Internet Appendix for
"Quotes, Trades and the Cost of Capital"

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1 Additional Tables

In this section, we provide tables that verify the robustness of some of our empirical results in the paper.

Table IA.1: Variable description

$N(quotes)_{i,t}$	Total number of quote updates in stock i over period t. (Source: TAQ)
$N(trades)_{i,t}$	Total number of trade executions in stock i over period t. (Source: TAQ)
$QT_{i,t} = \frac{N(quotes)_{i,t}}{N(trades)_{i,t}}$	Quote to trade ratio for stock i over period t. (Source: TAQ)
$R_{f,t}$	Risk free rate, one month Treasury bill rate. (Source: WRDS/Kenneth French Webpage)
$R_{m,t}$	Value weighted return on the market portfolio. (Source: WRDS/Kenneth French Webpage)
$R_{i,t}$, $R_{p,t}$	Return on stock i or portfolio p. (Source: WRDS/CRSP)
$r_{p,t} = R_{p,t} - R_{f,t}$	Excess return on portfolio p. (Source: WRDS/TAQ)
$r^a_{i,t}$	Risk-adjusted return on stock (or portfolio) i. (Source: WRDS/TAQ)
$r_{hml,t}$	Value factor constructed by Kenneth French. (Source: WRDS/Kenneth French Webpage)
$r_{smb,t}$	Size factor constructed by Kenneth French. (Source: WRDS/Kenneth French Webpage)
$r_{umd,t}$	Momentum factor (up-minus-down) constructed by Kenneth French. (Source: WRDS/Kenneth French Webpage)
$r_{liq,t}$	Liquidity factor constructed by Pástor and Stambaugh (2003). (Source: WRDS)
$r_{pin,t}$	PIN factor constructed by Easley et al. (2002). (Source: Sören Hvidkjaer Webpage)
$QSPREAD_{i,t}$	Quoted spread. Difference between best ask quote and best bid quote (measured in USD). (Source: TAQ)
$SPREAD_{i,t}$	Relative spread. The quoted spread divided by the mid-quote price (measured in %). (Source: TAQ)
$PRC_{i,t}$	Price in USD. (Source: WRDS/TAQ)
$USDVOL_{i,t}$	Trading volume in USD. (Source: WRDS/TAQ)
$VOLUME_{i,t}$	Share volume (measured in mill.). (Source: WRDS/TAQ)
$ILR_{i,t}$	Amihud (2002) illiquidity ratio for stock i over period t calculated as $ILR_{i,t} = [\sum(USDVOL_{i,t})/\lvert r_{i,t}\rvert] \cdot 10^6$. (Source: WRDS/TAQ)
$R1$	Previous month return (Source: WRDS)
$R212$	Cumulative return from month $t-2$ to $t-12$. (Source: WRDS)
$VOLAT_{i,t}$	Return volatility for stock i calculated as absolute return over period t. (Source: WRDS/TAQ)
$IDIOVOL_{i,t}$	Idiosyncratic volatility for stock i measured as the standard deviation of the residual from a three-factor Fama/French model on daily data as in Ang et al. (2009). (Source: WRDS/TAQ)
$MCAP_{i,t}$	Market Capitalization of a stock, calculated as the number of outstanding shares multiplied by price. (measured in mill. USD)
$BM_{i,t}$	Book-to-Market value for stock i calculated as the log of the book value of equity divided by the market value of equity measured for the previous fiscal year.
ANF_i	The number of analysts following firm i. (Source: IBES)
$INST_i$	Holdings of institutions in the equity of firm i at the end of the year constructed from 13F files. (Source: WRDS)

Table IA.2: Sample stock descriptives

The table presents the monthly time-series averages of the cross-sectional 25th percentiles, means, medians, 75th percentiles, and standard deviations of the variables for the sample stocks. The sample period is June 1994 through October 2012, and only NYSE/AMEX and NASDAQ listed stocks are included in the sample. Stocks with a price less than USD 2, above USD 1000, or with less than 100 trades in month t-1 are removed. Stocks that change listings exchange, CUSIP or ticker symbol are removed. The description of variables is in Table IA.1.

	p25	Mean	Median	p75	Std.dev
Number of sample stocks	2859	3126	3051	3367	390
MCAP (in mill. USD)	70	2569	252	1047	13417
PRC (Price in USD)	8	22	17	29	24
USDVOL (in mill. USD)	2	392	19	140	2261
VOLUME (in 1000 shares)	237	12330	1307	6390	69836
N(quotes) (in 1000)	1	166	9	110	504
N(trades) (in 1000)	0	28	2	16	111
QT (proxy for algorithmic trading)	0.80	25.03	3.13	9.88	162.90
SPREAD (%)	0.26	2.19	1.19	2.93	3.03
QSPREAD	0.04	0.28	0.18	0.39	0.48
ILR (%)	0.036	8.331	3.402	2.389	121.071
VOLA	0.006	0.027	0.012	0.029	0.066
BM (log)	0.32	0.74	0.56	0.89	1.03
r_i (indiv. stock midpoint excess returns, delist adj.)	-0.060	0.013	0.005	0.072	0.146
INST	0.198	0.456	0.435	0.692	0.297
R1 (lagged 1 month return in month t-1)	-0.058	0.014	0.003	0.071	0.153
R212 (cumulative returns month t-12 through t-2)	-0.129	0.129	0.094	0.334	0.489

Table IA.3: Additional determinants of the quote-to-trade ratio

The table shows panel regressions of the quote-to-trade ratio (QT) on different characteristics. The dependent variable is the monthly QT. The independent variables are: annual number of analysts following the stock (*ANF*), quarterly institutional ownership (*INST*), log-book-to-market as of the previous year end (*BM*); previous month return (*R*1); as well as contemporaneous (monthly) variables: log-market capitalization (*MCAP*), price (*PRC*), U.S. dollar trading volume (*VOLUME*), Amihud illiquidity ratio (*ILR*), relative bid-ask spread (*SPREAD*), volatility (*VOLAT*), number of NASDAQ market makers (*MM*), *AQ* is a dummy variable that takes the value one after the staggered introduction of Autoquote and zero otherwise, and *SSBAN* is a dummy variable that takes the value one during the 2008 U.S. short-selling ban and zero otherwise. Standard errors are double-clustered at the stock and month level.

	(1)	(2)	(3)
ANF	-0.03	-0.30***	-0.83***
	(-0.17)	(-3.98)	(-4.83)
INST	-61.45***	-29.25***	-41.00***
	(-9.84)	(-4.44)	(-6.23)
BM	-5.72	-2.26	-5.64
	(-1.56)	(-0.89)	(-1.54)
*R*1	-1.78	-6.49**	-7.91***
	(-0.83)	(-2.58)	(-2.82)
MCAP	-4.10	-2.60*	-7.74***
	(-1.59)	(-1.71)	(-2.73)
PRC	0.53***	0.38***	0.62***
	(3.01)	(4.32)	(3.24)
USDVOL	-1.18e-09***	-1.49e-09***	-1.24e-09***
	(-2.81)	(-6.53)	(-3.22)
ILR	0.20	-1.38	0.61
	(0.07)	(-0.60)	(0.20)
SPREAD	-255.24***	-148.28***	-281.92***
	(-3.78)	(-3.93)	(-4.43)
VOLAT	-2.85	-12.47***	-8.02**
	(-0.93)	(-2.94)	(-2.06)
MM	-1.53***		0.08
	(-9.11)		(0.31)
AQ		48.30***	71.08***
		(13.05)	(10.38)
SSBAN		-21.87***	-32.46***
		(-3.88)	(-3.96)
Stock FE	YES	YES	YES
Time FE	YES	NO	NO
N	385,098	672,888	385,100
Adj. R^2	0.226	0.182	0.205

Table IA.4: Risk-adjusted returns for quote-to-trade ratio portfolios

The table shows risk-adjusted monthly returns for various portfolios sorted on the quote-to-trade ratio (QT). The alphas reported in the table are the intercepts (risk-adjusted returns) of regressions of portfolio returns on risk factors. The monthly returns of the QT portfolios are risk-adjusted using several asset pricing models: CAPM, Fama and French (1993) model (FF3), a model that adds the Pástor and Stambaugh (2003) traded liquidity factor (FF3+PS), a five factor model that adds a momentum factor (FF3+PS+MOM), the Fama and French (2015) five factor model (FF5), and a model that adds the PIN factor for the period June 1994 to December 2002 (FF3+PS+MOM+PIN). We show the alpha for the lowest and highest QT portfolios and the alpha for the difference in returns between the low and high portfolios. In Panel A, stocks are assigned to 25 portfolios based on their QT level in month t. Then returns are calculated for each portfolio for month $t+1$. Panel B shows stocks assigned to 50 portfolios. ***, **, and * indicate rejection of the null hypothesis that the risk-adjusted portfolio returns are significantly different from zero at the 1%, 5%, and 10% level, respectively.

	Risk-adjusted returns (%)					
	CAPM	FF3	FF3+PS	FF3+PS +MOM	FF5	FF3+PS +MOM+PIN
	(1)	(2)	(3)	(4)	(5)	(6)
	Panel A: 25 QT portfolios					
α_1	0.89	1.10**	1.10**	1.88***	1.19***	1.91***
α_{25}	0.22	-0.22	-0.21	-0.03	-0.26*	-0.04
α_{1-25}	0.67	1.31***	1.31**	1.91***	1.45***	1.95***
	Panel B: 50 QT portfolios					
α_1	0.60	0.82*	0.81*	1.56***	1.21***	1.57***
α_{50}	0.08	-0.33	-0.34	-0.18	-0.33**	-0.19
α_{1-50}	0.52	1.15**	1.15**	1.74***	1.54***	1.76***

Table IA.5: FMB regressions using $t-2$ information

The table reports the Fama and MacBeth (1973) coefficients from a regression of risk-adjusted returns using the lagged quote-to-trade ratio (QT). The firm characteristics are measured in month $t-2$, except $R1$ and $R212$. The variables included are: relative bid/ask spread ($SPREAD$), Amihud illiquidity ratio (ILR), market value of equity ($MCAP$), book to market ratio (BM) calculated as the log of the book value of equity divided by the market value of equity measured for the previous fiscal year, previous month return ($R1$), and the cumulative return from month $t-2$ to $t-12$ ($R212$), idiosyncratic volatility ($IDIOVOL$) measured as the standard deviation of the residuals from a Fama and French (1993) three factor model regressed on daily raw returns within each month as in Ang et al. (2009), dollar volume ($USDVOL$), and price (PRC). All characteristics apart from returns are logged and all coefficients are multiplied by 100. The standard errors are corrected by using the Newey-West method with 12 lags. ***, **, and * indicate significance at the 1%, 5%, and 10% level, respectively. Panel A presents the results for information delay and Panel B presents the results on liquidity.

	(1)	(2)	(3)	(4)	(5)
Const.	0.004***	0.012***	0.009***	0.027***	0.030***
$QT_{i,t-2}$	-0.200***	-0.240***	-0.248***	-0.145***	-0.148***
$SPREAD_{i,t-2}$	0.132***		0.072*		0.034
$ILR_{i,t-2}$		0.088***	0.057*		-0.047*
$MCAP_{i,t-2}$				-0.060	-0.073
$BM_{i,t-2}$				0.063	0.063
$R1_{i,t-2}$				-5.111***	-5.111***
$R212_{i,t-2}$				0.100	0.129
$IDIOVOL_{i,t-2}$				-9.254***	-11.167***
$USDVOL_{i,t-2}$				0.034	0.004
$PRC_{i,t-2}$				-0.473***	-0.439***
R^2	0.01	0.01	0.01	0.03	0.04
Time series (months)	216	216	216	216	216

Table IA.6: Stock risk-adjusted returns and quote-to-trade ratio subsample

The table reports the Fama and MacBeth (1973) coefficients from regressions of risk-adjusted returns for single stocks, given by $r^a_{i,t} = r_{i,t} - \sum_{j=1}^{J} \beta_{i,j,t-1} F_{j,t}$ for two subsamples, before and after the introduction of algorithmic trading in 2002. Pre-2002 refer to the period from June 1994 to December 2002 and Post-2002 refers to the period from January 2003 to October 2013. The firm characteristics are measured in month $t-1$. The variables included are: relative bid/ask spread ($SPREAD$), Amihud illiquidity ratio (ILR), market value of equity ($MCAP$), book to market ratio (BM) calculated as the log of the book value of equity divided by the market value of equity measured for the previous fiscal year, previous month return ($R1$), and the cumulative return from month $t-2$ to $t-12$ ($R212$), idiosyncratic volatility ($IDIOVOL$) measured as the standard deviation of the residuals from a Fama and French (1993) three factor model regressed on daily raw returns within each month as in Ang et al. (2009), dollar volume ($USDVOL$), and price (PRC). All characteristics apart from returns are logged and all coefficients are multiplied by 100. The standard errors are corrected by using the Newey-West method with 12 lags. ***, **, and * indicate significance at the 1%, 5%, and 10% level, respectively.

