The Size of the Market

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Abstract

The size of the market is the number of buyers that can be reached effectively in a spatial market. We exhibit as determining factor the intercept of the demand function with production cost and with transportation cost. For linear demand and cost function explicit form has to be given. Market structures considered are isolated monopoly, monopolistic competition and perfect competition. When transportation costs fall, markets expand under monopoly, but shrink under monopolistic or perfect competition.

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1. Introduction

“The division of labor is limited by the size of the market”, says Adam Smith. But what determines the size of the market? As any regional scientist will tell you: transportation cost. In fact, the New Economic Geography claims an inverse relationship universally between transportation cost and market size. But it was Wilhelm Launhardt, famous location theorist and early mathematical economist, who, in 1885, under the impression of the dramatic impact of railroad transportation on markets and economic activity, first demonstrated the important relationship between market size and transportation cost.

But does a fall in transportation cost always mean an expansion of markets? The spread of shopping centers and shopping malls in our time with the implied shrinking of each center’s market territory would point to an opposite possibility. We propose a systematic examination of the determination of market size among alternative market scenarios. Section 2 considers the isolated monopolist: a single supplier or one of a set of suppliers at sufficient distances from each other so that their potential market areas do not interfere, i.e. overlap. Three styles of pricing are considered: mill pricing, uniform delivered pricing and perfectly discriminating pricing (Beckmann, 1976). Section 3 considers monopolistic suppliers close enough to find their market areas are restricted. When free entry is possible, we have the scenario of monopolistic competition, which was classically studied by August Lösch (1941).
Is perfect competition possible in space, that is, price taking, free entry and the elimination of profits? Yes, as we shall show in Section 4.

The demand function is assumed to be identical in each location, as customary in location theory, and, in fact, linear

\[ q = q(p) = a - p \]  

(1)

where \( q \) is quantity demanded and \( p \) is price paid by the customer. Here the price unit has been standardized to generate a slope of \(-1\). The intercept \( a \) is then an indicator of the strength of demand.

Under mill pricing the market price at distance \( r \) from the supplier is

\[ p = p_m + kr \]  

(2)

where \( p_m \) is the price at the supplier location, conventionally called the mill price, and \( k \) transportation cost per commodity unit and unit distance.

Alternatively, under uniform delivered pricing

\[ p = p_U \]  

(3)

the price is the same in all locations, but only those customers at distances \( r \) such that

\[ c + kr \leq p_U \]  

(3a)

will be served, where \( c \) is the marginal production cost.

Under perfectly discriminating pricing a separate price is set for each distance \( r \)

\[ p = p(r) \]  

(4)

As customary in location theory production cost is also assumed linear

\[ C = C(q) = cq + F \]  

(5)

with constant unit variable cost \( c \) and fixed cost \( F \). For simplicity we sometimes assume a constant density \( p(r) \) of buyers at all distances \( r \).

What is the absolute longest distance \( R \) at which buyers can be rewarded?

\[ c + kR = a \]

\[ R = \frac{a - c}{k} \]

which maximizes welfare, and leaves no profit margin to cover fixed cost.
2. Mill Pricing, Uniform Delivered Pricing and Perfectly Discriminating Pricing

For an isolated monopolistic supplier and mill pricing, the market radius \( R \) is determined by the disappearance of demand.

\[
q(p(R)) = 0 \quad \text{or} \quad p + kr = a
\]

\[
R = \frac{a - p}{k}
\]  
(6)

So that \( R \) is inversely proportional to \( k \) when there is no adjustment of the mill price \( p \). But the profit-maximizing mill price depends on \( k \).

For, consider

\[
\max_p (p - c) \int_{r}^{R} (a - p - kr) \rho(r) 2\pi r dr
\]

yielding

\[
2\pi \int_{r}^{R} (a + c - kr - 2p) r \rho(r) dr = 0
\]

\[
M[a + c - kr - 2p] = 0
\]

where \( M = \int_{U}^{R} 2r \rho(r) dr \)

\[
= M(R)
\]

is the mass of consumers, and

\[
\bar{r} M(R) = \int 2\pi r \rho(r) dr
\]

defines the average distance \( \bar{r} \) of customers for the supplier. Thus the optimal mill price is

\[
p_m = \frac{a + c - k\bar{r}}{2}
\]

Now for uniform density \( \rho(r) = \rho \)

\[
\bar{r} = \frac{2}{3} R
\]  
(7)
and so using (6)

\[
R = \frac{a \left( -a + c - \frac{2kR}{3} \right) - c}{k} \\
= \frac{a - \frac{1}{2} \left( a + c - \frac{2kR}{3} \right)}{k} \\
= \frac{1}{2} \frac{a - c}{k} + \frac{1}{3} R \\
\frac{2}{3} R = \frac{1}{2} \frac{a - c}{k} \\
R = \frac{3}{4} \frac{a - c}{k}
\]

(8)

So that the market radius as measure of market size is inversely proportional to transportation cost (Launhardt, 1885).

