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Real Exchange Rate Convergence:
The Roles of Price Stickiness and Monetary Policy

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Abstract

In many open-economy models based on Dornbusch (1976), the speed of convergence of the real exchange rate is tied to the stickiness of prices. The "purchasing power parity puzzle" concerns the empirical fact that real exchange rates appear to converge more slowly than nominal prices. In some New Keynesian models, when there is no interest-rate smoothing, the stickiness of prices does not matter at all for persistence, as Benigno (2004) showed. We show that in the presence of interest-rate smoothing, price stickiness does matter and endogenous real-exchange rate persistence is bounded above by the interest rate smoothing parameter and by the probability of a firm not changing prices under Calvo pricing. We also explain the relationship between the New Keynesian framework with Calvo pricing, and the Dornbusch framework where price stickiness is integral to real exchange rate convergence.

Rogoff (1996, p. 647-648) poses the "purchasing power parity puzzle" as: "How can one reconcile the enormous short-term volatility of real exchange rates with the extremely slow rate at which shocks appear to damp out...[Financial] shocks can have substantial effects on the real economy in the presence of sticky nominal wages and prices. Consensus estimates for the rate at which PPP damp, however, suggest a half-life of three to five years, seemingly far too long to be explained by nominal rigidities." For example, the first column of Table 1 records the serial correlation of the U.S. real consumer price exchange rate relative to each of the other G6 countries from January 1983 – December 2012. The average serial correlation reported is 0.990, implying a half-life of the real exchange rate of five and a half years.[1] Rogoff associates the slow adjustment of real exchange rates with price stickiness, though ultimately concludes that there must in addition be poor integration of international goods markets in order to explain real exchange rate persistence.[2]

Why is price stickiness associated with the slow adjustment of the real exchange rate to shocks? It is true that in the famous overshooting paper of Dornbusch (1976), the rate at which real exchange rates converge to PPP is governed at least in part by the degree of price stickiness.[3] However, Benigno (2004) demonstrates the open-economy equivalent of the familiar result in the closed-economy New Keynesian (NK) literature, that there is no endogenous persistence. In this case, Benigno shows that in a simple, canonical version of the open economy NK model, the pace at which PPP deviations damp out is not affected at all by the degree of price stickiness.

Any NK model could deliver a persistent real exchange rate if the exogenous driving variables were sufficiently persistent. However, one of the aims of Keynesian models is to provide a mechanism for shocks to be propagated into the economy in a way that leads to persistent business cycles, and in the open economy, to persistent real exchange rates. Endogenous persistence implies these variables are adjusting slowly toward some long-run values that might attain in a flexible-price economy. So it is a surprising failure that many NK models do not provide a rationale for slow adjustment even when there is nominal stickiness.

This note provides some insight into the role that price stickiness plays in real exchange rate adjustment in open-economy sticky-price models (and adjustment of output in closed-economy models.) One difference between the Dornbusch and Benigno models is a small but critical difference

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[1] The serial correlation is corrected for small sample bias using the Kendall adjustment.
[2] Carvalho and Nechio (2010) have found that real exchange rates even at a sectoral level adjust very slowly, so that the slow reversion of the real exchange rate cannot be attributed to an aggregation effect.
[3] In fact, in the version of the Dornbusch price adjustment considered here, based on Obstfeld and Rogoff's (1984) suggested modification, price stickiness is the only determinant of the speed of adjustment.

in the specification of price adjustment. In a sense, the Dornbusch model has built in features that ensure that price stickiness governs the speed of convergence of the real exchange rate.

The differences in the price adjustment models alone do not resolve the puzzle. If the other features of the Dornbusch model are left intact – a stable money demand function, and a random-walk money supply – but the Dornbusch price adjustment mechanism is replaced with the familiar Calvo pricing model used in Benigno, price stickiness still is important in determining the persistence of real exchange rates. Apparently the rule for monetary policy influences the extent to which price stickiness matters for the speed of real exchange rate convergence.

We consider monetary policy rules in which the interest rate is set to target inflation, but which allow for interest-rate smoothing. We show that the interest-rate smoothing parameter puts an upper bound on the persistence of the real exchange rate. However, another upper bound for the persistence of the real exchange rate is the probability of leaving prices intact in the Calvo pricing rule. In other words, if interest rates are smoothed to a very high degree, then the persistence of price adjustment puts an upper bound on real exchange rate persistence; but if prices are extremely persistent, then real exchange rates can be no more persistent than nominal interest rates.

Benigno (2004) shows that with no interest-rate smoothing, there is no endogenous real exchange rate persistence, and finds from numerical exercises that when the simple inflation-targeting rule is replaced by one in which the lagged interest rate appears in the rule, there is endogenous real exchange rate persistence and it depends on the degree of price stickiness. This raises another puzzle. With no interest rate smoothing, but serially correlated errors in the Taylor rule, the persistence of the real exchange rate is "exogenous", depending only on the persistence of the exogenous monetary shocks. Under interest rate smoothing, persistence is endogenous and depends on price stickiness. Why is the behavior of the real exchange rate apparently fundamentally different under serially correlated shocks versus interest-rate smoothing, when in fact those two types of policies are not very different? Our answer to this question follows from the demonstration that the degree of interest-rate smoothing puts an upper bound on the persistence of the real exchange rate. Indeed interest-rate smoothing does act like persistence of monetary policy shocks, leading to slow adjustment of the real exchange rate.

Benigno's theoretical result that with no interest-rate smoothing, there is no persistence of the real exchange rate can be understood as follows. Let q_t be the log of the real exchange rate, $q_t = s_t - p_t$, where p_t is the log of the home minus the log of the foreign price level. The real exchange rate can adjust not only through adjustment of nominal prices but also through the nominal

exchange rate under a floating exchange rate regime. Even if prices are sticky, the nominal exchange rate could respond to shocks in a way that eliminates persistent real exchange rate disequilibria. If there is no interest-rate smoothing, then the nominal exchange rate can facilitate full adjustment of the real exchange rate in one period. Price stickiness plays no role, because the nominal exchange rate can move to adjust the real exchange rate. Then real exchange rate persistence will arise only from exogenous persistence of shocks.

However, the monetary policy rules in both countries are important in determining the scope for nominal exchange rate movements to contribute to real exchange rate adjustment. In the extreme case in which nominal interest rates in both countries are smoothed forever, interest rates are constant. By uncovered interest parity, the rate of change of the exchange rate must then be constant. Therefore, the nominal exchange rate cannot play any role in the adjustment of the real exchange rate. In that case, the persistence of real exchange rates is determined by the persistence of nominal price adjustment - the real exchange rate can be no more persistent than nominal prices. At intermediate degrees of interest-rate smoothing, nominal exchange rates must adjust smoothly to satisfy interest parity, so they are less able to play a role in facilitating real-exchange rate adjustment.

