Effects of Transboundary Stock Pollution  
on the Mode of International Competition  

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Abstract  

This paper looks into potential determinants of the mode of international competition in a polluting good market by focusing on a strategic interaction between two environmentally concerned governments. From the analysis of our model based on a simple international duopoly model with transboundary stock pollution, we show how the resulting form of international competition depends on the magnitudes of the transboundary impacts of pollutant emissions and the decay rates of pollutant stocks in respective countries.

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\textit{Keywords:} transboundary pollution, stock pollution, international duopoly, endogenous timing

1. Introduction

Environmental considerations are becoming increasingly important in recent negotiations over trade liberalization. In political debates on NAFTA, environmental protection was advocated primarily against freer trade in North America. Some environmentalists persistently make similar arguments against globalization, of which a symbolic manifestation is their oppositions towards the WTO Round Talk in Seattle in 1999.

Reflecting these facts, the “trade and the environment” interrelationship has been attracting greater attention in economics. This growing literature provides many insights into different aspects of this relationship.\textsuperscript{1} The possible existence of gains from international trade in a polluting product depends on the market structure as well as the preferences of citizens. On top of these, when the pollution issue is transboundary

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\textsuperscript{1} Ulph (1997), Copeland and Taylor (2003), and Rauscher (2005) provide summaries of these findings.
by nature, the welfare impact of international trade could also depend on the physical characteristics of a pollution issue.

As for an international polluting oligopoly model, existing works mainly address the roles of environmental policies by taking market structures as exogenous (Barrett, 1994, Kennedy, 1994, and Ulph, 1996). Instead, this paper focuses on a governmental decision making that determines the possible mode of international duopoly in the presence of transboundary stock pollution. To this end, we employ a variation of an endogenous timing game developed by Hamilton and Slutsky (1990). In particular, we show that free trade under both Stackelberg and Cournot competition modes, as well as autarky, are possible equilibrium outcomes. The actual outcome is influenced by the magnitudes of not just economic variables, such as discount rates and the marginal damage cost of pollution, but also environmental variables, such as the transboundary impacts of pollutant emissions and the decay rates of pollutant stocks in respective countries. Especially, we have found that the Stackelberg competition can emerge as a subgame-perfect equilibrium outcome even between two completely symmetric countries, while this is not the case in the absence of transboundary pollution. This result suggests that transboundary pollution might make it contentious to take the potential mode of international competition for granted too readily, even for two identical nations.

This paper is structured as follows. Section 2 presents our basic model of two non-trading economies with transboundary stock pollution and derives its autarkic outcome. By extending the model to a world with two trading economies, the next section characterizes potential free trade outcomes in two different modes of international competition. Then, Section 4 describes a game between two governments and discusses its equilibrium results. The last section contains brief concluding remarks.

2. Autarky

This section develops our basic model and describes its autarkic outcome. The model consists of two countries (Home and Foreign), two goods (Goods 1 and 2), and one primary factor (labor). We assume that both countries share the identical preferences and production technologies, and produce both goods. In the following description of our model, we focus on Home since the Foreign country can be described symmetrically. We denote each Foreign variable by attaching an asterisk (*). Furthermore, Good 2 serves as a numeraire and is produced with a unitary input coefficient so that the wage rate is internationally equalized and fixed at unity. In the autarkic case, Good 1 is monopolistically supplied by a single domestic firm and \(c > 0\) units of labor are required to produce one unit of Good 1. Hence, the marginal cost of production is assumed to be constant at \(c\).

Now, let us assume that the intertemporal utility function of a representative consumer in Home is given by

\[V(t) = \max_{c(t)} \left\{ \int_{t}^{\infty} u(c(t), c(t+\delta)) e^{-\delta r} d\delta \right\}
\]

where \(u(c, s)\) is the utility function, \(c(t)\) is the consumption level at time \(t\), and \(r\) is the discount rate.