	Pre-2002	Post-2002
Const.	0.030**	0.041**
$QT_{i,t-1}$	-0.156**	-0.088*
$SPREAD_{i,t-1}$	0.068**	0.007
$ILR_{i,t-1}$	0.031	-0.033
$MCAP_{i,t-1}$	-0.351***	-0.115
$BM_{i,t-1}$	0.302***	-0.123*
$R1_{i,t-1}$	-4.144***	-4.643***
$R212_{i,t-1}$	0.566	-0.369
$IDIOVOL_{i,t-1}$	-17.318***	-8.464**
$USDVOL_{i,t-1}$	0.390***	-0.031
$PRC_{i,t-1}$	-0.516***	-0.361***
R^2	0.04	0.04
Time series (months)	100	116

Table IA.7: Stock risk-adjusted returns and quote-to-trade ratio and market makers

The table reports the Fama and MacBeth (1973) coefficients from regressions of risk-adjusted returns for single stocks, given by $r^a_{i,t} = r_{i,t} - \sum_{j=1}^{J} \beta_{i,j,t-1} F_{j,t}$ for the subsample with only market makers. The firm characteristics are measured in month $t-1$. The variables included are: relative bid/ask spread ($SPREAD$), Amihud illiquidity ratio (ILR), market value of equity ($MCAP$), book to market ratio (BM) calculated as the log of the book value of equity divided by the market value of equity measured for the previous fiscal year, previous month return ($R1$), and the cumulative return from month $t-2$ to $t-12$ ($R212$), idiosyncratic volatility ($IDIOVOL$) measured as the standard deviation of the residuals from a Fama and French (1993) three factor model regressed on daily raw returns within each month as in Ang et al. (2009), dollar volume ($USDVOL$), price (PRC) and the number of NASDAQ market makers (MM). All characteristics apart from returns are logged and all coefficients are multiplied by 100. The standard errors are corrected by using the Newey-West method with 12 lags. ***, **, and * indicate significance at the 1%, 5%, and 10% level, respectively.

Const.	0.027*
$QT_{i,t-1}$	-0.048*
$SPREAD_{i,t-1}$	0.066
$ILR_{i,t-1}$	0.045
$MCAP_{i,t-1}$	-0.280***
$BM_{i,t-1}$	0.099
$R1_{i,t-1}$	-4.727***
$R212_{i,t-1}$	0.108
$IDIOVOL_{i,t-1}$	-16.649***
$USDVOL_{i,t-1}$	0.355***
$PRC_{i,t-1}$	-0.723***
$MM_{i,t-1}$	0.000
R^2	0.04
Time series (months)	216

2 Random Trader Arrivals

In this section we consider the same setup as in the paper, but we no longer assume aggregate demands of the form

$$\begin{aligned} Q^b &= \frac{k}{2}(v-a) + \ell - m + \varepsilon^b, \quad \text{with} \quad \varepsilon^b \overset{IID}{\sim} \mathcal{N}(0, \Sigma_L/2), \\ Q^s &= \frac{k}{2}(b-v) + \ell + m + \varepsilon^s, \quad \text{with} \quad \varepsilon^s \overset{IID}{\sim} \mathcal{N}(0, \Sigma_L/2). \end{aligned} \tag{IA.1}$$

Instead, we allow traders to be selected at random from the population, in the spirit of Glosten and Milgrom (1985). Thus, at trading time τ a trader is randomly selected from the population described in Appendix B in the paper. The proofs are in Section 2.3 of this Internet Appendix.

2.1 Environment

At trading time τ a trader is selected at random and with equal probability he is an investor or a liquidity trader. If a liquidity trader is selected, he submits either a buy order or a sell order with equal probability, and the quantity is randomly chosen from the normal distribution $\mathcal{N}(\ell_L, \Sigma_L/2)$. If an investor is selected, his initial endowment is randomly chosen from the normal distribution $\mathcal{N}(M, \sigma_M^2)$, where M is the supply of the risky asset. Investors have CARA utility with coefficient A. Investors observe the asset value v before trading, and then trade on the exchange at the quotes set by the dealer: the ask quote a and the bid quote b. The asset liquidates at $v + u$, where u has a normal distribution $\mathcal{N}(0, \sigma_u^2)$. Finally, the dealer maximizes[1]

$$\mathsf{E}_\tau \Big(x_0 \, v \; + \; \big((v-b)Q^s + (a-v)Q^b \big) \; - \; \gamma \, x_\infty^2 \Big), \tag{IA.2}$$

where Q^s is the random quantity sold by the trader, Q^b is the random quantity bought by the trader, and x_∞ is the final inventory of the dealer:

$$x_\infty \; = \; x_0 - Q^b + Q^s. \tag{IA.3}$$

[1] We ignore the dealer's monitoring costs in this analysis. Also, recall that for simplicity of presentation in Appendix B we have assumed that the asset liquidates at $v + u$, while the investors observe the signal v. Since, however, u appears linearly in the objective function, it can be ignored.

2.2 Equilibrium

The investor's equilibrium behavior is the same as in Appendix B of the paper: his optimal trade depends on how his initial endowment is positioned relative to the *lower target* $\underline{X}$ and the *upper target* $\overline{X}$ defined by

$$\underline{X} \;=\; \frac{v-a}{A\sigma_u^2}, \qquad \overline{X} \;=\; \frac{v-b}{A\sigma_u^2}. \tag{IA.4}$$

The investor trades only when his initial endowment in the risky asset is outside of the target interval $[\underline{X}, \overline{X}]$. In that case, he trades exactly so that his final inventory is equal to the closest target. For instance, if the investor's initial endowment x is below $\underline{X}$, then the investor submits a buy order for $\underline{X} - x$. Ex ante, an initial endowment x occurs with probability density

$$\phi_M(x) \;=\; \frac{1}{\sigma_M}\,\phi\Big(\frac{x-M}{\sigma_M}\Big), \tag{IA.5}$$

where ϕ is the standard normal density.

The next result describes an approximation for the equilibrium behavior of the dealer when the volatility σ_M of the investors' initial endowment is large. For simplicity, we consider only the particular case when the dealer's initial inventory is x_0 is zero. Define

$$k \;=\; \frac{1}{A\sigma_u^2}, \qquad \ell \;=\; \frac{\ell_L + 2\phi(0)\gamma k \sigma_M}{6(1+\gamma k)}, \qquad m \;=\; \frac{M}{2}, \tag{IA.6}$$

where $\phi(0) = 1/\sqrt{2\pi}$.

Proposition IA.1. *Suppose the dealer has initial inventory $x_0 = 0$ and forecast w. Then the dealer's optimal quotes are*

$$a \;=\; w + h - \delta + O\Big(\frac{1}{\sigma_M}\Big), \qquad b \;=\; w - h - \delta + O\Big(\frac{1}{\sigma_M}\Big), \tag{IA.7}$$

where h and δ are given by

$$h \;=\; \frac{\ell}{k}, \qquad \delta \;=\; \frac{m}{k}\,\frac{1+2\gamma k}{1+\gamma k}, \tag{IA.8}$$

and k, ℓ, m are as in (IA.6).

Proposition IA.1 shows that the behavior of the dealer is the same as in the baseline model (see Proposition 1 in the paper). The only difference lies in the formulas for the coefficients k, ℓ, m. Recall that in Appendix B in the paper, we provide micro-foundations for the aggregate demand equations (IA.1). In that context, the coefficients are

$$\begin{aligned} k \;&=\; \frac{2\rho_1}{A\sigma_u^2}, \qquad \ell \;=\; \ell_L + \rho_0\sigma_M, \qquad m \;=\; \rho_1 M, \\ \rho_0 \;&=\; \frac{1}{\sqrt{8\pi}} \approx 0.1995, \qquad \rho_1 = \frac{1}{2\pi} + \frac{1}{4} \approx 0.4092 \end{aligned} \tag{IA.9}$$

(see equation (B4) in the paper). Note that the formulas in (IA.9) are similar to the formulas in (IA.6), indicating that the random arrival model in this section produces similar results to our baseline model in the paper.

2.3 Proofs of Results

Proof of Proposition IA.1. The dealer's choice variables are the quotes a and b, or equivalently the half spread $h = (a - b)/2$ and pricing discount $\delta = w - (a + b)/2$. The dealer's forecast error is $e = v - w$, which has a normal distribution $e \sim \mathcal{N}(0, G)$, where $G = 1/F > 0$ is the dealer's inverse precision function (given by monitoring). As in the baseline model, one can show that G does not influence the optimal choice of h and δ, and hence we can set from the beginning $G = 1$. Thus, we assume that the dealer's forecast error has a standard normal distribution

$$e \;=\; v - w \;\sim\; \mathcal{N}(0, 1). \tag{IA.10}$$

At time τ a trader arrives, which can be an investor with probability 1/2, or a liquidity trader with probability 1/2. Let x be the investor's initial endowment. Then, Lemma B.1 in Appendix B of the paper shows that the trader submits the following quantities Q^b and Q^s:

$$
\begin{aligned}
Q^b &= \ell_L + \varepsilon^b, \quad Q^s = 0, \quad \text{with probability } 1/2, \\
Q^b &= 0, \quad Q^s = \ell_L + \varepsilon^s, \quad \text{with probability } 1/2, \\
Q^b &= \underline{X} - x, \quad Q^s = 0, \quad \text{with probability } \tfrac{1}{2} \textstyle\int_{-\infty}^{\underline{X}} \phi_M(x) \mathrm{d}x, \\
Q^b &= 0, \quad Q^s = 0, \quad \text{with probability } \tfrac{1}{2} \textstyle\int_{\underline{X}}^{\overline{X}} \phi_M(x) \mathrm{d}x, \\
Q^b &= 0, \quad Q^s = x - \overline{X}, \quad \text{with probability } \tfrac{1}{2} \textstyle\int_{\overline{X}}^{\infty} \phi_M(x) \mathrm{d}x,
\end{aligned}
\tag{IA.11}
$$

where ε^b and ε^s are IID with normal distribution $\mathcal{N}(0, \Sigma_L/2)$, and $\phi_M(x)$ is the density function in (IA.5). Substituting the formulas for e, h and δ in (IA.2) and setting $x_0 = 0$, it follows that the dealer maximizes

$$
\mathsf{E}_{e,x} \Big(\big((h + \delta + e) Q^s + (h - \delta - e) Q^b \big) \; - \; \gamma \, (Q^s - Q^b)^2 \Big), \tag{IA.12}
$$

where $e \sim \mathcal{N}(0, G)$ and $x \sim \mathcal{N}(M, \sigma_M^2)$. Using the formulas in (IA.11), we recompute the dealer's objective function after multiplying by 2 and removing the terms that do not involve h or δ. Hence, the dealer maximizes

$$
\begin{aligned}
V \;=\; h\ell_L &+ \int_{-\infty}^{+\infty} \int_{-\infty}^{\underline{X}} \Big((h - \delta - e)(\underline{X} - x) - \gamma (x - \underline{X})^2 \Big) \, \phi_M(x) \mathrm{d}x \, \phi(e) \mathrm{d}e \\
&+ \int_{-\infty}^{+\infty} \int_{\overline{X}}^{+\infty} \Big((h + \delta + e)(x - \overline{X}) - \gamma (x - \overline{X})^2 \Big) \, \phi_M(x) \mathrm{d}x \, \phi(e) \mathrm{d}e
\end{aligned}
\tag{IA.13}
$$

where

$$
\underline{X} \;=\; k(\delta - h + e), \qquad \overline{X} \;=\; k(\delta + h + e), \qquad k \;=\; \frac{1}{A\sigma_u^2}. \tag{IA.14}
$$

By computing first the inner integral (with respect to x), we obtain a linear combination of terms of the form $\phi\big(\frac{\underline{X} - M}{\sigma_M}\big)$, $\Phi\big(\frac{\underline{X} - M}{\sigma_M}\big)$, $\phi\big(\frac{M - \overline{X}}{\sigma_M}\big)$ and $\Phi\big(\frac{M - \overline{X}}{\sigma_M}\big)$, with coefficients which are polynomial in e and the choice variables h and δ. Write

$$
\begin{aligned}
&\frac{\underline{X} - M}{\sigma_M} \;=\; \alpha_1 e + \beta_1, \qquad \frac{M - \overline{X}}{\sigma_M} \;=\; \alpha_2 e + \beta_2, \\
&\alpha_1 \;=\; \frac{k}{\sigma_M}, \quad \beta_1 \;=\; \frac{k(\delta - h) - M}{\sigma_M}, \quad \alpha_2 \;=\; -\frac{k}{\sigma_M}, \quad \beta_2 \;=\; \frac{M - k(\delta + h)}{\sigma_M}.
\end{aligned}
\tag{IA.15}
$$

