Speed Acquisition*

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Abstract

Speed has become a signature of modern financial markets. This paper studies investors' endogenous speed acquisition, alongside their information acquisition. In equilibrium, speed heterogeneity endogenously arises across investors, temporally fragmenting the process of price discovery. A deterioration in long-run price efficiency ensues. Intra- and intertemporal competition among investors drive speed and information to be either substitutes or complements. The model cautions the dysfunction of information aggregation in financial markets: An advancement in the information technology might worsen price efficiency, because it can endogenously complement investors' speed acquisition, further fragmenting the price discovery process and hurting price discovery.

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1 Introduction

Price discovery is a fundamental function of financial markets. It involves two steps: First, investors acquire information about the underlying security. Second, via trading, such information is incorporated in price. The first step determines the amount of information that the price can eventually reflect, i.e. the *magnitude* of price discovery. The second step is about the aggregation of information into the price, i.e. the *process* of price discovery.

Speed is an intrinsic characteristic underlying the process of price discovery. This is because trading takes time: Not all investors with information instantaneously gather together; neither do their trading orders. If the informed only slowly arrive in the market, the resulting price discovery process will also be slow. All else equal, the market is more efficient (in price) if the informed investors trade faster.

To date, the literature has mainly emphasized information acquisition, i.e. the magnitude aspect of price discovery, following the pioneering works by Grossman and Stiglitz (1980) and Verrecchia (1982). This paper complements this canonical perspective, with an enriched price discovery process, by studying investors' speed acquisition *alongside* their information acquisition.

Indeed, the notion of speed roots in the course of financial securities trading. Loosely speaking, there are three stages that affect trading speed: First, after acquiring information, investors can form a concrete trading idea sooner—based on the raw data—by hiring a larger analyst team or buying more computers. Second, before hitting the trading desk, investors' trading orders need to journey through middle/back offices for risk management, due diligence, and compliance. The tightening regulatory environment in recent years, e.g., Dodd-Frank Act and Volcker Rule, arguably has slowed down this second stage. Third, from the trading desk and onward, the speed of execution depends on technology investments on computer hardware, algorithms, and connection to exchange servers (co-location, fiber-optic cables, and microwave towers). This last aspect of trading speed has progressed drastically in the last decade, evidenced by the rise of machines—algorithmic and high-frequency trading technologies.

The above real-world aspects of trading speed raises a set of questions: How much speed technology should investors acquire? Is speed technology favored over information technology? How does speed acquisition affect investors' demand for information, and vice versa? Most importantly, what are the implications for the overall quality of price discovery and efficiency?

This paper develops a model to address these questions. The model builds on an economy populated by a fixed measure of atomless investors, who first invest in both speed and information technology and then trade a risky asset. The information technology determines an investor's private signal precision about the asset value, while the speed technology allows him to trade ahead of his peers. To fix the idea, consider a hedge fund for example. Its information acquisition involves investments in, e.g., sending analysts for firm visits or buying various datasets. The fund's speed acquisition covers a different aspect. It can invest in equipment, infrastructure, or simply more staff to speed up processing the acquired (raw) data, to streamline the compliance process, and to expedite order execution by its trading desk.

The rent-seeking investors in the model have incentive to acquire both technologies. The information technology directly adds to one's information rent (higher signal precision). Indirectly via the speed technology, the sooner an investor trades, the less price discovery has already occurred and the more rent can he extract from the same private signal—a "first-mover advantage". The equilibrium is found where each investor optimally acquires the two technologies to maximize his information rent, accounting for the investment costs and the competition from others.

A driving feature of the model is the *temporal fragmentation effect* of the speed technology. Due to endogenous speed acquisition, investors of different speed participate in the market at different times. Accordingly, the price discovery process also splits into parts, e.g., an early fragment with fast investors and a late fragment with slow investors. So long the speed technology is affordable, such fragmentation is a robust equilibrium feature: While all investors want to acquire speed to enjoy the "first-mover advantage", not everyone will be equally fast, for otherwise there is no "first-mover" and some will want to stay slow to save the speed acquisition cost. Speed heterogeneity thus naturally arises, temporally fragmenting the price discovery process.

The fragmented price discovery process delivers novel insights on the interaction of the two technologies and how both the magnitude and the process of price discovery might be affected. First, through the temporal fragmentation, the speed technology has a nonmonotonic impact on the magnitude of price discovery. With a more advanced (cheaper) speed technology, more investors acquire speed and become fast, increasing the magnitude of price discovery in the early fragment. At the same time, fewer investors remain slow and the late fragment of price discovery shrinks. The market's eventual price efficiency, therefore, can be either improved or hurt by the speed technology, depending on whether the boost in the early fragment or the decay in the late fragment dominates. This result holds even when information acquisition is shut down.

Second, the speed and the information technologies can be either substitutes or complements, as determined by the relative levels of the two. Consider, for example, a positive shock in the information technology, following which all investors acquire more information. How is the demand for speed affected? The answer depends on the relative change between fast and slow investors' rents. As everyone acquires more information, *intra*temporal competition intensifies, attenuating the rents, respectively, for the fast and for the slow investors, as in Grossman and Stiglitz (1980). New in this model, the increased early price discovery *inter*temporally hurts the slow ones' rent. (Intuitively, if the fast investors—"first movers"—have done almost all the price discovery, there will be little rent left for the slow.) Netting the *intra*- and *inter*temporal effects, if the fast are hurt more, some of them will want to stay slow instead, i.e., information acquisition substituting speed acquisition. If, however, the fast investors are not hurt much, more will want to trade early and the demand for speed will rise, complementing information acquisition. The model further characterizes the conditions for the complementarity or the substitution effect to dominate.

Third, when the two technologies exhibit complementarity, an advancement in the information technology—though, prompting all investors to acquire more information—can still hurt the long-run price efficiency. The reason is that, due to complementarity, the information technology also stimulates demand for speed, which fragments the price discovery process. As before, the fragmentation boosts the fast but shrinks the slow fragment of price discovery magnitude. When the decay in the slow fragment dominates, the overall magnitude of price discovery worsens. The key mechanisms at work—the endogenous complementarity and the temporal fragmentation by the speed technology—are the insights taught by the model.

The last result above cautions the dysfunction of information aggregation in financial markets. The "information technology" in the model can be interpreted broadly. For example, recent years have seen strengthened transparency and disclosure requirements by regulators. Policies like Sarbanes-Oxley, Regulation Fair Disclosure, and Rule 10b5-1 have arguably reduced the cost of information acquisition. In the meantime, there is evidence of speed acquisition complementing the accessibility of information. For example, Du (2015) finds that high-frequency traders are constantly crawling the website of U.S. SEC in order to trade on the information in latest company filings. To this extent, this paper argues that transparency and disclosure policies might generate unintended negative impact on information efficiency.

Some recent empirical evidence echoes this view. Weller (2016) shows that algorithmic trading has risen at the cost of long-run price discovery. Gider, Schmickler, and Westheide (2016) shows how high-frequency trading hurts the predictability of earnings in the far future. To emphasize, while the predictions are consistent, the mechanism put forward in this paper is new. For example, the argument by Weller (2016), and via equilibrium models by Dugast and Foucault (2017) and by Kendall (2017), is that short-run (early) price discovery can *crowd out* the acquisition of more precise information in the long-run (late)—a substitution effect. In contrast, the current paper emphasizes the endogenous *complementarity* between information and speed acquisition. As the information technology advances and incentivizes more investors to acquire speed, the price discovery process fragments at the cost of the (long-run) magnitude.

Different financial assets are exposed to different levels of information and speed technology.

The model thus also offers cross-sectional predictions of how technology advancement might affect different assets (e.g., stocks) differently. Bai, Philippon, and Savov (2016) finds a rising trend of the price informativeness of S&P 500 nonfinancial firms in a half-century sample period starting from the 1960s. The finding for firms beyond the S&P 500, however, is the opposite. Farboodi, Matray, and Veldkamp (2017) reproduce the patterns and explain these phenomena via investors' strategic information acquisition choice, facing attention constraints. This paper adds to the discussion that the distinction in different technologies—speed v.s. information—is important in determining individual stocks' respective price efficiency.

This paper further contributes to three strands of the literature. First, the vast literature on costly information acquisition largely focuses on the magnitude aspect of price discovery, following the seminal works by Grossman and Stiglitz (1980) and Verrecchia (1982). Recent studies explore other dimensions. Peress (2004, 2011) studies the wealth effect on information acquisition. Van Nieuwerburgh and Veldkamp (2009, 2010) analyze how investors acquire different types of information under limited attention. Goldstein and Yang (2015) explore the implication of information diversity. Banerjee, Davis, and Gondhi (2016) focus on how transparency affects information efficiency in a setting with price-elastic liquidity demand. Farboodi and Veldkamp (2016) look at the role of the financial sector and highlight the trade-off between the analysis of fundamental and order flow information. To compare, the aforementioned literature assumes that the market always clears with all investors trading at the same time—they have the same speed. Introducing endogenous speed acquisition, this paper allows to study the process of price discovery with investors arriving and trading asynchronously.

Second, the *temporal* fragmentation (due to speed technology) in this paper differs from the existing literature on *spatial* market fragmentation.[1] Regarding the focus on price discovery, an

[1] For example, Admati (1985), Pasquariello (2007), Boulatov, Hendershott, and Livdan (2013), Goldstein, Li, and Yang (2014), Cespa and Foucault (2014), among many others, study information and cross-market learning of correlated assets. Pagano (1989), Chowdhry and Nanda (1991), and Baruch, Karolyi, and Lemmon (2007) study trading of the same asset on different venues (e.g., dual-listed stocks). More recently, market fragmentation has been theorized in the context of dark v.s. lit trading mechanisms, as in Ye (2011), Zhu (2014), Brolley (2016), and Buti,

important feature of temporal fragmentation is that the information revealed in an early fragment naturally carries over to a late fragment—the market never forgets. Thus, slow investors' information rent is eroded away by fast investors, giving rise to the *inter*temporal competition. Such natural accumulation of information over time is critical in determining the complementarity or substitution between the two technologies. In a model of multiple venues (spatial fragmentation), there is no naturally directioned "flow" of information from one venue to another. More fundamentally, as investors trade simultaneously across venues, the notion of speed does not exist. Introducing the speed technology, this paper, therefore, studies a unique angle of market fragmentation.

Third, this paper lends equilibrium support to the literature with *endogenous bundling* of speed and information acquisition. As price discovery accumulates over time, the remaining information rent diminishes. Individual investors' incentive to acquire information, therefore, also increases with their trading speed: The model predicts that fast investors always acquire more information than slow investors. This insight justifies a popular connotation for fast traders that they are also more informed: See, among others, models by Hoffmann (2014), Biais, Foucault, and Moinas (2015), Budish, Cramton, and Shim (2015), and Bongaerts and Achter (2016); and evidence by Brogaard, Hendershott, and Riordan (2014) and Shkilko and Sokolov (2016).

The rest of the paper is organized as follows. Section 2 sets up the model and Section 3 derives its equilibrium. Section 4 then explores the model implications on investors' technology acquisition and on aggregate price efficiency. Discussions on model assumptions, robustness, and extensions are collated in Section 5. Section 6 then concludes.

2 Model

Assets. There is a risky asset and a risk-free numéraire. At the end of the game, each unit of the risky asset will pay off a normally distributed random amount V units of the numéraire. Without

Rindi, and Werner (2017). Finally, Chao, Yao, and Ye (2017a,b) study the competition among exchanges by zooming in on fee structure and tick size.

loss of generality, normalize $\mathbb{E}V$ to 0. Denoted by τ_0^{-1} (> 0) the unconditional variance of V.

Investors. There is a unity continuum of atomless investors, indexed by $i \in [0,1]$. They have constant absolute risk-aversion (CARA) preference with the same risk-aversion coefficient γ (> 0).

Speed technology. An investor i can invest in a speed technology to affect t_i, a set of time points when he can trade in the market (see "Timeline" below). Without investing in speed, all investors are slow, trading at $t_i = t_{\mathrm{S}} = \{2\}$ ("S" for slow). One can instead become fast and trade at $t_i = t_{\mathrm{F}}$ ("F" for fast) by paying $1/g_t$ units of the numéraire. The exogenous parameter g_t (> 0) measures the level of speed technology. The larger is g_t, the more advanced (cheaper) is the technology.

This paper explores two scenarios for fast investors: 1) They can only trade at $t_{\mathrm{F}} = \{1\}$, *"pure speed differential"*; or 2) they can trade at both dates with $t_{\mathrm{F}} = \{1, 2\}$, *"frequent fast trading"*. The first scenario applies to situations where, for example, fast active funds buy-and-hold (due to, e.g., transaction costs) some securities for long-term investment purposes. The second scenario speaks to funds that are more flexible in frequent, active trading strategies. The analysis will mainly focus on the first scenario to articulate the model's main intuition. The second scenario, studied in Section 5.1, serves as a robustness check for the main results.

Information technology. Before trading, each investor i observes a private signal S_i about the payoff V. Specifically, $S_i = V + \varepsilon_i$, where ε_i is independent of V, independent of any other $\varepsilon_{j \neq i}$, and normally distributed with zero mean and variance h_i^{-1} (> 0). The investor i can spend m_i (≥ 0) units of the numéraire on an information technology to improve his private signal precision:

$$h_i = g_h k_h(m_i),$$

where $k_h(\cdot)$ is twice-differentiable, concave, and strictly monotone increasing; and g_h (≥ 0) is a parameter measuring the marginal productivity of this information technology. Without investing in this technology, the investor gets no signal; i.e. $k_h(0) = 0$.

Due to the monotonicity of $k_h(\cdot)$, an investor's information acquisition can be referred to as either h_i (the precision) or m_i (the cost) interchangeably: There exists a weakly convex, monotone

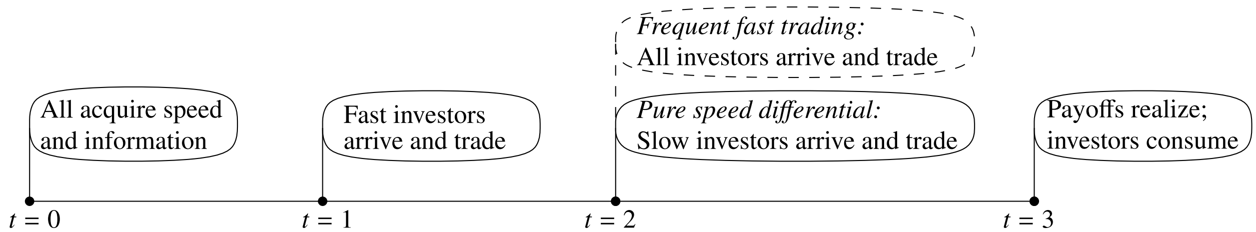

Figure 1: Timeline of the game. The model has four dates: $t \in \{0, 1, 2, 3\}$. At $t = 0$, all investors invest in technology; at $t \in \{1, 2\}$, investors arrive in the market at the time(s) according to their speed technology and submit their demand schedules to trade the risky asset; finally, at $t = 3$ the risky asset liquidates and all investors consume their terminal wealth. The figure outlines two scenarios: Under "pure speed differential", fast investors only trade once at $t = 1$ and there are only slow investors trading at $t = 2$. Under "frequent fast trading", fast investors can also trade at $t = 2$.

increasing information acquisition cost function $c(\cdot)$ such that $\forall h_i \geq 0$,

$$m_i = c(h_i) := {k_h}^{-1}(h_i/g_h).$$

To ensure that there is always some information in the market, let $\dot{c}(0) = 0$; equivalently, $\dot{k}_h(0) \to \infty$.

The information technology is assumed to be orthogonal to the speed technology (g_t and g_h are exogenous parameters, independent of each other). This is an intentional modeling choice, so that the comparative static analyses will help isolate the effect of one technology against the other. In reality, the two technologies will likely affect each other. Section 5.2 discusses such interdependence and its implications.

Timeline. There are four dates in the model: time $t \in \{0, 1, 2, 3\}$, as illustrated in Figure 1. At $t = 0$, all investors independently invest in technologies t_i and h_i. Time $t \in \{1, 2\}$ are trading rounds. The set of investors $\{i | \, t \in t_i\}$ arrive at t together and they independently submit demand schedules $\{x_i(p_t; \cdot)\}$ to trade the risky asset, based on his information set—private signal s_i, his existing holding of the asset (if any), and the public history of past prices. Specifically, at $t = 1$ only fast investors arrive and trade. At $t = 2$, only slow investors trade under "pure speed differential" (main model, Section 3 and 4), while all investors trade under "frequent fast trading" (robustness check, Section 5.1). Finally, at $t = 3$, the risky asset liquidates at V and all investors consume their terminal wealth.

Trading. In each trading round $t \in \{1, 2\}$ there is noise demand U_t, which is independent of all other random variables and is i.i.d. normally distributed with zero mean and variance τ_{U}^{-1}. (Section 5.3 discusses the robustness to time-varying noise trading.) The aggregate demand at t is

$$L_t(p) = \int_{i\in[0,1]} x_i(p; \cdot)\mathbb{1}_{\{t\in t_i\}}\mathrm{d}i + U_t. \tag{1}$$

There is a competitive market maker, who clears the market at all times at the efficient price given all historical public information (as in Kyle, 1985). Thus, the trading price in each round t is

$$P_t = \mathbb{E}\Big[V\Big|\{L(\cdot; r)\}_{r\le t, \forall r\in\{1,2\}}\Big]. \tag{2}$$

This way, the market price is always (semi-strong) efficient and suits the purpose of studying price efficiency. See, among others, Vives (1995); Hirshleifer, Subrahmanyam, and Titman (1994); and Cespa (2008). Section 5.4 discusses the motivation for and the robustness of this choice.

Strategy and equilibrium definition. To sum up, each investor maximizes his expected utility over the final wealth by optimizing his (cumulative) demand $x_i(\cdot)$ at each $t \in t_i$; and, backwardly, by choosing his technology pair $(t_i, h_i) \in \{t_{\mathrm{S}}, t_{\mathrm{F}}\} \times [0, \infty)$ at $t = 0$.

Denote by $\pi(t_i, h_i)$ the investor i's ex ante certainty equivalent (whose functional form will be derived below). Define $\mathcal{P} := \{(t_i, h_i)\}_{i\in[0,1]}$ as the collection of all investors' investment policies. A Nash equilibrium is a collection $\mathcal{P}$, such that for any investor i, fixing $\mathcal{P}\backslash(t_i, h_i)$, he has $\pi(t_i, h_i) \ge \pi(t, h), \forall(t, h) \in \{t_{\mathrm{S}}, t_{\mathrm{F}}\} \times [0, \infty)$.

3 Equilibrium analysis

This section studies the equilibrium under "pure speed differential", where the speed technology enables fast investors to trade early, but only, once at $t_{\mathrm{F}} = \{1\}$. This scenario has more tractability and allows the analysis to zoom in on the economic mechanisms behind the results. Section 5.1, "frequent fast trading", later demonstrates the robustness of the results when fast traders will be able to trade early and again at $t_F = \{1, 2\}$. Since in this section each investor only trades once, rather than a set of $t_i = \{1\}$ or $\{2\}$, the notations for investors' speed acquisition will be simplified to $t_i = 1$ for fast and $t_i = 2$ for slow.