The market radius \( R \) is still inversely proportional to \( k \). Under uniform pricing, equation (3a) shows that

\[
R = \frac{p - c}{k}
\]

(3b)

Here the profit-maximizing price is determined by

\[
\max_p (a - p) \int_0^R [p - c - kr] \rho(r) 2\pi r dr
\]

or

\[
0 = \int_0^R [a + c + kr - 2p] \rho(r) 2\pi r dr
\]

\[
p = \frac{1}{2} [a + c + kr]
\]

with uniform density \( \rho(r) = \rho \)

where again \( \bar{r} = \frac{2}{3} R \)

(7)

as before and, and so, using equation (3b)
\[ R = \frac{1}{k} \left[ \frac{1}{2} (a + c + kr) - c \right] \]

\[ R = \frac{1}{2} \frac{a - c}{k} + \frac{1}{3} R \]

\[ \frac{2}{3} R = \frac{1}{2} \frac{a - c}{k} \]

\[ R = \frac{3}{4} \frac{a - c}{k} \]  

(7)

the same as under mill pricing.

Under discriminatory pricing profit maximization requires

\[
\max \frac{\int_{0}^{R} [p(r) - c - kr][p(r) - c - kr]p(r)2\pi dr}{p(r)}
\]

or

\[
\int_{0}^{R} [a - 2p(r) + c + kr]p(r)2\pi dr
\]

\[
p(r) = \frac{1}{2}(a + c + kr)
\]

\[
= \frac{a + c}{2} + \frac{1}{2} kr
\]

so that, in effect, half the transportation cost is absorbed by the supplier (Singer, 1937).

At the market radius \( R \) customer demand just reaches zero

\[
a - \frac{a + c}{2} - \frac{krR}{2} = 0
\]

\[ R = \frac{a - c}{k} \]  

(9)

which is the largest economically feasible distance for profitable production and transportation. We remark in passing that perfect discrimination is easily shown to be the most profitable pricing scheme, while for linear demand functions, mill and uniform pricing turn out to be equally (but less) profitable (Beckmann, 1976).

In addition to transportation cost, it is the margin \( a-c \) between demand, intercept \( a \) (the strength of demand) and unit cost \( c \) that governs the size of the market: in direct proportion if size is defined as market radius.

When market size is defined as sales area, it is proportional to the square of \( \frac{a-c}{k} \).

When market size is measured by sales, it is inversely proportional to transportation cost factor \( k \) and directly proportional to the cube of the margin \( a - c \), as shown by an easy calculation.
Under uniform pricing monopoly profits are in fact

\[ G = -F + \left[ a + \frac{a + c - k\bar{r}}{2} \right] \int_0^R \left[ \frac{a + c + kR}{2} - c - kr \right] 2\pi \rho r dr \]

\[ = -F + \left[ a + \frac{a - c}{2} - k\bar{r} \right] \left[ \frac{a + c}{2} + k\bar{r} - k\bar{r} \right] \pi \rho R^2 \]

\[ = -F + \left( \frac{a - c}{2} - \frac{kR}{3} \right)^2 \pi \rho R^2 \]

A market exists therefore only when

\[ \left( \frac{a - c}{2} - \frac{kR}{3} \right)^2 R^2 \geq \frac{F}{\pi \rho} \]

or, using \( R = \frac{3}{4} \frac{a - c}{k} \)

\[ = \frac{a - c}{2} - \frac{a - c}{4} \]

\[ = \frac{a - c}{2} - \frac{kR}{3} \]

\[ = \frac{a - c}{4} \]

\[ \left( \frac{a - c}{4} \right)^2 \left( \frac{3}{4} \frac{a - c}{k} \right)^2 \geq \frac{F}{\pi \rho} \]

\[ F \leq \pi \rho \frac{9}{256} \frac{(a - c)^4}{k^2} \]

\[ k^2 F \leq 0.11045 \rho \frac{(a - c)^4}{k^2} \]

\[ a - c \gg 1 \]

The example of the Silk Road shows that high transportation allowed only high margin goods (silk, porcelain) to be traded between China and Europe.

The combination of fixed cost and transportation cost squared must be less than 0.11045 times density \( \rho \) times the spread \((a - c)^4\) to the fourth power.