Thus, to comput the outer integral (with respect to e), we need to be able to compute

$$I_n = \int_{-\infty}^{+\infty} u^n \phi(\alpha u + \beta)\phi(u)\mathrm{d}u, \qquad J_n = \int_{-\infty}^{+\infty} u^n \Phi(\alpha u + \beta)\phi(u)\mathrm{d}u. \tag{IA.16}$$

Note that $\phi'(u) = -u\phi(u)$. We now perform (i) direct computation for $n = 1$, and (ii) integration by parts to obtain recursive formulas for I_n and J_n.[2] We get

$$\begin{aligned} I_0 &= \frac{1}{\sqrt{\alpha^2+1}}\,\phi\left(\frac{\beta}{\sqrt{\alpha^2+1}}\right), \qquad I_n = \frac{(n-1)I_{n-2} - \alpha\beta I_{n-1}}{\alpha^2+1}, \\ J_0 &= \Phi\left(\frac{\beta}{\sqrt{\alpha^2+1}}\right), \qquad J_n = (n-1)J_{n-2} + \alpha I_{n-1}, \end{aligned} \tag{IA.17}$$

where $I_{-1} = J_{-1} = 0$. The formulas above imply

$$I_1 = -\frac{\alpha\beta}{\alpha^2+1} I_0, \qquad J_1 = \alpha I_0, \qquad J_2 = J_0 - \frac{\alpha^2\beta}{\alpha^2+1} I_0. \tag{IA.18}$$

Using the formulas above, we compute the dealer's objective function. Up to terms that do not depend on h and δ, this is equal to

$$V_1 = \big(2\phi(0)\gamma k\sigma_M + \ell_L\big)h - 3k(1+\gamma k)\big(h^2+\delta^2\big) + 3M(1+2\gamma k)\delta + O(1/\sigma_M). \tag{IA.19}$$

This is a linear-quadratic problem in h and δ, therefore up to terms of the order of $1/\sigma_M$, the unique solution is

$$h = \frac{\ell_L + 2\phi(0)\gamma k\sigma_M}{6k(1+\gamma k)}, \qquad \delta = \frac{M(1+2\gamma k)}{2k(1+\gamma k)}. \tag{IA.20}$$

Using the notations in (IA.6), we obtain the formulas in (IA.8), which finishes the proof. □

3 Model with Multiple Dealers

In this section we provide an extension of our baseline model (see Section 4 of the paper) to multiple dealers. The proofs are in Section 3.4 of this Internet Appendix.

[2] The formula for I_0 is computed by noticing that $\phi(u)$ and $\phi(\alpha u+\beta)$ are log-quadratic in u. The formula for J_0 is obtained by noticing that I_0 is the differential of J_0 with respect to β.

3.1 Environment

The market is composed of one risk-free asset and one risky asset. Trading in the risky asset takes place in a market exchange based on the mechanism described below. There are two types of market participants: (a) $N \geq 1$ market makers called *dealers* (individually referred to as "she") who monitor the market and set ask and bid quotes at which others trade, and (b) traders, who submit market orders.

Assets. The risk-free asset is used as a numeraire and has a return of zero. The risky asset has a net supply of $M > 0$. After trading, the risky asset liquidates at a fundamental value equal to v, which has a normal distribution $v \sim \mathcal{N}(v_0, \sigma_v^2)$, where σ_v is the *fundamental volatility.*

Trading. Trading occurs at at the first arrival τ in a Poisson process with frequency parameter normalized to one. If dealer $i = 1, \ldots, N$ submits an ask quote q_i and a bid quote b_i, traders submit aggregate buy market order Q^b and sell market order Q^s, which depend on the average quotes

$$a = \sum_{i=1}^{N} a_i, \qquad b = \sum_{i=1}^{N} b_i. \tag{IA.21}$$

The quantity Q^b is the *buy demand* and Q^s the *sell demand.* Together, Q^b and Q^s are called the *liquidity demand*, or the traders' *order flow.* Thus, upon observing the ask quote a and the bid quote b, traders submit at τ the following order flow:

$$\begin{aligned} Q^b &= \frac{k}{2}(v - a) + \ell - m + \varepsilon^b, \quad \text{with} \quad \varepsilon^b \overset{IID}{\sim} \mathcal{N}\left(0, \Sigma_L/2\right), \\ Q^s &= \frac{k}{2}(b - v) + \ell + m + \varepsilon^s, \quad \text{with} \quad \varepsilon^s \overset{IID}{\sim} \mathcal{N}\left(0, \Sigma_L/2\right), \end{aligned} \tag{IA.22}$$

The numbers k, ℓ, m and Σ_L are exogenous constants. The parameter k is the *investor elasticity*, ℓ is the *inelasticity parameter*, and m is the *imbalance parameter.* Microfoundations for the liquidity demand are provided in Appendix B in the paper.

Allocation Mechanism. Given the aggregate quantities in (IA.22), dealer $i =$

$1, \ldots, N$ trades the quantities

$$Q_i^b = \frac{1}{N}\left(\frac{k}{2}\Big((v-a)+\mu(a-a_i)\Big)+\ell-m+\varepsilon^b\right), \qquad \text{(IA.23)}$$
$$Q_i^s = \frac{1}{N}\left(\frac{k}{2}\Big((b-v)+\mu(b_i-b)\Big)+\ell+m+\varepsilon^s\right),$$

at the prices

$$a_{e,i} = a+\nu(a_i-a), \qquad b_{e,i} = b+\nu(b_i-b), \qquad \text{(IA.24)}$$

respectively, where $\mu, \nu \in [0,1]$. Both μ and ν as left as free parameters to allow for many possible allocation mechanisms.[3] Note that the quantities Q_i^b and Q_i^s indeed sum up to Q^b and Q^s, respectively.

Dealer Monitoring. Dealer $i = 1, \ldots, N$ monitors the fundamental value according to mutually independent Poisson processes with frequency $q_i > 0$ called the *monitoring frequency* (or *monitoring rate*). We assume that each dealer reveals her signal to the other dealers.[4] Thus, each dealer receives signals according to a Poisson process with frequency

$$q = \sum_{i=1}^{N} q_i. \qquad \text{(IA.25)}$$

Using Corollary 2 in the paper, we take a reduced form approach and replace the signals obtained from monitoring at the frequency q with a unique signal with precision

$$F(q) = \frac{1}{\mathsf{Var}(v-w)}, \qquad \text{(IA.26)}$$

where w is the dealer's forecast after observing the signal. We assume that $F(q)$ is increasing in q. Intuitively, an increase in the aggregate monitoring rate produces more precise forecasts for the dealers. The cost for dealer i of monitoring at the rate q_i is $C(q_i)$, and is paid only once at $t = 0$, before monitoring begins.

To simplify the equilibrium formulas, we assume that the precision function F and

[3] If μ is higher, the marginal effect of dealer i's quotes on the quantity traded is larger. If ν is higher, the marginal effect of dealer i's quotes on the trading price is larger.

[4] We do not specify the mechanism by which these signals become known by the other dealers. One possibility is that in equilibrium each dealer's quotes are in one-to-one correspondence to her last signal, which can therefore be inferred by the others. However, deviations from equilibrium could make the other dealers infer a different signal. We avoid this type of complication by simply assuming it away.

the monitoring cost C are linear increasing functions,

$$F(q) \;=\; f\,q, \qquad C(q_i) \;=\; c\,q_i, \tag{IA.27}$$

where f and c are positive constants.

Dealers' Quotes and Objective. Each time dealer i monitors, she sets the ask and bid quotes. We therefore interpret the monitoring rate q_i as dealer i's *quote rate*. Because monitoring is considered here in reduced form, we are interested only in the quotes (a_i, b_i) that are prevalent when trading occurs at τ. Thus, a quoting strategy for dealer i is a pair (a_i, b_i) where a_i is the ask quote and b_i is the bid quote. Each dealer starts with an initial inventory in the risky asset equal to x_0.[5] Let Q_i^b and Q_i^s be the buy and sell quantities, respectively, that dealer i trades according to the allocation mechanism described above. Then dealer i's inventory after trading is

$$x_{i,\text{end}} \;=\; x_0 - Q_i^b + Q_i^s. \tag{IA.28}$$

Then, given a quoting strategy (a_i, b_i) and a monitoring rate q_i, dealer' i's expected utility before trading at τ is equal to the expected profit minus the quadratic penalty in the inventory and minus the monitoring costs:

$$\mathsf{E}_\tau\left(x_0\,v \;+\; \big((v - b_{e,i})Q^s + (a_{e,i} - v)Q^b\big) \;-\; \gamma\,x_{i,\text{end}}^2 \;-\; C(q_i)\right), \tag{IA.29}$$

where the parameter $\gamma > 0$ is dealer i's *inventory aversion*, and $a_{e,i}$, $b_{e,i}$ are the effective quotes at which dealer i trades, as in (IA.24).

Equilibrium Concept. The structure of the game is as follow: First, each dealer $i = 1, \ldots, N$ chooses a constant monitoring rate q_i. Second, in the trading game dealer i chooses the ask quote a_i and the bid quote b_i such that objective function (IA.29) is maximized. After observing the dealers' quotes, the traders submit their order flow, which is then allocated to the dealers according to the mechanism described above.

[5] As in the paper, we first let the initial inventory x_0 as a free parameter, and later we set it equal to a particular value called the neutral (or preferred) inventory.

3.2 Equilibrium

We now describe the equilibrium behavior of the N dealers. As in the baseline model, the description of the equilibrium depends on the parameters of the order flow in (IA.22), and the dealers' forecast w of the fundamental value, which is the same for all dealers.

Proposition IA.2. *In the model with N dealers, suppose all dealers have the same initial inventory x_0. Then there exists a symmetric equilibrium in which dealer i's optimal quotes are*

$$a_i \;=\; w + h - \delta, \qquad b_i \;=\; w - h - \delta, \tag{IA.30}$$

and the half spread h, pricing discount δ, and total monitoring rate q satisfy

$$\begin{aligned} h \;&=\; \frac{\ell}{k}\,\frac{2+2(N-1)\nu}{2+(N-1)(\mu+\nu)}, \\ \delta \;&=\; \frac{2}{k}\,\frac{m\Big(1+(N-1)\nu+2\gamma k\big(1+(N-1)\mu\big)/N\Big)+\gamma k\big(1+(N-1)\mu\big)x_0}{2+(N-1)(\mu+\nu)+2\gamma k(1+(N-1)\mu)/N}, \\ q \;&=\; \sqrt{\frac{k}{N}\left(1+\frac{k\gamma}{N}\right)\frac{1}{fc}}. \end{aligned} \tag{IA.31}$$

To simplify presentation, we present the equilibrium for the *neutral inventory* $x_{0,\text{neutral}}$ at which dealers balance the order flow, i.e., they expect the buy and sell demands to be equal. Define the *neutral discount* to be the equilibrium discount δ_{neutral} corresponding to the case in which all dealers start with the neutral inventory.

Corollary IA.1. *In the model with N dealers, the neutral inventory, neutral pricing discount and neutral mid-quote price are given, respectively, by*

$$x_{0,\text{neutral}} \;=\; \frac{m}{\gamma k}, \qquad \delta_{\text{neutral}} \;=\; \frac{2m}{k}, \qquad p_{\text{neutral}} \;=\; w - \frac{2m}{k}. \tag{IA.32}$$

By visual inspection, we obtain the following corollary which provides comparative statics for the neutral discount δ, which is in one-to-one correspondence with the cost of capital

$$r \;=\; \frac{\mathsf{E}_\tau(v) - p_{\text{neutral}}}{p_{\text{neutral}}} \;=\; \frac{2m/k}{w-2m/k}. \tag{IA.33}$$

Corollary IA.2. *In the model with N dealers, the cost of capital is increasing in the elasticity parameter k, and is not affected by the inventory aversion γ.*

When the elasticity parameter k is higher, investors trade more aggressively on their information. Then, the dealers have an incentive to set a small risk premium δ (the difference between their forecast w and the mid-quote price), to reduce their expected inventory.

The fact that the neutral discount does not depend on inventory aversion has the same intuition as in the baseline model: this discount depends only on the properties of the order flow, and not on the dealers' inventory aversion.