3.1 Optimal trading

The equilibrium analysis begins with an investor's optimal trading demand at his trading round t_i. Fix all other investors' strategies and consider an investor i with technology (t_i, h_i). At $t = t_i$, he chooses his demand schedule x_i to maximize his expected utility over final wealth:

$$x_i \in \arg\max_{x_i} \mathbb{E}\left[-e^{-\gamma\cdot(V-P_t)x_i} \middle| V + \varepsilon_i = s_i, P_t = p, P_{t-1}, \ldots\right]$$

where P_t is given by the market maker's efficient pricing as in equation (2). Note that the investor also observes the price history $\{P_{t-1}, \ldots\}$ (with $P_0 = \mathbb{E}[V] = 0$). Standard conjecture-and-verify analysis as in Vives (1995) yields the following lemma.

Lemma 1 (Trading under "pure speed differential"). *For any technology pair (t_i, h_i), an investor i with signal s_i submits the optimal linear demand schedule at $t = t_i$:*

$$x_i = \frac{h_i}{\gamma}(s_i - p_{t_i}).$$

His certainty equivalent at the time of technology investment ($t = 0$) is

$$\pi(t_i, h_i) = \frac{1}{2\gamma}\ln\left(1 + \frac{h_i}{\tau_{t_i}}\right) - c(h_i) - \frac{2 - t_i}{g_t}, \tag{3}$$

where the price efficiency $\tau_t := \mathrm{var}[V | \{L_r(\cdot)\}_{\forall r \le t}]^{-1}$ satisfies the recursion of

$$\Delta\tau_t = \tau_t - \tau_{t-1} = \left(\int_{\{t_j = t\}} \frac{h_j}{\gamma}\mathrm{d}j\right)^2 \tau_U \tag{4}$$

with the initial value $\tau_0 = \mathrm{var}[V]^{-1}$*. The equilibrium price* P_t *satisfies the recursion of*

$$\Delta P_t = P_t - P_{t-1} = \frac{\Delta\tau_t}{\tau_t}\left(V + \frac{\gamma U_t}{\int_{\{t_j=t\}} h_j \mathrm{d}j} - P_{t-1}\right) \tag{5}$$

with the initial value $P_0 = \mathbb{E}V$ *(*$= 0$*).*

An investor's demand x_i scales with the difference between his private signal and the trading price, where the scaling factor h_i/γ—his trading aggressiveness—increases with the precision of his signal and decreases with his risk-aversion. His certainty equivalent has three components: The first term represents the information rent due to his private information, while the second and the third term correspond to the cost of information and speed acquisition, respectively.

Note that slow investors ($t_i = 2$) do *not* (directly) trade on the fast round price p_1, thanks to the competitive market maker who sets p_2 while recalling the information from $t = 1$ trading. As such, from a slow investor's perspective, observing only p_2 is as good as observing both p_1 and p_2.[2]

3.2 Optimal technology acquisition

The next step is to find investors' optimal technology investment (t_i, h_i) at $t = 0$. To proceed, define the population sizes as

$$\mu_{\mathrm{F}} := \int_{i\in[0,1]} \mathbb{1}_{\{t_i=1\}} \mathrm{d}i \text{ and } \mu_{\mathrm{S}} := \int_{i\in[0,1]} \mathbb{1}_{\{t_i=2\}} \mathrm{d}i,$$

where, by construction, $\mu_{\mathrm{F}} + \mu_{\mathrm{S}} = 1$. The search for equilibrium is a fixed-point problem: Given the population sizes μ_{F} and μ_{S}, what is each investor's optimal information acquisition h_i? Given information acquisition $\{h_i\}$, what is the "break-even" μ_{F} and μ_{S}, so that no individual investor wants to change his speed choice?

Consider first investors' optimal information acquisition h_i, by fixing their speed. That is, suppose there is an exogenous fraction $\mu_{\mathrm{F}} \in [0, 1]$ ($\mu_{\mathrm{S}} = 1 - \mu_{\mathrm{F}}$) of investors who are fast (slow).

[2] This feature inherits from Vives (1995) and dates back to the competitive market maker in Kyle (1985). In the dynamic equilibrium, also examined in Back (1992) among others, only the contemporaneous price p_t is relevant as a state variable, not the entire price history. See also Cespa (2008) for an application of dynamic information selling.

Note that an investor is atomlessly small and, hence, his individual information acquisition h_i does not affect the aggregate price efficiency τ_1 or τ_2. To maximize his certainty equivalent, an investor takes τ_{t_i} as given and chooses his information precision h_i according to the first-order condition of equation (3):

$$\frac{1}{2\gamma}\frac{1}{\tau_{t_i} + h_i} - \dot{c}(h_i) = 0, \tag{6}$$

which has a unique solution $h(\tau_{t_i})$, satisfying the second-order condition, thanks to the convexity of the cost $c(\cdot)$; see the "Information technology" paragraph in Section 2. By symmetry, therefore, all investors of the same speed $t_i = t \in \{1, 2\}$ acquire the same amount of information: $h_{\mathrm{F}} = h(\tau_1)$ for fast investors and $h_{\mathrm{S}} = h(\tau_2)$ for the slow.

The convexity of $c(\cdot)$ further implies that the unique solution to equation (6), $h(\tau)$, is decreasing in τ. As such, fast investors always acquire more information than slow ones:

$$h_{\mathrm{F}} \geq h_{\mathrm{S}}. \tag{7}$$

This is because the price discovery process is always cumulative: $\tau_2 \geq \tau_1$, as the market never forgets whatever has been revealed ($\Delta\tau_t \geq 0$ by equation 4). The earlier an investor can trade, the less price discovery the market has seen and the more valuable is his private information. To take this advantage, fast investors always have stronger incentive to acquire more information. This equilibrium result supports a popular connotation for fast traders that they are also more informed: A number of theory studies assume so exogenously: Hoffmann (2014), Biais, Foucault, and Moinas (2015), Budish, Cramton, and Shim (2015), and Bongaerts and Achter (2016). The empirical evidences by Brogaard, Hendershott, and Riordan (2014) and by Shkilko and Sokolov (2016) agree this perspective.

The second step is to fix investors information acquisition h_{F} and h_{S} and find the equilibrium speed acquisition μ_{F} and μ_{S}. The price efficiency τ_t recursion (equation 4) can be rewritten as

$$\Delta\tau_1 = \tau_1 - \tau_0 = \frac{\tau_{\mathrm{U}}}{\gamma^2} h_{\mathrm{F}}^2 \mu_{\mathrm{F}}^2 \text{ and } \Delta\tau_2 = \tau_2 - \tau_1 = \frac{\tau_{\mathrm{U}}}{\gamma^2} h_{\mathrm{S}}^2 \mu_{\mathrm{S}}^2. \tag{8}$$

These *increments* in price efficiency, $\Delta\tau_1$ and $\Delta\tau_2$, are referred to as the "early fragment" and the "late fragment" of price discovery, respectively. In contrast, the *cumulative* price efficiency, τ_1 and τ_2, are called the "short-run" and the "long-run price efficiency", respectively. An important observation is that the price discovery $\Delta\tau$ is *non*linear in the population size μ of the trading round. Under the current parametrization, fixing h_{F} and h_{S}, $\Delta\tau$ is convexly increasing in μ. Such nonlinearity underlies "the temporal fragmentation effect" of speed technology, as discussed in detail later in Section 4.2 (Proposition 4).

Equation (3) implies that investors of the same speed have the same ex ante certainty equivalent:

$$\begin{aligned} \pi_{\mathrm{F}} &= \frac{1}{2\gamma}\ln\left(1+\frac{h_{\mathrm{F}}}{\tau_1}\right) - c(h_{\mathrm{F}}) - \frac{1}{g_t}; \\ \pi_{\mathrm{S}} &= \frac{1}{2\gamma}\ln\left(1+\frac{h_{\mathrm{S}}}{\tau_2}\right) - c(h_{\mathrm{S}}). \end{aligned} \tag{9}$$

If $\pi_{\mathrm{F}} > \pi_{\mathrm{S}}$, all investors will acquire speed and become fast, leading to a corner solution of $\mu_{\mathrm{F}} = 1$ and $\mu_{\mathrm{S}} = 0$; and vice versa. In an interior equilibrium, it must be $\pi_{\mathrm{F}} = \pi_{\mathrm{S}}$ so that no investor has incentive to change his speed acquisition. The optimal population mix is determined via the break-even condition $\pi_{\mathrm{F}} = \pi_{\mathrm{S}}$.[3]

The following proposition summarizes the discussion above and states the equilibrium.

Proposition 1 (Equilibrium under "pure speed differential"). *There exists a unique equilibrium $\mathcal{P}$, depending on the speed technology g_t relative to a threshold $\hat{g}_t$ (> 0, see the proof):*

Case 1 (corner). *When $g_t \le \hat{g}_t$, all investors invest in $(t_i, h_i) = (2, h_{\mathrm{F}})$, where h_{F}, together with τ_2, uniquely solves the first-order condition (6) and the recursion (8) with $\mu_{\mathrm{F}} = 0$ and $\mu_{\mathrm{S}} = 1$.*

Case 2 (interior). *When $g_t > \hat{g}_t$, a mass $\mu_{\mathrm{F}} \in (0, 1)$ of investors invest in $(t_i, h_i) = (1, h_{\mathrm{F}})$, while the rest μ_2 investors invest in $(t_i, h_i) = (2, h_{\mathrm{S}})$, such that the equilibrium is uniquely solved*

[3] One can interpret the pair μ_{F} and μ_{S} as investors' ex ante probability mix between becoming fast or staying slow. That is, they play a symmetric mixed-strategy in speed acquisition: Each independently chooses to acquire speed, $t_i = 1$ (together with h_{F}), with probability μ_{F} or to stay slow, $t_i = 2$ (together with h_{S}), with probability $\mu_{\mathrm{S}} = 1 - \mu_{\mathrm{F}}$.

by $\{h_{\mathrm{F}}, h_{\mathrm{S}}, \mu_{\mathrm{F}}, \mu_{\mathrm{S}}\}$ *under the following equation system:*

$$\text{Optimal information acquisition:} \quad \frac{1}{2\gamma}\frac{1}{\tau_1 + h_{\mathrm{F}}} - \dot{c}(h_{\mathrm{F}}) = \frac{1}{2\gamma}\frac{1}{\tau_2 + h_{\mathrm{S}}} - \dot{c}(h_{\mathrm{S}}) = 0;$$

$$\text{Indifference in speed:} \quad \pi_{\mathrm{F}} = \pi_{\mathrm{S}};$$

$$\text{Population size identity:} \quad \mu_{\mathrm{F}} + \mu_{\mathrm{S}} = 1;$$

where the expressions of τ *and* π *are given by equations* (8) *and* (9).

The equilibrium depends on the level of speed technology: When $g_t \leq \hat{g}_t$, investing in speed is too costly for any investor and nobody acquires speed in equilibrium. Only for sufficiently advanced speed technology ($g_t > \hat{g}_t$) will there be some investors acquiring speed.[4] In fact, this same intuition holds in the other way:

Corollary 1. *Fixing the speed technology* g_t*, there exists a threshold* $\hat{g}_h$ *such that the equilibrium is interior if and only if* $g_h \geq \hat{g}_h$*.*

That is, when the information technology is too poor, the benefit in information rent of becoming fast is not sufficient to compensate for the cost of acquiring speed. As such, all investors stay slow.

3.3 Two constrained equilibria

In order to provide a clear contrast of the results, Section 4 will study two constrained versions of the model, where the acquisition of either one of the two technologies is shut down. The following two corollaries provide the existence and the uniqueness of equilibrium under these two constrained models. As both are special cases of Proposition 1, for brevity, their proofs are omitted.

[4] However, there are always non-zero mass of investors staying slow in equilibrium ($\mu_{\mathrm{S}} > 0$). To see the reason, suppose there is an equilibrium with all investors acquiring speed, i.e., $\mu_{\mathrm{F}} = 1$ and $\mu_{\mathrm{S}} = 0$. In this case there is no price discovery in the late fragment, i.e., $\tau_1 = \tau_2$. Equation (9) then suggests that the marginal fast investor is strictly better off if he instead does not invest in the speed technology, saving the speed acquisition cost $1/g_t$. Hence, some fast investors will deviate to staying slow.

Corollary 2 (Constrained equilibrium: exogenous speed). *Fix each investor's speed t_i with exogenous μ_{F} and μ_{S} ($= 1 - \mu_{\mathrm{F}}$). Then there exists a unique equilibrium in which fast and slow investors' information acquisition, h_{F} and h_{S}, solve the first-order conditions* (6).

When the speed technology is not available, only the interior case of Proposition 1 is relevant. Further, since the investors cannot choose speed, the indifference condition $\pi_{\mathrm{F}} = \pi_{\mathrm{S}}$ becomes irrelevant. Only the "optimal information acquisition" condition remains.

Corollary 3 (Constrained equilibrium: exogenous information). *Fix fast and slow investors' information acquisition at h_{F} and h_{S}, respectively. Then there exists a unique equilibrium, depending on the speed technology g_t relative to a threshold $\hat{g}_t$:*

Case 1 (corner). *When $g_t \leq \hat{g}_t$, all investors stay slow with $\mu_{\mathrm{F}} = 0$ and $\mu_{\mathrm{S}} = 1$.*

Case 2 (interior). *When $g_t > \hat{g}_t$, a mass $\mu_{\mathrm{F}} \in (0, 1)$ of investors acquire speed and become fast, while the rest μ_{S} stay slow. The equilibrium population sizes $\{\mu_{\mathrm{F}}, \mu_{\mathrm{S}}\}$ uniquely solve $\pi_{\mathrm{F}} = \pi_{\mathrm{S}}$ and $\mu_{\mathrm{F}} + \mu_{\mathrm{S}} = 1$.*

Corollary 3 is also a special case of Proposition 1, where the "optimal information acquisition" condition is dropped in the interior equilibrium as investors' signal precision are exogenously fixed.

4 Equilibrium properties

This subsection studies investors' speed and information acquisition and the effects on market quality. Three issues stand out: How does an advancement in one technology affect 1) investors' investment in it, 2) investors' investment in the other technology, and ultimately 3) the aggregate market quality—in particular, the price discovery.

In order to isolate the different implications of the speed and the information technology, the analysis begins by exploring two constrained model variants: Section 4.1 switches off speed acquisition and Section 4.2 information. Section 4.3 then studies the interaction of the two technologies.

The results of these sections are summarized in Table 1 in Appendix A. Finally, Section 4.4 introduces the new concept of "the speed of price discovery" and discusses its importance.

4.1 Information acquisition with exogenous speed

This subsection sets benchmark results under a constrained setting with only information acquisition: Investors' speed is exogenously given. Specifically, fix a mass $\mu_F \in [0, 1]$ of investors who are fast ($t_i = 1$), and the rest $\mu_S = 1 - \mu_F$ investors stay slow ($t_i = 2$). Thus, all results are with respect to the information technology g_h. The equilibrium corresponds to Corollary 2.

Proposition 2 (Information technology and information acquisition). *Fix the fast and the slow investors' sizes μ_F and μ_S. As the information technology g_h increases, both the fast and the slow investors individually acquire more information: $\partial h_i/\partial g_h > 0$ for $i \in \{F, S\}$.*

The result is not surprising. As g_h increases, each investor can acquire more precise information at the same expense. That is, information becomes relatively cheaper and all investors, fast or slow, acquire more of it. Panel (a) of Figure 2 illustrates this effect. The red-dashed line also plots the total information acquisition in the economy, $\int_{i \in [0,1]} h_i \mathrm{d}i = \mu_F h_F + \mu_S h_S$.

Intuitively, as all investors acquire more information, the price becomes more efficient as well:

Corollary 4 (Information technology and price efficiency). *Fix the fast and the slow investors' sizes μ_F and μ_S. As the information technology g_h increases, both the short-run and the long-run price efficiency improve. Mathematically, $\partial \tau_1/\partial g_h > 0$ and $\partial \tau_2/\partial g_h > 0$.*

Recall from equation (8) that $\Delta\tau = \tau_U h^2 \mu^2/\gamma^2$. Because the population sizes $\{\mu_F, \mu_S\}$ are exogenously fixed and because the individual information acquisition h_i monotonically increases with g_h, so does the price discovery $\Delta\tau$. Panel (b) of Figure 2 graphically illustrates the corollary.

Less trivial, perhaps, is the curvature patterned in Panel (a): When g_h is moderately large, both h_F and h_S are concavely increasing and, in particular, h_S grows slower than h_F. The reason is that investors' individual information acquisition "crowds out" each other (Grossman and Stiglitz,

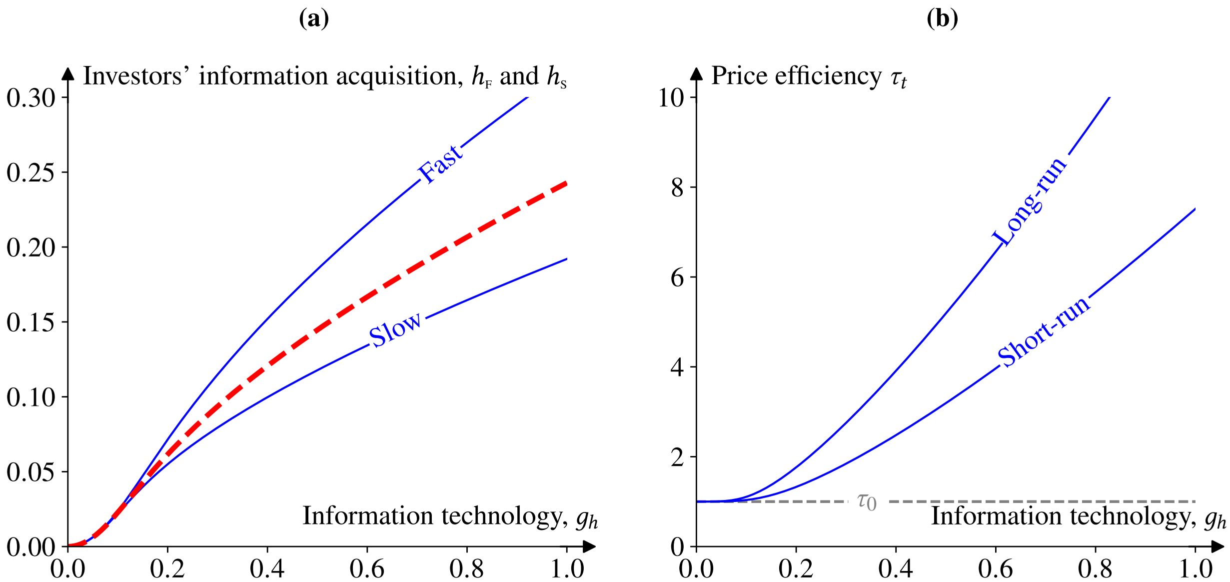

Figure 2: Information technology with exogenous speed. This figure shows how information technology g_h affects individual investors' information acquisition h_i in Panel (a) and the aggregate price efficiency τ_t in Panel (b). The red-dashed line in Panel (a) plots the aggregate demand for information in the economy, $\int_{i\in[0,1]} h_i \mathrm{d}i$. The primitive parameters used in this numerical illustration are: $\tau_0 = 1.0$, $\tau_U = 4.0$, $\gamma = 0.1$, and $k_h(m) = \sqrt{m}$. The fast investor's population size is fixed at $\mu_F = 0.4$; and, hence, $\mu_S = 0.6$.

