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A theoretical general equilibrium analysis of local teacher labor markets under different compensation regimes

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Abstract

Bargaining power of teacher unions over teacher wages has been either reduced or eliminated by several states in U.S. since 2011. This caused public school districts to move away from single salary schedules (fixed compensation regime) and adapt a flexible compensation regime at which teacher wage rises with quality (value added). In this paper, using a fully tractable general equilibrium model of local teacher labor markets, we theoretically analyze the effect of different compensation regimes on teacher efforts and average teacher quality in a district. In our model, teachers, heterogeneous in exogenously set quality, endogenously sort across two districts and also choose teaching efforts. Districts differ by endogenous teacher wages and exogenous revenues. The marginal disutility of effort for a teacher is different across the districts. Teacher labor markets clear in each district and teacher wages are determined. We solve for the unique equilibrium under each compensation regime and theoretically show that low(high) quality teachers exert the highest effort under fixed(flexible) compensation regime and exert the lowest effort under flexible(fixed) compensation regime. Also, we show that average teacher quality is highest in each district under flexible compensation regime and lowest in each district under fixed compensation regime. Our findings are consistent with several empirical studies.

JEL-Codes: I21, I28, J20

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1 Introduction

After the Great Recession, U.S. governmental agencies sought ways of using their funds more economically. For that sake, since 2011, several states including Idaho, Indiana, Iowa, Michigan, Tennessee, and Wisconsin passed legislation that either reduced or eliminated the collective bargaining power of teacher unions over teacher wages. This gave public school districts the opportunity to move away from single salary schedules and adapt a flexible compensation regime at which compensation rises with teacher's quality.[1] In this paper, we theoretically compare different teacher compensation regimes with regards to their effects on teacher efforts and average teacher quality in a district. Changes in teacher efforts and average teacher quality translates into change in average student achievement since they are essential inputs in producing achievement as found by empirical studies such as Rockoff (2004) and Rivkin et al. (2005).

Our model economy consists of two public school districts and a continuum of heterogeneous teachers. Teachers differ by exogenously set quality which is uniformly distributed. Teachers derive utility from consumption of the numeraire good, average student achievement in the classroom, and they derive disutility from teaching effort. School districts differ by exogenous revenues and endogenous teacher wages. School districts also differ by working conditions which is captured by a parameter that governs marginal disutility of teaching effort. Achievement depends on teacher's quality and effort and an exogenously given vector of inputs such as student's ability, peer effect, private educational spending, parent's education level, etc... Each teacher chooses effort for each student in the classroom. Teachers also make a choice among the two school districts which in turn yields the supply of teacher quality for each district. School districts determine their demand for teacher quality so as to equate their revenues with teachers' wage expenditures. Teacher labor market clears in each district and teachers' wages are determined. In our model, there exists a unique equilibrium under each compensation regime and we can find closed-form expressions for each endogenous variable.

Using our model, we analyze three teacher compensation regimes: i) flexible, ii) fixed, and iii) mixed. Under flexible compensation regime, a teacher's total compensation rises with quality and wage rate per unit of quality differs across districts. Under fixed compensation regime, a teacher's total compensation is fixed in a district and is independent of quality. Fixed wage differs across districts as in data. In the traditional single salary schedule observed in reality, the teacher wage is unrelated to quality as noted in Hanushek (2007) and Podgursky (2007). Therefore, we think of fixed compensation regime in our model as corresponding to the observed single salary schedule. Under mixed compensation regime, one of the districts implements fixed compensation regime and

[1] What we mean by "quality" is teacher's value added.

the other district implements flexible compensation regime. We find that, in equilibrium, low quality teachers exert highest effort under fixed compensation regime and exert lowest effort under flexible compensation regime. On the other hand, high quality teachers exert highest effort under flexible compensation regime and exert lowest effort under fixed compensation regime We also find that average teacher quality is highest in both districts under flexible compensation regime. And it is lowest in both districts under fixed compensation regime. The average qualities in the districts under mixed compensation regime lies between the first two regimes.