3.3 Representative Dealer

We now compare the equilibrium for $N \geq 1$ dealers, each having inventory aversion γ, with the equilibrium for one dealer with inventory aversion

$$\gamma^{(1)} = \frac{\gamma}{N}. \tag{IA.34}$$

Intuitively, we verify whether we can replace N dealers with a representative dealer with proportionally smaller inventory aversion.

Corollary IA.3. *Let the superscript (N) describe variables in the equilibrium with N dealers. Then the equilibrium half spread, monitoring rate, neutral inventory and neutral discount satisfy, respectively,*

$$\begin{aligned} h^{(N)} &= \frac{2+2(N-1)\nu}{2+(N-1)(\mu+\nu)}\, h^{(1)}, \qquad q^{(N)} = \frac{q^{(1)}}{\sqrt{N}} \\ x_{0,\text{neutral}}^{(N)} &= \frac{x_{0,\text{neutral}}^{(1)}}{N}, \qquad \delta_{\text{neutral}}^{(N)} = \delta_{\text{neutral}}^{(1)}. \end{aligned} \tag{IA.35}$$

One implication of Corollary IA.3 is that the neutral inventory of a representative dealer is N times larger than the neutral inventory of each dealer in the N-dealer equilibrium. This implies that the aggregate neutral inventory is the same in the two models, which justifies our choice of the representative dealer inventory aversion in (IA.34).

The first equation in (IA.35) implies that the equilibrium bid-ask spread in the two models depends on the market allocation policy, that is, on the allocation coefficients μ and ν. If the allocation coefficients are equal, then the equilibrium spread is the same in the two models.

From (IA.35) we also see that the neutral discount, and therefore the cost of capital, does not depend on the number of dealers, but only on the properties of the order flow, and in particular on the ratio between the imbalance parameter m and the investor elasticity k. This is in line with Prediction 3 in Section 4.5 of the paper: the number of market makers in a stock does not affect its cost of capital.

The only important difference between the two model arises for the total monitoring rate. The monitoring rate in the N-dealer model is lower than in the representative dealer model by the square root of N. Intuitively, this is because each dealer exerts a positive externality on the other dealers: when she monitors, she reveals her signal via her quotes and thus improves every dealer's estimate of the fundamental value. Because of this externality, each dealer under-monitors the market in equilibrium.

This last result justifies Prediction 1 in Section 4.5 of the paper: a large number of market makers in a stock is associated to a low quote-to-trade ratio. In that section, a large number of market makers is interpreted as a low value of the parameter γ (the inventory aversion of the representative market maker), which implies that the representative dealer can afford to monitor less often and thus set a lower QT ratio. In this Internet Appendix, we have an additional interpretation for Prediction 1: each market maker's public quotes exert a positive externality on the other market makers and therefore reduce everyone's incentive to monitor the market.

3.4 Proofs of Results

Proof of Proposition IA.2. Fix the monitoring rates q_i, $i = 1, \dots, N$. Let $\mathcal{I}_1$ be the dealers' information set just before trading at t, and by E_1 the expectation operator conditional on $\mathcal{I}_1$. Let $w = \mathsf{E}_1(v)$ be the dealers' forecast of the fundamental value, and G the variance of the forecast error:

$$G \;=\; \mathsf{Var}(v - w). \tag{IA.36}$$

We now compute dealer i's expected utility coming from a quoting strategy (a_i, b_i). Recall that if a and b are the average quotes, the total liquidity demand is given by (Q^b, Q^s), where

$$Q^b = \frac{k}{2}(v-a) + \ell - m + \varepsilon^b, \qquad Q^s = \frac{k}{2}(b-v) + \ell + m + \varepsilon^s, \tag{IA.37}$$

ε^b and ε^s are independent and normally distributed by $\mathcal{N}\big(0, \Sigma_L/2\big)$. Recall that the allocation mechanism requires that dealer i trades the quantities

$$\begin{aligned} Q_i^b &= \frac{1}{N}\left(\frac{k}{2}\Big((v-a) + \mu(a-a_i)\Big) + \ell - m + \varepsilon^b\right), \\ Q_i^s &= \frac{1}{N}\left(\frac{k}{2}\Big((b-v) + \mu(b_i-b)\Big) + \ell + m + \varepsilon^s\right), \end{aligned} \tag{IA.38}$$

at the prices

$$a_{e,i} = a + \nu(a_i - a), \qquad b_{e,i} = b + \nu(b_i - b), \tag{IA.39}$$

respectively, where $\mu, \nu \in [0,1]$. Denote the sum of the other traders' quotes as

$$a_{-i} = \sum_{j\neq i} a_j, \qquad b_{-i} = \sum_{j\neq i} b_j. \tag{IA.40}$$

To further simplify notation, let

$$\begin{aligned} &\tilde{a}_i = \frac{a_i}{N}, \quad \tilde{a}_{-i} = \frac{a_{-i}}{N}, \quad \tilde{b}_i = \frac{a_i}{N}, \quad \tilde{b}_{-i} = \frac{a_{-i}}{N}, \\ &\tilde{k} = \frac{k}{N}, \quad \tilde{\ell} = \frac{\ell}{N}, \quad \tilde{m} = \frac{m}{N}, \quad \tilde{\Sigma}_L = \frac{\Sigma_L}{N^2}, \\ &h_i = \frac{a_i - b_i}{2}, \quad \delta_i = w - \frac{a_i + b_i}{2}, \quad e = v - w. \end{aligned} \tag{IA.41}$$

With this notation, the liquidity demand satisfies

$$\begin{aligned} Q_i^b &= \frac{\tilde{k}}{2}\Big((v-a) + \mu(a-a_i)\Big) + \tilde{\ell} - \tilde{m} + \frac{\varepsilon^b}{N}, \\ Q_i^s &= \frac{\tilde{k}}{2}\Big((b-v) + \mu(b_i-b)\Big) + \tilde{\ell} + \tilde{m} + \frac{\varepsilon^s}{N}. \end{aligned} \tag{IA.42}$$

where

$$a = \tilde{a}_i + \tilde{a}_{-i}, \qquad b = \tilde{b}_i + \tilde{b}_{-i}, \tag{IA.43}$$

and dealer i's inventory at liquidation is $x_{i,\text{end}} = x_0 - Q_i^b + Q_i^s$. Dealer i solves

$$\max_{a_i, b_i} \mathsf{E}_\tau \Big(x_0 v + \big(v - b_{e,i}\big) Q_i^s + \big(a_{e,i} - v\big) Q_i^b - \gamma x_{i,\text{end}}^2 \Big), \tag{IA.44}$$

where $a_{e,i}$ and $b_{e,i}$ are the effective quotes at which dealer i trades (see equation (IA.24)).

We also require that the equilibrium is symmetric, i.e., we impose that the equilibrium quotes satisfy $a_{-i} = (N-1)a_i$ and $b_{-i} = (N-1)b_i$. We note that the quoting strategy (a_i, b_i) is equivalent to choosing (h_i, δ_i). It is straightforward (although computationally tedious) to obtain the optimal strategy of dealer i in the symmetric equilibrium:

$$\begin{aligned} h_i &= \frac{\ell}{k} \frac{2 + 2(N-1)\nu}{2 + (N-1)(\mu+\nu)}, \\ \delta_i &= \frac{2}{k} \frac{m\Big(1 + (N-1)\nu + 2\gamma k\big(1 + (N-1)\mu\big)/N\Big) + \gamma k\big(1 + (N-1)\mu\big)x_0}{2 + (N-1)(\mu+\nu) + 2\gamma k(1 + (N-1)\mu)/N}. \end{aligned} \tag{IA.45}$$

This proves the first two formulas in (IA.31).

Regardless of the initial inventory, dealer i's maximum expected utility (not accounting for monitoring costs) is of the form

$$U_{\max} = D - \frac{k}{N}\Big(1 + \frac{k\gamma}{N}\Big) G, \tag{IA.46}$$

where D is a constant that does not depend on the dealer i's monitoring rate q_i. With the usual notation, we assume the other dealers choose the monitoring rates q_j which sum to q_{-i}. Then we have

$$G = \mathsf{Var}(v - w) = \frac{1}{f(q_i + q_{-i})}. \tag{IA.47}$$

If we account for the monitoring costs $C(q_i) = cq_i$, equation (IA.46) implies that dealer i's maximum expected utility is

$$U_{\max} = D - \frac{k}{N}\Big(1 + \frac{k\gamma}{N}\Big) \frac{1}{f(q_i + q_{-i})} - cq_i. \tag{IA.48}$$

The first order condition in q_i implies that the optimum q_i satisfies $(q_i + q_{-i})^2 = \frac{k}{N}\big(1 +$

$\frac{k\gamma}{N}\big)\frac{1}{fc}$. Because the total monitoring rate is $q = q_i + q_{-i}$, we obtain $q^2 = \frac{k}{N}\big(1 + \frac{k\gamma}{N}\big)\frac{1}{fc}$, which proves the last formula in (IA.31). □

Proof of Corollary IA.1. We use the notation from the proof of Proposition IA.2. Note that the symmetry of the equilibrium implies $a = a_i$ and $b = b_i$. From (IA.42) it follows that in equilibrium $Q_i^b - Q_i^s = \tilde{k}\big(v - \frac{a_i+b_i}{2}\big) - 2\tilde{m} + \frac{\varepsilon^b - \varepsilon^s}{N}$. Since $\mathsf{E}_\tau(v) = w$ and $w - \frac{a_i+b_i}{2} = \delta_i$, we obtain

$$\mathsf{E}_\tau(Q_i^b - Q_i^s) \;=\; \tilde{k}\delta_i - 2\tilde{m}. \tag{IA.49}$$

We now compute the neutral inventory $x_{0,\text{neutral}}$ at which $\mathsf{E}_\tau(Q_i^b) = \mathsf{E}_\tau(Q_i^s)$. Equation (IA.49) implies that $\delta_i = \delta_{\text{neutral}}$ satisfies

$$\delta_{\text{neutral}} \;=\; \frac{2m}{k}, \tag{IA.50}$$

which also proves the formula $p_{\text{neutral}} = w - \delta_{\text{neutral}} = w - \frac{2m}{k}$. Substituting the neutral discount in (IA.45), we compute

$$x_{0,\text{neutral}} \;=\; \frac{m}{\gamma k}. \tag{IA.51}$$

□

Proof of Corollary IA.3. The proof follows from equations (IA.31) and (IA.32). □

4 Model with Multiple Trading Rounds

This section builds an extension with multiple trading rounds of the baseline model in Section 4 of the paper. One notable difference from the baseline model is that in the multi-trade extension we assume that trading takes place at deterministic times (in event time) rather than at random times.[6] This extension is closely related to the price pressures model of Hendershott and Menkveld (2014, henceforth HM2014). The proofs of the results are given in Section 4.6 of this Internet Appendix.

[6]See Section 5 of this Internet Appendix for more discussion regarding this choice.

4.1 Environment

The market is composed of one risk-free asset and one risky asset. Trading in the risky asset takes place in a market exchange, at discrete dates $t = 0, 1, 2, \ldots$ such that the trading frequency is normalized to one. There are two types of market participants: (a) one monopolistic market maker called the *dealer* ("she") who monitors the market and sets the quotes at which others trade, and (b) traders, who submit market orders.

Note that here the trading times are deterministic rather than follow a Poisson process with frequency equal to one (as in the model with a single trading round in the paper). To justify this choice, note that Corollary 2 in the paper shows that the baseline model with Poisson monitoring is essentially the same as in a static model where the dealer receives only one signal (with appropriate precision) at a deterministic time. Section 5 of this Internet Appendix provides additional justification by considering an actual monitoring process at fractional deterministic times.

Assets. The risk-free asset is used as a numeraire and has a return of zero. The risky asset has a net supply of $M > 0$. It pays a dividend D before each trading date. The ex-dividend fundamental value v_t follows a continuous random walk process for which the increments have variance per unit of time equal to $\Sigma_v = \sigma_v^2$, where σ_v is the *fundamental volatility*. One possible interpretation for v_t is that it is the cash value that shareholders receive at liquidation, an event which can occur in each trading round with a fixed probability.[7]

Trading. At trading date $t = 1, 2, \ldots$, after observing the ask quote a_t and the bid quote b_t, traders submit the following aggregate market orders:

$$\begin{aligned} Q_t^b &= \frac{k}{2}(v_t - a_t) + \ell - m + \varepsilon_t^b, \quad \text{with} \quad \varepsilon_t^b \overset{IID}{\sim} \mathcal{N}\left(0, \Sigma_L/2\right), \\ Q_t^s &= \frac{k}{2}(b_t - v_t) + \ell + m + \varepsilon_t^s, \quad \text{with} \quad \varepsilon_t^s \overset{IID}{\sim} \mathcal{N}\left(0, \Sigma_L/2\right), \end{aligned} \tag{IA.52}$$

where Q_t^b is the *buy demand* and Q_t^s is the *sell demand*. The numbers k, ℓ, m and Σ_L are exogenous constants. Together, Q_t^b and Q_t^s are called the *liquidity demand*, or the traders'

[7]Suppose there exists $\pi \in (0, 1)$ such that the asset liquidates in each period with probability π, in which case the shareholders receive v_t per share. Then it can be showed that the expected profits of a trader with quantities bought and sold at t equal to $-Q_t^b$ and $-Q_t^s$, respectively, has the form described in equation (IA.56) with $\beta = 1 - \pi$, and $\gamma = C(q) = 0$.

order flow. The parameter k is the *investor elasticity*, ℓ is the *inelasticity parameter*, and m is the *imbalance parameter*. Micro-foundations for the liquidity demand are provided in Appendix B in the paper.