1980): As the information technology improves, investors individually acquire more information, improving the aggregate price efficiency τ, which, in turn, discourages investors' information acquisition h_i. (The optimal $h(\tau)$ is, all else equal, a decreasing function per equation 6.)

Notably, such crowding out takes two forms. First, *intra*temporally, all fast investors crowd out each other's information acquisition at $t = 1$; and all slow investors at $t = 2$. This yields the concavity of h_F and h_S in g_h. Second, *inter*temporally, the fast investors crowd out the slow, because, naturally, the price efficiency cumulatively grows over time ($\tau_2 \geq \tau_1$). It is this intertemporal crowding-out effect that makes h_S even more concave in g_h, compared to h_F.

This ***inter*temporal** crowding-out effect is a novel insight revealed by the model. Its distinction versus the conventional ***intra*temporal** crowding-out effect (when all investors trade at the same time) bears great significance. Once both the speed and the information technology are made available to investors, the two forces drive substitution/complementarity between the two technologies in contrasting directions (Section 4.3). Before that, it is useful to look at the other constrained equilibrium, where investors can acquire speed but not information.

4.2 Speed acquisition with exogenous information

This subsection exogenizes investors' information acquisition. Specifically, each investor has an endowed signal with precision fixed at the same level of $h_i = h_\circ > 0$, $\forall i \in [0,1]$. They cannot acquire additional information but can still acquire speed: Their speed choice $t_i \in \{1,2\}$ and, consequently, the aggregate population sizes $\{\mu_{\mathrm{F}}, \mu_{\mathrm{S}}\}$ are endogenous.[5] The equilibrium corresponds to Corollary 3.

In this equilibrium, a better speed technology g_t reduces investors' cost to become fast. The usual demand effect applies: Demand rises when price drops, as illustrated in Panel (a) of Figure 3 and formally stated in the proposition below.

Proposition 3 (Speed technology and speed acquisition). *Fix all investors' signal precision at $h_i = h_\circ$ (> 0). In the interior equilibrium, as the speed technology g_t advances, more investors acquire speed: $\partial\mu_{\mathrm{F}}/\partial g_t > 0$.*

An advancement in the speed technology g_t, however, has different implications on the short-run and the long-run price efficiency. The following proposition states the result and Panel (b) of Figure 3 illustrates the patterns.

Proposition 4 (Speed technology and price efficiency). *Fix all investors' signal precision at $h_i = h_\circ$ (> 0). In the interior equilibrium, as the speed technology g_t advances, the short-run price efficiency τ_1 monotonically increases, while the long-run price efficiency τ_2 first decreases and then increases.*

[5] More generally, one can bundle the speed and the information technology: When an investor acquires speed, he gets the pair $(t_i, h_i) = (1, h_{\mathrm{F}})$ and instead if he stays slow, he gets $(2, h_{\mathrm{S}})$, with $h_{\mathrm{S}} \leqslant h_{\mathrm{F}}$. The special case of $h_{\mathrm{F}} = h_{\mathrm{S}} = h_\circ$ simplifies the exposition to highlight the effect of speed acquisition. What matters for this subsection is that both h_{F} and h_{S} are fixed and investors cannot acquire more information—the information acquisition channel is shut down.

(a) **(b)**

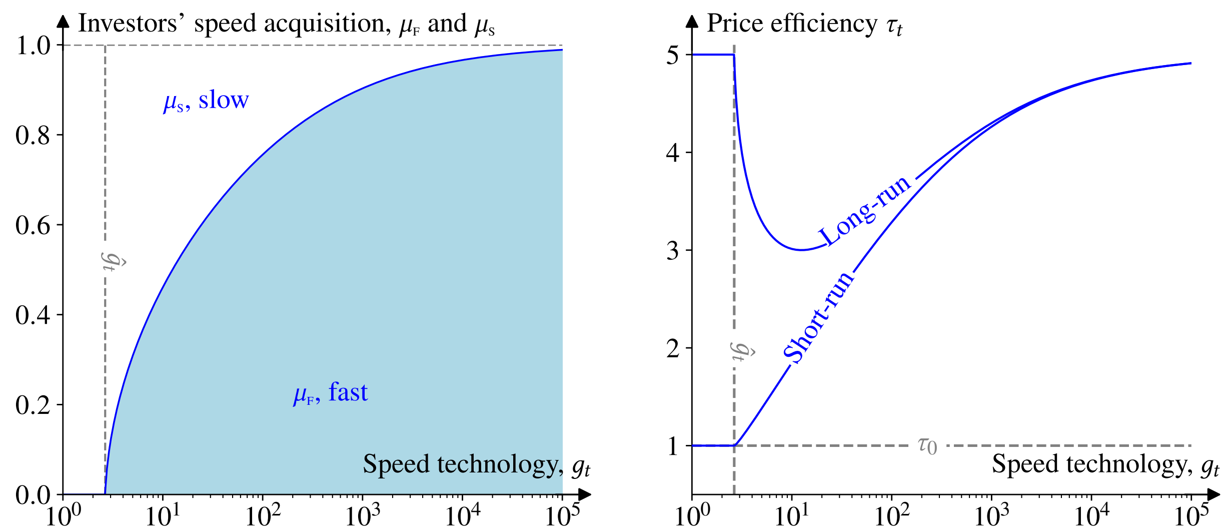

Figure 3: Speed technology with exogenous information. This figure shows how speed technology g_t affects individual investors' speed acquisition t_i in Panel (a) and the aggregate price efficiency τ_t in Panel (b). The horizontal axis shows the speed technology level g_t. To the right of the vertical dashed line, the equilibrium is interior—there are both fast and slow investors. In Panel (a), the vertical axis indicates the population sizes of the fast (shaded area) and the slow investors (white area). The primitive parameters used in this numerical illustration are: $\tau_0 = 1.0$, $\tau_U = 4.0$, $\gamma = 0.1$, and $k_h(m) = \sqrt{m}$. Investors' common signal precision is fixed at $h_\circ = 0.1$.

The driver of this result is the **temporal fragmentation** effect of the speed technology—it temporally fragments investors' participation. When the speed technology is affordable (beyond the threshold $\hat{g}_t$), the unity of investors no longer trade at the same time. A fraction μ_F of them becomes fast and trade at $t = 1$, while the rest μ_S ($= 1 - \mu_F$) still trade slowly at $t = 2$.

Just like the investors, so is the price discovery process fragmented into an early fragment $\Delta\tau_1$ and a late $\Delta\tau_2$. From equation (8), it follows that the early fragment increases with g_t:

$$\Delta\tau_1 = \frac{\tau_U}{\gamma^2} h_\circ^2 \mu_F^2,$$

as μ_F is increasing with g_t (Proposition 3). However, the late fragment drops with g_t:

$$\Delta\tau_2 = \frac{\tau_U}{\gamma^2} h_\circ^2 \mu_S^2 = \frac{\tau_U}{\gamma^2} h_\circ^2 \cdot (1 - \mu_F)^2.$$

The long-run $\tau_2 = \tau_0 + \Delta\tau_1 + \Delta\tau_2$ is subject to the joint force of both fragments of price discovery and, therefore, exhibits a nonmonotonic trend in the speed technology g_t.

Further, Proposition 4 states that the long-run price efficiency τ_2 is U-shape in the speed technology. This U-shape arises from the fact that each fragment of price discovery, $\Delta\tau$, is a *convex function* in the population size μ.[6] Because of such convexity, the impact of a marginal change in μ (due to speed technology) on τ depends on the initial level of μ. For example, when g_t is close to the threshold of $\hat{g}_t$, most of the investors are slow—μ_F closer to zero and μ_S to one. Suppose a small increase in the speed technology $\mathrm{d}g_t$ prompts a small population $\mathrm{d}\mu_\mathrm{F}$ to move from slow to fast. The resulting loss in price efficiency in the late fragment $\Delta\tau_2$ is much larger than the gain in the early $\Delta\tau_1$:

$$\mathrm{d}\tau_2 = \frac{\partial\tau_2}{\partial\mu_\mathrm{F}}\mathrm{d}\mu_\mathrm{F} = \left(\frac{\partial\Delta\tau_1}{\partial\mu_\mathrm{F}} + \frac{\partial\Delta\tau_2}{\partial\mu_\mathrm{F}}\right)\mathrm{d}\mu_\mathrm{F} = \frac{\tau_\mathrm{U}}{\gamma^2}h_\circ^2 \underbrace{\frac{\partial}{\partial\mu_\mathrm{F}}\left(\mu_\mathrm{F}^2 + \mu_\mathrm{S}^2\right)}_{=2(2\mu_\mathrm{F}-1)<0 \text{ for } \mu_\mathrm{F} \text{ close to } 0} \mathrm{d}\mu_\mathrm{F}.$$

The reverse holds true when μ_F is close to one and μ_S close to zero.

Finally, to reinforce/the understanding of the temporal fragmentation effect, note that the level of the long-run price efficiency $\tau_2(g_t)$ is the same at the either extreme of g_t:

$$\lim_{g_t \downarrow \hat{g}_t} \tau_2(g_t) = \lim_{g_t \uparrow \infty} \tau_2(g_t) = \tau_0 + \frac{\tau_\mathrm{U}}{\gamma^2}h_\circ^2.$$

This equality should not come as a surprise because in either of the two extremes, the investors are no longer fragmented: When $g_t \downarrow \hat{g}_t$, all investors remain slow and trade at $t = 2$. When $g_t \uparrow \infty$, all investors become fast and trade at $t = 1$. That is, the temporal fragmentation of speed only manifests for moderate levels of speed technology, which in turn affects price efficiency nonmonotonically.

[6] The convexity of price discovery $\Delta\tau$ in population size μ is a universal feature in the literature. See, among many others, Grossman and Stiglitz (1980), Hellwig (1980), Verrecchia (1982), for example. The source of such convexity—more specifically, the quadratic term μ^2—is the conventional choice of the price efficiency measure: the reciprocal of the conditional *variance*, a statistic of the second moment, of the risky asset value.

4.3 Interaction between speed and information technology

In this subsection, both speed and information technologies are made available to investors. The unconstrained equilibrium stated in Proposition 1 holds, together with Corollary 1. Suppose there has been an advancement in one technology. The discussion below focuses its three effects: 1) investors' acquisition in this advancing technology; 2) investors' acquisition in the other technology; and 3) the market's price discovery function.

4.3.1 Own-price effect: Acquisition in the advancing technology

The level of a technology are modeled as, effectively, the marginal cost of acquiring that technology. As such, when a technology advances, the first-order effect is essentially its own-price effect: investors' demand responding to a cheaper technology. Unsurprisingly and consistent with Proposition 2 and 3, demand increases when a technology improves.

> **Proposition 2 (continued).** *Whether investors' speed acquisition is exogenous or endogenous, in the interior equilibrium, investors' information acquisition monotonically increases with the information technology: $\partial h_i/\partial g_h > 0$ for $i \in \{F, S\}$.*

> **Proposition 3 (continued).** *Whether investors' information acquisition is exogenous or endogenous, in the interior equilibrium, as the speed technology advances, more investors acquire speed: $\partial \mu_{\mathrm{F}}/\partial g_t > 0$.*

For completeness, Panel (a) and (b) of Figure 4 illustrate this intuitive own-price effect of the speed and the information technology.

4.3.2 Cross-price effect: Are speed and information substitutes or complements?

Perhaps more interesting is investors' demand for one technology when the other improves. The cross-price effects are graphed in Panel (c) and (d) in Figure 4. Panel (c) shows how investors' (aggregate) demand for speed, $\int_{[0,1]} \mathbb{1}_{\{t_i=1\}} \mathrm{d}i = \mu_{\mathrm{F}}$, changes when the information technology g_h

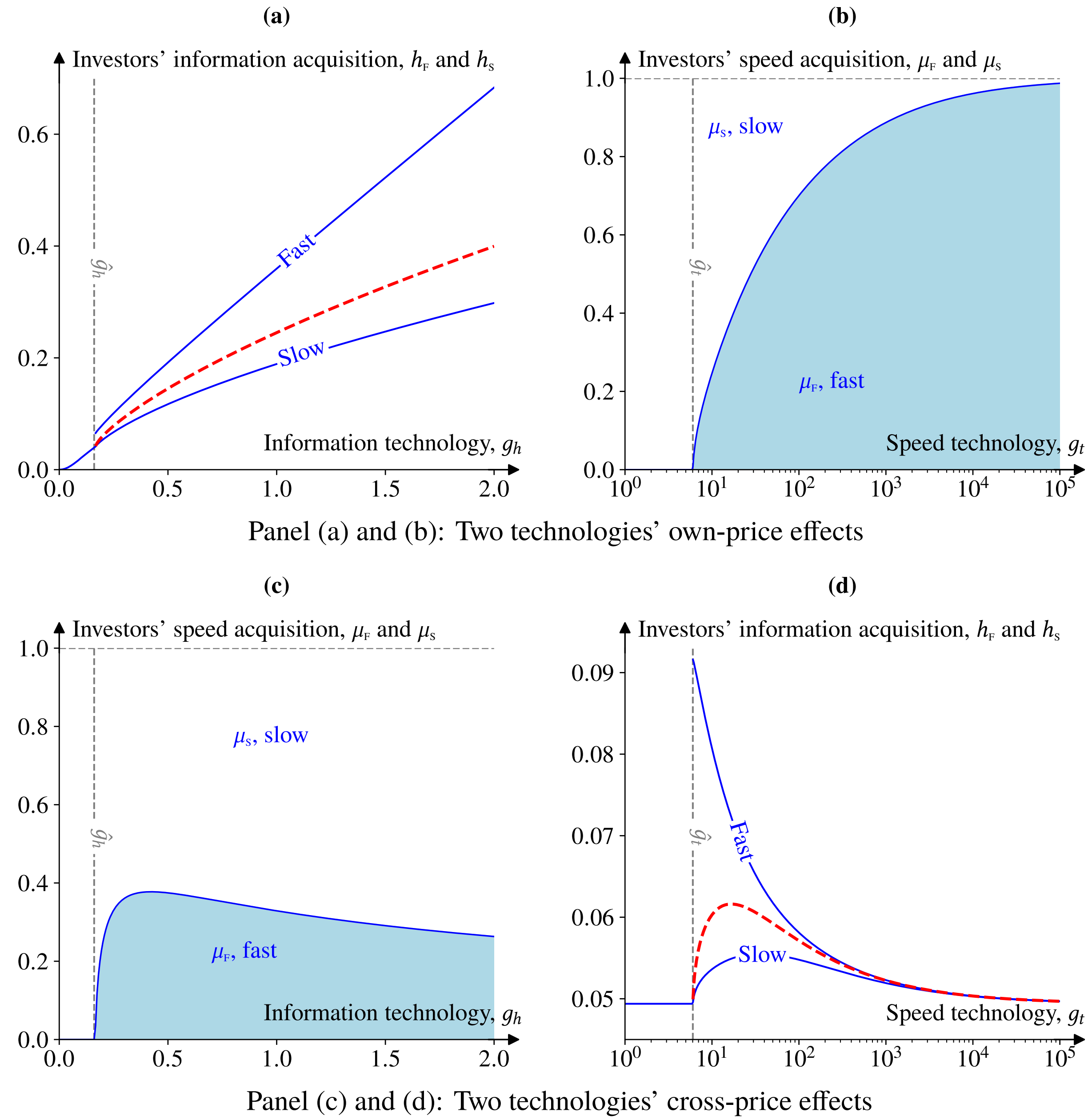

Figure 4: Technology acquisition. This figure illustrates how investors' technology acquisition (demand for speed and for information) are affected differently by levels of technologies. Panel (a) and (b) show the technologies' own-price effect. Panel (c) and (d) show the cross-price effect. The vertical dashed lines indicate the thresholds of the corresponding technology, below which all investors stay slow. The red-dashed lines in Panel (a) and (d) are the aggregate demand for information in the economy, $\int_{i\in[0,1]} h_i \mathrm{d}i$. The primitive parameters used in this numerical illustration are: $\tau_0 = 1.0$, $\tau_U = 4.0$, $\gamma = 0.1$, and $k_h(m) = \sqrt{m}$. For Panel (b) and (d), $g_h = 0.2$. For Panel (a) and (c), $g_t = 10.0$.

increases. Panel (d) plots three lines: a fast investor's individual demand for information, h_{F}; a slow investor's, h_{S}; and the aggregate demand, $\int_{[0,1]} h_i \mathrm{d}i = \mu_{\mathrm{F}} h_{\mathrm{F}} + \mu_{\mathrm{S}} h_{\mathrm{S}}$ (the red-dashed line). In both panels, it can be seen that the aggregate demand for one technology, when the other improves, is first increasing but eventually decreasing, cateris paribus. *The speed and the information technology can be either complements or substitutes, depending on their relative levels.*

> **Proposition 5 (Complementarity and substitution between speed and information).** *Fixing the speed technology g_t, as the information technology g_h increases, an investor's speed and information acquisition are initially complements but eventually substitutes. The same holds true when g_t increases, fixing g_h.*

To understand such cross-price effects, it is useful to recall the different "crowding-out effects" discussed in Section 4.1. Consider an advancement in the information technology g_h (as in Panel (c)), which stimulates both fast and slow investors to acquire more information (Proposition 2 above). Three crowding-out effects arise: *intra*temporal crowding-out among fast investors at $t = 1$, *intra*temporal crowding-out among slow investors at $t = 2$, and *inter*temporal crowding-out from fast investors to slow investors. The first effect hurts fast investors' information rent, making them less willing to acquire speed—reducing demand for speed. The second and the third effects hurt slow investors, incentivizing them to leave $t = 2$ and to compete with fast investors at $t = 1$ instead—raising demand for speed.

It is these countervailing crowding-out effects that drive the net demand for the speed technology to increase or to decrease with the information technology improves. When initially the information technology is low (close to $\hat{g}_h$), there are very few fast investors (μ_{F} close to zero; Corollary 1). As such, the slow investors' *intra*temporal crowding-out effect dominates, stimulating them to acquire speed and move to $t = 1$, forming complementarity between speed and information. As more investors have acquired speed, they yield additional *inter*temporal crowding-out effect on the remaining slow ones, further strengthening their incentive to move to $t = 1$. (The two forces jointly make the initial rise of μ_{F} with g_t very steep.) Eventually, however, when there are too many fast investors, the *intra*temporal crowding-out effect at $t = 1$ dominates: Fast investors' rent is hurt too much by the advancement of information technology, their competition too fierce, no longer profitable to acquire speed.

Panel (d) can be intuitively explained with the three crowding-out effects as well. As the speed technology g_t increases, more and more investors acquire speed (Proposition 3 above). The increase in μ_{F} intensifies the fast investors' competition and their *intra*temporal crowding-out at $t = 1$ reduces their individual information acquisition h_{F}. For the slow investors, the pattern of h_{S} looks different because it is subject to both the intratemporal crowding-out at $t = 2$ and the intertemporal crowding-out by fast investors. Initially, when the speed technology is low (close to $\hat{g}_t$), most of the investors are slow (μ_{F} close to zero) and, hence, the dominating effect is the reduction of *intra*temporal crowding-out at $t = 2$ (as investors move to fast, fewer remain slow). As a result, with less crowding-out, the remaining slow investors individually acquire more information. However, when a lot of investors are already fast, the *inter*temporal crowding-out is no longer negligible. It eventually becomes the dominant effect that drives down slow investors' information acquisition h_{S}.