We show that the Dornbusch price-adjustment equation is designed in such a way that real exchange rate adjustment is achieved entirely through price adjustment. The nominal exchange rate does not have to bear any of the burden of adjustment, so only the stickiness of prices determines the speed of adjustment of the real exchange rate. If we replace the Dornbusch price adjustment equation with the Calvo price adjustment equation but keep the rest of Dorbusch's model intact, then price stickiness may still matter for real exchange rate adjustment to the extent that exchange rates do not play a role. For example, if the nominal exchange rate follows a random walk (because, in the Dornbusch setting, relative money supplies follow a random walk), then real exchange rate persistence is bounded by the persistence of prices.

In this note, we are concerned only with endogenous persistence, and so we examine persistence under the assumption that shocks are i.i.d.[4] Some of the recent literature has examined the interaction of persistent shocks and price stickiness in leading to very persistent real exchange rates. It is helpful to defer the discussion of the literature until after the presentation of the New Keynesian and Dornbusch models. Section 4 explores the contributions of West (1988), Bergin and Feenstra (2001), Kollmann (2001), Steinsson (2008), Martinez-Garcia and Sondegaard (2012) and Carvalho

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[4] Chari et. al. (2002) generate some persistence in the real exchange rate by assuming highly serially correlated monetary policy shocks – that is, through exogenous persistence. Groen and Matusmoto (2004) find the interaction of persistent shocks and interest-rate smoothing can increase real exchange rate persistence. Monacelli (2004) makes a similar finding.

and Nechio (2010, 2013). Many of these studies are concerned with the interaction of persistent shocks and market structure to produce slow-adjusting real exchange rates.

Carvalho and Nechio (2010) considers a multi-sector model of real exchange rates, and shows quantitatively how real exchange rate persistence is increased relative to simpler one-sector models. One extension of our simple framework is related to this finding. We introduce two sectors of production – a sticky-price and a flexible-price sector. We show that the addition of the flexible-price sector induces endogenous persistence in the real exchange rate for sticky price goods, even when there is no interest rate smoothing. This arises because, with a flexible price sector, the inflation targeting rule for monetary policy induces serial correlation in the change in the exchange rate.

There is a simple closed-economy analog to the open economy model considered here. The analysis of real exchange rate persistence here can be applied to understanding persistence of the output gap and inflation in a closed economy. We simply interpret each of the "relative" variables (home relative to foreign inflation or home relative to foreign interest rates) as the corresponding variable in the closed economy. Under complete markets in the open economy, $q_t = \phi c_t$, where ϕ is the inverse of the intertemporal elasticity of substitution under standard preferences, and c_t is the log of home minus foreign consumption. Since consumption equals output in a simple closed-economy NK model with no investment or government, we can interpret q_t / ϕ as output in a closed economy model. That is, the difference between the real exchange rate and its equilibrium value in the open economy model translates into ϕ times the real output gap in the closed economy. The nominal exchange rate translates into ϕ times nominal output in the closed economy. In simple closed economy models, the log of the output gap is proportional to the percent mark-up of the price over the unit labor cost, or the inverse of the ratio of the real wage to labor productivity. So we can also map the real exchange rate in the open economy model into the real wage in the closed economy, and the nominal exchange rate into the nominal wage. See Appendix A for details.

Section 1 presents the simple New Keynesian model. In that section, we derive the key results that real exchange rate persistence is bounded above both by the persistence of prices in the Calvo price-setting framework and the degree of interest-rate smoothing in the Taylor rule. This implies that with no interest rate smoothing, there is no real exchange rate persistence. Section 2 then compares the New Keynesian model to the model with Dornbusch price setting. In section 3, we examine some extensions to the basic model. We then place the simple New Keynesian model in the context of the recent literature that has attempted to account for real exchange rate persistence in section 4.

1. The New Keynesian Model

Take a canonical open economy New Keynesian model. The model is derived in Appendix A. There are three elements: interest parity, Taylor rules, and open-economy Phillips curves. According to uncovered interest parity:

(1) $i_{t+1} = E_t s_{t+1} - s_t - \varepsilon_t$.

In this equation, i_{t+1} is the short term home less foreign interest differential. We adopt the somewhat nonstandard notation of using $t+1$ time subscript on the interest differential so the model is easier to fit into the Blanchard-Kahn (1980) framework. The interest rate differential refers to a riskless deposit acquired at time t, and paying off at time $t+1$. In the Blanchard-Kahn framework, this is a time $t+1$ variable that is "predetermined", which means it is known at time t. We allow for a risk premium in the form of an ex ante excess return on the foreign deposit, given by ε_t, which is assumed to be an exogenous random variable.

The next component is an open-economy Phillips curve. Assume that the Phillips curves are symmetric in the home and foreign countries, so we can take the difference between them and write:

(2) $\pi_t = \delta\left(q_t - \overline{q}_t\right) + \beta E_t \pi_{t+1}$. $\delta > 0$, $0 < \beta < 1$.

Here, π_t is home inflation (between $t-1$ and t) minus foreign inflation. β is the discount factor in utility. This type of Phillips curve can be derived in a number of contexts. This relationship can be derived under local currency pricing (LCP) with no home bias in preferences, or under producer currency pricing (PCP) with home bias.[5] $\overline{q}_t$ is an exogenously given random variable that represents the equilibrium value for the real exchange rate that would prevail if prices were perfectly flexible. This term may arise – there may be deviations from purchasing power parity in the long run – because of pricing to market by monopolistic firms, or because of changes in the terms of trade. The appendix derives this equation under both the LCP and PCP assumptions, and explains the driving forces behind $\overline{q}_t$ in each case.

The Phillips curve is derived from the Calvo model of price setting, which is examined in more detail in section 3. From (2), we can see that prices adjust more slowly when δ is smaller.

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[5] For example, this is the Phillips curve for CPI inflation under LCP in Benigno (2004) when all prices adjust at the same speed, or in Engel (2011) when there is no home bias in preferences under LCP and no cost-push shocks. It is the form of the Phillips curve in Engel (2011) under PCP when there is home bias in preferences.

Prices adjust slowly in the Calvo model because each period, a fraction θ of firms leave their price unchanged. The relation between δ and θ is given by $\delta \equiv (1-\theta)(1-\theta\beta)/\theta$.

Finally, there is a Taylor rule:

(3) $i_{t+1} = \sigma \pi_t + \alpha i_t + u_t$. $0 \leq \alpha \leq 1$.

Here we assume that policymakers in each country target the inflation rate, raising the nominal interest rate when current inflation is higher. There also may be nominal interest rate smoothing when $\alpha > 0$. Shocks to the economic system arise from the shocks to monetary policy. We assume the parameters of the Taylor rule are identical in the home and foreign countries, so (3) expresses relative interest rates, i_{t+1}, as a function of the relative inflation rate, π_t, the relative lagged interest rates, i_t, and the relative error, u_t.

Model Solutions and Speed of Convergence

We begin by examining the model in the simple case of no interest rate smoothing before moving on to richer models. In the models considered here, the endogenous persistence is generated by monotonic convergence of the real exchange rate to its equilibrium value, with the rate of convergence determined by the eigenvalues of the dynamic system. This section presents the basic NK models and derives some key results, but the intuition and comparison the Dornbusch model are left to section 2.