Our predictions stated above are consistent with the findings of empirical studies. For instance, using a rich structural econometric model for Wisconsin's districts after Act 10, Biasi (2018) finds that average teacher quality and teacher efforts rise in those districts that switched to flexible pay scheme relative to the districts that kept the single salary schedule. Lovenheim and Willen (2019) analyzes the state duty-to-bargain law in U.S., which increased the bargaining power of teacher unions, and finds that education quality, teacher quality and effort decreased as a result leading to decreases in annual earnings of males. Roth (2019) and Anderson et al. (2019) also find positive effects on education quality and achievement following the state laws that decreased collective-bargaining power of teacher unions.

In a dynamic general equilibrium framework, Tamura (2001) shows that per capita incomes converge across states in U.S. if teacher quality is more important than class size in producing human capital. Gilpin and Kaganovich (2012) using a dynamic general equilibrium model compares different teacher pay regimes in terms of teacher quality distribution over time, income inequality, and economic growth. Different from these papers, our static model is silent about economic growth and inequality. On the positive side, in our model, teachers choose effort for each student and choose among the districts different from these papers. As far as we are concerned, the closest theoretical model to ours is studied in Biasi (2018) at which teachers' salaries are exogenously given and teachers cannot choose effort.

In our model, economy-wide teacher quality distribution is exogenous and does not change with teacher compensation regime. More specifically, teacher quality distribution may be expected to improve as the economy moves from a fixed compensation regime to a flexible one. However, according to OECD (2014), the earning of an average teacher is 68% of the earning of an average college graduate in U.S. in 2012. This implies that the average teacher's earning should rise by approximately 50% in order to attract an average college graduate into the teaching profession. We believe that such an increase in average teacher's earning is unlikely in practice when the economy moves to a flexible payment regime. Moreover, as noted in Corcoran et al. (2004) and Hoxby and Leigh (2004), the fraction of highly qualified teachers is decreasing in U.S. over time. Based on these, we believe that our model is not an oversimplification of reality.

This paper is organized as follows. Section 2 explains the model. Equilibrium is defined in Section 3 and its theoretical properties are analyzed in Section 4. Section 5 concludes.

2 Benchmark Model

In a static environment, we consider two school districts and a continuum of teachers with measure one. Teachers differ by exogenously set quality. Teachers' choice among school districts determines the supply of quality in each district. Teachers also choose effort for each student in their classrooms. School districts differ by exogenously given revenues, working conditions, and endogenous teacher wages. A school district's revenue is used solely to finance teacher wage compensations. The budget constraint of a district determines its demand for teacher quality. The teacher labor market in each district clears and equilibrium wages are determined. There are three possible teacher compensation regimes in each district: i) flexible (ℓ), ii) fixed (d), and iii) mixed (m).

2.1 Teachers

The set of students in the classroom of teacher j in district i is denoted with S_{ij} which is exogenously given. The exogenous probability density function of students over the support S_{ij} is denoted with $f_{ij}(s)$. Economy-wide measure of all students is one. The measure of set S_{ij} is denoted with $\mu(S_{ij})$. We assume S_{ij}, $f_{ij}(s)$, and $\mu(S_{ij})$ are independent of the teacher compensation regime.

2.1.1 Preferences

Teachers have identical preferences. Following Yeşilırmak (2019), the utility function for teacher j in district i with compensation regime $r \in \{\ell, d, m\}$ is:

$$u(c_{ijr}, \overline{a}_{ijr}, \{e_{ijsr}\}_{s\in S_{ij}}) = c_{ijr}\overline{a}_{ijr} - \int_{S_{ij}} \frac{e_{ijsr}^{\gamma_i}}{\gamma_i} \mu(S_{ij}) f_{ij}(s) ds, \qquad \gamma_i > 1 \quad \forall i,$$

where c_{ijr} denotes the consumption of the numeraire good, $\overline{a}_{ijr}$ denotes the mean student achievement in the classroom, and e_{ijsr} denotes the teaching effort put by the teacher on student type $s \in S_{ij}$. Moreover, as clarified below, $\overline{a}_{ijr}$ depends on teacher's quality, effort, and other exogenously given inputs. Teachers getting utility from the mean achievement is consistent with the findings of previous studies such as Hanushek et al. (2004), Boyd et al. (2005), and Clotfelter et al. (2011).