Not all stocks enjoy the same levels of speed technology and information technology. For example, in terms of speed, it is known that algorithmic traders concentrate on large stocks. In terms of information, some firms have higher analysts' coverage, more media exposure, and higher institutional holdings than others. The above predictions on investors' technology acquisition thus speak to the cross-section of financial markets. Further, the model also relates to investor demographics in different securities. For example, one can interpret the fast investors as hedge funds, who trade more often and possibly have information advantage over slow investors like pension funds and retail investors. Proposition 5 implies that securities with moderate information acquisition cost, e.g., medium cap and moderate analyst coverage, should see most concentrated trading by (fast and informed) hedge funds, holding everything else the same. In particular, they should trade sooner (faster) on their information.

4.3.3 Technology and price discovery

The effects of the technologies on the aggregate price efficiency τ_t are illustrated in Figure 5. The patterns shown in Panel (a), where the speed technology g_t varies with a fixed information technology g_h, are qualitatively similar to those shown in Panel (b) of Figure 3. This suggests that even with endogenous information acquisition, the speed technology's temporal fragmentation effect dominates.The following result extends Proposition 4.

> **Proposition 4 (continued).** *Whether investors' information acquisition is exogenous or endogenous, in the interior equilibrium, as the speed technology g_t advances, the short-run price efficiency τ_1 monotonically increases, while the long-run price efficiency τ_2 initially decreases but eventually increases.*

The speed technology's temporal fragmentation effect highlights the contribution of this paper. For example, this mechanism differs from Dugast and Foucault (2017) and Kendall (2017), who show that the acquisition of shallow information in the short-run can crowd out the acquisition of deep information, therefore hurting the long-run price efficiency. Banerjee, Davis, and Gondhi (2016) also argue that cheaper fundamental information could still worsen price efficiency, because investors might want to learn more about a price-elastic liquidity shock component in asset prices.

Panel (b) of Figure 5 contrasts Panel (b) of Figure 2 with varying information technology g_h. While the short-run price efficiency τ_1 monotonically increases in both cases, the long-run price efficiency τ_2 is no longer monotone when investors can endogenously acquire speed. Surprisingly, advancements in the information technology might hurt overall price efficiency:

> **Proposition 6 (Information technology and price efficiency).** *In the interior equilibrium, advancements in the information technology always improves short-run price efficiency τ_1. However, with endogenous speed acquisition, long-run price efficiency τ_2 is initially hurt but eventually improved. Mathematically, $\partial\tau_1/\partial g_h > 0$ for all g_h; and $\partial\tau_2/\partial g_h < 0$ (> 0) when g_h is small (large).*

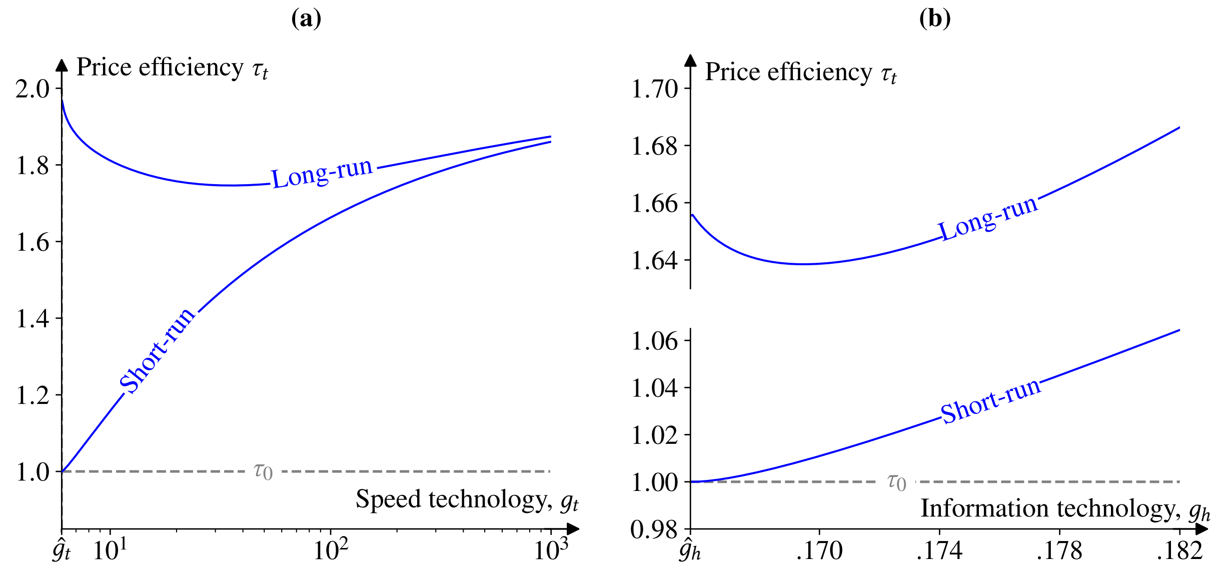

Figure 5: Price efficiency. This figure illustrates how the aggregate price efficiency τ_t is affected differently by different technologies. Panel (a) shows the response to varying speed technology g_t and Panel (b) to information technology g_h. To manifest the patterns, only the range with interior equilibrium is shown; i.e. $g_t > \hat{g}_t$ in Panel (a) and $g_h > \hat{g}_h$ in Panel (b). Further, the vertical axis in Panel (b) is split into two ranges, respectively, for the long-run and the short-run price efficiency. The primitive parameters used in this numerical illustration are: $\tau_0 = 1.0$, $\tau_U = 4.0$, $\gamma = 0.1$, and $k_h(m) = \sqrt{m}$. For Panel (a), $g_h = 0.2$. For Panel (b), $g_t = 10.0$.

To understand how information technology might hurt price efficiency, one needs to recall from the discussion above 1) that the two technologies can exhibit complementarity (when g_h is close to the threshold $\hat{g}_h$); and 2) that speed technology temporally fragments the price discovery process. Combining these two sheds intuition on the U-shape τ_2: Start from a very low level of information technology (g_h close to $\hat{g}_h$), at which most investors stay slow (trading concentrates at $t = 2$). When g_h improves, because of the complementarity, investors acquire both speed and information. In turn, those who have acquired speed fragment the price discovery process and such fragmentation hurts the long-run price efficiency τ_2 (Proposition 4). Eventually, as the information technology improvement furthers (large g_h), few investors acquire speed because the two technologies become substitutes, the population again concentrates at $t = 2$, the fragmentation effect diminishes, and the aggregate price efficiency improves.

Technology has always been evolving but has price efficiency improved alongside the technology? Proposition 4 and 6 both predict that long-run price efficiency has a U-shape in either technology, and there is empirical evidence supporting such nonmonotonicity. For example, Figure 6 of Morck, Yeung, and Yu (2000) shows that the firm-specific component of stock returns exhibits a U-shape trend after World War II. They argue that the firm-specific component of stock return variation reflects the firm-level information.

It is tempting, looking at the U-shapes in Figure 5, to draw the conclusion that the potential dysfunction of the price discovery function of the financial market is not necessarily relevant: Since the decreases in the long-run price efficiency only occurs to a local range of parameters, if the technologies are advanced enough, the price efficiency would only monotonically increase. Such a conclusion is partial. The panels of Figure 5 only show the effect of one technology, *holding the other technology constant*. (Equivalently, both Proposition 4 and 6 study comparative statics, *cateris paribus*.) In reality, both technologies advance continuously, and it is the joint force of the two that determines whether price efficiency increases or decreases. This point is further illustrated in Figure 8, a contour plot of (g_t, g_h), later in Section 5.2.

4.4 The speed of price discovery

A noteworthy feature of the model is the temporal fragmentation of the price discovery, due to investors' endogenous speed technology acquisition. This feature enables researchers to study, for example, "given certain information and speed technology, how *fast* does price discovery occur?" This novel angle of "price discovery speed" differs from the conventional focus on the magnitude and is of great importance for market quality. Compare two market environments, both of which eventually (in the long-run) will lead to the same level of price efficiency, τ_2. All else equal, the one that achieves "faster" price discovery is more efficient than the other, as the end-users of the financial market can utilize such information more timely for purposes like hedging, real investment, and production.

To see the importance of price discovery speed, consider an economy with only the speed technology, as studied in Section 4.2. The effect of g_t is illustrated in Panel (b) of Figure 3. There is a firm who learns from the asset's price to make real investment decisions. It can decide in the short-run at $t = 1$ (e.g., end of the current trading day) or wait for more information in the long-run at $t = 2$ (e.g., end of the week). The firm needs at least three units of price efficiency, i.e. $\tau_t \geq 3$, to deem the asset's price reliably informative. When the speed technology is low, roughly $g_t < 60$, the firm will have to wait till the long-run ($t = 2$) because $\tau_1 < 3 < \tau_2$. (Note the horizontal-axis is in log-scale.) If waiting is costly, a boost in the speed technology, e.g. to $g_t > 60$, can help the firm expedite decision making sooner at $t = 1$. The real efficiency benefits from such speed technology.

There is a catch, however, due to the non-monotone effect on the magnitude of price discovery. The long-run price efficiency τ_2 initially decreases (and eventually increases) with the speed technology. This could create problem for the firm in the example above, especially when it requires a more informative asset price, e.g. $\tau_t \geq 4$. If the speed technology is still very low (g_t close to $\hat{g}_t$), the firm will be able to make its decision eventually in the long-run ($t = 2$) as $\tau_2 > 4$. However, as the speed technology (moderately) improves, to around $g_t = 10$, the speed technology fragments price discovery and hurts the magnitude of price efficiency. Now even in the long-run, $\tau_2 < 4$ and the firm is never able to rely on the asset price confidently. The real efficiency is hurt.

The above example shows that real-decision makers (hedgers, producers, etc.) value both the speed and the magnitude of price discovery. The magnitude τ_t has been well-define and extensively studied in the literature. This paper formally introduces the price discovery speed as:

$$\frac{\Delta\tau_1}{\Delta\tau_1 + \Delta\tau_2},$$

i.e., the percentage of price discovery that is achieved in the short-run over the total price discovery in the long-run. Such a ratio isolates the magnitude of price discovery on the speed. The higher (lower) is the ratio, the faster (slower) is price discovery, as most of the discovery is achieved in the short-run (long-run). The following proposition describes how this perspective of the quality

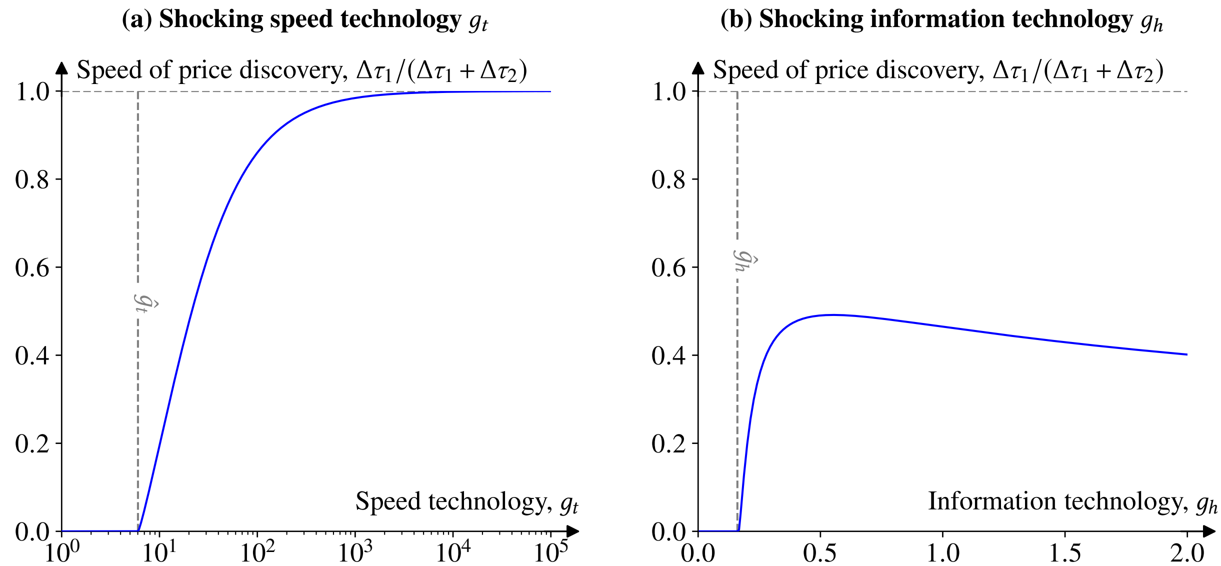

Figure 6: Speed of price discovery. This figure illustrates how the speed of price discovery is affected differently by different technologies. The level of the speed technology g_t varies in Panel (a), while the level of the information technology g_h varies in Panel (b). The primitive parameters used in this numerical illustration are: $\tau_0 = 1.0$, $\tau_U = 4.0$, $\gamma = 0.1$, and $k_h(m) = \sqrt{m}$. For Panel (a), $g_h = 0.2$. For Panel (b), $g_t = 10.0$.

of price discovery is affected by technological advancement.

Proposition 7 (Speed of price discovery and technology). *In the interior equilibrium, the speed of price discovery increases with the speed technology. However, an improvement in the information technology initially increases, but eventually decreases, the speed of price discovery.*

Figure 6 numerically illustrates the patterns. Panels (a) shows that as the speed technology increases, the speed of price discovery monotonically increases as well. This is unsurprising as the dominating effect of speed technology is to drive up μ_F, the size of the fast investors, which in turn adds to the early price discovery $\Delta\tau_1 = \tau_U\mu_F^2 h_F^2/\gamma^2$, relative to the late price discovery $\Delta\tau_2 = \tau_U \cdot (1-\mu_F)^2 h_S^2/\gamma^2$.

This monotone increasing pattern, however, no longer holds with respect to the information technology g_h, as shown in Panel (b). While initially speeding up, the price discovery process eventually slows down for sufficiently high g_h. This is because of the complement and the substitution effect between the speed and the information technology, as discussed in Proposition 5. Initially, when g_h increases from a relatively low level, there is complementarity between investors' speed and information acquisition: Not only do they acquire more information, but also more speed. The increase in μ_{F}, drives up $\Delta\tau_1$ more than $\Delta\tau_2$, speeding up the price discovery process. When the information technology is advanced enough, however, the intratemporal competition among fast investors is so intense that fewer of them are willing to stay fast, reducing the equilibrium μ_{F}; see Panel (c) of Figure 4. Information acquisition eventually substitutes speed acquisition: As μ_{F} reduces (and μ_{S} increases), price discovery slows down and occurs more at $t = 2$.

Further analysis of how the price discovery speed would affect real-efficiency (Bond, Edmans, and Goldstein, 2012) will be an interesting extension from the current model. In particular, the analysis will likely reveal new channels of how technologies could create (or destroy) real social value, due to real agents' value in price discovery speed. As such extension will require a formal model of the real sector (and its interaction with the financial market), which is beyond the current paper's focus on the financial market, it is left for future research.

5 Discussion and robustness

This section discusses some choices in setting up the model. The robustness of the model predictions as well as potential alternative interpretations of the results are also offered.

5.1 Frequent fast trading

The model so far has analyzed the scenario of "pure speed differential", in which the speed technology allows fast investors trade at $t = 1$, sooner than slow investors at $t = 2$. Under the alternative "frequent fast trading", fast investors can trade at $t_{\mathrm{F}} = \{1, 2\}$. That is, the speed technology in addition allows fast investors to trade more frequently. It turns out that the main results studied in Section 4 qualitatively remain the same when fast investors are given this additional trading opportunity.

To begin with, the following lemma establishes investors' optimal trading.

Lemma 2 (Trading under "frequent fast trading"). *An investor i's* cumulative *demand in round t is $x_{it} = \frac{h_i}{\gamma}(s_i - p_t)$, where h_i is his information acquisition, s_i is his private signal, and p_t is the round t trading price set by the competitive market maker. The price discovery $\Delta\tau_t$ and the trading price p_t satisfy the same recursions* (4) *and* (5) *as stated in Lemma 1. At $t = 0$, fast and slow investors' certainty equivalent are given, respectively, by*

$$\begin{aligned} \pi_{\mathrm{F}} &= \frac{1}{2\gamma}\ln\left(1 + \frac{h_{\mathrm{F}}}{\tau_1} + \frac{h_{\mathrm{F}}}{\tau_2^2}\frac{\Delta\tau_2}{\tau_1}\right) - c(h_{\mathrm{F}}) - \frac{1}{g_t}; \\ \pi_{\mathrm{S}} &= \frac{1}{2\gamma}\ln\left(1 + \frac{h_{\mathrm{S}}}{\tau_2}\right) - c(h_{\mathrm{S}}). \end{aligned} \tag{10}$$

Lemma 2 above outlines investors' optimal trading strategy in each round, the dynamics of price efficiency τ_t, as well as investors' ex ante certainty equivalent. Two observations are worth highlighting when comparing with Lemma 1: First, inheriting from Vives (1995), the recursions of price efficiency τ_t and of the price p_t *remain exactly the same*, in spite of fast investors' frequent trading. The reason is that the competitive investors only acquire information once. While the fast investors trade repeatedly, they do not reveal additional information to the market. To see this, note that a fast investor's *cumulative* demand at t is $x_{it} = \frac{h_i}{\gamma}(s_i - p_t)$. His net demand in round $t = 2$, therefore, is $x_{i2} - x_{i1} = \frac{h_i}{\gamma}(p_1 - p_2)$, independent of his private signal s_i. (He simply rebalances his position based on the new price p_2.) As such, a fast investor contributes his private signal to price discovery once and only once, at $t = 1$.

Second, a fast investor's ex-ante certainty equivalent sees an extra term of $\frac{h_{\mathrm{F}}^2}{\tau_2^2}\frac{\Delta\tau_2}{\tau_1}$ inside the $\ln(\cdot)$ operator. In fact, this is the only difference of this extension compared to "pure speed differential". This positive term represents fast investors' additional information rent from repeated trading. It is increasing in h_{F} as this extra information rent still relies on the precision of his private information.