Solution under No Interest-Rate Smoothing

First, consider the model when there is no interest rate smoothing, so $\alpha = 0$. With a few steps of algebra, the system can be written as:

(4) $$\begin{bmatrix} E_t \pi_{t+1} \\ E_t q_{t+1} \end{bmatrix} = \begin{bmatrix} 1/\beta & -\delta/\beta \\ \sigma - (1/\beta) & 1+(\delta/\beta) \end{bmatrix} \begin{bmatrix} \pi_t \\ q_t \end{bmatrix} + \begin{bmatrix} (\delta/\beta)\overline{q}_t \\ u_t - (\delta/\beta)\overline{q}_t + \varepsilon_t \end{bmatrix}.$$

In the language of Blanchard-Kahn, neither π_{t+1} nor q_{t+1} are predetermined, so stability requires that both eigenvalues be greater than one. The necessary and sufficient condition for that to be true is the familiar "Taylor condition", $\sigma > 1$. In this model, there is no "endogenous" persistence. The persistence of inflation and the real exchange rate is entirely determined by the persistence of the exogenous random shocks. For example, if u_t is a first-order autoregression with autocorrelation coefficient ρ, and the other shocks are set to zero, then inflation and the real exchange rate will also be first-order autoregressions with the same autocorrelation coefficient. Notably, the degree of price

stickiness (δ) does not matter at all for the persistence of either inflation or the real exchange rate. This is the result in Benigno (2004), and corresponds to the well-known result in the closed-economy New Keynesian literature that there is no "endogenous persistence" in the canonical model.

Solution under Interest-Rate Smoothing

Now consider the model with interest rate smoothing ($\alpha > 0$). In this system, there is endogenous persistence, which means that the real exchange rate and the inflation are can be persistent even if u_t, ε_t and $\bar{q}_t$ are i.i.d. – that is, even if the shocks themselves are not persistent. Moreover, the degree of price stickiness does matter for the persistence (as shown below, and which Benigno (2004) shows numerically.) We can write the system as:

$$(5) \quad \begin{bmatrix} E_t \pi_{t+1} \\ E_t q_{t+1} \\ i_{t+1} \end{bmatrix} = \begin{bmatrix} 1/\beta & -\delta/\beta & 0 \\ \sigma - (1/\beta) & 1 + (\delta/\beta) & \alpha \\ \sigma & 0 & \alpha \end{bmatrix} \begin{bmatrix} \pi_t \\ q_t \\ i_t \end{bmatrix} + \begin{bmatrix} (\delta/\beta)\bar{q}_t \\ u_t - (\delta/\beta)\bar{q}_t + \varepsilon_t \\ u_t \end{bmatrix}.$$

Since there is one predetermined variable in this system, the Blanchard-Kahn result requires that one eigenvalue be less than one and two be greater than one for stability. It is easy to show that the stability condition is the familiar condition when there is interest-smoothing: $\sigma + \alpha > 1$. Since there is only one eigenvalue that is less than one, it must be real. Call that root μ. With a stable root, there is endogenous persistence, meaning that even if shocks are i.i.d., the real exchange rate will still be persistent. In fact, in this case, $E_t q_{t+1} = \mu q_t$, so the speed of convergence is given by μ.

As shown below, when prices are stickier (δ in the price-adjustment equation is smaller), the persistence μ is typically larger. This is somewhat puzzling. Why does price stickiness not matter at all when there is no interest rate smoothing, but it does matter when there is? In fact, in the case of no-interest rate smoothing, if $u_t = \rho u_{t-1} + \zeta_t$, where ζ_t is an i.i.d. mean-zero shock and $0 < \rho < 1$, we can rewrite the Taylor rule as

$$(6) \quad i_{t+1} = \sigma \pi_t + \rho(i_t - \sigma \pi_{t-1}) + \zeta_t.$$

This does not look much different at all from the Taylor rule with interest-rate smoothing, equation (3), when u_t is an i.i.d., mean-zero shock. Why does price stickiness matter in one case but not the other?

The answer can be seen from this proposition:

Proposition: $\mu \leq \alpha$ and $\mu \leq \theta$

Proof: See Appendix A.

The significance of the proposition is this: In the case of no interest rate smoothing ($\alpha = 0$), there is no endogenous persistence as we have seen. (This result is actually a special case of the proposition.) We have noted that, holding other shocks constant, if u_t is an AR(1), the persistence of the real exchange rate is determined by the persistence of u_t. A Taylor rule with no interest-rate smoothing but a persistent u_t looks like a Taylor rule with interest smoothing but no persistence in u_t. What this proposition makes clear is that they are similar in that ultimately with interest rate smoothing, the endogenous persistence is bounded above by the degree of interest-rate smoothing given by α.

The degree of price stickiness does matter for the endogenous persistence when $\alpha > 0$. Appendix A shows that μ must satisfy the equation:

(7) $\qquad \left(\dfrac{1}{\beta} - \mu \right)(1-\mu)(\alpha - \mu) - \dfrac{\delta \mu}{\beta}(\alpha + \sigma - \mu) = 0$

Differentiate to get (with a little work):

(8) $\qquad \dfrac{d\mu}{d\delta} = -\dfrac{\mu}{\beta D}(\alpha + \sigma - \mu) \le 0$, where

$\qquad D = (1-\mu)(\alpha-\mu) + (\dfrac{1}{\beta} - \mu)(\alpha - \mu) + (\dfrac{1}{\beta} - \mu)(1-\mu) + \dfrac{\delta}{\beta}(\alpha + \sigma - 2\mu) \ge 0$.

More price stickiness increases the persistence of the real exchange rate: A lower δ increases μ, though μ is bounded above by α.[6]

It is also true that the greater the interest rate smoothing, the more persistent is the real exchange rate:

(9) $\qquad \dfrac{d\mu}{d\alpha} = \left(\dfrac{1}{\beta} - \mu \right)(1-\mu)\left(\dfrac{\sigma}{\alpha + \sigma - \mu} \right) \ge 0$.

Proposition 1 shows that the endogenous persistence is always less than the degree of interest-rate smoothing in the Calvo-pricing model. Conversely, the rate of convergence must also be less than the stickiness of prices.

We postpone the intuition for the proposition until we compare the New Keynesian model to the Dornbusch model, with particular attention to the price adjustment equations.

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[6] Appendix A proves $D > 0$. From inspection of (7) it is clear that we cannot have $\mu < 0$, hence $\mu \ge 0$.