Dealer Monitoring. The dealer monitors the market according to an independent Poisson process with frequency parameter $q > 0$ called the *monitoring frequency* (or *monitoring rate*). In the spirit of Corollary 2 in the paper, we take a reduced form approach and replace the signals obtained from monitoring at the frequency q with signals that summarize the dealer's information just before trading at each t. Denote by w_t the dealer's forecast of the fundamental value v_t just before trading occurs at t. The forecast is the expected fundamental value of the asset conditional on all the information available until t. We define the *precision function* F_t as the inverse variance of the forecast error $v_t - w_t$. We assume that the precision function does not depend on t, and is an increasing function of the monitoring rate q:[8]

$$F(q) \;=\; \frac{1}{\mathsf{Var}(v_t - w_t)}. \qquad \text{(IA.53)}$$

The intuition is that an increase in the monitoring rate produces more precise forecasts for the dealer. Per unit of time, the cost of monitoring at the rate q is $C(q)$, which is an increasing function of q.

To simplify the equilibrium formulas, we assume that the precision function $F(q)$ and the monitoring cost $C(q)$ are linear increasing functions,

$$F(q) \;=\; f\, q, \qquad C(q) \;=\; c\, q, \qquad \text{(IA.54)}$$

where f and c are positive constants.[9]

Dealer's Quotes and Objective. As in the baseline model in the paper, we interpret the monitoring rate q as the *quote rate*. Because monitoring is considered here in reduced form, we are interested only in the quotes (a_t, b_t) that are prevalent when trading occurs at integer times t.

[8]In Section 5 of this Internet Appendix we show how to generate $F(q)$ using a specific signal structure obtained by monitoring at fractional times $1/q$.

[9]In the proof of Proposition IA.4, we describe equilibrium conditions for more general F and C.

Thus, a quoting strategy for the dealer is a set of processes a_t (the ask quote) and b_t (the bid quote) which are measurable with respect to the dealer's information set. Let x_t be the dealer's inventory in the risky asset just before trading at t.[10] If Q_t^b is the aggregate buy market order at t, and Q^s is the aggregate sell market order at t, the dealer's inventory evolves according to

$$x_{t+1} \;=\; x_t - Q_t^b + Q_t^s. \tag{IA.55}$$

Then, for a given quoting strategy, the dealer's expected utility at τ is equal to the expected profit from date τ onwards, minus the quadratic penalty in the inventory, and minus the monitoring costs:

$$\mathsf{E}_\tau \sum_{t=\tau}^{\infty} \beta^{t-\tau} \Big(x_t\, D \;+\; \big((v_t - b_t) Q_t^s + (a_t - v_t) Q_t^b \big) \;-\; \gamma\, x_t^2 \;-\; C(q) \Big), \tag{IA.56}$$

where $\beta \in (0,1)$ and $\gamma > 0$. Thus, the dealer maximizes expected profit, but at each t faces a utility loss that is quadratic in the inventory. Note that except for the dividend payment this utility function is essentially the same as the one specified in HM2014.[11]

Equilibrium Concept. The structure of the game is as follow: First, before trading begins (before $t = 0$), the dealer chooses a constant monitoring rate q. Second, in the trading game the dealer continuously chooses the quotes (the ask quote a_t and the bid quote b_t) such that objective function (IA.56) is maximized.

4.2 Optimal Quotes

We solve for the equilibrium in two steps. In the first step (Section 4.2), we take the dealer's monitoring rate q as given and describe the optimal quoting behavior. In the second step (Section 4.3), we determine the optimal monitoring rate q as the rate which

[10] We let the initial inventory x_0 as a free parameter, although later (in Section 4.5) we set it equal to the parameter $\bar{x}$ from equation (IA.62), which is the long-term mean of the dealer's equilibrium inventory.

[11] This penalty can be justified either by the dealer facing external funding constraints, or by her being risk averse. The latter explanation is present in HM2014 (Section 3). There, the dealer maximizes quadratic utility over non-storable consumption. To solve the dynamic optimization problem, HM2014 consider an approximation of the resulting objective function (see their equation (16)). This approximation coincides with our dealer's expected utility in (IA.56) when $C(q) = 0$.

maximizes the dealer's expected utility.

We thus start by fixing the monitoring rate q. Consider the game described in Section 4.1, with positive parameters $D, k, \ell, m, \Sigma_L, \gamma$. Define the following constants:

$$\begin{aligned} h &= \frac{\ell}{k}, \quad \omega = \frac{1-\beta}{\beta k}, \quad \alpha = \beta \, \frac{(\gamma - \omega) + \sqrt{(\gamma-\omega)^2 + \frac{4\gamma}{\beta k}}}{2}, \\ \lambda &= \frac{\alpha}{1+k\alpha} = \frac{-(\gamma+\omega) + \sqrt{(\gamma-\omega)^2 + \frac{4\gamma}{\beta k}}}{2}, \\ \delta &= \frac{1-\beta+2k\alpha}{k(1-\beta+k\alpha)}\, m - \frac{\beta}{2(1-\beta+k\alpha)}\, D. \end{aligned} \tag{IA.57}$$

The optimal behavior of the dealer in the trading game is described by the next result.

Proposition IA.3. *The dealer's optimal quotes at $t = 0, 1, \ldots$ are*

$$a_t = w_t - \lambda x_t + h - \delta, \qquad b_t = w_t - \lambda x_t - h - \delta, \tag{IA.58}$$

where w_t is the dealer's value forecast, and x_t is her inventory. The mid-quote price $p_t = (a_t + b_t)/2$ satisfies

$$p_t = w_t - \lambda x_t - \delta = w_t - \lambda x_t - \frac{1-\beta+2k\alpha}{k(1-\beta+k\alpha)}\, m + \frac{\beta}{2(1-\beta+k\alpha)}\, D. \tag{IA.59}$$

To get intuition for this result, suppose the imbalance parameter m and the dividend D are both zero (hence $\delta = 0$). Consider first the particular case when the dealer is risk-neutral: $\gamma = 0$. In that case, both α and λ are equal to zero, and the dealer's inventory x_t does not affect her strategy. Equation (IA.58) implies that the dealer sets her quotes at equal distance around her forecast w_t. Hence, the ask quote at t is $a_t = w_t + h$, and the bid quote is $b_t = w_t - h$, where h is the constant half spread. The equilibrium value $h = \ell/k$ reflects two opposite concerns for the dealer: If she sets too large a half spread, then investors (whose price sensitivity is increasing in k) submit a smaller expected quantity at the quotes.[12] If she sets too small a half spread, this decreases the part of the profit that comes from the inelastic part ℓ of traders' order flow.

[12]For instance, equation (IA.52) implies that the expected quantity traded at the ask is $\mathsf{E}_t(Q_t^b) = \frac{k}{2}(w_t - a_t) + \ell$, which is decreasing in a_t.

When the dealer has inventory concerns ($\gamma > 0$), her inventory affects the optimal quotes: according to equation (IA.58), the quotes are equally spaced around an inventory-adjusted forecast ($w_t - \lambda x_t$). The effect of the dealer's inventory on the mid-quote price is in fact the *price pressure* mechanism identified by HM2014. To understand this phenomenon, suppose that before trading at t the dealer has zero inventory, and at t traders submit a net demand Q. The dealer's inventory then becomes negative ($-Q$). To avoid the inventory penalty, the dealer must bring back the inventory to zero. For that, the dealer must raise the quotes to convince more sellers to arrive. Quantitatively, according to (IA.58) the dealer must increase both quotes by λQ, with the coefficient λ as in equation (IA.57). This makes the corresponding slope coefficient λ essentially a price impact coefficient, in the spirit of Kyle (1985).[13]

According to (IA.59), the mid-quote price is decreasing in the imbalance parameter m, and increasing in the dividend D. To understand why, suppose the imbalance parameter m is large, yet the dealer sets the mid-quote price equal to her forecast (that is, $p_t = w_t$). The dealer then expects the sell demand to be much larger than the buy demand. Thus, in order to avoid inventory buildup and to attract more buyers, she must lower her price well below her forecast. A similar intuition works when the dividend D is large, but the above argument reverses: because investors prefer getting a large dividend, to attract more sellers the dealer must set a price higher than the forecast.

4.3 Optimal Monitoring and the QT Ratio

We now discuss the dealer's optimal monitoring rate q. Because the trading rate is normalized to one, we identify the *quote-to-trade ratio* as the monitoring rate q:

$$q = \text{Quote-to-Trade Ratio.} \tag{IA.60}$$

[13]We stress that in our model price impact is caused by inventory considerations and not by adverse selection between the dealer and the traders. Nevertheless, adverse selection occurs as long as the dealer's signal precision f is not infinite. The interested reader can separate the effect of inventory and information by analyzing more carefully the dealer's signal structure described in Appendix 5.3. There we see that the informativeness of trading depends on the noise parameter Σ_L. The signal structure, however, is chosen there to justify the reduced-form assumption in (IA.53). Under that structure, the dealer is only concerned about her forecast just before trading, and not on what effect trading has on this forecast. But under a different signal structure this fact is no longer true, e.g., if we set $\tilde{V}_\eta = V_\eta$ and $\tilde{V}_\psi = V_\psi$ (see the discussion before equation (IA.94)).

Thus far, the description of the equilibrium does not depend on a particular specification for the precision function $F(q)$ or the monitoring function $C(q)$. To provide explicit formulas, however, we now assume that both functions are linear: $F(q) = fq$ and $C(q) = cq$. In the proof of Proposition IA.4, we describe the equilibrium conditions for more general F and C. Proposition IA.4 shows how to compute the dealer's optimal monitoring rate, which as we discussed above is the equilibrium QT ratio.

Proposition IA.4. *The dealer's optimal monitoring rate q satisfies*

$$q^2 \;=\; \frac{k(1+k\alpha)}{fc} \;=\; \frac{k\beta}{fc}\,\frac{(\gamma-\omega)+\sqrt{(\gamma-\omega)^2+\frac{4\gamma}{\beta k}}}{-(\gamma+\omega)+\sqrt{(\gamma-\omega)^2+\frac{4\gamma}{\beta k}}}. \tag{IA.61}$$

Using the formula in (IA.61), we provide some comparative statics for q.

Corollary IA.4. *The QT ratio q is increasing in investor elasticity k and inventory aversion γ, and is decreasing in signal precision f and in monitoring cost c.*

If the investor elasticity k is larger, investors are more sensitive to the quotes, and the dealer increases her monitoring rate to prevent both adverse selection and large fluctuations in inventory. Indeed, there are two reasons for this increase, which can be understood by writing equation (IA.61) as a sum: $q^2 = \frac{k}{fc} + \frac{k^2\alpha}{fc}$. The first term (which does not depend on the dealer's inventory aversion γ) simply reflects that by increasing her monitoring rate, the dealer reduces the adverse selection that comes from trading with investors with superior information. The second term depends on the parameter α, which is increasing in the inventory aversion γ (see the proof of the Corollary). If γ is larger, the dealer is relatively more concerned about her inventory than about her profit. She then increases her monitoring rate to stay closer to the fundamental value, such that her inventory is not expected to vary too much.

If the signal precision parameter f is smaller, the dealer gets noisier signals each time she monitors, hence she must monitor the market more often in order to avoid getting a large inventory. As a result, in neglected stocks where we expect dealer's signals to be noisier, the QT ratio q should be larger. This result is consistent with the stylized empirical fact SF1 in the paper, that the QT ratio is larger in neglected stocks, i.e., stocks with low market capitalization, institutional ownership, analyst coverage, trading volume, and volatility.

Similarly, if the monitoring cost parameter c is smaller, the dealer can afford to monitor more often in order to maintain the same precision, which increases the QT ratio. There is much evidence that the costs of monitoring have decreased dramatically in recent times (see Hendershott, Jones, and Menkveld, 2011). Accordingly, our stylized empirical fact SF2 documents a sharp rise in the QT ratio, especially in the second part of our sample (2003–2012).