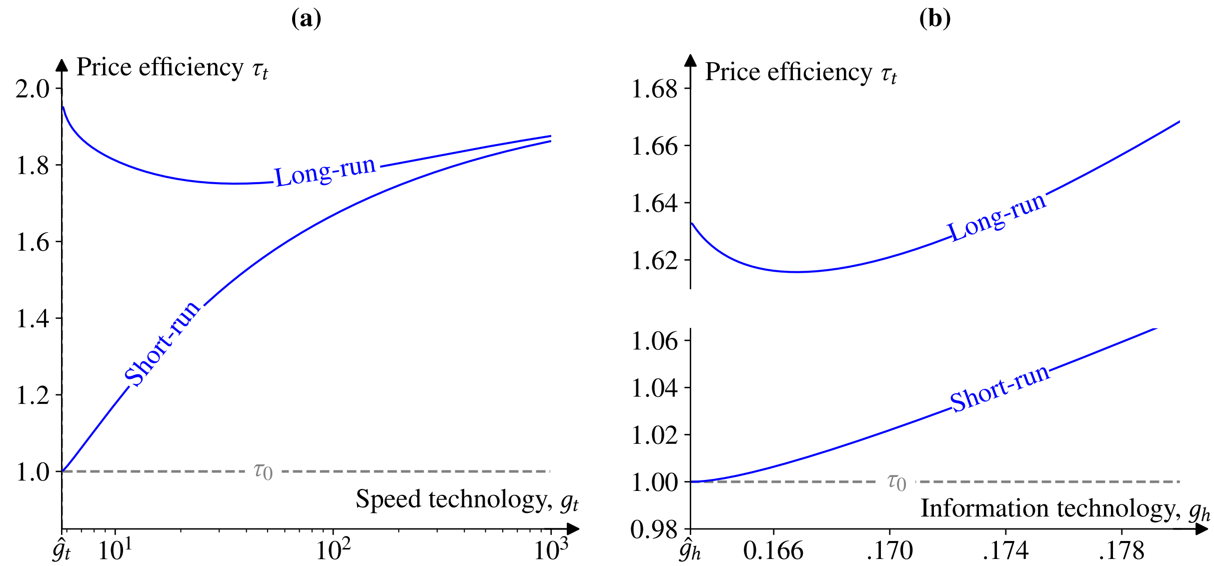

Figure 7: Price efficiency under repeated fast trading. This figure replicates the patterns shown in Figure 5 to illustrate how the aggregate price efficiency τ_t is affected differently by different technologies, under the model extension where fast investors can trade at both $t = 1$ and $t = 2$. The primitive parameters used in this numerical illustration are: $\tau_0 = 1.0$, $\tau_U = 4.0$, $\gamma = 0.1$, and $k_h(m) = \sqrt{m}$. For Panel (a), $g_h = 0.2$. For Panel (b), $g_t = 10.0$.

It decreases in both $\Delta\tau_1$ and $\Delta\tau_2$ because more price discovery in either $t = 1$ or $t = 2$ reduces the advantage of his private signal from repeated trading.

To solve the equilibrium, it remains to pin down investors' optimal information acquisition h_F and h_S for fast and slow investors, respectively, together with the equilibrium population sizes μ_F of fast investors (and $\mu_S = 1 - \mu_F$ of slow investors) so that $\pi_F = \pi_S$. Unfortunately, the additional term $\frac{h_i^2}{\tau_2^2}\frac{\Delta\tau_2}{\tau_1}$ in fast investor's certainty equivalent limits the analytic tractability. Nevertheless, the properties of the equilibrium can be numerically examined. It turns out, after very extensive numerical exploration, that the patterns found under "pure speed differential" remain robust when fast investors trade more frequently. (That is, the frequent trading advantage of the speed technology only provides a second-order effect.) To demonstrate the robustness, Figure 7 reproduces Figure 5 to show that both speed and information technologies have U-shaped nonmonotonic effect on the long-run price efficiency. Other numerical results are omitted for brevity.

It is worth emphasizing that allowing fast investors to trade more frequently does *not* invalidate the two main findings: 1) that speed technology has a temporal fragmentation effect on price discovery; and 2) that different (inter/intratemporal) crowding-out effects drive the speed and the information technology to be either substitutes or complements. In fact, these two findings only depend on the price discovery component, $\Delta\tau_t$, which, as Lemma 2 shows, remains the same as under "pure speed differential".

5.2 Dependence between the two technologies

In the current model, the acquisition of one technology does not affect the other. The independence between the two technologies need not necessarily be the case. On the one hand, the two can complement each other. The complementarity can arise from the common hardware needed, e.g. processing capacity (CPUs), bandwidth (cables and optical fiber), etc. Thus, having invested in such hardware for one technology can reduce the cost for the other (e.g., g_h increases in g_t). This feature is often seen in the algorithmic trading and high-frequency trading literature, where an investor's technology investment gives him a "bundled" advantage in both information and speed. Examples include Hoffmann (2014); Biais, Foucault, and Moinas (2015); Bongaerts and Achter (2016); among others. In contrast, the current model predicts "endogenous bundling" of the two technologies, as fast investors always acquire more information than slow ones (equation 7).

On the other hand, the two technologies can exhibit certain substitution. Dugast and Foucault (2017) argue that because information processing is time consuming, the speed of the "deeply" informed investors is limited and they can only trade after the "shallowly" informed. For example, sending analysts for firm visits is a time-consuming way of acquiring information. That is, investing in one technology might increase the (marginal) cost for the other (e.g., g_h decreases in g_t).

Exactly how speed and information technologies interfere with each other is perhaps a question of engineering and computer science. The current model specification sets a benchmark with independent technologies—an agnostic view. The outcomes of the model, therefore, offer a

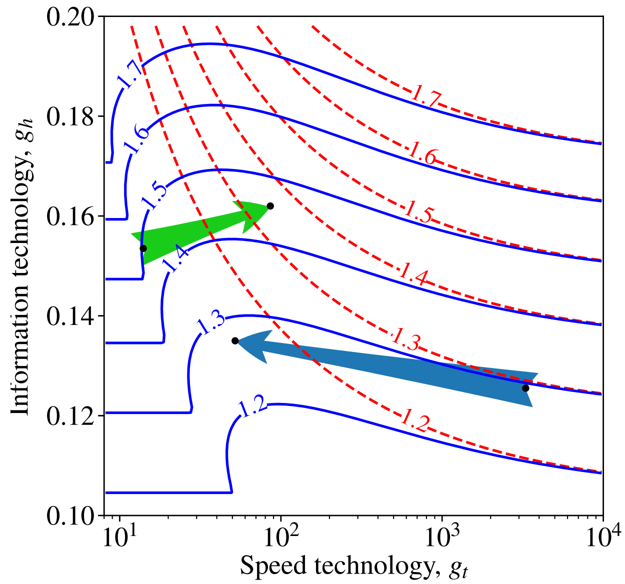

Figure 8: Price efficiency plotted against both technologies. This contour graph plots how the long-run price efficiency τ_2, in blue-solid line, and the short-run price efficiency τ_1, in red-dashed line, vary with the two technologies, g_t and g_h. The two arrows illustrates the different effects of an information technology advancement. The left arrow (green) shows complementarity between the two, while the right arrow (blue) shows substitution. The primitive parameters used in this numerical illustration are: $\tau_0 = 1.0$, $\tau_U = 4.0$, $\gamma = 0.1$, and $k_h(m) = \sqrt{m}$.

clean set of predictions on investors' *endogenous* demand for the two technologies, as opposed to exogenous substitution/complementarity built in the cost functions.

In fact, with independent technologies, the current model offers a starting point to study situations where the two technologies exhibit exogenous substitution of complementarity. Suppose the market quality of interest is price efficiency τ_t. Figure 8 plots τ_1 (blue-solid line) and τ_2 (red-dashed line) on a contour of (g_t, g_h).[7] When the two technologies exhibit complementarity, the effect of an increase in one technology can be examined by, e.g., the left (green) arrow in the figure

[7] Note the pattern shown is consistent with Figure 5: Moving right on a horizontal cut of Figure 8, the information technology g_h is fixed and as the speed technology g_t improves, the short-run price efficiency τ_1 nonmonotonically increases, while the long-run price efficiency τ_2 first decreases and then increases. Moving upward on a vertical cut, g_t is fixed and as g_h increases, τ_1 monotonically increases but τ_2 first decreases and then increases.

(g_h increases from 0.15 to 0.16, while g_t increases from about 10 to 100). If instead the substitution of the technologies dominate, the effect can be shown by, e.g., the right (blue) arrow (g_h mildly increases from 0.125 to 0.135, while g_t drops sharply from about 4,000 to 50). In both examples, note that the long-run price efficiency τ_2 (blue-solid contour lines) drops. Note that the right (blue) arrow is consistent with Dugast and Foucault (2017), who show that when processing information takes time, better information might hurt price efficiency.

5.3 The amount of noise trading

Introducing noise trading U_t in each round is a standard practice to avoid a fully revealing equilibrium. The current setup assumes that the magnitude of noise trading in each round is the same: $\mathrm{var}[U_t] = \tau_{\mathrm{U}}^{-1}$ for all $t \in \{1, 2\}$. This need not be the case. It is straightforward to extend the model to account for time-varying noise trading by assuming time-dependent $\mathrm{var}[U_t] = \tau_{\mathrm{U}_t}^{-1}$. Such an extension will only quantitatively change the equilibrium. The key economic insights of the model are unaffected. First, irrespective of the relative sizes of noise trading, the speed technology creates temporal fragmentation in investors' participation and in the price discovery process. Adapting the price efficiency recursion (equation 8) yields

$$\tau_2 = \tau_0 + \frac{\tau_{\mathrm{U}1}}{\gamma^2}\mu_{\mathrm{F}}^2 h_{\mathrm{F}}^2 + \frac{\tau_{\mathrm{U}2}}{\gamma^2}\mu_{\mathrm{S}}^2 h_{\mathrm{S}}^2$$

and it can be seen that the speed technology still fragments the price discovery process into the early and the late fragments (but with different τ_{U} in each fragment). Second, the result that the speed and the information technologies can be either substitutes or complements depends only on the relative strength of *intra*temporal competition and *inter*temporal crowding out. Having different sizes of noise trading would only affect the threshold of when which effect dominates. Indeed, all the analysis in Section 4 qualitatively go through. For example, Figure 9 illustrates that the qualitative predictions of Proposition 4 and 6 remain robust about the long-run price efficiency τ_2.

The underlying assumption for such (possibly time-varying) exogenous noise trading is that

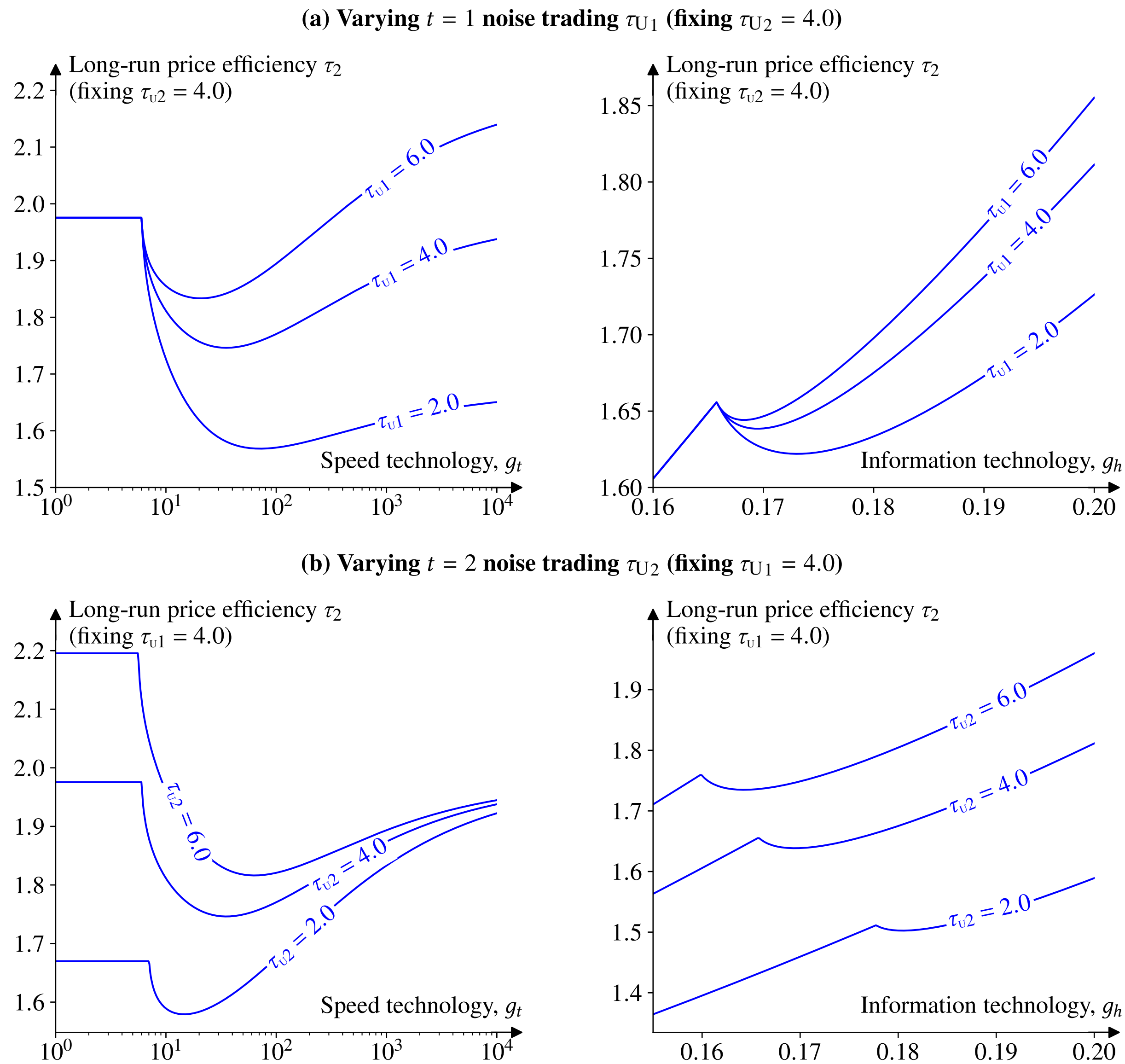

Figure 9: Time varying noise trading. This figure illustrates how different amount of noise trading τ_{U1} and τ_{U2} affects the long-run price discovery, τ_2. Three levels of noise trading are illustrated: $\tau_{Ut} \in \{2.0, 4.0, 6.0\}$. Panel (a) varies τ_{U1} while fixing $\tau_{U2} = 4.0$. Panel (b) varies τ_{U2} while fixing $\tau_{U1} = 4.0$. In each panel, the left and the right graph increases the speed and the information technology, respectively. The other primitive parameters used in this numerical illustration are: $\tau_0 = 1.0$, $\gamma = 0.1$, and $k_h(m) = \sqrt{m}$.

some investors in the economy (unmodeled) have no flexibility at all in terms of how much and when to trade. Endogenizing such "noise trading", making such "noise" demand either price-elastic or timing sensitive, will lead to richer predictions. For example, the aggregate noise can arise from investors' hedging of their endowment shocks, as in Diamond and Verrecchia (1981). More recently, Goldstein, Ozdenoren, and Yuan (2013) and Banerjee, Davis, and Gondhi (2016) assume price-elastic liquidity trading. Such extensions are beyond the scope of the current paper and are left for future research.

5.4 The market clearing mechanism

In each trading round $t \in \{1, 2\}$, investors' demand schedules are cleared by a competitive market maker, who can take any position at the efficient price. The key advantage of having such a market maker is that he helps ensure the trading price p_t is always semi-strong efficient (as in Kyle, 1985). This way, price efficiency in a later round is naturally higher than in earlier rounds, as the market never forgets the information already discovered.

A market maker is not the only way to facilitate trading. An alternative is to determine the price p_t by market clearing, as in Grossman and Stiglitz (1980) and Verrecchia (1982). Under this alternative setup, investors will trade for two reasons: 1) their private information and 2) providing liquidity for the noise demand U_t. The liquidity provision motive for trading is not in the current model: The competitive market maker can take any position needed to clear the market and investors only trade for their information advantage. Cespa and Vives (2012, 2015) study these two different motives in details.

The current model is set up intentionally with the competitive market maker, so that all the results clearly go only through the information channel, uncontaminated by liquidity provision motives. Including the latter will be an interesting extension, but beyond the current paper, to study how speed technology affects liquidity provision in the market. Importantly, irrespective of the market maker, this paper's two key mechanisms—the temporal fragmentation by the speed technology and the novel *inter*temporal crowding out effect— remain robust.

6 Conclusion

There are two aspects of price discovery: the magnitude and the process. The magnitude aspect (investors' information acquisition) has been a key focus of the existing literature. Studying a model with investors' endogenous speed acquisition (alongside their information acquisition), this paper turns the focus to the process of price discovery, i.e., the process through which acquired information is incorporated into price. The analysis reveals that these two aspects of price discovery are intrinsically connected via investors' competition. There are two key mechanisms at work: First, with heterogeneous speed, the investors participate in the market at different points of time and the price discovery is accordingly fragmented temporally. Such temporal fragmentation allows the model to differentiate investors' well-known *intra*temporal competition (e.g., Grossman and Stiglitz, 1980) from a novel *inter*temporal crowding out effect. Second, investors' information and speed acquisition can be either complements or substitutes of each other, depending on the relative strengths of the *intra*- and the *inter*temporal competition. Based on the interaction of these two mechanisms, the model generates testable implications on how advancement in technology would affect market quality. Most notably, when either the speed or the information technology improves, through the negative impact on the price discovery process, the aggregate magnitude of price discovery can be hurt. This provides a cautionary tale of the disruptive effects of how technological advancement, as seen in recent years, might negatively affect aggregate price efficiency in financial markets.

Appendix

A Summary of equilibrium properties

The model predictions from Section 4.1 to 4.3 are summarized in the form of "regressions" in Table 1. The "dependent variables", shown in columns, include investors' (individual and aggregate) information acquisition and speed acquisition, as well as the effects on short-run and long-run price discovery. The "regressors" are (positive) shocks to the technology levels, g_h and g_t.

	(1) Information acquisition			(2) Speed acquisition			(3) Price discovery	
	h_{F}	h_{S}	$\int_0^1 h_i \mathrm{d}i$	μ_{F}	μ_{S}	$\int_0^1 \mathbb{1}_{\{t_i=1\}}\mathrm{d}i$	τ_1	τ_2
(a) Exogenous speed and endogenous information								
g_h:	↗	↗	↗				↗	↗
(b) Exogenous information and endogenous speed								
g_t:				↗	↘	↗	↗	↘↗
(c) Endogenous speed and endogenous information								
g_h:	↗	↗	↗	↗↘	↘↗	↗↘	↗	↘↗
g_t:	↘	↗↘	↗↘	↗	↘	↗	↗	↘↗

Table 1: Summary of effects of technology shocks. This table summarizes how technology shocks affect different aspects of the market: (1) investors' information acquisition; (2) speed acquisition; and (3) price discovery. For each of these three aspects, both the short-run (h_{F}, μ_{F}, and τ_1) and the long-run (h_{S}, μ_{S}, and τ_2) effects are shown. In addition, investors' aggregate demand for information ($\int_0^1 h_i \mathrm{d}i$) and for speed ($\int_0^1 \mathbb{1}_{\{t_i=1\}}\mathrm{d}i$) are also tabulated. Three setting are considered: investors (a) have exogenous speed but can endogenously acquire information; (b) have exogenous information but can endogenously acquire speed; and (c) can endogenously acquire both speed and information. Each row represents a positive shock in the respective technology, g_h for information and g_t for speed. A monotone increasing (decreasing) response to the technology shock is indicated by ↗ (↘), while a hump-shape (U-shape) by ↗↘ (↘↗).

B Proofs

For notation simplicity, the proofs will often use $\mu_1 = \mu_{\mathrm{F}}$, $\mu_2 = \mu_{\mathrm{S}}$, $h_1 = h_{\mathrm{F}}$, $h_2 = h_{\mathrm{S}}$, $\pi_1 = \pi_{\mathrm{F}}$, and $\pi_2 = \pi_{\mathrm{S}}$. This way, the subscript $t = 1$ can handily refer to both the time $t = 1$ and the "F"ast investors; and similarly, $t = 2$ refers to both the time $t = 2$ and the "S"low investors.

