thus observe that these five parameters have an unambiguous effect on $CV$:

$$CV(\omega) = CV\left(\frac{q, \tau_r, \tau_w, \varphi, z}{+ - - + +}\right)$$

Making use of $\tau_w = 1 - qz$, the above $CV$ can be further reduced to:

$$CV(\omega) = CV\left(\frac{q, \tau_r, \varphi, z}{+ - + +}\right)$$

References