The second term in teachers' utility function is the total cost of effort which is a convex function. The parameter γ_i captures the marginal disutility of effort and differs across districts to reflect the empirical fact that working conditions differ across districts (Hanushek and Rivkin (2007)). Specifically, without loss of generality, we assume $\gamma_1 > \gamma_2$ implying that marginal disutility of effort is higher in the first district than the second for any teacher.[2] Equivalently, working conditions are worse in the first district.[3]

In the teacher's utility function, we assume complementarity between consumption and mean achievement. As an alternative, one could assume an additively separable utility function. However, in that case, optimal student specific effort chosen by a teacher does not depend on teacher's income which is not desirable in our case since we want to compare optimal effort under different compensation regimes. Moreover, in the additively separable utility case, when consumption is zero teacher can still get positive utility which is not plausible. On the other hand, the complementarity implies zero indirect utility when consumption is zero since teacher would set optimal effort to zero.

2.1.2 Incomes

Teachers differ by exogenously set quality λ. The exogenous economy-wide probability density function of teacher quality is denoted with $t(\cdot)$ which is assumed to be a uniform distribution over $[0, q]$. Total income of teacher j working in district i with compensation regime $r \in \{\ell, d, m\}$ is denoted with I_{ijr}. Under flexible compensation regime, $I_{ij\ell} = \lambda_j w_{i\ell}$ where $w_{i\ell}$ is the wage rate per unit of quality in district i. Under fixed compensation regime, $I_{ijd} = w_{id}$ where wid is the total wage paid for any teacher in district i. Under mixed compensation regime, we assume fixed compensation regime prevails in the first district and flexible compensation prevails in the second district. Under this regime, $I_{1jm} = w_{1m}$ where w_{1m} is the total wage paid for any teacher in the first district and $I_{2jm} = \lambda_j w_{2m}$ where w_{2m} is the wage rate per unit of quality in the second district.

2.2 School Districts

The exogenously given revenue of district i in any compensation regime is denoted with R_i. All revenue in a district is spent on the teachers' wages which is the unique expenditure. Under compensation regime r, given I_{ijr}, school district i demands those teachers with qualities belonging to the set $\Omega_{ir} \subset [0, q]$.[4] Then the budget constraint for district i is,

$$R_i = \int_{\Omega_{ir}} I_{ijr} t(\lambda) d\lambda \qquad \forall i. \tag{1}$$

[2]One can also assume working conditions are worse in the second district. This is just a labeling issue.

[3]In our model, we don't need to impose any restrictions on the mean incomes of the districts. However, in reality, richer districts have better working conditions than poorer districts. Thus, one could visualize first district as poor and second district as rich district.

[4]Clearly, Ω_{ir} depends on the district wage which is suppressed in the notation.

2.3 Student Achievement

The achievement of student type s in teacher j's class in district i with compensation regime r is determined as follows:

$$a_{ijsr} = \lambda_j e_{ijsr}^{\alpha} h(\upsilon_{ijs}), \qquad \alpha \in (0,1)$$

where $h(\cdot)$ is a function of exogenously given vector of inputs (υ_{ijs}) not captured in the model such as student's ability, classroom peer effect, private educational spending, student's effort, parent's education level, etc... Based on this, the average achievement in teacher j's class in district i with compensation regime r can be expressed as:

$$\overline{a}_{ijr} = \int\limits_{S_{ij}} \lambda_j e_{ijsr}^{\alpha} h(\upsilon_{ijs}) f_{ij}(s) ds.$$

In our model, student-teacher ratio (or class size) is not considered as an input in determining achievement. However, as in Tamura (2001), we think of class size as inverse of teacher's time which is proxied by teacher's effort on a student in our model.