Lemma 1

Proof. The proof proceeds by conjecture-and-verify (as in Vives, 1995). Conjecture that a fast investor i's demand schedule is $x_i = a_{i,1}s_i - b_{i,1}p_1$ and that a slow investor i's demand schedule is $x_i = a_{i,2}s_i - b_{i,2}p_1 - c_{i,2}p_2$. At $t = 1$, with only the fast investors, the aggregate demand is

$$L_1(p_1) = \int_{i\in[0,1]} x_i(p_1, s_i)\mathbb{1}_{\{t_i=1\}}\mathrm{d}i + U_1 = \left(\int_{t_i=1} a_{i,1}\mathrm{d}i\right)V - \left(\int_{t_i=1} b_{i,1}\mathrm{d}i\right)p_1 + U_1,$$

where the convention $\int \varepsilon_i \mathrm{d}i = 0$ is used. From the market maker's perspective, the sufficient summary statistic, therefore, is the intercept of the above linear demand, which can be transformed into $z_1 := V + U_1/\left(\int_{t_i=1} a_{i,1}\mathrm{d}i\right)$. Therefore, using standard property of normal distribution,

$$\tau_1 = \mathrm{var}[V|\,L_1(\cdot)\,]^{-1} = \tau_0 + \left(\int_{t_i=1} a_{i,1}\mathrm{d}i\right)^2 \tau_{\mathrm{U}}. \tag{B.1}$$

The incremental price discovery is $\Delta\tau_1 = \left(\int_{t_i=1} a_{i,1}\mathrm{d}i\right)^2 \tau_{\mathrm{U}}$. The maker maker sets the efficient price

$$p_1 = \mathbb{E}[V|\,L_1(\cdot)\,] = \mathbb{E}[V||\,z_1\,] = \frac{\tau_0}{\tau_1}p_0 + \frac{\Delta\tau_1}{\tau_1}z_1. \tag{B.2}$$

As such, the trading price p_1 is an equivalent statistic of z_1. From a fast investor's perspective, $\mathrm{var}[V|\,s_i, p_1\,]^{-1} = \mathrm{var}[V|\,s_i, z_1\,]^{-1} = h_i + \tau_1$ and $\mathbb{E}[V|\,s_i, p_1\,] = \mathbb{E}[V|\,s_i, z_1\,] = (\tau_0 p_0 + h_i s_i + \Delta\tau_1 z_1)/(\tau_1 + h_i)$. Using the above, a CARA fast investor i's optimal demand is

$$x_i = \frac{\mathbb{E}[V|\,s_i, p_1\,] - p_1}{\gamma\mathrm{var}[s_1, p_1]} = \frac{1}{\gamma}(h_i s_i + \Delta\tau_1 z_1 - (\tau_0 + h_i + \Delta\tau_1)p_1) = \frac{h_i}{\gamma}(s_i - p_1).$$

(Recall the normalization $p_0 = 0$.) The conjectured linear demand $x_i = a_{i,1}s_i - b_{i,1}p_1$ for fast investors has thus been verified with coefficients $a_{i,1} = b_{i,1} = h/\gamma$.

At $t = 2$, only slow investors trade and the aggregate demand is

$$\begin{aligned} L_2(p_2; p_1) &= \int_{i\in[0,1]} x_i(p_2, s_i; p_1)\mathbb{1}_{\{t_i=2\}}\mathrm{d}i + U_2 \\ &= \left(\int_{t_i=2} a_{i,2}\mathrm{d}i\right)V - \left(\int_{t_i=2} b_{i,2}\mathrm{d}i\right)p_1 - \left(\int_{t_i=2} c_{i,2}\mathrm{d}i\right)p_2 + U_2, \end{aligned}$$

Recalling p_1, the market maker updates his information set to $\{p_1, z_2\}$, where $z_2 := V + U_2 / \left(\int_{t_i=2} a_{i,2} \mathrm{d}i\right)$ summarizes the new information in $L_2(\cdot)$. Then,

$$\text{(B.3)} \qquad \tau_2 = \text{var}[V | p_1, L_2(\cdot)]^{-1} = \text{var}[V | z_1, z_2]^{-1} = \tau_1 + \left(\int_{t_i=2} a_{i,2} \mathrm{d}i\right)^2 \tau_{\mathrm{U}},$$

where the incremental price discovery $\Delta\tau_2 = \left(\int_{t_i=2} a_{i,2} \mathrm{d}i\right)^2 \tau_{\mathrm{U}}$. The market maker then sets the efficient price

$$\text{(B.4)} \qquad p_2 = \mathbb{E}[V | p_1, L_2(\cdot)] = \mathbb{E}[V | z_1, z_2] = \frac{\tau_0}{\tau_2} p_0 + \frac{\Delta\tau_1}{\tau_2} z_1 + \frac{\Delta\tau_2}{\tau_2} z_2.$$

A slow investor updates $\text{var}[V | s_i, p_1, p_2]^{-1} = \text{var}[V | s_i, z_1, z_2]^{-1} = h_1 + \tau_2$ and $\mathbb{E}[V | s_i, p_1, p_2] = \mathbb{E}[V | s_i, z_1, z_2] = (\tau_0 p_0 + \Delta\tau_1 z_1 + \Delta\tau_2 z_2 + h_i s_i)/(\tau_2 + h_i)$. Solving a quadratic optimization problem, a CARA slow investor's optimal demand is

$$x_i = \frac{\mathbb{E}[V | s_i, p_1, p_2] - p_2}{\gamma \text{var}[s_1, p_1, p_2]} = \frac{1}{\gamma}(h_i s_i + \Delta\tau_1 z_1 + \Delta\tau_2 z_2 - (\tau_0 + \Delta\tau_1 + \Delta\tau_2 + h_i) p_2) = \frac{h_i}{\gamma}(s_i - p_2).$$

Thus the conjectured linear demand for slow investors is also verified with coefficients $a_{i,2} = c_{i,2} = h_i/\gamma$ and $b_{i,2} = 0$. That is, the slow investor's demand is independent of p_1.

The analysis so far has proved the investors' optimal demand as stated in the lemma. In the meantime, equations (B.1) through (B.4) verify the recursion systems of p_t and $\Delta\tau_t$. It remains to compute the investors' ex ante certainty equivalent. Consider a fast investor. Before accounting for the technology acquisition cost, his expected utility at $t = 0$ is $-\mathbb{E}\left[\exp\left\{-\frac{[\mathbb{E}[V | s_i, p_1] - p_1]^2}{2\text{var}[V | s_i, p_1]}\right\}\right]$. The expressions derived earlier yield the following: $\mathbb{E}[V | s_i, p_1] - p_1 = \frac{h_i}{\tau_1 + h_i}\left(\frac{\tau_0}{\tau_1} V + \varepsilon_i - \frac{\Delta\tau_1}{\tau_1} \frac{U_1}{\int_{t_j=1}(h_j/\gamma)\mathrm{d}j}\right)$ and $\text{var}[V | s_i, p_1]^{-1} = \tau_1 + h_i$. Plug the above into the $t = 0$ expected utility for a fast investor, simplify, and the resulting ex ante certainty equivalent *before technology acquisition costs* is $\frac{1}{2\gamma} \ln\left(1 + \frac{h_i}{\tau_1}\right)$. Subtracting the information acquisition cost and the speed acquisition cost gives the expression stated in the lemma. The calculation for slow investors repeats the above and is omitted. □

Lemma 2

Proof. An investor i's terminal wealth is $(p_2 - p_1)x_{i,1} + (V - p_2)x_{i,2}$, where $x_{i,t}$ is his *cumulative* position by round t. In particular, a slow investor has $x_{i,1} = 0$. Thus, at $t = 2$, each investor i solves $\max_{x_{i,2}} \mathbb{E}[-\exp\{-\gamma \cdot [(P_2 - P_1)x_{i,1} + (V - P_2)x_{i,2}]\} | s_i, p_1, p_2, x_{i,1}]$, or, equivalently,

$$\text{(B.5)} \qquad -\exp\{-\gamma \cdot (p_2 - p_1)x_{i,1}\} \max_{x_{i,2}} \mathbb{E}[-\exp\{-\gamma \cdot (V - P_2)x_{i,2}\} | s_i, p_1, p_2].$$

Hence, the optimization problem reduces to the same one as the one faced by the slow investors in the main model. (The position $x_{i,1}$ is irrelevant for the $t = 2$ optimization.) The same conjecture-and-verify analysis as in Lemma 1 applies and gives the optimal linear *cumulative* demand, $x_{i,2} = (h_i/\gamma)(s_i - p_2)$, for both the fast and the slow investors.

Now consider fast investors' optimization at $t = 1$. Substituting his optimal demand $x_{i,2}$ into optimization (B.5), at $t = 1$, a fast investor i solves

$$\max_{x_{i,1}} \mathbb{E}\left[-\exp\left\{-\gamma \cdot (P_2 - P_1)x_{i,1} - \frac{h_i^2}{2(\tau_2 + h_i)}(S_i - P_2)^2\right\} \Big| \, s_i, p_1\right],$$

where $\mathrm{var}[V|\, s_i, P_2]^{-1} = \tau_2 + h_i$. Note that the second term in the exponential does *not* affect the optimization. Further, due to the competitive market maker, $\mathbb{E}[V|\, s_i, p_1] = \mathbb{E}[\mathbb{E}[V|\, s_i, p_1, P_2]|\, s_i, p_1] = \mathbb{E}[P_2|\, s_i, p_1]$; and, similarly, $\mathrm{var}[V|\, s_i, p_1] = \mathrm{var}[P_2|\, s_i, p_1]$. Hence, the fast investor equivalently solves $\max_{x_{i,1}} \mathbb{E}[-\exp\{-\gamma \cdot (V - P_1)x_{i,1}\}|\, s_i, p_1]$. The optimization problem is equivalent to the one for fast investors solved in the proof of Lemma 1 and the same conjecture-and-verify analysis gives $x_{i,1} = (h_i/\gamma)(s_i - p_1)$.

The recursions of τ_t and p_t can be found using the above optimal demand functions. At $t = 1$, since the fast investors' optimal demand is the same as shown in Lemma 1, the same results hold: $\Delta\tau_1 = \tau_1 - \tau_0 = \left(\int_{t_j=1} \frac{h_j}{\gamma}\mathrm{d}j\right)^2 \tau_{\mathrm{U}}$ and $p_1 = p_0 + \frac{\Delta\tau_1}{\tau_1}\left(V + \frac{\gamma U_1}{\int_{t_j=1} h_j \mathrm{d}j}\right)$. At $t = 2$, the market maker observes the aggregate demand

$$\begin{aligned} L_2(p_2) &= \int_{t_j=1} \Big(x_{j,2}(s_j, p_2) - x_{j,1}(s_j, p_1)\Big)\mathrm{d}j + \int_{t_j=2} x_{j,2}(s_j, p_2)\mathrm{d}j + U_2 \\ &= p_1 \int_{t_j=1} \frac{h_j}{\gamma}\mathrm{d}j - p_2 \int_{\forall j} \frac{h_j}{\gamma}\mathrm{d}j + V \int_{t_j=2} \frac{h_j}{\gamma}\mathrm{d}j + U_2, \end{aligned}$$

where the second equality follows the optimal demand schedules derived earlier. Observe how the fast investors' signals are exactly offset, not contributing to the price discovery in the second fragment ($t = 2$). Intuitively, this is because their signals are already reflected in the first fragment (the $t = 1$ trading) and the only new information arises from the slow investors' signals. Again, the market maker sets the price exactly the same as in Lemma 1 and the resulting recursions hold: $\Delta\tau_2 = \tau_2 - \tau_1 = \left(\int_{t_j=2} \frac{h_j}{\gamma}\mathrm{d}j\right)^2 \tau_{\mathrm{U}}$ and $p_2 = p_1 + \frac{\Delta\tau_2}{\tau_2}\left(V + \frac{\gamma U_2}{\int_{t_j=2} h_j \mathrm{d}j} - p_1\right)$.

Finally, consider investors' ex ante certainty equivalent. Since slow investors only trade once at $t = 2$, they expect the same certainty equivalent π_{S} as solved in Lemma 1. A fast investor i's unconditional expected utility, before paying the technology cost, is

$$
\begin{aligned}
&\mathbb{E}\left[-\exp\left\{-\gamma\cdot(P_2-P_1)x_{i,1}-\frac{h_i^2}{2(\tau_2+h_i)}(S_i-P_2)^2\right\}\right]\\
=&\mathbb{E}\left[-\exp\left\{-\gamma\cdot(S_i-P_1)x_{i,1}+\gamma\cdot(S_i-P_2)x_{i,1}-\frac{h_i^2}{2(\tau_2+h_i)}(S_i-P_2)^2\right\}\right]\\
=&\mathbb{E}\left[-\exp\left\{-h_i\cdot(S_i-P_1)^2+h_i\cdot(S_i-P_2)(S_i-P_1)-\frac{h_i^2}{2(\tau_2+h_i)}(S_i-P_2)^2\right\}\right]
\end{aligned}
$$

where the last equality follows the optimal demand $x_{i,1}(\cdot)$ derived above. Define $Y := [S_i-P_1; S_i-P_2]$ as a bivariate normal (column) random vector, with

$$
\mathbb{E}Y=\begin{bmatrix}0\\0\end{bmatrix}\quad\text{and}\quad \mathrm{var}Y=\begin{bmatrix}\tau_1^{-1}+h_i^{-1} & \tau_2^{-1}+h_i^{-1}\\ \tau_2^{-1}+h_i^{-1} & \tau_2^{-1}+h_i^{-1}\end{bmatrix}.
$$

Then the above expected utility can be rewritten as $\mathbb{E}[-e^{Y^TAY}]$ where the coefficient matrix A is given by $A=[-h_i, h_i/2; h_i/2, -h_i^2/(2(\tau_2+h_i))]$. Evaluating the expectation with the density of the bivariate normal Y yields the expected utility of $-\tau_1\tau_2/\sqrt{\tau_1\cdot(h_i+\tau_2)(-h_i\tau_1+(h_i+\tau_1)\tau_2)}$. Solving the certainty equivalent yields the form of π_{F} stated in the lemma. □

Proposition 1

Proof. The proof begins by writing investors' certainty equivalent π_1 and π_2 as functions of the fast population size μ_1 in $[0,1]$. To do this, first note that from the first-order condition (6), investors' endogenous choice of h_i can be written as a monotone function of τ_{t_i}. By Lemma 1, $\tau_1=\tau_0+\Delta\tau_1$ and $\tau_2=\tau_0+\Delta\tau_2$, where $\Delta\tau_t=\tau_{\mathrm{U}}\mu_t^2h_t^2/\gamma^2$. Hence, τ_1 is effectively a function of μ_1, while τ_2 of both μ_1 and μ_2. Finally, note that $\mu_2=1-\mu_1$. As such, investors' certainty equivalent π_t are functions of μ_1. Then, depending on μ_1, there are three cases.

Case 1: First, suppose $\mu_1=1$ and $\mu_2=0$; i.e. all investors pay the speed technology cost $1/g_t$ and become fast. If this is the case, then in equilibrium $\pi_1\geq\pi_2$ must hold. Consider an investor i's unilateral deviation to not investing in the speed technology, saving the cost of $1/g_t$ and becomes slow. By equation (4), the price efficiency remains the same, $\tau_1=\tau_2$, because a single investor's deviation has zero population measure. Then i's optimal technology investment h_i, by the first-order condition (6), remains the same as if he were fast: $h_i=h(\tau_2)=h(\tau_1)=h_1$. As a result, his certainty equivalent $\pi_2=\pi_1+1/g_t>\pi_1$ and he indeed will deviate. Such a case of $\mu_1=1$ and $\mu_2=0$, therefore, can never be an equilibrium.

Case2: Second, consider the case of $\mu_1 = 0$ and $\mu_2 = 1$. (This will correspond to the corner equilibrium stated in the proposition.) If this is an equilibrium, it has to be the case that $\pi_1 \leq \pi_2$, i.e., all investors stay slow. The argument below shows that fixing all other exogenous parameters, $\pi_1 \leq \pi_2$ holds if and only if $g_t < \hat{g}_t$, for some threshold $\hat{g}_t$. At $\mu_1 = 0$, $\tau_1 = \tau_0 < \tau_2$ and thus a slow investor's unilateral deviation to fast yields $\pi_1|_{\mu_1=0} = \frac{1}{2\gamma}\ln\left(1 + \frac{h_1}{\tau_0}\right) - \dot{c}(h_1) - \frac{1}{g_t}$, where h_1 is the unique solution implied by the first-order condition (6) with $\tau_1 = \tau_0$. By envelope theorem, $\partial\pi_1/\partial g_t = 1/g_t^2 > 0$. Therefore, $\pi_1|_{\mu_1=0}$ is monotone increasing in g_t with limits $\lim_{g_t\downarrow 0}\pi_1 = -\infty < 0 < \pi_2 < \lim_{g_t\uparrow\infty}\pi_1$. (Note that $\pi_2|_{\mu_1=0}$ is a finite number unaffected by g_t.) By continuity, therefore, there exists a unique $\hat{g}_t$ such that $\pi_1 = \pi_2$ when $\mu_1 = 0$. As such, $\pi_1 \leq \pi_2$, supporting $\mu_1 = 0$ and $\mu_2 = 1$, if and only if $g_t \leq \hat{g}_t$. When instead $g_t > \hat{g}_t$, this corner equilibrium does not exist.

Case3: Third, consider the interior case of $\mu_1 \in (0, 1)$, implying $\pi_1 = \pi_2$. The key is to show the following result: The difference $\pi_1 - \pi_2$ strictly decreases in μ_1. Evaluate the partial derivative of $\pi_1 - \pi_2$ with respect to μ_1 and after some simplification,

$$\frac{\partial(\pi_1 - \pi_2)}{\partial\mu_1} \cdot 2\gamma = \left[\frac{h_2/\tau_2}{\tau_2 + h_2} - \frac{h_1/\tau_1}{\tau_1 + h_1}\right]\frac{\partial\tau_1}{\partial\mu_1} + \frac{h_2/\tau_2}{\tau_2 + h_2)}\frac{\partial\Delta\tau_2}{\partial\mu_1}.$$

Note that the term in the square-brackets is non-positive, because $\tau_2 \geq \tau_1$ by construction and because $h_t = h(\tau_t)$ decreases in τ_t as implied by the first-order condition (6).