2. Price Setting: Calvo versus Dornbusch

We can contrast two different approaches to sticky price setting. The first is the Calvo price-setting model, which is used above. In this model, a randomly chosen fraction θ of price setting firms leave their price unchanged between period $t-1$ and t in each period, while $1-\theta$ set their price optimally, taking into account the fact that in each period in the future the probability of being able to adjust their price optimally will again be $1-\theta$. If all prices were flexible, we would have $p_t = s_t - \bar{q}_t$. Instead, the price for the firms that adjust their prices at time t, denoted $\bar{p}_t$, satisfies (up to a first-order log-linear approximation) the equation:

(10) $\bar{p}_t = (1-\theta\beta)(s_t - \bar{q}_t) + \beta\theta E_t \bar{p}_{t+1}, \qquad 0 < \theta < 1.$

The overall price level is a weighted average (again, to a first-order approximation) of the prices of firms that change their prices and those that do not:

(11) $p_t = \theta p_{t-1} + (1-\theta)\bar{p}_t.$

With some manipulation, $\bar{p}_t$ can be eliminated from these two equations to arrive at:

(12) $\pi_t = \dfrac{(1-\theta)(1-\theta\beta)}{\theta}(q_t - \bar{q}_t) + \beta E_t \pi_{t+1},$

an open-economy Phillips curve. This is the equation we have used above.

Dornbusch price setting is not derived, but simply specified in an ad hoc way. Under Dornbusch price setting, the price level at time t is set a period in advance. It takes the form:

(13) $p_t - p_{t-1} = (1-\theta)(\tilde{p}_{t-1} - p_{t-1}) + E_{t-1}\tilde{p}_t - \tilde{p}_{t-1}, \quad 0 < \theta < 1$

$\tilde{p}_t$ is the level that prices would take under price flexibility, so the first term on the right-hand-side of equation (13), $(1-\theta)(\tilde{p}_{t-1} - p_{t=1})$, represents partial adjustment of prices. The larger is θ, the slower the adjustment of prices. The second term, $E_{t-1}\tilde{p}_t - \tilde{p}_{t-1}$, represents a trend term.[7] Obstfeld and Rogoff (1984) show how it is important to add this term to the price adjustment equation. For example, it implies that if prices are already in equilibrium at time t, $p_{t-1} = \tilde{p}_{t-1}$, firms allow prices to rise at the equilibrium expected rate of inflation.

Clearly, any system that includes this price adjustment equation will have one root that is equal to θ, and so will have endogenous persistence. This can be seen by rewriting the equation as:

$p_t - E_{t-1}\tilde{p}_t = \theta(p_{t-1} - \tilde{p}_{t-1})$

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[7] In Dornbusch (1976), the price adjustment equation does not include this term. However, it would not be necessary to include it because in Dornbusch (1976), $E_{t-1}\tilde{p}_t - \tilde{p}_{t-1}$ is zero because the analysis considers only permanent changes in relative money supplies.

The Dornbusch specification for price setting does not specify the solution for $\tilde{p}_t$. One possibility is to allow $\tilde{p}_t$ to equal $\overline{p}_t$, as determined in equation (10). If we follow Dornbusch (1976) more closely and assume

(14) $\tilde{p}_t = s_t - \overline{q}_t$,

then this price setting equation "hardwires" the link between the real exchange rate and the speed of price adjustment. Equations (13) and (14) together imply $E_t(q_{t+1} - \overline{q}_{t+1}) = \theta(q_t - \overline{q}_t)$, irrespective of the monetary policy rule.[8]

It is interesting to compare the Calvo price setting equation (11) with the Dornbusch equation, (13). One difference is that the Dornbusch model has prices set one period in advance. More importantly, under the Calvo price setting mechanism, even when $p_{t-1} = \overline{p}_{t-1}$, inflation of actual prices is not equal to inflation of $\overline{p}_t$. We can rewrite (11) as:

(15) $p_t - p_{t-1} = (1-\theta)(\overline{p}_{t-1} - p_{t-1}) + (1-\theta)(\overline{p}_t - \overline{p}_{t-1})$,

which can be compared to the Dornbusch equation (13). In contrast to the Dornbusch model of price adjustment, under Calvo pricing if the price level is at its equilibrium value in time $t-1$, actual inflation will not follow equilibrium inflation (even in expectation), because only the fraction $1-\theta$ firms adjust their price.

Alternatively, the Dornbusch price setting equation can be understood as one in which a fraction $1-\theta$ set their price at the optimal level (in expectation, since prices are set one period in advance), but the fraction θ of firms that do not set price optimally still adjust their price at the expected rate of inflation of equilibrium prices, $E_{t-1}\tilde{p}_t - \tilde{p}_{t-1}$. In other words, (13) can be written as:

(16) $p_t - p_{t-1} = (1-\theta)(E_{t-1}\tilde{p}_t - p_{t-1}) + \theta(E_{t-1}\tilde{p}_t - \tilde{p}_{t-1})$.

Under this interpretation, a fraction $1-\theta$ adjust their price to the expected equilibrium price, but a fraction θ adjust their prices so that the gap between their price and the equilibrium price remains constant. Contrasting (15) and (16), if no firms adjust their prices, so $\theta = 1$, under Calvo pricing, inflation is zero, but there is a role for the exchange rate to adjust to bring $\overline{p}_t$ closer to p_t. However, under Dornbusch pricing, if $\theta = 1$, then the expected gap between the price level and its equilibrium

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[8] Obstfeld and Rogoff (1984) call the case in which $\tilde{p}_t = \overline{p}_t$ the Barro-Gordon price adjustment equation, and when $\tilde{p}_t = s_t - \overline{q}_t$, the Mussa price adjustment equation. They show that the dynamics of the equilibrium model are qualitatively identical under the two specifications.

level cannot adjust, $p_t - p_{t-1} = E_{t-1}\left(\tilde{p}_t - \tilde{p}_{t-1}\right)$. Only price adjustment can lead to adjustment in the real exchange rate under Dornbusch pricing.

Intuition

Why does price stickiness matter for the persistence of the real exchange rate under some monetary policy regimes, but matters less or not at all under other regimes? To sharpen intuition, assume the risk premium and the log of the equilibrium real exchange rate are constant and equal to zero, $\varepsilon_t = \overline{q}_t = 0$. Suppose the real exchange rate is out of equilibrium at time t, which in our formulation that assumes long run purchasing power parity means $s_t - p_t \neq 0$. The adjustment of the real exchange rate between time t and $t+1$ could occur from either nominal price adjustment, or from movement in the nominal exchange rate. Real exchange rate convergence is pinned to the speed of adjustment of nominal prices only to the extent that monetary policy does not allow adjustment to occur through the nominal exchange rate channel.

Consider the question of why the persistence of nominal prices puts an upper bound on real exchange rate adjustment when there is interest-rate smoothing in the New Keynesian model. Take the limiting case in which the relative interest rate is smoothed so much it is expected to be kept constant over time: $E_t i_{t+j} = i_{t+1}$ for $t > 1$. Then, because interest parity holds, the expected change in the nominal exchange rate is constant: $E_t\left(s_{t+j+1} - s_{t+j}\right) = E_t\left(s_{t+j} - s_{t+j-1}\right)$ for $t > 0$. If the expected change in the nominal exchange rate is constant forever, it must equal the expected long run inflation rate (which is zero.) Hence, the nominal exchange rate does not bear any of the load of the real exchange rate. Prices must do all of the adjustment. It follows that the real exchange rate can adjust no faster than nominal prices.