Syropoulos (1994), Ohkawa, Okamura and Tawada (2001), and Supasri and Tawada (2007) apply similar endogenous timing frameworks to trade policy issues. Our model differs from theirs in that the outcome of a governmental interaction dictates the roles of oligopolists in an international market.
$V = \int_0^\infty e^{-rt} \left[ AC_1 - \frac{C_1^2}{2} + C_2 - sZ \right] dt,$  \hspace{1cm} (1)

where $V$ is the consumer’s utility level, $r$ is its discount rate, $C_i, i=1,2$, is the consumption of each good, $A$ is a positive parameter with $A > c$, $s$ is the constant marginal damage cost of its pollutant stock, and $Z$ is the pollution stock level in this country. We suppose that the generation of the pollutant emissions associated with the production of Good 1 is treated as a negative externality by this consumer and, therefore, out of its control. Hence, the consumer optimizes his or her instantaneous utility by ignoring the adverse effects of the pollution completely.

Letting $p$ denote the price of Good 1 measured by the price of Good 2, this consumer’s utility maximization problem subject to the budget constraint yields the demand function of Good 1 as $C_1 = A - p$. Under autarky, the market-clearing condition is $A - p = x$, where $x$ is the output of Good 1. Hence, the inverse demand function and the monopoly firm’s profit at each time instant, $\pi$, are respectively

$$p = A - x,$$  \hspace{1cm} (2)

$$\pi = (A - c - x) x.$$  \hspace{1cm} (3)

Using (2) and (3), the long-term social welfare of the nation, $U$, which is the sum of the consumer surplus, the monopolist’s profit, and the environmental damage cost of the pollution over the infinite time horizon, can be expressed as

$$U = \int_0^\infty e^{-rt} \left[ \frac{(A-p)^2}{2} + \pi - sZ \right] dt.$$  \hspace{1cm} (4)

In the subsequent analysis, (4) gives the payoff of the government under respective situations.

As for the pollution issue, we suppose that the production of one unit of Good 1 emits one unit of pollutant and, furthermore, that one unit of domestic pollutant emissions causes a unit increase in the domestic pollutant flow. Concerning the transboundary effects of the pollutant emissions, we assume that Foreign’s (resp. Home’s) pollutant emissions $x^*$ (resp. $x$) increase Home’s (resp. Foreign’s) current pollutant flow by $\alpha x^*$ (resp. $\alpha^* x$), where $\alpha \in [0, 1]$ and $\alpha^* \in [0, 1]$ are parameters which we call ‘pollutant import coefficients’ in order to stress the directions of the pollutant flow. In the cases of global pollutants, such as CO$_2$ that is a culprit of the global warming problem, we have $\alpha = \alpha^* = 1$, while both $\alpha$ and $\alpha^*$ are generally strictly smaller than one and take different values in so-called regional environmental issues, such as the transboundary acid rain problem in Northern Europe. When $\alpha = \alpha^* = 0$, on the other hand, the export of a pollutant to another country (or the import by one country of another’s pollutant) is completely void.

Indeed, it is possible to have $\alpha > 1$, for instance, when a firm is located close to the border and the wind is blowing quite predominantly toward the other country. However, since the inclusion of such a possibility provides no significant additional insight into the outcome, we simply restrict our attention to $\alpha \in [0, 1]$.\footnote{Indeed, it is possible to have $\alpha > 1$, for instance, when a firm is located close to the border and the wind is blowing quite predominantly toward the other country. However, since the inclusion of such a possibility provides no significant additional insight into the outcome, we simply restrict our attention to $\alpha \in [0, 1]$.}
hand, the pollution problem is completely localized. In sum, the pollution levels in the respective countries are described as

$$\dot{Z} = x + \alpha x^* - kZ,$$

(5)

$$\dot{Z}^* = x^* + \alpha^* x - k^* Z^*,$$

(6)

where $\dot{Z}$ and $\dot{Z}^*$ respectively denote the time derivatives of the pollutant stocks, and $k$ and $k^*$ are the decay rates of the pollutant stocks in the respective countries. In this article, we assume that, while the preference-related variables of the consumers, such as discount rates and marginal costs of the pollution, and firms’ production costs are exactly symmetric across the two countries, the environmental variables, i.e., pollutant import coefficients and decay rate of the pollutants, can take different values. Hence, we mainly examine the impacts of the latter variables on the equilibrium outcomes.