### 3. Monopolistic Competition

When monopolists are not isolated, since their potential market areas overlap, they are strictly speaking oligopolists. When neighboring suppliers are sufficiently numerous, say six, as in the classical model of Lösch (1941) they may be styled monopolistic competitors. Following Lösch we assume that they ignore opportunities for aggressive market expansion by price cutting or area defense by price following. But simply take
the inherited market radii $R$ as given and maximize profits on this basis. Now, with mill pricing, profits, $G$, are dependent on market radius $R$.

While Lösch deduced that free entry would squeeze the markets into a system of hexagons, we will operate with the simpler circular, but approximately valid, market shapes the same as for isolated monopolists, but now with each firm’s market radius $R$ considered as given.

Under mill pricing profit maximization still yields an optimal mill price

$$P_M = \frac{a + c}{2} - \frac{k}{2} \bar{r}$$  \hspace{1cm} (10)

And for uniform density $\rho(r) = \rho$

$$\bar{r} = \frac{2}{3} rR$$  \hspace{1cm} (7)

$$P_M = \frac{a + c}{2} - \frac{kR}{3}$$  \hspace{1cm} (11)

the resulting profit clearly depends on $R$, the market radius competitive free entry has imposed on each firm.

$$G = -F + (p - c) \int_0^R (a - p - kr) \rho 2\pi r dr$$  

$$= -F + \left(\frac{a + c}{2} - \frac{k}{3} R - c\right) \int_0^R \left[a - \left(\frac{a + c}{2} - \frac{kR}{3}\right) - kr\right] \rho 2\pi r dr$$  

$$= -F + \left[\frac{a - c}{2} - \frac{k}{3} R\right]^2 \pi \rho R^2$$  

$$= -F + \frac{\pi \rho}{4} k^2 \left[\frac{a - c}{k} - \frac{2}{3} R\right]^2 R^2$$

in equilibrium with profits eliminated  \hspace{1cm} $G = 0$

$$\left(\frac{a - c}{k} - \frac{2}{3} R\right)^2 R^2 = \frac{4F}{\pi \rho k^2}$$

$$kR \left(\frac{a - c}{k} - \frac{2}{3} R\right) = 2 \sqrt{\frac{F}{\pi \rho}}$$

write this as

$$\varphi(k, R) = 2 \sqrt{\frac{F}{\pi \rho}}$$  

$$\varphi = (a - c)R - \frac{2kR^2}{3}$$
implicit differentiation gives

$$\frac{dR}{dk} = \frac{\rho \psi}{\rho k} = - \frac{\rho \psi}{\rho R}$$

Now

$$\frac{\delta \varphi}{\delta k} = -\frac{2R^2}{3} < 0$$

$$\frac{\delta \varphi}{\delta R} = \frac{a - c}{k} - \frac{4kR}{3} > 0 \quad \text{for} \quad R < \frac{3}{4} \frac{a - c}{k}$$

which was the market radius chosen by the unconstrained isolated monopolist.

Hence \( \text{sign} \frac{dR}{dk} = -\frac{\delta \varphi}{\delta k} = -\frac{\delta \varphi}{\delta R} = + \)

Falling transportation cost reduces market size by enabling more firms to enter a more profitable industry. An example of this is shopping malls.

4. Is Perfect Competition Possible in Space?

For completeness’ sake we consider also the scenario of perfect spatial competition. Eliminating monopoly requires uniform pricing and a price just sufficient so that sales to the most distant customers at \( R \) just cover cost \( c + kR = p_u \).

Profits are then

$$G = \int_0^{kR} (a - p)(p - c - kr) \pi \rho r dr - F$$

$$= \int_0^R [a - c - kr][c + kR - c - kr] \pi \rho r dr - F$$

$$= [a - c - kR] 2\pi \rho k \int_0^R (R - r) r dr - F$$

$$= [a - c - kR] 2\pi \rho k R^3 \left( \frac{1}{2} - \frac{1}{3} \right) - F$$

$$= \frac{\pi \rho k}{3} [akR - kR] R^2 - F$$

Now profits have a maximum with respect to \( k \) when

$$k = \frac{a - c}{2R} \quad \text{or} \quad R = \frac{1}{2} \frac{a - c}{k}$$
and a maximum with respect to $R$ when

$$R = \frac{3}{4} \frac{a - c}{k}$$

As before, in the domain

$$\frac{1}{2} \frac{a - c}{k} \leq R \leq \frac{3}{4} \frac{a - c}{k}$$

falling profitability with respect to $k$ requires increasing market size as $k$ rises, in order to meet fixed cost, corresponding with the monopolistic competition case.

To conclude, market size as the fundament of the division of labor requires a detailed analysis of the spatial side of an economy. While time has had a clearly recognized place in economic theory, the proper regard for space has long been missing.

References