4.4 Neutral Inventory and Pricing Discount

In this section, we study the equilibrium evolution of the dealer's inventory. As we see in Proposition IA.3, the dealer's inventory is an important state variable. Corollary IA.5 computes its long-term mean and describes the equilibrium quotes by considering deviations of the dealer's inventory from its long-term mean.

Corollary IA.5. *The dealer's inventory is an* $AR(1)$ *process:*

$$x_{t+1} - \bar{x} \;=\; \frac{1}{1+k\alpha}\big(x_t - \bar{x}\big) + \varepsilon_t, \qquad \bar{x} \;=\; \frac{1+k\alpha}{k\alpha}\,\frac{(1-\beta)m + \beta k D/2}{1-\beta+k\alpha}. \tag{IA.62}$$

where ε_t *is IID with mean zero and variance* $\frac{k^2}{fq} + \Sigma_L$. *The mid-quote price satisfies*

$$p_t \;=\; w_t - \lambda\big(x_t - \bar{x}\big) - \bar{\delta}, \qquad \bar{\delta} \;=\; \frac{2m}{k}. \tag{IA.63}$$

The mean inventory $\bar{x}$ represents the dealer's bias in holding the risky asset. In HM2014 both m and D are zero, and therefore the mean inventory $\bar{x}$ is also zero. In our case both m and D are positive, hence $\bar{x}$ is also positive. Intuitively, the case when m is positive corresponds to the case when investors are risk averse and the risky asset is in positive net supply (see the micro-foundations in Appendix B). But the dealer also behaves approximately as a risk averse investor because of the quadratic penalty in inventory (see Footnote 11). Therefore, our model becomes essentially a risk sharing problem, in which the dealer holds a positive inventory on average.[14]

[14]Even if $m = 0$, the dealer tends to hold inventory when the dividend D is positive. Indeed, in that

If we write the mid-quote equation (IA.63) at both t and $t+1$, we compute

$$p_{t+1} - p_t \; = \; w_{t+1} - w_t + \psi \left(x_t - \bar{x} \right) - \lambda \varepsilon_{t+1}, \qquad \psi \; = \; \frac{\lambda k \alpha}{1 + k\alpha}. \tag{IA.64}$$

We define the *neutral state* the situation in which the dealer's inventory is at its long-term mean ($x_t = \bar{x}$). In this state, equation (IA.64) implies that the expected change in price is zero, which in the language of HM2014 means that there is no price pressure.

In general, we define the *pricing discount* as the difference between the dealer's forecast and the mid-quote price,

$$\delta_t \; = \; w_t - p_t. \tag{IA.65}$$

From (IA.63) it follows that the pricing discount in the neutral state is the same as its long-term average, and is equal to $\bar{\delta} = 2m/k$. Note that this value is independent on the characteristics of the dealer, that is, on the inventory aversion γ, the signal precision f, or the monitoring cost c. We have thus proved the main result of this section.

Corollary IA.6. *The average pricing discount is $\bar{\delta} = 2m/k$, and does not depend on dealer characteristics.*

In particular, the average pricing discount does not depend on the dealer's inventory aversion γ. This is because in the neutral state there is no price pressure and the dealer just needs to balance the order flow such that the inventory does not accumulate in either direction. This result is surprising, because one may expect the discount to be larger if the dealer has a larger inventory aversion γ. But while a larger coefficient γ just increases the speed of convergence of the pricing discount to its mean, it does not change the mean itself, which depends only on the properties of the order flow.[15]

The average pricing discount $\bar{\delta}$ does depend on the properties of the order flow: the imbalance parameter m and the investor elasticity k. If the imbalance parameter m is larger, the dealer expects the difference between the sell and buy demands to be

case the dealer must increase her quotes to attract sellers (see equation (IA.59)), which tends to raise her inventory and thus increase the dividend collected.

[15] According to (IA.63), the equilibrium discount satisfies $\delta_t - \bar{\delta} = \lambda(x_t - \bar{x})$, and thus δ_t and x_t are both $AR(1)$ processes with the same autoregressive coefficient: $1/(1 + k\alpha)$. From (IA.57), α is increasing in γ, therefore the speed of mean reversion of both processes is also increasing in γ.

larger. To compensate, the dealer must lower price to encourage demand, and therefore increase the discount. If the investor elasticity k is larger, investors are more sensitive to mispricing and therefore trade more intensely when the price is different from the fundamental value. To prevent an expected accumulation of inventory, the dealer must then set the price closer to her forecast, which implies a lower discount.

4.5 Cost of Capital

In this section, we define and analyze the cost of capital in the context of our model. We consider the point of view of an econometrician that has access to the quote and trade information, but not necessarily to the dealer's inventory and forecast (in practice, dealers' inventories and forecasts are not public information). The expected return (including dividends) at date t is then

$$r_t \;=\; \frac{\mathsf{E}_t(p_{t+1}) + D - p_t}{p_t}, \tag{IA.66}$$

where E_t be the expectation operator conditional on the past information, p_t is the mid-quote price, and D is the dividend per share.

To simplify the presentation, we assume that the dealer's inventory starts at its long-term mean, that is, we set $x_0 = \bar{x}$. In this neutral state the price does not change in expectation (see Section 4.4). We define the *cost of capital* to be the expected return in the initial state.[16] Denote the initial dealer forecast by $w_0 = \overline{w}$. Then, the cost of capital is

$$r \;=\; \frac{D}{\overline{w} - \bar{\delta}} \;=\; \frac{D}{\overline{w} - \frac{2m}{k}}. \tag{IA.67}$$

Note that the cost of capital is in one-to-one correspondence with the pricing discount $\bar{\delta} = 2m/k$. Thus, the cost of capital does not depend on the dealer characteristics either. The cost of capital does depend on the characteristics of the order flow: the imbalance parameter m and the investor elasticity k. The intuition for this dependence is the same

[16]We define the cost of capital only in the initial (neutral) state, because we want to avoid price pressures that appear later in other states. Another reason is that in general it is difficult to analyze risk premia in dynamic microstructure models. Indeed, if the expected return is constant, return compounding implies that the price process grows exponentially on average, and to keep up the fundamental value should also follow a geometric Brownian motion. But to maintain a tractable model we need the fundamental value to follow an arithmetic Brownian motion.

as in the discussion after Corollary IA.6.

The next result connects the cost of capital to the equilibrium QT ratio.

Corollary IA.7 (QT Effect)**.** *Holding all parameters constant except for the investor elasticity k, there is an inverse relation between the cost of capital and the QT ratio.*

Thus, the key driver of the QT effect in our model is investor elasticity. When k is larger, Corollary IA.4 shows that the QT ratio is also larger: because traders are more sensitive to the quotes, in order to prevent large fluctuations in inventory the dealer must monitor more often. At the same time, when k is larger, the cost of capital is smaller: because investors trade more intensely when the price differs from the fundamental value, in order to prevent an expected accumulation of inventory the dealer must set the price closer to her forecast, which implies a lower discount and hence a lower cost of capital.

In Appendix B of the paper we provide micro-foundations for the order flow, and we show that the investor elasticity k is larger when traders are less risk averse. Therefore, trader risk aversion drives the QT effect: less risk averse traders cause both a larger QT ratio and a smaller cost of capital.

4.6 Proofs of Results

Proof of Proposition IA.3. Fix the monitoring rate $q > 0$. Let $\mathcal{I}_t$ be the dealer's information set just before trading at t, and by E_t the expectation operator conditional on $\mathcal{I}_t$. Let $w_t = \mathsf{E}_t(v_t)$ be the dealer's forecast of the fundamental value, and G the variance of the forecast error. From (IA.53), we have

$$G \;=\; \mathsf{Var}(v_t - w_t) \;=\; \frac{1}{fq}. \tag{IA.68}$$

We now compute the dealer's expected utility coming from a quoting strategy (a_t, b_t). If we define

$$h_t \;=\; \frac{a_t - b_t}{2}, \qquad \delta_t \;=\; w_t - \frac{a_t + b_t}{2}, \qquad e_t \;=\; v_t - w_t, \tag{IA.69}$$

then the quoting strategy is equivalent to choosing (h_t, δ_t). Equation (IA.52) implies that traders' buy and sell demands at t are given, respectively, by $Q_t^b = \frac{k}{2}(v_t - a_t) + \ell - m + \varepsilon_t^b$ and $Q_t^s = \frac{k}{2}(b_t - v_t) + \ell + m + \varepsilon_t^s$, with $\varepsilon_t^b, \varepsilon_t^s \sim \mathcal{N}(0, \Sigma_L/2)$. Let $\varepsilon_t = -ke_t + \varepsilon_t^s - \varepsilon_t^b$. This is uncorrelated with the past information and has a normal distribution $\mathcal{N}(0, k^2G + \Sigma_L)$. If x_t is the dealer's inventory before trading at t, equation (IA.55) shows that x_t describes the recursive equation $x_{t+1} = x_t - Q_t^b + Q_t^s$, which translates into

$$x_{t+1} \;=\; x_t - k\delta_t + 2m + \varepsilon_t \quad \text{with} \quad \varepsilon_t \overset{IID}{\sim} \mathcal{N}\big(0, k^2G + \Sigma_L\big). \tag{IA.70}$$

Substituting Q_t^b and Q_t^s in the dealer's objective function from (IA.56), and ignoring the monitoring costs $C(q)$, we get $\mathsf{E}_\tau \sum_{t=\tau}^{\infty} \beta^{t-\tau}\, \mathsf{E}_t \Big(Dx_t - \frac{k}{2}(a_t - v_t)^2 - \frac{k}{2}(v_t - b_t)^2 + (\ell - m)(a_t - v_t) + (\ell + m)(v_t - b_t) - \gamma x_t^2 \Big)$. We decompose $\mathsf{E}_t(v_t - b_t)^2 = \mathsf{E}_t(v_t - w_t + w_t - b_t)^2 = G + (w_t - b_t)^2$, and similarly $\mathsf{E}_t(a_t - v_t)^2 = G + (a_t - w_t)^2$. Using the notation in (IA.69), it follows that the dealer's maximization problem at τ is

$$\max_{(h_t, \delta_t)_{t \geq \tau}} \; \mathsf{E}_\tau \sum_{t=\tau}^{\infty} \beta^{t-\tau} \Big(Dx_t - kG - k\delta_t^2 - kh_t^2 + 2\ell h_t + 2m\delta_t - \gamma x_t^2 \Big), \tag{IA.71}$$

where x_t evolves according to (IA.70). Using the Bellman principle of optimization, we reduce the dynamic optimization in (IA.71) to the following static optimization problem:

$$V(x_t) \;=\; \max_{h_t, \delta_t} \Big(2dx_t - kG - k\delta_t^2 - kh_t^2 + 2\ell h_t + 2m\delta_t - \gamma x_t^2 + \beta\, \mathsf{E}_t\, V(x_{t+1}) \Big), \tag{IA.72}$$

where $d = \frac{D}{2}$. We guess that $V(x)$ is a quadratic function of the form

$$V(x) \;=\; W_0 - 2W_1 x - W_2 x^2 \tag{IA.73}$$

for some constants W_0, W_1, W_2. Substituting x_{t+1} from (IA.70), the problem becomes

$$\begin{aligned} V(x_t) \;=\; & \max_{h_t, \delta_t} \Big(2dx_t - kG - k\delta_t^2 - kh_t^2 + 2\ell h_t + 2m\delta_t - \gamma x_t^2 \\ & + \beta W_0 - 2\beta W_1 (x_t - k\delta_t + 2m) - \beta W_2 (x_t - k\delta_t + 2m)^2 - \beta W_2 (k^2G + \Sigma_L) \Big). \end{aligned} \tag{IA.74}$$

The first order condition in (IA.74) with respect to h_t implies $h_t = \frac{\ell}{k}$, which shows that the optimal $h_t = h$, the constant defined in (IA.57). The first order condition in (IA.74) with respect to δ_t implies $\delta_t = \frac{\beta W_2}{1+k\beta W_2}\, x_t + \frac{m+k\beta W_1+2km\beta W_2}{k(1+k\beta W_2)}$, which shows that the optimal $\delta_t = \lambda x_t + \Delta$, where

$$\lambda \;=\; \frac{\alpha}{1+k\alpha}, \qquad \Delta \;=\; \frac{m+k\alpha_1+2km\alpha}{k(1+k\alpha)}, \qquad \alpha_1 \;=\; \beta W_1, \qquad \alpha \;=\; \beta W_2. \quad \text{(IA.75)}$$

Because $V(x_t) = W_0 - 2W_1 x_t - W_2 x_t^2$, we solve for W_0, W_1, W_2:

$$\begin{aligned} W_0 \;&=\; \frac{1}{1-\beta}\Big(\frac{\ell^2}{k} - k(1+k\alpha)G - \alpha\Sigma_L + \frac{(1+k\alpha)\big((1-\beta)m+\beta k d\big)^2}{k(1-\beta+k\alpha)^2}\Big), \\ W_1 \;&=\; \frac{\alpha}{1-\beta+k\alpha}\, m - \frac{1+k\alpha}{1-\beta+k\alpha}\, d, \qquad W_2 \;=\; \frac{\beta W_2}{1+k\beta W_2} + \gamma. \end{aligned} \quad \text{(IA.76)}$$

For a maximum, we need to have $W_2 > 0$. The quadratic equation for W_2 in (IA.76) has a unique positive solution,

$$W_2 \;=\; \frac{\gamma - \omega + \sqrt{(\gamma-\omega)^2 + 4\frac{\gamma}{\beta k}}}{2}, \quad \text{with} \quad \omega \;=\; \frac{1-\beta}{\beta k}. \quad \text{(IA.77)}$$

This implies that $\alpha = \beta W_2$ indeed satisfies (IA.57).