Let us now formulate the optimization problem of each country’s firm.⁴ Again, we focus on the Home firm, since its Foreign counterpart acts in exactly the same fashion. Specifically, the Home firm solves the following problem in the autarkic case:

$$\max_x (A - c - x)x,$$

whose solution can be immediately obtained as

$$x^A = \frac{A - c}{2},$$

(7)

where the superscript $A$ represents the autarkic outcome. Also, the autarkic price is derived from the demand function as

$$p^A = \frac{A + c}{2}.$$  

(8)

Substituting (7) into (5), we have⁵

$$\dot{Z}^A = (1 + \alpha)\frac{A - c}{2} - kZ^A,$$

where $Z^A$ is the pollutant stock under autarky. This is a simple first-order linear ordinary differential equation, which can be solved easily and yields the path of the pollutant stock over time in Home as

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⁴ For the simplicity of exposition, we write the behavior of the firms in both countries in a completely static fashion although the firms’ actual behaviors would not change at all if they maximized their respective profits dynamically since the firms do not care about the stock pollution issue, quite similarly to the consumer who regards the pollution problem as an externality.

⁵ Note that the firm in Foreign produces the same amount of Good 1 as Home’s firm in the autarkic case. This is due to our symmetry assumptions on the characteristics of the firms as well as the representative consumers across the countries.
\[ Z^* = e^{-kt}Z_0 + (1 - e^{-kt})\frac{(1 + \alpha)(A - c)}{2k}, \quad (9) \]

where \( Z_0 \) is the initial pollutant stock level in Home.

Throughout this paper, let us express the total value of the “national payoff” of each country by excluding the influence of the initial pollutant stock level from the value of its social welfare. This is acceptable for our main purpose here, i.e., to analyze a strategic interaction between the governments since the size of \( Z_0 \) does not affect the relative significance of their individual payoffs under respective situations.\(^6\) Hence, by substituting the time-path of the pollutant stock in (9) except the term involving \( Z_0 \), as well as (3), (7) and (8) into (4), the payoff of the Home government in the autarkic outcome can be written as

\[
U_A = \frac{3}{8} (A - c)^2 \int_0^\infty e^{-rt} dt - \frac{s(1 + \alpha)(A - c)}{2k} \int_0^\infty e^{-rt} \left[1 - e^{-rt}\right] dt
\]

\[= \frac{3}{8r} (A - c)^2 - \frac{s(1 + \alpha)}{2r(k + r)} (A - c) \quad (10)\]

It should be noted that, even under autarky, the national welfare of Home is affected by the production level in Foreign through the transboundary pollution in (5). Hence, there exists a negative externality associated with the production of Good 1 across the two countries. The next section extends this model to an international duopoly in two different modes of competition.

3. International duopoly

When the two domestic markets of Good 1 described above are fully integrated internationally, the new market-clearing condition becomes

\[ C_1 + C_1^* = 2(A - p) = x + x^*, \]

which is inverted to yield

\[ p = A - \frac{x + x^*}{2}. \quad (11) \]

This defines the inverse demand function for Good 1 in the international market of Good 1 and each firm’s profit function becomes

\[ \pi = (A - c - \frac{x + x^*}{2})x. \]

\(^6\) This property stems from our assumption that the marginal damage cost of the pollution is constant. If the damage cost function were nonlinear, the initial pollutant stock level would matter.
First, we consider the case where the two firms are engaged in a Cournot-type competition in this aggregated market. In essence, these firms determine their respective output supply levels concurrently. Specifically, these two firms respectively attempt to solve the following optimization problems:

\[
\max_x \pi = (A - c - \frac{x + x^*}{2})x,
\]

\[
\max_{x^*} \pi^* = (A - c - \frac{x + x^*}{2})x^*.
\]

We can immediately obtain the first-order conditions:

\[
A - c - \frac{x^*}{2} - x = 0,
\]

\[
A - c - \frac{x}{2} - x^* = 0,
\]

which lead to their respective reaction functions:

\[
x = A - c - \frac{x^*}{2}, \quad (12)
\]

\[
x^* = A - c - \frac{x}{2}. \quad (13)
\]

Solving these equations simultaneously, we can obtain the Cournot equilibrium levels of output supply for the respective firms:

\[
x^c = x^* = \frac{2(A - c)}{3}. \quad (14)
\]

Furthermore, the equilibrium price becomes

\[
p^c = \frac{A + 2c}{3}. \quad (15)
\]

Comparing (8) and (15), we can easily confirm \( p^c < p^A \), which implies that the procompetitive effect of the opening of international trade surfaces under the Cournot competition.