One still needs to sign both $\partial\tau_1/\partial\mu_1$ and $\partial\Delta\tau_2/\partial\mu_1$. To do so, rearrange the first-order condition (6) for $t = 1$ as $(\tau_0 + \Delta\tau_1 + g_h k_h(m_1))/\dot{k}_h(m_1) = g_h/(2\gamma)$ with $\Delta\tau_1 = \mu_1^2 g_h^2 k_h(m_1)^2 \tau_{\mathrm{U}}/\gamma^2$ following equation (4). It can then immediately be concluded that m_1 strictly decreases in μ_1, as otherwise the left-hand side of the above equation is always increasing in μ_1, unable to maintain the equality. (Recall that $k_h(\cdot)$ is concavely increasing.) Similarly, it is also known that τ_1 $(= \tau_0 + \Delta\tau_1)$ decreases in m_1. Hence, τ_1 (and $\Delta\tau_1$) increases in μ_1. For $t = 2$, $(\tau_0 + \Delta\tau_1 + \Delta\tau_2 + h_2)/\dot{k}_h(m_2) = g_h/(2\gamma)$ with $\Delta\tau_2 = (1 - \mu_1)^2 g_h^2 k_h(m_2)^2 \tau_{\mathrm{U}}/\gamma^2$. Note that $\frac{\partial\Delta\tau_2}{\partial\mu_1} = \left(-2(1 - \mu_1)h_2^2 + 2(2 - \mu_1)^2 h_2 \frac{\partial h_2}{\partial\mu_1}\right)\frac{\tau_{\mathrm{U}}}{\gamma^2}$. As such, if $\Delta\tau_2$ increases in μ_1, then it has to be the case that $\partial h_2/\partial\mu_1 > 0$. Because $h_2 = g_h k_h(m_2)$, m_2 is also increasing in μ_1. It then follows that the left-hand side of the above equation strictly increases in μ_1—$\Delta\tau_1$, $\Delta\tau_2$, and m_2 all increase with μ_1, invalidating the equality. Therefore, it must be $\Delta\tau_2$ decreases in μ_1.

As τ_1 increases in μ_1 but $\Delta\tau_2$ decreases in μ_1, one can conclude from the above partial derivative that the difference $\pi_1 - \pi_2$ indeed strictly decreases in μ_1.

To sum up, recall from the first cases that at $\mu_1 = 1$, $\pi_1 < \pi_2$. From the second case, at $\mu_1 = 0$, $\pi_1 > \pi_2$ if and only if $g_t > \hat{g}_t$. Hence, when $g_t \leq \hat{g}_t$, the equilibrium with interior μ_1 does not exist due to the above monotonicity of $\pi_1 - \pi_2$ in μ_1. When $g_t > \hat{g}_t$, there exists a unique $\mu_1 \in (0, 1)$ such that $\pi_1 = \pi_2$, sustaining this equilibrium. This completes the proof of this proposition. □

Proposition 2 and Corollary 4

Proof. First, the following shows that h_t is monotonically increasing in g_h for both $t = 1$ and $t = 2$. The first-order condition (6) can be written as $\dot{k}_h(m_t)/(2\gamma) - k_h(m_t) = \tau_t/g_h$, which uniquely solves m_t. Holding g_h (and γ) constant,

$$\frac{\partial m_t}{\partial \tau_t} = \frac{1}{g_h}\left(\frac{\ddot{k}_h(m_t)}{2\gamma} - \dot{k}_h(m_t)\right)^{-1} \leq 0 \tag{B.6}$$

where the inequality follows the concavity of $k_h(m)$. (Note that this also implies that $m_1 \geq m_2$ because $\tau_2 \geq \tau_1$.) In addition,

$$\frac{\partial m_t}{\partial g_h} = -\frac{\tau_t}{g_h^2}\left(\frac{\ddot{k}_h(m_t)}{2\gamma} - \dot{k}_h(m_t)\right)^{-1} = -\frac{\tau_t}{g_h}\frac{\partial m_t}{\partial \tau_t} \geq 0. \tag{B.7}$$

From the definition of $h_t = g_h k_h(m_t)$, $\partial h_t/\partial g_h = k_h(m_t) + g_h \dot{k}_h(m_t) \partial m_t/\partial g_h \geq 0$. Therefore, in any case, the equilibrium h_t is increasing in the information technology g_h.

The rest of this proof only deals with the case of exogenous speed acquisition, i.e., with fixed μ_1 and μ_2. The proof for the case of endogenous μ_t is deferred to proof of Proposition 6. Consider the short-run of $t = 1$. While g_h increases, h_1 increases to satisfy the first-order condition, as shown above. It then follows that $\partial\tau_1/\partial g_h > 0$ because $\tau_1 = \tau_0 + \tau_{\mathrm{U}} h_1^2 \mu_1^2/\gamma^2$ with μ_1 exogenous.

Consider the long-run of $t = 2$ now. Suppose the opposite, $\partial\tau_2/\partial g_h < 0$, is true. Then h_2 should be decreasing with g_h because $\tau_2 = \tau_1 + \tau_{\mathrm{U}} h_2^2 \mu_2^2/\gamma^2$ with τ_1 is increasing in g_h. However, the transformation of first-order condition 6, $g_h/(2\gamma) = (\tau_2 + h_2) {k_h}^{-1}(h_2/g_h)$, shows that it is impossible for both τ_2 and h_2 to be decreasing with g_h at the same time. Thus, the assumed inequality is wrong and τ_2 increases with g_h. □

Proposition 3

Proof. To avoid repetition, the proof only considers the full equilibrium where the information acquisition is available. A similar argument can be constructed for the special case where all investors have the same exogenous information $h_\circ$. In the interior equilibrium, $\pi_1 - \pi_2 = 0$ and, following the proof of Proposition 1, the equality implies an implicit function of μ_1 in terms of the speed technology g_t, which implies: $\frac{\mathrm{d}\mu_1}{\mathrm{d}g_t} = -\frac{\partial\pi_1/\partial g_t}{\partial(\pi_1-\pi_2)/\partial\mu_1}$, where the denominator of the fraction is negative as shown in Case 3 of the proof for Proposition 1. The numerator equals $1/g_t^2 > 0$ by envelope theorem. Therefore, μ_1 increases in g_t. □

Proposition 4

Proof. This proof deals with two cases. The first case is where all investors' information precision is exogenously given at $h_\circ$. The second case is where investors endogenously acquire information.

In the first case, as shown in the proof of Proposition 3, μ_1 is increasing with g_t, which directly implies that τ_1 is increasing with g_t. For the long-run price efficiency τ_2, by the implicit function theorem, $\partial\tau_2/\partial\mu_1 = 2\tau_{\mathrm{U}} h_o^2 \mu_1/\gamma^2 - 2\tau_{\mathrm{U}} h_o^2 \mu_2/\gamma^2$, or $\partial\tau_2/\partial g_t = 2(\tau_{\mathrm{U}} h_o^2 \mu_1/\gamma^2 - 2\tau_{\mathrm{U}} h_o^2 \mu_2/\gamma^2)(\partial g_t/\partial\mu_1)$. It is clear that $\partial\tau_2/\partial g_t < 0$ when μ_1 is close to zero and μ_2 close to one (i.e., g_t is small), and $\partial\tau_2/\partial g_t > 0$ when μ_1 is close to one and μ_2 close to zero (i.e., g_t is large).

For the second case, two steps are involved. The first step is to prove that $\partial\tau_1/\partial g_t > 0$. In the interior equilibrium, the first-order condition (6) for $t = 1$, together with $\tau_1 = \tau_0 + \tau_{\mathrm{U}} h_1^2 \mu_1^2/\gamma^2$, implies an implicit function of $h_1 = g_h k_h(m_1)$ and μ_1, from which $\partial h_1/\partial\mu_1 = -\frac{2\tau_{\mathrm{U}}\mu_1 h_1^2/\gamma^2}{2\tau_{\mathrm{U}}\mu_1^2 h_1/\gamma^2 + 1 - \ddot{k}_h(m_1)/\dot{k}_h(m_1)} < 0$, where the inequality follows because $k_h(\cdot)$ is concavely increasing. From the effect of speed technology and population of sizes, $\partial\mu_1/\partial g_t > 0$. Therefore, by chain rule, $\partial h_1/\partial g_t < 0$. The first-order condition (6) also implies that τ_1 decreases with m_1 and, hence, also with h_1, yielding $\partial\tau_1/\partial g_t > 0$.

The second step is to prove that τ_2 first decreases and then increases with g_t. In the interior equilibrium, the first-order condition (6) for $t = 2$ always holds. Recall $\tau_2 = \tau_0 + \tau_{\mathrm{U}}\tau_1^2\mu_1^2/\gamma^2 + \tau_{\mathrm{U}}\tau_2^2\mu_2^2/\gamma^2$. By implicit function theorem, it implies

$$\frac{\partial h_2}{\partial\mu_2} = -\frac{4\tau_{\mathrm{U}}}{\gamma}\frac{\mu_2 h_2^2 - \mu_1 h_1^2 - \mu_1^2 h_1 \partial h_1/\partial\mu_1}{-\ddot{k}_h(m_2)/\dot{k}_h(m_2) + 2\gamma + 4\tau_{\mathrm{U}}\mu_2^2\tau_2/\gamma}.$$

As done in the proof of step 1, the idea is to first sign the above partial derivative and then sign $\partial h_2/\partial g_t$ using chain rule: $\frac{\partial h_2}{\partial g_t} = \frac{\partial h_2}{\partial\mu_2}\frac{\partial\mu_2}{\partial\mu_1}\frac{\partial\mu_1}{\partial g_t}$, where $\partial\mu_2/\partial\mu_1 = -1$ following the identity $\mu_1 + \mu_2 = 1$ and $\partial\mu_1/\partial g_t > 0$. In particular, consider the limits of $\partial h_2/\partial\mu_2$ as $g_t \uparrow \infty$ and $g_t \downarrow \hat{g}_t$, respectively. To evaluate these limits, one needs to show that h_1, h_2, and $\partial h_1/\partial\mu_1$ are have finite bounds.

The finite bounds for h_t can be easily established by noting from the first-order condition (6) that τ_t in equilibrium is monotone decreasing in τ_t. From the model setting, it is known that τ_t has strictly positive lower bound τ_0. Therefore, both h_1 and h_2 have finite upper bounds. (They also have lower bounds of zero by construction.) Finally, from the expression of $\partial h_1/\partial\mu_1$ derived in the proof of the previous step, it can be seen that $\mu_1 \cdot (\partial h_1/\partial\mu_1) = -\frac{2\tau_{\mathrm{U}}\mu_1^2 h_1/\gamma^2}{2\tau_{\mathrm{U}}\mu_1^2\tau_1/\gamma^2 + 1 - \ddot{k}_h(m_1)/\dot{k}_h(m_1)} h_1 > -h_1$ is also bounded.

Now the limits can be evaluated. When speed technology $g_t \uparrow \infty$, almost all investors become fast and $\mu_2 \downarrow 0$ and $\lim_{\mu_2 \downarrow 0}(\frac{\partial h_2}{\partial \mu_2}) = -\frac{4\tau_U}{\gamma} \frac{-\mu_1 h_1^2 - \mu_1^2 h_1 \partial h_1/\partial \mu_1}{-\ddot{k}_h(m_2)/\dot{k}_h(m_2)+2\gamma} > 0$. Similarly, when speed technology $g_t \downarrow \hat{g}_t$, almost all investors stay slow, $\mu_1 \downarrow 0$, and $\lim_{\mu_1 \downarrow 0}(\frac{\partial h_2}{\partial \mu_2}) = -\frac{4\tau_U}{\gamma} \frac{\mu_2 h_2^2}{-\ddot{k}_h(m_2)/\dot{k}_h(m_2)+2\gamma+4\tau_U \mu_2^2 h_2/\gamma} < 0$. As the above shows, for sufficiently large (low) g_t, h_2 increases (decreases) in μ_2 and hence decreases (increases) in g_t by the chain rule expression above. The first-order condition (6) implies that τ_2 decreases with τ_2 and the stated results are proved. □

Proposition 5

Proof. **Fixing g_t, g_h increases from $\hat{g}_h$ to ∞.** The aggregate demand for speed in the economy is $\int_{[0,1]} \mathbb{1}_{\{t_i=1\}} \mathrm{d}i = \mu_1$. From $\Delta\tau_1 = \tau_U h_1^2 \mu_1^2/\gamma^2$, by implicit function theorem,

$$\frac{\partial \mu_1}{\partial g_h} = \frac{\gamma^2}{2\tau_U \mu_1 h_1^2}\left(\frac{\partial \Delta\tau_1}{\partial g_h} - \frac{2\tau_U \mu_1^2 h_1}{\gamma^2}\frac{\partial h_1}{\partial g_h}\right). \tag{B.8}$$

Hence, the sign of $\partial\mu_1/\partial g_h$ depends on the difference between the two terms in the brackets. Consider first the case of a very small g_h. Corollary 1 establishes the existence of a lower bound $\hat{g}_h$ for g_h, such that the equilibrium is interior if and only if $g_h \geq \hat{g}_h$. In particular, when $g_h \downarrow \hat{g}_h$, the marginal investor is just indifferent between becoming fast or not, implying $\mu_1 \downarrow 0$. The first-order condition (6) at this limit gives $1/(2(\tau_0 + h_1)\gamma) - \dot{c}(h_1) = 0$, which has interior solution of $0 < h_1 < \infty$, thanks to the assumption of $\dot{c}(0) = 0$. By differentiability, therefore, $\partial h_1/\partial g_h$ is finite in this limit as well. Taken together, the second term in the above brackets has limit zero as $\mu_1 \downarrow 0$, when $g_h \downarrow \hat{g}_h$. The remaining term is $\partial\Delta\tau_1/\partial g_h$, which is shown by Proposition 6 to be strictly positive. Thus, $\partial\mu_1/\partial g_h$ is positive in the case of a very small g_h, close to the lower bound of $\hat{g}_h$.

Consider next the case of a very large g_h, i.e. $g_h \uparrow \infty$. First, there exists an upper bound for investors' expense on information acquisition, m_t. To see this, note from the first-order condition (6):

$$\frac{1}{2\gamma}\dot{k}_h(m_t) > \frac{1}{2\gamma}\dot{k}_h(m_t) - \frac{1}{g_h}\tau_t = k_h(m_t) \geq k_h(0) + m_t\dot{k}_h(m_t) = m_t\dot{k}_h(m_t) \tag{B.9}$$

where the first inequality holds because $\tau_t \geq \tau_0 > 0$ and the last inequality holds by concavity of $k_h(\cdot)$ and by $k_h(0) = 0$. Therefore, for $t \in \{1, 2\}$, there exists an upper bound for $m_t \leq 1/(2\gamma)$, an upper bound for $k_h(m_t) \leq k_h(1/(2\gamma))$, and a lower bound for $\dot{k}_h(m_t) \geq \dot{k}_h(1/(2\gamma)) > 0$. Second, in the limit of $g_h \uparrow \infty$, the equilibrium is always interior (following Corollary 1). Hence, the limit of the fast investor's ex ante certainty equivalent $\lim_{g_h \uparrow \infty} \pi_1 = \frac{1}{2\gamma} \lim_{g_h \uparrow \infty} \ln\left(1 + \frac{h_1}{\tau_1}\right) - \lim_{g_h \uparrow \infty} m_1 - \frac{1}{g_t}$ exists and must be nonnegative to sustain the interior equilibrium. Since m_1 is bounded from above, it follows that $\lim_{g_h \uparrow \infty}(h_1/\tau_1)$ also exists and is strictly positive. That is, there exists some $a \in (0, \infty)$, such that $\lim_{g_h \uparrow \infty}(\tau_1/h_1) = a$. Equivalently, as τ_0 is a finite constant, $\lim_{g_h \uparrow \infty}(\Delta\tau_1/h_1) = a$. Further, a fast investor's first-order condition (6) can be rewritten as $\frac{1}{2\gamma}\frac{g_h}{\tau_1+h_1} - \dot{c}(h_1/g_h) = 0$. Since the above holds under $g_h \uparrow \infty$, it follows that $h_1 \sim g_h$; or $\lim_{g_h \uparrow \infty}(h_1/g_h) = b \in (0, \infty)$. (If h_1 is of higher magnitude than g_h, the limit of the first term above falls to zero, while the limit of the second term is strictly positive as $c(\cdot)$ is strictly convex. If instead h_1 is of lower magnitude than g_h, the limit of the first term approaches infinity, while the second term falls to zero.) Now consider the limit of the difference in the brackets of equation (B.8):

$$\lim_{g_h \uparrow \infty}\left(\frac{\partial \Delta\tau_1}{\partial g_h} - 2\frac{\tau_U \mu_1^2 h_1}{\gamma^2}\frac{\partial h_1}{\partial g_h}\right) = \lim_{g_h \uparrow \infty}\left(\frac{\partial \Delta\tau_1}{\partial g_h} - 2\frac{\Delta\tau_1}{h_1}\frac{\partial h_1}{\partial g_h}\right) = (ab - 2ab) < 0$$

where the last equality follows L'Hôpital's rule. Therefore, in the limit of $g_h \uparrow \infty$, $\partial\mu_1/\partial g_h < 0$. Finally, consider the value of μ_1 in this limit. Note that $\Delta\tau_1 = \tau_0 + \tau_U \mu_1^2 h_1^2/\gamma^2$. Therefore, in order for $\lim_{g_h \uparrow \infty}(\Delta\tau_1/h_1) = a \in (0, \infty)$ to hold, it must be such that $\lim_{g_h \uparrow \infty}(\mu_1^2 h_1) = c \in (0, \infty)$, i.e., μ_1 in this limit is of magnitude $h_1^{-1/2}$. As $h_1 \uparrow \infty$, this also implies that $\mu_1 \downarrow 0$ in this limit.

Fixing g_h, g_t increases from $\hat{g}_t$ to ∞. The aggregate demand for information is $\bar{h} := \mu_1 h_1 + \mu_2 h_2$. Since μ_1 is monotone in g_t (Proposition 3), it is sufficient to examine the partial derivative of the above aggregate demand with respect to μ_1: $\partial\bar{h}/\partial\mu_1 = h_1 - h_2 + \mu_1 \cdot (\partial h_1/\partial\mu_1) - \mu_2 \cdot (\partial h_2/\partial\mu_2)$. At the initial extreme of $g_t \downarrow \hat{g}_t$, the proof of Proposition 4 has shown that 1) $\mu_1 \downarrow 0$, 2) $\mu_1 \cdot \partial h_1/\partial\mu_1$ is bounded, and 3) $\partial h_2/\partial\mu_2 < 0$. Taking these into the above partial derivative yields $\partial\bar{h}/\partial\mu_1 \to h_1 - h_2 - \mu_2 \cdot (\partial h_2/\partial\mu_2) > 0$, recalling that $h_1 \geq h_2$ from equation (7). At the eventual extreme of $g_t \uparrow \infty$, the proof of Proposition 4 has shown that 1) $\mu_2 \downarrow 0$, 2) $\partial h_1/\partial\mu_1 < 0$, and 3) $\partial h_2/\partial\mu_2 > 0$. In addition, since $\mu_2 \downarrow 0$, $\Delta\tau_2 = \mu_2^2 h_2^2 \tau_U/\gamma^2 \downarrow 0$ (h_2 is bounded), implying $\tau_2 \downarrow \tau_1$ and 4) $h_2 \uparrow h_1$. Taking the above into $\bar{h}$ yields $\partial\bar{h}/\partial\mu_1 \to \mu_1 \cdot (\partial h_1/\partial\mu_1) - \mu_2 \cdot (\partial h_2/\partial\mu_2) < 0$. □

Proposition 6

Proof. By construction, $\tau_1 = \tau_0 + \Delta\tau_1$ and $\tau_2 = \tau_0 + \Delta\tau_1 + \Delta\tau_2$. The first-order condition implicitly has m_1 and m_2 as functions of $m_1(\Delta\tau_1)$ and $m_2(\Delta\tau_1, \Delta\tau_2)$. Further, $\Delta\tau_t = \tau_U g_h^2 k_h(m_t)^2 \mu_t^2/\gamma^2$, or $\mu_t = \frac{\gamma}{\sqrt{\tau_U}}\frac{\sqrt{\Delta\tau_t}}{g_h k_h(m_t)}$. Therefore, the unconstrained equilibrium (with endogenous acquisition of both speed and information) is pinned down by a two-equation, two-unknown system: $\pi_1 - \pi_2 = 0$ and $\mu_1 + \mu_2 - 1 = 0$; or, equivalently, with a vector function $F(\Delta\tau_1, \Delta\tau_2; g_h)$,

$$F = \begin{bmatrix} \left(\frac{1}{2\gamma}\ln\left(1 + \frac{g_h k_h(m_1)}{\tau_1}\right) - m_1 - \frac{1}{g_t}\right) - \left(\frac{1}{2\gamma}\ln\left(1 + \frac{g_h k_h(m_2)}{\tau_2}\right) - m_2\right) \\ \frac{\sqrt{\Delta\tau_1}}{k_h(m_1)} + \frac{\sqrt{\Delta\tau_2}}{k_h(m_2)} - \frac{\sqrt{\tau_U}}{\gamma} g_h \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \tag{B.10}$$

where $\{m_t\}_{t\in\{1,2\}}$ are functions of $\Delta\tau_1$ and $\Delta\tau_2$ following the first-order condition (6), which can be rewritten as $\dot{k}_h(m_t)/(2\gamma) - k_h(m_t) = \tau_t/g_h$.