Algebraically, from interest parity, if interest rates are completely smoothed, so $i_{t+1} = 0$ and is constant forever, then $E s_{t+j} = s_t$ for $j > 0$. Substituting this relationship into (10), the equilibrium price is given by $\overline{p}_t = s_t$. In turn, substituting this into the price adjustment equation (11), we find $p_t = \theta p_{t-1} + (1-\theta)s_t$, or, considering the expected change in prices between t and $t+1$:

(17) $\quad E_t p_{t+1} - p_t = (1-\theta)(s_t - p_t)$.

Inflation is expected to erase a fraction $1-\theta$ of the disequilibrium in the real exchange rate in this case, while the exchange rate does none of the work. Subtracting $E_t s_{t+1} - s_t = 0$ from (17), we then

get that the persistence of the real exchange rate is determined by the fraction of the firms that do not adjust their prices, $E_t q_{t+1} = \theta q_t$.

When interest rates are not completely smoothed, the nominal exchange rate can bear part of the load of real exchange rate adjustment. In fact, in the absence of interest rate smoothing, the real exchange rate will be in disequilibrium in the period of the shock, but then is expected to return the following period to equilibrium, so at any time t, $E_t q_{t+1} = 0$. The real exchange rate does have to change contemporaneously at the time of the shock. That is, it is not the case that the nominal price level and exchange rate can adjust in such a way instantaneously so that PPP holds at all times and $q_t = 0$ always. If the real exchange rate were always in equilibrium, then the Phillips curve would imply inflation was zero at all times. When a monetary shock occurs, the interest rate changes, and therefore the nominal exchange rate must be expected to change. If inflation is expected to be zero, this implies the real exchange rate is expected to change, which contradicts the assumption that $q_t = 0$ always.

The preceding develops the logic of why the speed of adjustment of nominal prices in the New Keynesian framework sets an upper bound on real exchange rate persistence. When there is complete interest rate smoothing the persistence of real exchange rates is determined by the persistence of nominal prices, but real exchange rates can adjust in a single period when there is no interest rate smoothing. In the former case, the nominal exchange rate bears none of the load of adjustment of the real exchange rate, but in the latter case, the nominal exchange rate can move to adjust the real exchange rate. When there is some interest-rate smoothing, the real exchange rate does not adjust fully in one period. But why does the rate of convergence of the interest rate set an upper bound on real exchange rate convergence?

Suppose that nominal prices did not adjust at all. The fraction of firms that reset their prices each period is zero, so $p_t = p_{t-1}$. For simplicity, assume the log of the price level is zero. In this case, nominal prices do none of the adjustment, and the exchange rate must do all of it, so $E_t s_{t+1} = (1-\mu)s_t$. Then, $E_t s_{t+2} = (1-\mu)E_t s_{t+1}$. If we use these two equations, and rearrange, we find that $E_t(s_{t+2} - s_{t+1}) = -\mu(E_t s_{t+1} - s_t) - \mu s_t = (1-\mu)(E_t s_{t+1} - s_t)$, or $E_t i_{t+2} = (1-\mu)E_t i_{t+1}$. The speed of convergence of the real exchange rate is determined by the speed of convergence of the nominal interest rate. When prices are able to adjust, convergence happens more quickly, so the interest rate persistence puts an upper bound on real exchange rate persistence.

We have seen that the Dornbusch price adjustment equation builds in that the speed of adjustment of the real exchange rate is tied to the speed of adjustment of prices. There is no role for the nominal exchange rate to adjust to equilibrate the real exchange rate, because it is intrinsic in the Dornbusch price adjustment equation that prices do all of the adjustment. In general, both prices and exchange rates may play a part in the real exchange rate adjustment process under Calvo pricing. In this case, both the stickiness of prices and the degree of interest-rate smoothing play a role in the speed of convergence of the real exchange rate. As section 1 showed, an increase in price stickiness or an increase in interest-rate smoothing will increase the persistence of the real exchange rate.

3. Extensions of the New Keynesian Model

Exogenous Money Growth Rules

We can tie models with money supply growth rules into this framework. Suppose instead of the Taylor rule, we had these equations:

(18) $m_t - p_t = -\lambda i_{t+1}$, $\lambda > 0$

(19) $m_t - m_{t-1} = v_t$.

Here, m_t is the home less foreign log money supplies, and v_t is an exogenous error term. The first equation represents home relative to foreign money demand equations. The second is a money growth rule, again home relative to foreign. Then we can take first differences of (18), and substitute (19) to get

(20) $i_{t+1} = (1/\lambda)\pi_t + i_t - (1/\lambda)v_t$.

Equation (20) is actually a special case of (3). $1/\lambda$ and $-(1/\lambda)v_t$ in (20) corresponds to σ and u_t, respectively, in (3). Equation (20) is the case of (3) in which the interest rate smoothing parameter is equal to one ($\alpha = 1$). That is, an exogenous money growth rule is exactly like an inflation targeting rule, with extreme interest smoothing. The system with (20) replacing (3) is just a special case of (5). The stability condition is always met: $(1/\lambda)+1>1$.

Consider what this case means for Proposition 1. In general, price stickiness matters for persistence but Proposition 1 shows α is an upper bound on the persistence of the real exchange rate. Here, $\alpha = 1$, so the upper bound does not bind the extent to which price stickiness can influence the speed of reversion of the real exchange rate.

Money Demand Function Derived from Money in the Utility Function

Equation (18) is a special case of a money demand function with no activity variable. In Obstfeld and Rogoff (2003), if utility depends on the log of real balances and markets are complete (see Appendix A), we get:

(21) $m_t - p_t = q_t - \lambda i_{t+1}$.

This changes (20) to

(22) $i_{t+1} = (1/\lambda)\pi_t + (1/\lambda)(q_t - q_{t-1}) + i_t - (1/\lambda)v_t$

Now we have a four-variable system that can be written as:

(23)
$$
\begin{bmatrix} E_t\pi_{t+1} \\ E_t q_{t+1} \\ i_{t+1} \\ x_{t+1} \end{bmatrix} = \begin{bmatrix} 1/\beta & -\delta/\beta & 0 & 0 \\ (1/\lambda)-(1/\beta) & 1+(1/\lambda)+(\delta/\beta) & 1 & -1/\lambda \\ 1/\lambda & 1/\lambda & 1 & -1/\lambda \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \pi_t \\ q_t \\ i_t \\ x_t \end{bmatrix} + \begin{bmatrix} (\delta/\beta)\overline{q}_t \\ -v_t/\lambda - (\delta/\beta)\overline{q}_t + \varepsilon_t \\ -v_t/\lambda \\ 0 \end{bmatrix},
$$

where $x_t \equiv q_{t-1}$. Recall that $\delta \equiv (1-\theta)(1-\theta\beta)/\theta$, a relationship that is derived in section 3 below. The eigenvalues of this system are 0, θ, $\dfrac{1+\lambda}{\lambda}$, and $1/\theta\beta$. There are two variables that are predetermined (i_{t+1} and x_{t+1}) and two roots less than one, so the system is stable.