Moreover, the payoff of the Home government can be obtained in the exact same way as in the autarkic case above. Consequently, we have the Home government’s payoff, except the effect of the initial pollutant stock level as

\[
U^c = \frac{4}{9r} (A - c)^2 - \frac{2s(1 + \alpha)}{3r(k + r)} (A - c). \quad (16)
\]
As another possible mode of international duopoly under free trade, we also consider the case where the two firms are engaged in a Stackelberg type competition. In a Stackelberg duopoly game, the leading firm is somehow able to make a credible commitment with respect to the supply level of Good 1 prior to its follower.

Substituting (13) into the definition of $\pi$ above, the Home firm’s profit function, when it acts as the Stackelberg leader, can be described as

$$\pi = \left( A - c - \frac{x + x^*}{2} \right) x$$

$$= \left( \frac{A - c}{2} - \frac{x}{4} \right) x.$$

Thus, from the profit maximization problem of this Stackelberg leader, we can easily derive the following levels of output supply in a Stackelberg equilibrium:

$$x^L = A - c,$$  \hspace{1cm} (17)

$$x^F = \frac{A - c}{2},$$  \hspace{1cm} (18)

where $x^L$ and $x^F$ respectively denote the output levels of the leader and the follower. Furthermore, the equilibrium price, $p^S$, becomes

$$p^S = A - \frac{x^S + x^*}{2} = \frac{A + 3c}{4}.$$  \hspace{1cm} (19)

Comparing (15) and (19), we can observe $p^S < p^C$, i.e., the price of Good 1 is lower under the Stackelberg competition than under the Cournot competition. Hence, the procompetitive effect of international trade is strengthened further in the Stackelberg case.

In a similar way to the autarkic case above, substituting (17) and (18) into (5), we can obtain the path of the pollutant stock over time. Then, by substituting this time-path of the pollutant stock as well as (3), (17), (18) and (19) into (4), we have the payoffs of the countries with the leader firm and the follower firm, respectively, as

$$U^L = \frac{17}{32r} (A - c)^2 - \frac{s(2 + \alpha)}{2r(k + r)} (A - c),$$  \hspace{1cm} (20)

$$U^F = \frac{13}{32r} (A - c)^2 - \frac{s(1 + 2\alpha)}{2r(k + r)} (A - c).$$  \hspace{1cm} (21)

Once again, these payoff values are described by excluding the effects of the initial pollutant stocks just for the simplicity of exposition.
4. Endogenous Timing Game

In this section, we introduce a game between the two governments over their respective firms’ roles in the international market of Good 1. This game takes place, once and for all, prior to the international duopoly game described above. Each government decides only on the timing of its own firm’s participation in the international market, and it does not possess any other kinds of policy measures. For simplicity, the strategy space of this inter-governmental game is restricted to {1, 2, no trade} and we consider pure strategies alone. The number, 1 or 2, associates with a particular timing when the government approves the entry of its own firm into the international market for the first time. These numbers reflect the eagerness of the government to engage its firm in the potential international market, with 1 indicating its utmost willingness. Moreover, each government’s action effectively commits its own firm to a specific timing of entry.

When one government chooses 1 and the other chooses 2, the former nation’s firm assumes the role of the Stackelberg leader in the international duopoly while the firm in the latter country becomes the Stackelberg follower. When the two governments choose the same number, whether it is 1 or 2, the mode of competition becomes Cournot. A firm cannot export its product when its government decides to close the domestic market to the import from the other country. Hence, when at least one of the two governments chooses “no trade”, the autarkic situation must arise in each country in our two-country model. The payoff matrix of this inter-governmental game is described in Figure 1, with all the payoff values corresponding to the ones described in the previous sections.