Take total derivatives with respect to g_h on the equilibrium condition $F = 0$ to get $\begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix}\begin{bmatrix} \mathrm{d}\Delta\tau_1 \\ \mathrm{d}\Delta\tau_2 \end{bmatrix} + \begin{bmatrix} F_{1g} \\ F_{2g} \end{bmatrix}\mathrm{d}g_h = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$. One can easily evaluate, using envelope theorem,

$$F_{1g} = \frac{1}{2\gamma}\frac{k_h(m_1)}{\tau_1 + g_h k_h(m_1)} - \frac{1}{2\gamma}\frac{k_h(m_2)}{\tau_2 + g_h k_h(m_2)} = \frac{1}{g_h}\left(\frac{k_h(m_1)}{\dot{k}_h(m_1)} - \frac{k_h(m_2)}{\dot{k}_h(m_2)}\right) > 0,$$

where the second equality follows the first-order condition (6), while the last inequality follows the concavity of $k_h(m)$, knowing that $m_1 > m_2$. Also,

$$\begin{aligned} F_{2g} &= -\frac{\sqrt{\Delta\tau_1}}{k_h(m_1)^2}\dot{k}_h(m_1)\frac{\partial m_1}{\partial g_h} - \frac{\sqrt{\Delta\tau_2}}{k_h(m_2)^2}\dot{k}_h(m_2)\frac{\partial m_2}{\partial g_h} - \frac{\sqrt{\tau_{\mathrm{U}}}}{\gamma} \\ &= -\frac{\sqrt{\tau_{\mathrm{U}}}}{\gamma}\mu_1 g_h\frac{\dot{k}_h(m_1)}{k_h(m_1)}\frac{\partial m_1}{\partial g_h} - \frac{\sqrt{\tau_{\mathrm{U}}}}{\gamma}\mu_2 g_h\frac{\dot{k}_h(m_2)}{k_h(m_2)}\frac{\partial m_2}{\partial g_h} - \frac{\sqrt{\tau_{\mathrm{U}}}}{\gamma} < -\frac{\sqrt{\tau_{\mathrm{U}}}}{\gamma} < 0, \end{aligned}$$

where the equality uses the expression of μ_t and the inequality holds because $\partial m_t/\partial g_h$ is derived earlier to be positive (inequality B.7).

The elements in the Jacobian matrix can also be evaluated. Using envelope theorem,

$$F_{11} = -\frac{k_h(m_1)}{\dot{k}_h(m_1)\tau_1} + \frac{k_h(m_2)}{\dot{k}_h(m_2)\tau_2} \leq 0$$

where the inequality holds because $k_h(m_1)/\dot{k}_h(m_1) \geq k_h(m_2)/\dot{k}_h(m_2)$ (concavity) and $\tau_1 \leq \tau_2$. Similarly,

$$F_{12} = \frac{k_h(m_2)}{\dot{k}_h(m_2)\tau_2} > 0.$$

Now consider the partial derivatives with respect to F_2:

$$\begin{aligned} F_{21} &= \frac{1}{2\sqrt{\Delta\tau_1}k_h(m_1)} - \frac{\sqrt{\Delta\tau_1}}{k_h(m_1)^2}\dot{k}_h(m_1)\frac{\partial m_1}{\partial\tau_1}\cancel{\frac{\partial\tau_1}{\partial\Delta\tau_1}} - \frac{\sqrt{\Delta\tau_2}}{k_h(m_2)^2}\dot{k}_h(m_2)\frac{\partial m_2}{\partial\tau_2}\cancel{\frac{\tau_2}{\Delta\tau_1}} \\ &= \frac{1}{2\sqrt{\Delta\tau_1}k_h(m_1)} - \frac{\sqrt{\tau_{\mathrm{U}}}}{\gamma}\mu_1 g_h\frac{\dot{k}_h(m_1)}{k_h(m_1)}\frac{\partial m_1}{\partial\tau_1} - \frac{\sqrt{\tau_{\mathrm{U}}}}{\gamma}\mu_2 g_h\frac{\dot{k}_h(m_2)}{k_h(m_2)}\frac{\partial m_2}{\partial\tau_2} > 0 \end{aligned}$$

where the equality follows the expression of μ_t and the inequality holds because $\partial m_t/\partial\tau_t \leq 0$ as shown before (inequality B.6). Similarly,

$$F_{22} = \frac{1}{2\sqrt{\Delta\tau_2}k_h(m_2)} - \frac{\sqrt{\Delta\tau_2}}{k_h(m_2)^2}\dot{k}_h(m_2)\frac{\partial m_2}{\partial\tau_2}\cancel{\frac{\partial\tau_2}{\partial\Delta\tau_2}} > 0.$$

By Cramer's rule,

$$\frac{\partial \Delta\tau_1}{\partial g_h} = \frac{\begin{vmatrix} -F_{1g} & F_{12} \\ -F_{2g} & F_{22} \end{vmatrix}}{\begin{vmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{vmatrix}} \text{ and } \frac{\partial \Delta\tau_2}{\partial g_h} = \frac{\begin{vmatrix} F_{11} & -F_{1g} \\ F_{21} & -F_{2g} \end{vmatrix}}{\begin{vmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{vmatrix}}.$$

The sign of the denominator is easy to shown: $F_{11}F_{22} - F_{12}F_{21} < 0$. It remains to examine the numerators. For τ_1, it can be seen that $-F_{1g}F_{22} + F_{12}F_{2g} < 0$; hence $\partial\tau_1/\partial g_h = \partial\Delta\tau_1/\partial g_h > 0$.

To sign $\partial\tau_2/\partial g_h$ is equivalent to signing the sum of the numerators of $\partial\Delta\tau_1/\partial g_h$ and $\partial\Delta\tau_2/\partial g_h$:

$$(-F_{1g}F_{22} + F_{12}F_{2g}) + (-F_{11}F_{2g} + F_{1g}F_{21}) = (F_{21} - F_{22})F_{1g} + (F_{12} - F_{11})F_{2g}.$$

To prove the statement made in the proposition, the objective is to show that under the limits of $g_h \uparrow \infty$ and of $g_h \downarrow \hat{g}_h$, the sign of the above term is negative and positive, respectively (recall that the determinant for the denominator is negative). The proof of Proposition 5 shows that in the upper limit, $\mu_1 \downarrow 0$ and $\mu_2 \uparrow 1$. The proof of Corollary 1 shows that in the lower limit, investors are just indifferent between acquiring the speed or not, implying again $\mu_1 \downarrow 0$ and $\mu_2 \uparrow 1$. Using these limiting values of μ_1 and μ_2, the above simplifies to

$$\left(\frac{1}{2\sqrt{\Delta\tau_1}k_{h1}} - \frac{1}{2\sqrt{\Delta\tau_2}k_{h2}}\right)F_{1g} + \frac{k_{h1}}{\dot{k}_{h1}\tau_1}F_{2g}, \tag{B.11}$$

where, simplifying the notation, $k_h(\cdot)$ and $\dot{k}_h(\cdot)$ are replaced by subscripts of $t \in \{1, 2\}$.

Consider the limit of $g_h \uparrow \infty$ first. Equation (B.11) satisfies the following inequality:

$$\left(\frac{1}{2\sqrt{\Delta\tau_1}k_{h1}} - \frac{1}{2\sqrt{\Delta\tau_2}k_{h2}}\right)F_{1g} + \frac{k_{h1}}{\dot{k}_{h1}\tau_1}F_{2g} < \frac{F_{1g}}{2\sqrt{\Delta\tau_1}k_{h1}}$$

because $F_{1g} > 0$ and $F_{2g} < -\sqrt{\tau_U}/\gamma < 0$. The proof of Proposition 5 establishes that $\Delta\tau_1 \to \infty$. In addition, the inequality (B.9) establishes that in equilibrium, both m_1 and m_2 have finite upper and lower bounds, implying that both k_{h1} and F_{1g} is also finite (since $k_h(\cdot)$ is twice-differentiable). Therefore, $\lim_{g_h\uparrow\infty}(F_{1g}/(2\sqrt{\Delta\tau_1}k_{h1}) = 0$ and

$$\lim_{g_h\uparrow\infty}\left[\left(\frac{1}{2\sqrt{\Delta\tau_1}k_{h1}} - \frac{1}{2\sqrt{\Delta\tau_2}k_{h2}}\right)F_{1g} + \frac{k_{h1}}{\dot{k}_{h1}\tau_1}F_{2g}\right] < \lim_{g_h\uparrow\infty}\frac{F_{1g}}{2\sqrt{\Delta\tau_1}k_{h1}} = 0.$$

This proves that in this upper limit, τ_2 is increasing with g_h.

Finally, consider the limit of $g_h \downarrow \hat{g}_h$. As $g_h \downarrow \hat{g}_h$, clearly F_{1g} and F_{2g} are finite. However, $\mu_1 \downarrow 0$, $\Delta\tau_1 \downarrow 0$, and the first term of equation (B.11) approaches $+\infty$. The sum of numerators above therefore has a positive sign. Given the negative sign of the denominator, it can be concluded that

$\partial\tau_2/\partial g_h < 0$ in the limit of $g_h \downarrow \hat{g}_h$. □

Proposition 7

Proof. To prove the proposition, it is equivalent to sign the difference between the partial derivatives of $\Delta\tau_1$ and of $\Delta\tau_t$ with respect to the two technology g_t and g_h. For example, if $\Delta\tau_1$ increases faster than $\Delta\tau_2$, then it follows that the speed of price discovery is increasing; and vice versa. This proof proceeds with the two types of technology shocks separately.

Shocking the speed technology g_t. From the proof of Proposition 6, it can be seen that the equilibrium is characterized by the vector function $F(\Delta\tau_1, \Delta\tau_2; g_t, g_h) = 0$ (equation B.10). The partial derivatives of F with respect to the speed technology g_t are $F_{1g} = 1/g_t^2$ and $F_{2g} = 0$. As in the proof of Proposition 6, by Cramer's rule,

$$\text{sign}\left(\frac{\partial\Delta\tau_1}{\partial g_t}\right) = -\text{sign}(-F_{1g}F_{22} + F_{12}F_{2g}) = \text{sign}\left(\frac{F_{22}}{g_t^2}\right) > 0,$$
$$\text{sign}\left(\frac{\partial\Delta\tau_2}{\partial g_t}\right) = -\text{sign}(-F_{11}F_{2g} + F_{1g}F_{21}) = -\text{sign}\left(\frac{F_{21}}{g_t^2}\right) < 0.$$

Therefore, $\partial\Delta\tau_1/\partial g_t - \partial\Delta\tau_2/\partial g_t > 0$, implying that $\Delta\tau_1$ is increasing in the speed technology g_t, while $\Delta\tau_2$ is decreasing. The speed of price discovery thus increases with the speed technology.

Shocking the information technology g_h. The objective is to sign the difference of $\partial\Delta\tau_1/\partial g_h - \partial\Delta\tau_2/\partial g_h$ under the limits of $g_h \downarrow \hat{g}_h$ and $g_h \uparrow \infty$ respectively. Using the expressions derived from the proof of proposition 6, the sign of the above difference is the same as the sign of $(F_{21} + F_{22})F_{1g} - (F_{11} + F_{12})F_{2g}$.

Consider the limit of $g_h \downarrow \hat{g}_h$ first. Clearly, in this limit, F_{11}, F_{12}, F_{22}, F_{1g}, and F_{2g} are all finite. (In particular, $\partial m_t/\partial g_h$ is finite following the expression B.7 and the upper bound of $m_t \leq 1/(2\gamma)$ established in the proof of Proposition 5.) However, Corollary 1 establishes that $\mu_1 \downarrow 0$ and there is no price discovery in the short-run, i.e., $\Delta\tau_1 \downarrow 0$. Thus, the first term in F_{21} approaches positive infinity, driving the above difference expression also to positive infinity. In this limit, therefore, $\Delta\tau_1$ increases faster than $\Delta\tau_2$ and the speed of price discovery is increasing in g_h.

Consider next the limit of $g_h \uparrow \infty$. Observe that because $F_{21} > 0$, $F_{22} > 0$, and $F_{1g} > 0$, the following inequality always holds: $(F_{21}+F_{22})F_{1g} - (F_{11}+F_{12})F_{2g} < (F_{11}+F_{12})F_{2g} = \left(-\frac{k_{h1}}{\dot{k}_{h1}\tau_1} + \frac{2k_{h2}}{\dot{k}_{h2}\tau_2}\right)F_{2g}$, where the last equality simply uses the expressions for F_{11} and F_{22}. Recall that m_t is bounded from above by $1/(2\gamma)$, hence, F_{2g} is always finite and so are k_{ht} and $\dot{k}_{ht}$. Yet, $\tau_t \uparrow \infty$ in the limit of $g_h \uparrow \infty$. Therefore, $\lim_{g_h\uparrow\infty}\Big((F_{21} + F_{22})F_{1g} - (F_{11} + F_{12})F_{2g}\Big) < \lim_{g_h\uparrow\infty}\left(-\frac{k_{h1}}{\dot{k}_{h1}\tau_1} + \frac{2k_{h2}}{\dot{k}_{h2}\tau_2}\right)F_{2g} = 0$. That is, in this upper limit of g_h, $\Delta\tau_1$ is growing slower than $\Delta\tau_2$, or the speed of price discovery becomes decreasing with the information technology g_h. This completes the proof. □

Corollary 1

Proof. Consider the threshold $\hat{g}_t$, at which the benefit of investing in speed to trade at $t = 1$ is just small enough, so that the marginal investor is just willing to stay slow. Therefore, at this threshold $\mu_1 = 0$ and $\mu_2 = 1$, implying $\pi_1 = \frac{1}{2\gamma}\ln\left(1 + \frac{g_h k_h(m_1)}{\tau_0}\right) - m_1 - \frac{1}{\hat{g}_t}$ and $\pi_2 = \frac{1}{2\gamma}\ln\left(1 + \frac{g_h k_h(m_2)}{\tau_2}\right) - m_2$, where $\tau_2 = \tau_0 + \tau_U g_h^2 k_h(m_2)^2/\gamma^2$. In equilibrium, it has to be such that $\pi_1 = \pi_2 = \pi^*$, which implies $\tau_2/(\tau_2 + g_h k_h(m_2)) > \tau_0/(\tau_0 + g_h k_h(m_1))$ because $m_1 + 1/\hat{g}_t > m_1 > m_2$. Subtract by 1 on both sides and rearrange to get $k_h(m_2)/(\tau_2 + g_h k_h(m_2)) < k_h(m_1)/(\tau_0 + g_h k_h(m_1))$.

Next, from the expression of π_1, by envelope theorem, $\frac{\partial \pi^*}{\partial g_h} = \frac{1}{2\gamma}\frac{k_h(m_1)}{\tau_0 + g_h k_h(m_1)} + \frac{1}{\hat{g}_t^2}\frac{\partial \hat{g}_t}{\partial g_h}$. Similarly, from the expression of π_2, $\frac{\partial \pi^*}{\partial g_h} = \frac{1}{2\gamma}\frac{1}{\tau_2 + g_h k_h(m_2)}\left(1 - \frac{2h g_h^2 k_h(m_2)^2}{\gamma^2 \tau_2}\right) k_h(m_2) < \frac{1}{2\gamma}\frac{k_h(m_2)}{\tau_2 + g_h k_h(m_2)} < \frac{1}{2\gamma}\frac{k_h(m_1)}{\tau_0 + g_h k_h(m_1)} = \frac{\partial \pi^*}{\partial g_h} - \frac{1}{\hat{g}_t^2}\frac{\partial \hat{g}_t}{\partial g_h}$. Therefore, $\partial \hat{g}_t/\partial g_h < 0$.

Further, consider the extremes of $g_h \downarrow 0$ and $g_h \uparrow \infty$. Toward the lower bound 0, from the expression of π_1 it can be seen that the first term in π_1 drops down to zero. Since an investor always has the option not to trade, π_1 is bounded below by zero. This leads to $m_1 \downarrow 0$ and $1/\hat{g}_t \downarrow 0$, implying $\lim_{g_h \downarrow 0} \hat{g}_t = \infty$. On the other hand, the first-order condition (6) applied to π_1 implies $0 = \frac{\dot{k}_h(m_1)}{2\gamma} - k_h(m_1) - \frac{\tau_0}{g_h} < \left(\frac{1}{2\gamma} - m_1\right)\dot{k}_h(m_1)$, where the inequality follows because $\tau_0/g_h > 0$ and because $k_h(m) \geq \dot{k}_h(m)m$ by concavity. Hence, m_1 is always bounded from above by $1/(2\gamma)$. From the first-order condition, with τ_1 fixed at τ_0, it follows the concavity of $k_h(\cdot)$ that m_1 monotone increases in g_h, and so does $k_h(m_1)$. Taken together, $\lim_{g_h \uparrow \infty} \pi_1 > \frac{1}{2\gamma}\lim_{g_h \uparrow \infty} \ln\left(1 + \frac{g_h k_h(m_1)}{\tau_0}\right) - \frac{1}{2\gamma} - \lim_{g_h \uparrow \infty} \frac{1}{\hat{g}_t}$. If $\lim_{g_h \uparrow \infty} \hat{g}_t > 0$, then the above limit of π_1 shoots to infinity. In that case, the assumed equilibrium will not hold, however, because all slow investors will have incentive to acquire speed by paying $1/\hat{g}_t$ to earn infinite profit. Therefore, it has to be the case that $\lim_{g_h \uparrow \infty} \hat{g}_t = 0$.

Finally, the above concludes that $\hat{g}_t$ is a strictly decreasing function in g_h, with $\hat{g}_t(0) \to \infty$ and $\hat{g}_t(\infty) \to 0$. As the strict monotonicity implies invertibility, there exists $\hat{g}_h(g_t)$ for all $g_t \in (0, \infty)$ such that the equilibrium is interior if and only if $g_h \geq \hat{g}_h(g_t)$. □

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