If the shocks are i.i.d., mean-zero, we find $E_t q_{t+1} = \theta q_t$. That is, the persistence of the real exchange rate is determined entirely by the stickiness of the prices. In fact, Gali (1996) has shown that with the money demand given by (21) and when the log of the money supply follows a random walk, the nominal interest rate differential is zero, which implies $E_t s_{t+1} - s_t = 0$. As explained above, when the nominal exchange rate follows a random walk, prices carry the burden of adjustment.

Notice that the persistence of the real exchange rate does not depend on λ. Carvalho and Nechio (2010) use what amounts to a money demand specification in which $\lambda = 0$, and can be considered a special case of the model here. When the log of money growth follows a random walk, since $m_t - p_t = q_t$, we have $E_t\pi_{t+1} = E_t q_{t+1} - q_t$, or $E_t s_{t+1} - s_t = 0$. We find still that $E_t q_{t+1} = \theta q_t$, as in Carvalho and Nechio.

A New Keynesian Model with a Flexible-Price Sector

Carvalho and Nechio (2010) examine a model with multiple sectors, each of which have slow price adjustment, but a model in which the speed of adjustment may vary across sectors. Their concern is whether the aggregate real exchange rate converges more slowly than the average rate of convergence of the sectoral real exchange rates. Here, we ask whether differential speeds of

adjustment influence the endogenous persistence of the real exchange rate. We look at a simple model to highlight the mechanism through which persistence is affected. Our baseline model is an NK model with no interest-rate smoothing. In the single-sector model we have laid out, there is no endogenous persistence in this case. We introduce a flexible-price sector into this model, and find that this addition induces persistence in the sticky price sectoral real exchange rate.

The model, which is derived in detail in Appendix A, assumes that there are goods with sticky prices and goods with flexible prices produced in each country. The two countries are symmetric, and households in the two countries have identical preferences. Purchasing power parity does not hold because there are deviations from the law of one price in the sticky-price sector. There is local-currency price stickiness and there may also be equilibrium pricing to market. This leads to a Phillips curve for the sticky-price sector that is identical to the Phillips curve in the basic model presented in section 1:

(24) $\qquad \pi_t^S = \delta\left(q_t^S - \overline{q}_t^S\right) + \beta E_t \pi_{t+1}^S.$ $\qquad \delta > 0,\ 0 < \beta < 1.$

Here, π_t^S refers to home minus foreign inflation in the consumer price sub-index for the sticky-price goods. The sectoral real exchange rate, q_t^S, may be non-zero in the long-run because $\overline{q}_t^S$ may not be zero due to equilibrium deviations from the law of one price across the countries. The consumer price indexes put equal weight on home and foreign goods in both countries.

Because the law of one price holds for flexible price goods, and the price indexes for flexible price goods put equal weight on home and foreign goods, the home minus foreign inflation difference for these goods (π_t^F) is simply equal to the rate of depreciation of the home currency:

$\qquad \pi_t^F = s_t - s_{t-1}.$

The overall difference in home and foreign consumer price inflation is a weighted average of the inflation in the two sectors, with the weights given by the expenditure shares on each sector:

(25) $\qquad \pi_t = b\pi_t^S + \left(1-b\right)\pi_t^F = \pi_t^S + \left(1-b\right)\left(q_t^S - q_{t-1}^S\right),$

where q_t^S is the real exchange rate for the sticky-price sector.

The model is completed with the interest-parity condition (1) and the Taylor rule given by equation (3), but with no interest-rate smoothing ($\alpha = 0$). In particular, the monetary policy is assumed to target overall CPI inflation, π_t, rather than just inflation in one of the sectors.

We can write out the dynamic system as:

$$(26) \quad \begin{bmatrix} E_t\pi_{t+1}^S \\ E_t q_{t+1}^S \\ z_{t+1} \end{bmatrix} = \begin{bmatrix} 1/\beta & -\delta/\beta & 0 \\ \sigma-(1/\beta) & 1+(\delta/\beta)+\sigma(1-b) & -\sigma(1-b) \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \pi_t^S \\ q_t^S \\ z_t \end{bmatrix} + \begin{bmatrix} (\delta/\beta)\bar{q}_t^S \\ u_t-(\delta/\beta)\bar{q}_t^S+\varepsilon_t \\ 0 \end{bmatrix}.$$

In this system, z_t is equal to the lagged real exchange rate, q_{t-1}. This lag appears in the dynamic system because monetary policy targets current inflation of the consumer basket. That basket includes the flexible price goods, and the relative inflation is given by the change in the nominal exchange rate, $s_t - s_{t-1}$. The dynamic system includes the inflation rate of sticky-price goods, so targeting consumer price inflation amounts to targeting a linear function of sticky-price inflation and the change in the real exchange rate (see equation (25).) This induces a smoothing of exchange rate changes much as does interest-rate smoothing, which then means that price adjustment plays a role in real-exchange rate adjustment.

The dynamics of sticky-price inflation depends on the adjustment of the real exchange rate, which introduces endogenous persistence into the model. There is one predetermined variable in the system (26), so we can derive that the stability condition is the Taylor condition, $\sigma>1$. Again, we use μ to designate the root that is less than one, and, if shocks are i.i.d., the real exchange rate will still be persistent and satisfly $E_t q_{t+1} = \mu q_t$.

Appendix 1 demonstrates $\mu<\sigma(1-b)$. To interpret this condition, first suppose that the measure of flexible price goods is small, so let $b \to 1$. Then we have $\mu \to 0$, so the model with very few flexible price goods is similar to the model in which all goods have sticky prices, and in this case of no interest rate smoothing, there is little endogenous persistence.

When there are a large measure of flexible price firms ($b \to 0$), then q_t^S has endogenous persistence that depends on the frequency of price resetting. q_t^S may converge very slowly if θ is large. However, because purchasing power parity holds for the flexible price firms, as $b \to 0$, the weight of flexible-price goods in the consumption basket in both countries goes to one. Even though q_t^S can be very persistent, it is stationary, and so has finite variance. But $\text{var}(q_t)=b^2 \text{var}(q_t^S)$, so as the measure of the flexible price firms increases toward one, the variance of the real exchange rate falls to zero.

4. The Literature

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West (1988) is an early paper that derives conditions under which output is persistent in a sticky price model. At a glance, it appears that the results of this paper are closely related to this one. In particular, West finds that output is persistent and nearly a random walk when there is a high degree of interest rate smoothing and prices are very sticky. However, while the conclusions are the same, the model and the economics that lead to the persistence are completely different. The model is one in which prices are set by Taylor overlapping contracts. Output is assumed to be proportional to the ex ante real interest rate. The nominal interest rate is assumed to follow an exogenous autoregressive process. This model has multiple solutions, one of which is stable. West focuses on that solution. When the nominal interest rate is nearly a random walk, and prices adjust very slowly so that inflation is nearly zero, then the real interest rate will also be nearly a random walk. Because output is assumed to be proportional to the real interest rate, it then also follows a random walk. That mechanism is clear and straightforward, but also clearly not related to the one described in this paper. In the West set-up, the nominal interest rate is exogenously determined, and its persistence determines the persistence of the real interest differential if prices are very slow to adjust. In the New Keynesian models considered here, the persistence of all of the endogenous variables depends on the persistence both of prices and interest rates.