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7 In our simple setup where the environmental damage cost is assumed to be linear, there is no incentive for either government to alter its decision at a later stage.

8 We consider this “no environmental policy” case mainly to focus on the impacts of trade interventions by the governments upon the mode of the international competition. This setup may not be too farfetched when environmental policy is difficult to implement for informational and/or institutional reasons while the government is able to control the openness of its market relatively easily.

9 This can be interpreted as follows. The relative reluctance of a government in allowing its own firm to export the product essentially delays the required preparation for increasing its production level and forces this nation’s firm to assume the role of the second-mover under a possible Stackelberg outcome.
From Figure 1, we can obtain the subgame perfect Nash equilibria, which can include a duopoly case unless the outcome of the inter-governmental game dictates autarky. As we discuss below, the subgame-perfect Nash equilibrium outcomes of this whole game can be categorized into several different classes, depending upon the values of the economic and environmental variables. In particular, we focus upon the magnitudes of the two environmental variables, i.e., \( k \), the decay rate of the pollutant stock, and \( \alpha \), the transboundary pollutant import coefficient, as well as upon whether the two countries are symmetric or not in these respects.

### 4.1 The symmetric case

We begin our analysis of the inter-governmental game with a simple case where the two countries share the same values of \( k \) and \( \alpha \). That is, we suppose that \( k=k^* \) and \( \alpha=\alpha^* \) in this subsection. Despite the fact that the two countries are completely symmetric in both environmental and economic aspects, we have indeed various possibilities as equilibrium outcomes of this game, depending on the magnitudes of these variables. The first finding is summarized in the following statement.

**Proposition 1.** For any value of the pollutant import coefficient, free trade in the Cournot mode can be a subgame-perfect Nash equilibrium outcome for a sufficiently large value of the pollutant decay rate.

**Proof.** By construction, the autarkic situation always constitutes a subgame-perfect Nash equilibrium of the whole game. This is because, whatever action it may take, a government’s payoff is the same autarkic one when the other government chooses “no trade” in the inter-governmental game. We now attempt to show that the Cournot competition can also be an equilibrium outcome under certain parameter values. In order for the Cournot type competition to be a subgame-perfect Nash equilibrium in addition to the autarkic situation, the government must, at least, prefer the Cournot outcome to the autarkic one. Taking the ratio of \( U^A \) in (10) and \( U^C \) in (16), and setting \( U^A/U^C < 1 \) yields

\[
k + r > 12s(1 + \alpha) \frac{1}{5(A - c)}.
\]

Moreover, we need to exclude the circumstances under which one government wishes its firm to be the Stackelberg leader and the other wishes its firms to be the
Stackelberg follower at the same time, in comparison with having their firms compete in the Cournot fashion. Taking the ratio of \( U^L \) in (20) and \( U^C \) and setting \( U^L/U^C < 1 \) leads to

\[
k + r < \frac{48s(2 + \alpha)}{25(A - c)}.
\]

(23)

If (23) is met, the government prefers to have its firm become one of the Cournot competitors to having its firm become the Stackelberg leader. On the other hand, the government would be better off by having its firm become a Cournot competitor rather than the Stackelberg follower if \( U^F/U^C < 1 \) holds. Taking the ratio of \( U^F \) and \( U^C \) and setting \( U^F/U^C < 1 \) leads to\(^{10}\)

\[
k + r > \frac{48s(1 - 2\alpha)}{11(A - c)}.
\]

(24)

In sum, if \( k \) and \( \alpha \) satisfy (22) and, at least, one of (23) and (24), the Cournot duopoly is an equilibrium outcome. Q.E.D.

Proposition 1 implies that, if the actual values of the variables fall in the region \( C \) in Figure 2 with \( \alpha \) on the horizontal axis and \( (k + r) \) on the vertical axis,\(^ {11}\) the Cournot competition in the international market can bring net gain from trade to each country.

\(^{10}\) Condition (24) is meaningful only if \( \alpha < 1/2 \) because \( U^F > U^C \) trivially holds for any \( \alpha > 1/2 \).