If the dealer has an inventory of $x_t = x$, from equation (IA.73) it follows that the maximum expected utility she can achieve at t is $V(x) = W_0 - 2W_1 x - W_2 x^2 = \frac{1}{1-\beta}\big(\frac{\ell^2}{k} - \alpha\Sigma_L - k(1+k\alpha)G + \frac{(1+k\alpha)((1-\beta)m+\beta k d)^2}{k(1-\beta+k\alpha)^2}\big) - 2W_1 x - W_2 x^2$. Since $G = \frac{1}{fq}$, we get

$$U(q) \;=\; \frac{1}{1-\beta}\left(\widetilde{W}_0 - \frac{k(1+k\alpha)}{fq}\right) - 2W_1 x - W_2 x^2, \quad \text{(IA.78)}$$

where $\widetilde{W}_0$, W_1 and W_2 do not depend on q. Also, using $\alpha_1 = \beta W_1$, we compute $\Delta = \frac{1-\beta+2k\alpha}{k(1-\beta+k\alpha)}\, m - \frac{\beta}{1-\beta+k\alpha}\, d$. Since $d = \frac{D}{2}$, this proves that the formula for Δ in (IA.57). □

Proof of Proposition IA.4. Consider a more general function $F(q) = 1/\mathsf{Var}(v_t - w_t)$ that is increasing in the monitoring rate q. If $G(q) = 1/F(q)$, we have showed in the proof of Proposition IA.3 that the dealer's maximum expected utility is of the form $V(x_t) = W_0 - 2W_1 x_t - W_2 x_t^2$, where W_0, W_1 and W_2 are as in (IA.76). This formula, however, does not include the monitoring costs per unit of time, $C(q)$. If we include these costs, the dealer's maximum utility is $W_0 - 2W_1 x_t - W_2 x_t^2 - \frac{C(q)}{1-\beta}$. But up to a constant that does not depend on q, this utility is equal to $\frac{-k(1+k\alpha)G(q) - C(q)}{1-\beta}$. The first order condition with respect to q is equivalent to $-k(1+k\alpha)G'(q) - C'(q) = 0$. Thus, the optimal monitoring rate satisfies

$$-\frac{C'(q)}{G'(q)} = \frac{C'(q)F^2(q)}{F'(q)} = k(k\alpha + 1). \tag{IA.79}$$

The second order condition for a maximum is $k(k\alpha+1)G''(q) + C''(q) > 0$, which is satisfied if the functions G and C are convex, with at least one of them strictly convex.

We now use the linear specification $C(q) = cq$ and $F(q) = fq$, and compute the optimal monitoring rate q. Since $G(q) = \frac{1}{fq}$, from (IA.79) it follows that q satisfies $fcq^2 = k(k\alpha+1)$, which proves the first part of equation (IA.61). Because the function G is strictly convex, note that the second order condition is satisfied.

The second part of (IA.61) follows by using the expression for α in (IA.57). □

Proof of Corollary IA.4. We first prove that α is decreasing in k and increasing in γ. Equation (IA.76) implies that $\alpha = \beta W_2$ satisfies the equation $\frac{\alpha}{\beta} - \gamma = \frac{\alpha}{1+k\alpha}$. Differentiating this equation with respect to k, we get $\frac{\partial\alpha}{\partial k} = -\frac{\beta\alpha^2}{(1+k\alpha)^2-\beta} < 0$. Similarly, differentiation with respect to γ implies $\frac{\partial\alpha}{\partial\gamma} = \frac{\beta(1+k\alpha)^2}{(1+k\alpha)^2-\beta} > 0$.

Equation (IA.61) implies that q and the term $Q = k(1+k\alpha)$ have the same dependence on the parameters k and γ. Using the formula above for $\frac{\partial\alpha}{\partial k}$, we compute $\frac{\partial Q}{\partial k} = \frac{(1+k\alpha)^2(1-\beta+2k\alpha)}{(1+k\alpha)^2-\beta} > 0$. Finally, Q is increasing in α, which (as proved above) is increasing in γ, hence Q is also increasing in γ.

By visual inspection of equation (IA.61), it is clear that the quote-to-trade ratio q is decreasing in f and increasing in σ_v. □

Proof of Corollary IA.5. Using equation (IA.70) and the fact that in equilibrium $\delta_t = \lambda x_t + \Delta$, it follows that the dealer's inventory evolves according to $x_{t+1} = (1-k\lambda)x_t - k\Delta + 2m + \varepsilon_t$, with $\varepsilon_t \sim \mathcal{N}(0, k^2G + \Sigma_L)$ and $G = \frac{1}{F} = \frac{1}{fq}$. From (IA.57), the coefficient $\phi = 1 - k\lambda = \frac{1}{1+k\alpha} \in (0,1)$, hence $x_{t+1} = \frac{1}{1+k\alpha}x_t - k\Delta + 2m + \varepsilon_t$. Thus, x_t follows an $AR(1)$ process with auto-regressive coefficient ϕ, mean $\bar{x} = (2m - k\Delta)/(1-\phi)$, and variance $\Sigma_x = \left(\frac{k^2}{fq} + \Sigma_L\right)/(1-\phi^2)$. Using the formula for Δ in (IA.57), it is straightforwad to prove the formula for $\bar{x}$ in (IA.62). One can also show that $\Sigma_x = \frac{k\alpha(2+k\alpha)}{(1+k\alpha)^2}\left(\frac{k^2}{fq} + \Sigma_L\right)$. □

Proof of Corollary IA.6. This has already been proved in the discussion that precedes the statement of the Corollary. Alternatively, Proposition IA.3 implies that the pricing discount at t is equal to $w_t - p_t = \lambda x_t + \Delta$, whose average equals $\lambda \bar{x} + \Delta$. Using (IA.57), we compute the average discount to be $2m/k$, which is the same as $\bar{\bar{\delta}}$. □

Proof of Corollary IA.7. First, we prove rigorously equation (IA.67). Since the system is initially in the neutral state ($x_0 = \bar{x}$), according to (IA.64) the expected price change $\mathsf{E}_0\, p_1 - p_0$ is zero. But, if $\overline{w}$ is the initial forecast, by definition $\overline{w} - p_0$ is the pricing discount. Since in the neutral state the pricing discount is $\bar{\bar{\delta}} = 2m/k$, it follows that $p_0 = \overline{w} - \bar{\bar{\delta}}$, which proves (IA.67). Suppose now we hold all parameters constant except for k. Clearly, the cost of capital is decreasing in k, as the pricing discount $\bar{\bar{\delta}}$ is decreasing in k. At the same time, Corollary IA.4 implies that the QT ratio q is increasing in k. This proves the inverse relation between r and q. □

5 Monitoring and Signals

5.1 Preliminaries

The purpose of this section is to provide micro-foundations for the dealer's precision function $F(q)$ in the multi-trade model of Section 4 in this Internet Appendix. For simplicity, we assume that both trading and monitoring occur at deterministic times, equal to the averages of the corresponding random times.[17] This is equivalent to timing trades in event time (at equally spaced intervals of length one), and timing the *average* number of monitoring times that occur between subsequent trading rounds. The downside of this approach is that in principle the monitoring rate q must be an integer. Nevertheless, the description of the equilibrium remains valid also for non-integer q, and thus we can think of the equilibrium being valid for all values of the monitoring rate.

Recall that in that model trading takes place at integer times $t = 0, 1, 2, \ldots$, and the

[17] We already know that in the baseline (single-trade) model in the paper we can replace the signals obtained from monitoring at the frequency q with signals that summarize the dealer's information just before trading at each t. But to extend this result to a multi-trade extension would be very difficult, because one would have to keep track of the different numbers of monitoring rounds between all the trading rounds. To avoid this difficulty, we essentially consider only the average outcome of the trading and monitoring processes.

monitoring rate is $q > 0$. We now assume that monitoring takes place at fractional times $\frac{0}{q}, \frac{1}{q}, \frac{2}{q}, \ldots$, where q is a positive integer. To simplify notation, we index monitoring times by $\tau = 0, 1, 2, \ldots$ rather than by the corresponding fractional times. With this notation, trading takes place at $\tau = 0, q, 2q, \ldots$, which are integer multiples of the monitoring rate. By convention, we assume that when a trading date coincides with a monitoring date, monitoring occurs before trading. Equations (IA.52) imply that traders' order flow satisfies

$$\begin{aligned} Q_\tau^b &= \frac{k}{2}(v_\tau - a_\tau) + \ell - m + \varepsilon_\tau^b, \quad \text{with} \quad \varepsilon_\tau^b \overset{IID}{\sim} \mathcal{N}\left(0, \Sigma_L/2\right), \\ Q_\tau^s &= \frac{k}{2}(b_\tau - v_\tau) + \ell + m + \varepsilon_\tau^s, \quad \text{with} \quad \varepsilon_\tau^s \overset{IID}{\sim} \mathcal{N}\left(0, \Sigma_L/2\right), \end{aligned} \tag{IA.80}$$

5.2 Uninformative Trading

We first analyze the simpler case when the trading process is uninformative to the dealer. Formally, this occurs when the trading noise measured by Σ_L is sufficiently large (see equation (IA.96) below). In this case, we ignore the trading process altogether and focus instead on the monitoring process. Denote by $\mathcal{I}_\tau$ the dealer's information set after monitoring at τ, and by $w_\tau = \mathsf{E}(v_\tau | \mathcal{I}_\tau)$ the dealer's forecast at τ.

We now show that any positive function $F(q)$, not necessarily linear, can arise as the dealer's precision function for a certain set of signals. Define

$$G \;=\; G(q) \;=\; \frac{1}{F(q)}. \tag{IA.81}$$

Fix $q > 0$, and define $V_\eta = V_\eta(q) > 0$ as follows: if $F(q) \leqq /\Sigma_v$, choose any $V_\eta > 0$; and if $F(q) > q/\Sigma_v$, choose any $V_\eta \in \left(0, \frac{1}{F(q) - q/\Sigma_v}\right)$. Also, define $V_v = V_v(q)$ and $V_\psi = V_\psi(q)$ by

$$V_v \;=\; \frac{\Sigma_v}{q}, \qquad V_\psi \;=\; G^2 \, \frac{V_\eta + V_v}{V_\eta V_v} - G. \tag{IA.82}$$

Clearly, $V_v > 0$. We show that $V_\psi > 0$ as well. Indeed, from the definition of V_η, we see that $\big(F(q) - q/\Sigma_v\big)V_\eta < 1$ for all $q > 0$. Using the notation above, this is the same as $\left(\frac{1}{G} - \frac{1}{V_v}\right)V_\eta < 1$, which is equivalent to $\frac{1}{G} < \frac{1}{V_v} + \frac{1}{V_\eta}$. Thus, $G\frac{V_\eta + V_v}{V_\eta V_v} > 1$ or equivalently

$V_\psi = G\big(G\frac{V_\eta+V_v}{V_\eta V_v} - 1\big) > 0$. Note that equation (IA.82) implies

$$\frac{G^2}{G+V_\psi} \;=\; \frac{V_v V_\eta}{V_v+V_\eta}. \tag{IA.83}$$

We define the signal observed by the dealer at $\tau = 0$. Since we can choose freely the initial variance $\mathsf{Var}(v_0) = \Sigma_{v_0}$, consider $\Sigma_{v_0} > G$, and suppose that at $\tau = 0$ the dealer observes $s_0 = v_0 + \nu$, with $\nu \sim \mathcal{N}\big(0, \frac{G\Sigma_{v_0}}{\Sigma_{v_0}-G}\big)$. Then, the dealer's forecast is $w_0 = \mathsf{E}(v_0|s_0) = \beta_0 s_0$, where $\beta_0 = G/\Sigma_{v_0}$. A direct computation shows that indeed $\mathsf{Var}(v_0 - w_0) = G$. Thus, if we define

$$G_\tau \;=\; \mathsf{Var}(v_\tau - w_\tau), \quad \tau \;\geq\; 0, \tag{IA.84}$$

we have $G_0 = G$.