It is helpful to present a simplified version of the model West (1988) considers, but which has the same underlying intuition. The simplification comes by assuming a price adjustment equation of the form:

$$p_{t+1} - p_t = \gamma E_t q_{t+1},$$

so inflation is determined by partial adjustment to the expected future misalignment of the exchange rate. The parameter γ determines the speed of adjustment of prices. This simplified Phillips curve shares the key properties of the Taylor price adjustment equation actually considered by West, which are that price setting is forward looking and prices for time $t+1$ are fully determined at time t. Then assume that the real exchange rate is proportional to the real interest rate differential:

$$q_t = -\delta \left(i_t - \left(E_t p_{t+1} - p_t \right) \right).$$

Assume the interest rate is an exogenous autoregressive process with persistence given by λ. Then we find:

$$q_t = -\frac{\delta}{1 - \delta \gamma \lambda} i_t,$$

so the real exchange rate inherits the properties of the nominal interest differential. In contrast to the New Keynesian model we have examined, the real exchange rate is as persistent as the interest

differential, irrespective of the stickiness of prices. If the interest rate differential has a root close to one, then so will the real exchange rate, even if prices adjust very quickly. The model of overlapping price contracts in West introduces moving average dynamics into the interest rate and the real exchange rate. But the model does not share the feature of the New Keynesian model we investigate here, that price stickiness puts an upper bound on the persistence of the real exchange rate.[9]

Bergin and Feenstra (2001) address real exchange rate persistence from a different perspective. They build a model in which pass-through of exchange rates to prices would be imperfect even if prices were flexible. That setting leads to greater persistence of the real exchange rate when prices are sticky. They begin with a simple model of nominal exchange rate determination. Nominal exchange rates are determined by relative money supplies, which are assumed to follow a random walk, which in turn makes the nominal exchange rate follow a random walk. Under Calvo price setting with standard CES preferences, we have seen that the persistence of the real exchange rate is tied to the probability of leaving prices unchanged in each period when the exchange rate is expected not to adjust. Bergin and Feenstra show that, in contrast, under translog preferences, prices converge more slowly than the probability of leaving prices unchanged. Each time prices are reset, under these preferences, price adjustment is smaller compared to the CES case because firms are concerned about the effect of their price adjustment on market share. Bouakez (2005) introduces a more general set of preferences in which the desired mark-up depends on the firm's relative price, and demonstrates that this can increase real exchange rate persistence.

Cheung and Lai (2000) and Steinsson (2008) have made the observation that the reversion of the real exchange rate is not monotonic, as we have generally assumed in the examples of the simple models considered here. When the real exchange rate is shocked away from equilibrium, it tends to initially diverge further from equilibrium before converging. That type of dynamics requires a richer model than the ones considered here. Steinsson (2008) takes a model that is quite similar in structure to the Taylor-rule models with Calvo pricing we have considered above, but adds a "cost-push" shock to the Phillips curve. Serial correlation of the cost-push shock can contribute to the "hump shape" in the impulse response function of the real exchange rate.

It might be the case that the persistence of the real exchange rate is attributable not to the persistence of $q_t - \overline{q}_t$ addressed here, but instead to persistence of the equilibrium component, $\overline{q}_t$.

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[9] A different viewpoint on West (1988) is that, because the interest rate process is exogenous, the persistence of the real exchange rate (actually, output in his closed economy model) is generated by exogenous persistence rather than endogenous dynamics. West presents an alternative version of the model with exogenous persistence in the real money supply.

Engel (2000) casts some doubt on this possibility. Models of the equilibrium real exchange rate, $\bar{q}_t$, generally rely on movements in some internal relative prices: the price of nontraded goods relative to traded goods, or the relative price of imported goods to locally-produced goods. To see this, suppose the log of the home price level is a weighted average of the prices of two traded goods, and a nontraded good:

$$p_t = (\alpha + \eta) p_{Tt} + (1 - \alpha - \eta) p_{Nt}, \qquad \text{where } p_{Tt} = (\alpha p_{1t} + \eta p_{2t})/(\alpha + \eta).$$

Suppose in the foreign country, the weights on the first two goods are reversed, so good 1 receives weight $\eta/(\alpha + \eta)$. Assume, however, that the law of one price holds for traded goods. Then:

$$\tilde{q}_t = \frac{\alpha - \eta}{\alpha + \eta}(p_{2t} - p_{it}) + (1 - \alpha - \eta)\left[(p_{Nt}^* - p_{Tt}^*) - (p_{Nt} - p_{Tt})\right]$$

The equilibrium real exchange rate is determined by relative prices internal to each country in this case. In the data, the volatility of those relative prices are generally low (and their importance in the real exchange rate is weighted by an expenditure share which must be less than one), so they are a small contributor to the movements in q_t. $q_t - \bar{q}_t$ is much more volatile, than $\bar{q}_t$. As Engel (2000) demonstrates, the volatility of $\bar{q}_t$ is so low relative to $q_t - \bar{q}_t$ that the dynamics of the real exchange rate are dominated by the $q_t - \bar{q}_t$ component. Estimates of the persistence of q_t inherit the persistence of $q_t - \bar{q}_t$, even when $\bar{q}_t$ actually has a unit root, so the puzzle in the estimated persistence of q_t arises from persistence in $q_t - \bar{q}_t$. This accords with Rogoff's (1996) observation that models of the equilibrium real exchange rate can account for the persistence but not the volatility of the real exchange rate. Martinez-Garcia and Sondegaard (2012) find that a New Keynesian model with capital accumulation can produce a persistent real exchange rate if the adjustment of the capital stock to real shocks is sufficiently slow, but the real exchange rate in the model is much less volatile than actual real exchange rates.

Carvalho and Nechio (2013) find results similar to Martinez-Garcia and Sondegaard (2012) in a multi-sector model with sector-specific capital. Compared to a model with no capital, real exchange rate persistence increases but volatility declines. A key insight is that the model reproduces the hump-shaped adjustment dynamics discussed above. In fact, their numerical exercises find the real exchange rate is *less* persistent when there is interest-rate smoothing, because under interest-rate smoothing the adjustment is monotonic.

One clear conclusion from our analysis is that if the nominal exchange rate follows a random walk, real exchange rate persistence must arise from the persistence of nominal price adjustment. In

fact, many take as a stylized fact that the logs of nominal exchange rates among high income countries are simple random walks. However, an alternative view can be gleaned from studies of non-linear adjustment. Taylor et. al. (2000) find that when the real exchange rate is far from its equilibrium value, convergence is fast, but is much slower as the real exchange rate approaches its unconditional mean. Nam (2011), in a threshold cointegration framework, investigates the relative contribution of s_t and p_t to real exchange rate convergence. Nam finds that when the real exchange rate lies inside a symmetric band around the unconditional mean of the real exchange rate, there is no statistically significant evidence of reversion. Outside of the band, there is convergence, and it is driven almost entirely by nominal exchange rate movements. The exchange rate is not a random walk, because its movements are predictable when the real exchange rate is far from its mean.