\(^{11}\) Since \( k \) and \( r \) effectively play the same roles in our results, we mainly express them in such a combined form.
This outcome would be fairly intuitive as it implies that free trade can materialize more easily among nations whose environmental bodies are inherently more resilient against the pollutant accumulation in terms of having a greater decay rate, $k$. Also, the geographical or climatic conditions matters qualitatively in the sense that a smaller transboundary impact leads to the greater likelihood that the gain from trade realizes as a less significant decay rate is required for its realization.

In addition to the Cournot type competition, there is a possibility that a Stackelberg-type duopoly emerges as an equilibrium outcome even with two completely symmetric countries.

**Proposition 2.** When pollution is sufficiently localized, it is possible that the Stackelberg-type competition emerges as a subgame-perfect Nash equilibrium outcome even when the two countries are completely symmetric.

**Proof.** In order for a Stackelberg competition to be a Nash equilibrium outcome of the inter-governmental game, first of all, the Stackelberg outcome has to be superior to the autarkic outcome for both countries. Such conditions are given by $U^A/U^L < 1$ and $U^A/U^F < 1$, which are respectively translated into

$$k + r > \frac{16s}{5(A – c)},$$

(25)
Moreover, these two countries must simultaneously be better off under the Stackelberg equilibrium in comparison with the Cournot equilibrium. Hence, it must be the case that $U^L/U^C > 1$ and $U^F/U^C > 1$, which are respectively transformed into

$$k + r > \frac{16s\alpha}{A - c}. \tag{26}$$

Therefore, if $k$ and $\alpha$ satisfy (25), (26), (27) and (28) simultaneously, the Stackelberg type competition becomes an equilibrium outcome. Q.E.D.

The region of having a Stackelberg-type competition as a subgame perfect Nash equilibrium outcome is depicted as region $S$ in Figure 2. The likelihood of having a Stackelberg-type competition as an equilibrium outcome per se would be quite slim in this symmetric case, as we can infer from the very limited area of region $S$ in Figure 2. Still, it is somewhat intriguing to find the possibility of a Stackelberg outcome even between two completely symmetric countries. We can easily see that the Stackelberg competition will never be an equilibrium outcome in the absence of transboundary pollution by fixing $s$ at zero and conducting a similar analysis as above.

In addition, the Stackelberg leader is relatively better off than the second mover if we have $U^L/U^F > 1$, which translates into

$$k + r > \frac{48s(2 + \alpha)}{25(A - c)}, \tag{27}$$

$$k + r < \frac{48s(1 - 2\alpha)}{11(A - c)}. \tag{28}$$

Hence, we have the case of the first mover advantage when (29) is met. This result is quite intuitive since the nation with a Stackelberg leader would rather be environmentally more resilient as it experiences a significant increase in the pollutant emissions in comparison with the other nation.

When neither set of the conditions described in the proofs of Proposition 1 and Proposition 2 holds, autarky is the only equilibrium outcome. The region that satisfies the conditions for the realization of such an outcome is depicted as the non-shaded region $A$ in Figure 2. Irrespective of the pollutant import coefficient ($\alpha$), as long as the decay rate of the pollutant stock ($k$) is sufficiently low given the discount rate ($r$), the government would be strictly better off by remaining in the autarkic situation and, as a consequence, the international trade of Good 1 may not materialize between the two countries. In other words, if a country is sufficiently vulnerable to the pollution issue, in terms of a slow decay of the pollutant stock, it rationally opts for autarky for the fear of the long-term effect of the pollution, even though free trade in the good itself could be mutually gainful.
In one extreme case of $\alpha = \alpha^* = 0$, where the pollution problem is completely localized, all of the three types of outcomes, i.e., autarky, Cournot and Stackelberg competitions in the aggregated market, are possible, depending on the magnitude of the decay rate of the pollutant stock, $k$. In particular, if the pollutant decay rate is sufficiently high, i.e., $k > 12s/5(A - c) - r$, the gain from trade can be captured by each country in this completely localized pollution problem.