2.4 Teacher's Problem

Teacher j in district i with compensation regime r solves the following problem given I_{ijr}, $\mu(S_{ij})$, $h(\upsilon_{ijs})$, and $f_{ij}(s)$:

$$V_{ijr} = \max_{\substack{c_{ijr} \\ \{e_{ijsr}\}_{s \in S_{ij}}}} u(c_{ijr}, \overline{a}_{ijr}, \{e_{ijsr}\}_{s \in S_{ij}}) \tag{2}$$

$$s.t. \qquad c_{ijr} = I_{ijr},$$

$$\overline{a}_{ijr} = \int\limits_{S_{ij}} \lambda_j e_{ijsr}^{\alpha} h(\upsilon_{ijs}) f_{ij}(s) ds,$$

where V_{ijr} denotes the indirect utility of the teacher. Teacher j chooses to work in that district at which indirect utility is highest. The set of teachers that choose district i with compensation regime r is denoted by $L_{ir} \subset [0, q]$.[5]

3 Equilibrium

An equilibrium under compensation regime r is a collection of c_{ijr}, e_{ijsr}, I_{ijr}, w_{ir}, Ω_{ir}, and L_{ir} for each district i, for each teacher j, and for each student s such that:

1. c_{ijr}, $\{e_{ijsr}\}_{s \in S_{ij}}$ solves the problem (2).

[5]Clearly, L_{ir} depends on the wages in both districts which is suppressed in the notation.

2. Teacher j chooses to work in the district that yields higher indirect utility.

3. Budget of each district is balanced.

4. Teacher labor market clears ($L_{ir} \equiv \Omega_{ir}$) and w_{ir} is determined in each district.

4 Characteristics of Equilibrium

In this section, we solve for the unique equilibrium under each compensation regime. The teacher's problem (2) can be reexpressed as:

$$V_{ijr} = \max_{\{e_{ijsr}\}_{s \in S_{ij}}} I_{ijr} \int_{S_{ij}} \lambda_j e_{ijsr}^{\alpha} h(\upsilon_{ijs}) f_{ij}(s) ds - \int_{S_{ij}} \frac{e_{ijsr}^{\gamma_i}}{\gamma_i} \mu(S_{ij}) f_{ij}(s) ds \tag{3}$$

Lemma 1. *There exists a unique solution for the problem* (3) *which is given by:*

$$e_{ijsr} = \left(\frac{\alpha \lambda_j I_{ijr} h(\upsilon_{ijs})}{\mu(S_{ij})} \right)^{\frac{1}{\gamma_i - \alpha}}.$$

Proof. The objective function is strictly concave since $\alpha \in (0,1)$ and $\gamma_i > 1$. This implies existence of a unique solution. The solution is characterized by the following first order condition:

$$\alpha \lambda_j I_{ijr} e_{ijsr}^{\alpha - 1} h(\upsilon_{ijs}) f_{ij}(s) = e_{ijsr}^{\gamma_i - 1} \mu(S_{ij}) f_{ij}(s),$$

solving which implies the desired result. □

Substituting e_{ijsr} into (3) implies that:

$$V_{ijr} = \frac{\lambda_j^{\frac{\gamma_i}{\gamma_i - \alpha}} I_{ijr}^{\frac{\gamma_i}{\gamma_i - \alpha}}}{(\mu(S_{ij}))^{\frac{\alpha}{\gamma_i - \alpha}}} \left(\int_{S_{ij}} \left(\alpha^{\frac{\alpha}{\gamma_i - \alpha}} - \frac{\alpha^{\frac{\gamma_i}{\gamma_i - \alpha}}}{\gamma_i} \right) (h(\upsilon_{ijs}))^{\frac{\gamma_i}{\gamma_i - \alpha}} f_{ij}(s) ds \right). \tag{4}$$