Nam's findings support the notion that an answer to the puzzle of the slow convergence of real exchange rates lies in understanding the behavior of nominal exchange rates. Also, the existence of a band where there is little or no reversion echoes Rogoff's (1996) conclusion that barriers to trade in goods may help account for the PPP puzzle. A micro-founded model of these bands of inaction, based on a model of limit pricing, can be found in Atkeson and Burstein (2007) and Burstein and Jaimovich (2012).

5. Conclusion

In the open economy, we see that under Calvo pricing, both the nominal exchange rate and the relative home to foreign nominal price levels can adjust to eliminate real exchange rate disequilibria. When there is no interest-rate smoothing, the exchange rate can do all of the adjustment and there is no endogenous persistence in the real exchange rate. The analog to the nominal exchange rate in the closed economy is the nominal wage (or nominal GDP). With no interest rate smoothing, the nominal wage can bear the burden of real wage adjustment, and so price stickiness does not govern the speed of convergence of real wages, the output gap, or inflation. A natural way in the closed economy to build in some endogenous persistence would be to allow for nominal wage stickiness as well as price stickiness, as in Erceg, et. al. (2000). Nominal wage stickiness is a palatable assumption in a closed economy, but the analogous assumption in the open economy is sticky nominal exchange rates which clearly is not a good assumption for flexible-exchange rate economies.[10]

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[10] Nominal wage stickiness does lead to more real exchange rate persistence in a model with exogenous persistence of shocks. See Kollmann (2001), for example. However, that paper finds that even with wage and price stickiness, a basic New Keynesian model cannot match the persistence of real exchange rates in the data.

Does interest rate persistence solve the PPP puzzle? Probably not. In the models with Calvo pricing, the endogenous persistence of the real exchange rate cannot be greater than the sluggishness of prices, as the Proposition shows. However, in the simple model of section 1, setting risk premium shocks and shocks to the equilibrium real exchange rate to zero, if the error in the Taylor rule, u_t, is an AR(1) with serial correlation of ρ, then the real exchange rate is given by $q_t = \dfrac{-\delta}{(\delta + \beta)(1 - \rho)} u_t$, and will also have a persistence of ρ. A highly persistent shock to the Taylor rule can lead to a very persistent real exchange rate in this simple model. Table 1 presents persistence statistics for the U.S. relative to the other six G7 countries. These are estimates of the first-order serial correlation of the real exchange rate, the nominal interest differential, and the residual from four different Taylor rule specifications (two which include only the inflation differential, and two which also include a measure of cyclical unemployment.) The point estimates for the persistence of the real exchange rate are generally greater than those for the measures of monetary policy persistence.[11] Monetary policy shocks are estimated to be strongly serially correlated, which under uncovered interest parity implies the nominal exchange rate does not adjust quickly to achieve real exchange rate equilibrium, but this monetary persistence alone does not seem to account for the sluggishness of real exchange rate adjustment.

The intuition drawn from the simple models in this paper are that both price adjustment and nominal exchange rate determination play roles in real exchange rate convergence, but the richer models explored in the literature are needed to resolve the PPP puzzle.

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[11] One exception is the estimated monetary persistence for the U.S. relative to Japan in the 1996-2007 sub-period is greater than the estimated persistence of the real exchange rate.

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Table 1
Serial Correlation of Real Exchange Rate and Monetary Shocks

January 1983 – December 2012

Country	q_t	$i_t - i_t^j$	u_t^{T1}	u_t^{T2}	u_t^{T3}	u_t^{T4}
Canada	0.995 (0.008)	0.951 (0.019)	0.942 (0.023)	0.922 (0.025)	0.937 (0.020)	0.908 (0.029)
France	0.983 (0.013)	0.893 (0.046)	0.911 (0.037)	0.907 (0.034)	0.854 (0.045)	0.857 (0.044)
Germany	0.983 (0.013)	0.990 (0.012)	0.942 (0.024)	0.950 (0.022)	0.924 (0.029)	0.905 (0.028)
Italy	0.985 (0.014)	0.961 (0.020)	0.919 (0.036)	0.916 (0.035)	0.906 (0.034)	0.890 (0.040)
Japan	0.980 (0.012)	0.984 (0.010)	0.973 (0.013)	0.936 (0.025)	0.969 (0.013)	0.893 (0.035)
United Kingdom	0.966 (0.024)	0.967 (0.017)	0.909 (0.030)	0.897 (0.027)	0.858 (0.038)	0.881 (0.035)

January 1983 – August 1996

Country	q_t	$i_t - i_t^j$	u_t^{T1}	u_t^{T2}	u_t^{T3}	u_t^{T4}
Canada	0.994 (0.011)	0.907 (0.039)	0.892 (0.045)	0.909 (0.032)	0.876 (0.042)	0.894 (0.033)
France	0.981 (0.019)	0.811 (0.074)	0.859 (0.055)	0.875 (0.047)	0.733 (0.062)	0.786 (0.063)
Germany	0.984 (0.018)	0.990 (0.014)	0.944 (0.034)	0.967 (0.026)	0.931 (0.034)	0.950 (0.027)
Italy	0.981 (0.019)	0.879 (0.050)	0.881 (0.058)	0.881 (0.053)	0.771 (0.063)	0.824 (0.056)
Japan	0.983 (0.014)	0.982 (0.017)	0.910 (0.031)	0.906 (0.032)	0.876 (0.042)	0.842 (0.049)
United Kingdom	0.962 (0.032)	0.959 (0.025)	0.893 (0.039)	0.879 (0.039)	0.851 (0.047)	0.848 (0.048)

September 1996 – December 2012

Country	q_t	$i_t - i_t^j$	u_t^{T1}	u_t^{T2}	u_t^{T3}	u_t^{T4}
Canada	1.006 (0.015)	0.975 (0.021)	0.883 (0.037)	0.920 (0.026)	0.846 (0.047)	0.840 (0.049)
European Area	0.985 (0.018)	0.990 (0.012)	0.893 (0.030)	0.978 (0.015)	0.917 (0.026)	0.975 (0.032)
Japan	0.943 (0.025)	0.987 (0.013)	0.985 (0.014)	0.975 (0.014)	0.979 (0.018)	0.956 (0.025)
United Kingdom	0.974 (0.019)	0.976 (0.019)	0.968 (0.023)	0.959 (0.025)	0.908 (0.044)	0.957 (0.028)

Notes to Tables
Serial correlation estimates (Newey-West standard errors in parentheses).

q_t is the log of the real CPI exchange rate relative to U.S.

$i_t - i_t^j$ is the U.S. interest rate less interest rate of country j. Interest rates are 1-month Eurodeposit rates.

$u_t^{T,i}$ is the residual of the U.S. less foreign Taylor rule residual for various Taylor rules. See Appendix B for details.

European Area refers to a simple average of France, Italy, and Germany. See Appendix B for details.

See Appendix B for data description.