In the other extreme case of $\alpha = \alpha^* = 1$, on the other hand, the Stackelberg-type competition never occurs since no country would be content to allow its firm to become a Stackelberg follower. If $k + r < 24s/5(A - c)$ holds, autarky is the only equilibrium outcome and, otherwise, the Cournot type competition can also be an equilibrium outcome. As we can easily imagine, the gain from trade is more likely to materialize as the pollution issue is more localized since the line, $U^A = U^C$, is monotone increasing in $\alpha$ in Figure 2.

4.2 The asymmetric case

When the two countries are asymmetric in terms of having different values of decay rates of their respective pollutant stocks ($k$) and pollutant import coefficients ($\alpha$), we need to examine each government’s preferred outcomes separately. In order to simplify the following exposition, as a possible form of Stackelberg-type competition, we focus on the case where Home’s firm is the Stackelberg leader and Foreign’s firm is the follower. It should be noted that exactly the same argument holds even when the roles in a Stackelberg equilibrium are reversed between the two firms.

Our finding can be summarized in the following statement:

**Proposition 3.** In the case of asymmetric countries, the likelihood of a Stackelberg equilibrium significantly expands, compared to the symmetric case.

**Proof.** In an analogous manner to the symmetric case above, we can derive the conditions under which the Cournot competition is an equilibrium outcome. When $k$, $\alpha$, $k^*$, and $\alpha^*$ satisfy the following two conditions:

\[
k + r > \frac{12s}{5(A - c)} (1 + \alpha), \tag{30}
\]

\[
k^* + r > \frac{12s}{5(A - c)} (1 + \alpha^*), \tag{31}
\]

When (30) and (31) are satisfied for the respective countries, both of them can gain by moving from autarky to the Cournot-Nash type competition under free trade. However, a Stackelberg outcome may be even more beneficial than the Cournot-Nash outcome to both nations. Similarly to the symmetric case, we can safely exclude such a possibility if either $U^H/U^C < 1$ or $U^F/U^C < 1$ is satisfied. Each of these conditions can be expressed as (32) and (33) just below.
and, in addition, one of the following two conditions, the Cournot competition becomes an equilibrium outcome and both countries can potentially gain from trade:\textsuperscript{13}

\[ k + r > \frac{48s(2 + \alpha)}{25(A - c)}, \tag{32} \]

\[ k^\ast + r > \frac{48s(1 - 2\alpha^\ast)}{11(A - c)}, \tag{33} \]

On the other hand, when \( k, \alpha, k^\ast, \) and \( \alpha^\ast \) satisfy the following four conditions simultaneously, the Stackelberg-type competition with Home’s firm being the leader and Foreign’s firm being the follower emerges as a subgame-perfect Nash equilibrium outcome:\textsuperscript{14}

\[ k + r > \frac{16s}{5(A - c)}, \tag{34} \]

\[ k + r < \frac{48s(2 + \alpha)}{25(A - c)}, \tag{35} \]

\[ k^\ast + r > \frac{16s\alpha^\ast}{A - c}, \tag{36} \]

\[ k^\ast + r > \frac{48s(1 - 2\alpha^\ast)}{11(A - c)}, \tag{37} \]

As we show using the diagram below, the region for the Stackelberg competition to become a subgame-perfect Nash equilibrium expands significantly, compared to the symmetric case. Q.E.D.

Again, as in the symmetric case above, the autarkic situation always constitutes a subgame perfect Nash equilibrium outcome of the whole game. However, this equilibrium is weakly dominated by free trade outcomes described above.

In particular, the region of having a Stackelberg competition as a subgame-perfect Nash equilibrium outcome is depicted as region \( S \) and \( S^\ast \) in Figure 3.

\textsuperscript{13} Recall that (33) is meaningful only if \( \alpha^\ast < 1/2 \) since any value of \( \alpha^\ast > 1/2 \) leads to \( U^\ast_F > U^F_{tr} \) trivially.