At each $\tau = 1, 2, \ldots$, the dealer observes two signals:

$$\begin{cases} r_\tau = (v_{\tau-1} - w_{\tau-1}) + \psi_\tau, \text{ with } \psi_\tau \overset{IID}{\sim} \mathcal{N}(0, V_\psi), \text{ and} \\ s_\tau = (v_\tau - v_{\tau-1}) + \eta_\tau \text{ with } \eta_\tau \overset{IID}{\sim} \mathcal{N}\,(0, V_\eta). \end{cases} \tag{IA.85}$$

Since the forecast is $w_\tau = \mathsf{E}(v_\tau | r_\tau, s_\tau, r_{\tau-1}, s_{\tau-1}, \ldots)$, its increment is $\Delta w_\tau = w_\tau - w_{\tau-1} = \mathsf{E}(v_\tau - w_{\tau-1}|r_\tau, s_\tau) = \mathsf{E}(v_\tau - v_{\tau-1}|s_\tau) + \mathsf{E}(v_{\tau-1} - w_{\tau-1}|r_\tau)$. Then,

$$\Delta w_\tau \;=\; \frac{V_v}{V_v+V_\eta}\, s_\tau + \frac{G_{\tau-1}}{G_{\tau-1}+V_\psi}\, r_\tau. \tag{IA.86}$$

We compute $v_\tau - w_\tau = v_{\tau-1} - w_{\tau-1} + \Delta v_\tau - \Delta w_\tau = \frac{V_\psi}{G_{\tau-1}+V_\psi}(v_{\tau-1} - w_{\tau-1}) - \frac{G_{\tau-1}}{G_{\tau-1}+V_\psi}\psi_\tau + \frac{V_\eta}{V_v+V_\eta}\Delta v_\tau - \frac{V_v}{V_v+V_\eta}\eta_\tau$. Taking variance on both sides, we obtain the recursive equation

$$G_\tau \;=\; \frac{G_{\tau-1}V_\psi}{G_{\tau-1}+V_\psi} + \frac{V_v V_\eta}{V_v+V_\eta}. \tag{IA.87}$$

From (IA.83), we substitute $\frac{V_v V_\eta}{V_v+V_\eta}$ by $\frac{G^2}{G+V_\psi}$, and the recursive equation (IA.87) becomes

$$G_\tau - G_{\tau-1} \;=\; \left(1 - \frac{V_\psi^2}{(G+V_\psi)(G_{\tau-1}+V_\psi)}\right)(G - G_{\tau-1}). \tag{IA.88}$$

Because $G_0 = G$, equation (IA.88) implies that G_τ is constant and equal to G for all τ.[18] Since $G = \frac{1}{F(q)}$, this finishes the proof.

For future reference, we use equation (IA.86) to compute $\mathsf{Var}(\Delta w_\tau) = \frac{V_v^2}{V_v+V_\eta} + \frac{G^2}{G+V_\psi}$. Equation (IA.83) then implies that $\mathsf{Var}(\Delta w_\tau) = \frac{V_v^2}{V_v+V_\eta} + \frac{V_v V_\eta}{V_v+V_\eta} = V_v$. Thus, we have proved that

$$\mathsf{Var}(\Delta w_\tau) \;=\; \mathsf{Var}(\Delta v_\tau) \;=\; V_v \;=\; \frac{\Sigma_v}{q}. \tag{IA.89}$$

5.3 Informative Trading

We now analyze the general case when the trading process is informative, meaning that the noise parameter Σ_L can be any positive real number. Thus, beside the monitoring times, we also need to analyze the dealer's inference at the trading times $\tau = 0, q, 2q, \ldots$, where q is the monitoring rate and is a positive integer. (Recall that on the these dates monitoring occurs before trading.)

We now show that any linear function $F(q) = fq$ that satisfies a mild condition (see equation (IA.93) below) can arise as the dealer's precision function for a set of signals. As before, given $F(q)$ we define $G = G(q) = \frac{1}{F(q)} = \frac{1}{fq}$. Denote by $\mathcal{I}_\tau$ the dealer's information set after monitoring at τ, by $w_\tau = \mathsf{E}(v_\tau | \mathcal{I}_\tau)$ the dealer's forecast at τ, and by $e_\tau = v_\tau - w_\tau$ her forecast error. Then, equations (IA.80) become

$$\begin{aligned} Q_\tau^b &= \frac{k}{2} e_\tau - (a_\tau - w_\tau) + \ell + \varepsilon_\tau^b, \quad \text{with} \quad \varepsilon_\tau^b \overset{IID}{\sim} \mathcal{N}\left(0, \Sigma_L/2\right), \\ Q_\tau^s &= -\frac{k}{2} e_\tau - (w_\tau - b_\tau) + \ell + \varepsilon_\tau^s, \quad \text{with} \quad \varepsilon_\tau^s \overset{IID}{\sim} \mathcal{N}\left(0, \Sigma_L/2\right), \end{aligned} \tag{IA.90}$$

At trading time $t = 0, q, 2q, \ldots$, define also

$$w_{\tau_+} \;=\; \mathsf{E}(v_\tau \mid \mathcal{I}_\tau, Q_\tau^b, Q_\tau^s), \qquad G_{\tau_+} \;=\; \mathsf{Var}(v_\tau - w_{\tau_+}). \tag{IA.91}$$

As in the informative case, we look for a stationary equilibrium, which here means that we want the dealer to have a periodic signal precision with periodicity equal to the

[18]Note that the coefficient in front of $G - G_{\tau-1}$ in equation (IA.88) is a number in the interval $(0, 1)$. It is then straightforward to show that G_τ converges monotonically to the constant G regardless of the initial value G_0.

monitoring rate q. Thus, the signal precision follows a periodic sequence of the form

$$G_0\ ,\ G_{0_+}\ ,\ G_1\ ,\ \cdots\ ,\ G_q\ =\ G_0\ ,\ G_{q+}\ ,\ G_{q+1}\ ,\ \cdots \tag{IA.92}$$

We show that there is a simple solution for which G_τ are equal to $G = \frac{1}{fq}$, as long as the following condition is satisfied:

$$f\ >\ \frac{1}{\Sigma_v} \qquad \text{or} \qquad \frac{\Sigma_L f^2}{k^2}\ >\ \frac{1}{\Sigma_v} - f. \tag{IA.93}$$

To understand intuitively the role played by this condition, suppose (IA.93) fails to hold. This means that the noise component of trading, measured by Σ_L, is small. Then, the increase in precision $(1/G_0 - 1/G_{0_+})$ that comes from the information content of trading is also small. By contrast, the decrease in precision $(1/G_{0_+} - 1/G_1)$ that comes from the diffusion in fundamental value during the interval $[0, 1]$ is large, and thus the equation $G_0 = G_1$ cannot hold when (IA.93) fails. Note that the condition (IA.93) also translates into the requirement that the dealer's monitoring precision f is sufficiently high.

Suppose now condition (IA.93) is satisfied. We then assume that the dealer receives the same signals r_τ and s_τ as in the uninformative case, except for the monitoring times that come just after trading: $\tau = 1, q+1, 2q+1, \ldots$. At those times, we modify the variance of r_τ and s_τ, by defining new values for V_ψ and V_η. To see how this is done, consider the following cases:

- If $f > 1/\Sigma_v$, we multiply by q to obtain $fq = F = 1/G > 1/V_v$, where $V_v = \Sigma_v/q$. In this case, we choose $\frac{1}{\tilde{V}_\eta}$ in the positive interval $\left(\frac{1}{G} - \frac{1}{V_v}, \frac{\Sigma_L}{k^2 G^2} + \frac{1}{G} - \frac{1}{V_v}\right)$.

- If $f \leq 1/\Sigma_v$, we have $1/G \leq 1/V_v$. Because q is a positive integer, condition (IA.93) implies $\frac{\Sigma_L f^2}{k^2} q^2 > \left(\frac{1}{\Sigma_v} - f\right) q$, which is equivalent to $\frac{\Sigma_L}{k^2 G^2} > \frac{1}{V_v} - \frac{1}{G}$. In this case, we choose $\frac{1}{\tilde{V}_\eta}$ in the interval $\left(0, \frac{\Sigma_L}{k^2 G^2} + \frac{1}{G} - \frac{1}{V_v}\right)$. Since $1/G - 1/V_v \leq 0$, it follows that $\frac{1}{\tilde{V}_\eta}$ also belongs to the larger interval $\left(\frac{1}{G} - \frac{1}{V_v}, \frac{\Sigma_L}{k^2 G^2} + \frac{1}{G} - \frac{1}{V_v}\right)$.

Thus, in both cases $\frac{1}{\tilde{V}_\eta}$ lies in the interval $\left(\frac{1}{G} - \frac{1}{V_v}, \frac{\Sigma_L}{k^2 G^2} + \frac{1}{G} - \frac{1}{V_v}\right)$, or equivalently

$\frac{1}{\tilde{V}_\eta} + \frac{1}{V_v} - \frac{1}{G}$ lies in the interval $\left(0, \frac{\Sigma_L}{k^2 G^2}\right)$. Now define

$$\tilde{V}_\psi \;=\; \frac{\Sigma_L}{k^2} \frac{\left(\frac{\Sigma_L}{k^2 G^2} + \frac{1}{G} - \frac{1}{V_v}\right) - \frac{1}{\tilde{V}_\eta}}{\tilde{V}_\eta - \left(\frac{1}{G} - \frac{1}{V_v}\right)}. \tag{IA.94}$$

From the above discussion, it follows that both $\tilde{V}_\eta$ and $\tilde{V}_\psi$ are positive, and hence when $\tau = 1, q+1, 2q+1, \ldots$, the modified signals r_τ and s_τ are well defined.

We show $G_\tau = G$ for all $\tau \geq 0$. Because the only difference between the informative and the uninformative case occurs at $\tau = 1, q+1, 2q+1, \ldots$, without loss of generality we only need to prove that $G_1 = G$. Since trading at $\tau = 0$ is informative for the dealer, her forecast after trading is $w_{0_+} = \mathsf{E}(v_0 | \mathcal{I}_0, Q_0^b, Q_0^s) = w_0 + \mathsf{E}(e_0 | Q_0^b, Q_0^s)$, where $e_0 = v_0 - w_0$ and

$$\mathsf{E}(e_0 | Q_0^b, Q_0^s) \;=\; \frac{kG}{k^2 G + \Sigma_L}\left(Q_0^b - Q_0^s\right), \qquad \mathsf{Var}(e_0 | Q_0^b, Q_0^s) \;=\; \frac{G \Sigma_L}{k^2 G + \Sigma_L}. \tag{IA.95}$$

We apply the recursive formula (IA.87) for $\tau = 1$, by replacing (i) V_η with $\tilde{V}_\eta$, (ii) V_ψ with $\tilde{V}_\psi$, and (iii) G_0 with $G_{0_+} = \frac{G\Sigma_L}{k^2 G + \Sigma_L}$. Then, a direct computation shows that $G_1 = G$. Since all G_τ are equal to G, it follows that $F(q) = fq$.

We can now determine when trading is uninformative for the dealer. From the above analysis, this translates into the update $w_{0_+} - w_0$ being much smaller than a generic increment $w_\tau - w_{\tau-1}$ (for τ not of the form $1, q+1, 2q+1, \ldots$). This translates into the condition that the variance Σ_L is sufficiently large:

$$\Sigma_L \;\gg\; \frac{k^2}{f^2 \Sigma_v}. \tag{IA.96}$$

Indeed, using equations (IA.89) and (IA.95), the condition $\mathsf{Var}(w_{0_+} - w_0) \ll \mathsf{Var}(\Delta w_\tau)$ becomes $\frac{k^2 G^2}{k^2 G + \Sigma_L} \ll \frac{\Sigma_v}{q}$, which translates to $\frac{\Sigma_L}{k^2 G^2} \gg \frac{q}{\Sigma_v}$, or since $G = \frac{1}{fq}$, to $\Sigma_L \gg \frac{k^2 \Sigma_v}{q f^2 \Sigma_v}$. But the monitoring rate q is a positive integer, hence the condition is equivalent to $\Sigma_L \gg \frac{k^2}{f^2 \Sigma_v}$.

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