\textsuperscript{14} In order for this Stackelberg outcome to be a Nash equilibrium of the inter-governmental game, first of all, the Stackelberg outcome has to be superior to the autarkic outcome for both countries. Such conditions are given by \( U^L/A > U^F \) and \( U^\ast L/A > U^\ast F \), respectively. Moreover, the respective countries must simultaneously be better off under the Stackelberg equilibrium in comparison with the Cournot-Nash equilibrium. Hence, it must be the case that \( U^L/A > 1 \) or \( U^\ast L/A > 1 \), which are respectively transformed into (35) and (37).
Similarly to the symmetric case above, when the decay rate of its pollutant stock is sufficiently low given the pollutant import coefficient, the government tends to be strictly better off by remaining in the autarkic situation and, as a consequence, the international trade of Good 1 may not materialize between the two countries. Even when the Cournot outcome is dominated by the autarkic outcome for both countries, however, the Stackelberg outcome can also be an equilibrium and free trade is beneficial to the two countries in this asymmetric case as long as certain conditions are met. In the context of our analytical model, therefore, we can state that gain from trade is more likely to be captured by each country with stronger dissimilarity in the environmental characteristics across the countries.
Let us look into the characteristics of a Stackelberg outcome in the asymmetric case. In order for a Stackelberg outcome to be an equilibrium, there needs to be a country with a fairly small import coefficient of the transboundary pollution.\(^{15}\)

This country must also have a relatively low value of the decay rate. If these two conditions are met concurrently, this environmentally-vulnerable country would be willing to have its firm become the Stackelberg follower since its consumer benefits from a lower price due to the expanded supply of Good 1 but does not have to suffer too greatly from the transboundary pollution due to the associated expansion of the polluting good production in the other nation as long as the value of the pollutant import coefficient is sufficiently small. On the other hand, the country whose firm is the Stackelberg leader is going to experience a significant increase in its domestic production of the polluting good. These observations imply that, if a Stackelberg equilibrium exists, the relatively downstream country, not the upstream country, is more likely to have the Stackelberg leader firm provided that this country is sufficiently resistant to the pollutant flow. Most importantly, somewhat unidirectional pollutant flow actually contributes to the realization of the trade gain even when the upstream country is environmentally sensitive in terms of having a small decay rate.

In one extreme case of \(\alpha = \alpha^* = 0\), i.e., when the pollution problem is completely localized, all of the three outcomes are possible, depending on the magnitudes of the decay rates of the pollutant stocks, \(k\) and \(k^*\). Unlike the symmetric case above, even when the pollutant decay rate in one country is quite small, the gain from trade can be captured by each country in the Stackelberg outcome in the asymmetric case, provided that the other country’s pollutant decay rate is sufficiently high. In the other extreme case of \(\alpha = \alpha^* = 1\), on the other hand, the Stackelberg-type competition never occurs since no country would be content with allowing its firm to become the Stackelberg follower. Nevertheless, the gain from trade can realize to each country in the Cournot fashion even in this global pollution case, as long as \(k\) and \(k^*\) are sufficiently large.\(^{16}\)

5. Concluding remarks

In our particular game model, the governments intervene in the market with respect to the timing of their firms’ entering the international market and, consequently, determine the roles of their respective firms there. The results of our analysis indicate that the decay rates of pollutant stocks and transport coefficients of transboundary pollution might play significant roles in determining not only the existence of net gain from trade, but also the type of competition in the international market of such a product. Although the government tends to forgo the trade opportunity when the stock pollution exerts environmental damages for a prolonged period of time, a smaller pollutant import coefficient and dissimilarity between the two countries on these environmental aspects could nevertheless create an opportunity to bring gains from trade to both nations.

\(^{15}\) Indeed, it is necessary that \(\alpha \leq 9/51\).

\(^{16}\) Specifically, this occurs when \(24s/5(A - c) < k + r\) and \(24s/5(A - c) < k^* + r\) simultaneously hold just as in the symmetric case.
We must admit that our analysis rests on several simplifying assumptions. Most significantly, we have not allowed any role of environmental policies by individual governments. In addition to deciding on the timing of entries by their own firms into the international market, the governments might try to influence the equilibrium outcome by implementing environmental policies, which produces a three-stage game model where the governments determine the timing of entries in stage 1 and subsequently determine the time-path of emission taxes/quotas in stage 2, and the firms play a duopoly game in stage 3. It is our future research agenda to explore the consequences of such an extended model.

References


In order to analyze the outcome of the last two stages, we need to resort to a differential game framework (Long, 1992, and Yanase, 2007).