It should be noted that $V_{ijr} \geq 0$ for any teacher in any district with any compensation regime since $\left(\alpha^{\frac{\alpha}{\gamma_i - \alpha}} - \frac{\alpha^{\frac{\gamma_i}{\gamma_i - \alpha}}}{\gamma_i} \right) > 0$ given that $\alpha \in (0,1)$ and $\gamma_i > 1$. Given V_{ijr}, the following lemma characterizes a teacher's district choice under flexible and fixed compensation regimes. For that sake, let us define κ as follows:

$$\kappa := \left[\frac{(\mu(S_{2j}))^{\frac{\alpha}{\gamma_2 - \alpha}} \left(\int_{S_{1j}} \left(\alpha^{\frac{\alpha}{\gamma_1 - \alpha}} - \frac{\alpha^{\frac{\gamma_1}{\gamma_1 - \alpha}}}{\gamma_1} \right) (h(\upsilon_{1js}))^{\frac{\gamma_1}{\gamma_1 - \alpha}} f_{1j}(s) ds \right)}{(\mu(S_{1j}))^{\frac{\alpha}{\gamma_1 - \alpha}} \left(\int_{S_{2j}} \left(\alpha^{\frac{\alpha}{\gamma_2 - \alpha}} - \frac{\alpha^{\frac{\gamma_2}{\gamma_2 - \alpha}}}{\gamma_2} \right) (h(\upsilon_{2js}))^{\frac{\gamma_2}{\gamma_2 - \alpha}} f_{2j}(s) ds \right)} \right].$$

Lemma 2. a) *Under flexible compensation regime, teacher j chooses first school district if and only if* $\lambda_j \leq \widetilde{\lambda}_\ell := \frac{w_{1\ell}^{\frac{\gamma_1(\gamma_2-\alpha)}{2\alpha(\gamma_1-\gamma_2)}}}{w_{2\ell}^{\frac{\gamma_2(\gamma_1-\alpha)}{2\alpha(\gamma_1-\gamma_2)}}}\Gamma_\ell$ *where* $\Gamma_\ell = \kappa^{\frac{(\gamma_1-\alpha)(\gamma_2-\alpha)}{2\alpha(\gamma_1-\gamma_2)}}$.

b) *Under fixed compensation regime, teacher j chooses first school district if and only if* $\lambda_j \leq \widetilde{\lambda}_d := \frac{w_{1d}^{\frac{\gamma_1(\gamma_2-\alpha)}{\alpha(\gamma_1-\gamma_2)}}}{w_{2d}^{\frac{\gamma_2(\gamma_1-\alpha)}{\alpha(\gamma_1-\gamma_2)}}}\Gamma_d$ *where* $\Gamma_d = \kappa^{\frac{(\gamma_1-\alpha)(\gamma_2-\alpha)}{\alpha(\gamma_1-\gamma_2)}}$.

c) *Under mixed compensation regime, teacher j chooses first school district if and only if* $\lambda_j \leq \widetilde{\lambda}_m := \frac{w_{1m}^{\frac{\gamma_1(\gamma_2-\alpha)}{\gamma_1\gamma_2+\gamma_1\alpha-2\gamma_2\alpha}}}{w_{2m}^{\frac{\gamma_2(\gamma_1-\alpha)}{\gamma_1\gamma_2+\gamma_1\alpha-2\gamma_2\alpha}}}\Gamma_m$ *where* $\Gamma_m = \kappa^{\frac{(\gamma_1-\alpha)(\gamma_2-\alpha)}{\gamma_1\gamma_2+\gamma_1\alpha-2\gamma_2\alpha}}$.

Proof. Under any compensation regime r, teacher j chooses to work in the first district if and only if $V_{1jr} \geq V_{2jr}$. For flexible compensation regime, substituting $I_{1j\ell} = \lambda_j w_{1\ell}$ and $I_{2j\ell} = \lambda_j w_{2\ell}$ into $V_{1j\ell}$ and $V_{2j\ell}$ and solving the inequality implies that:

$$\lambda_j^{\frac{2\gamma_2}{\gamma_2-\alpha}-\frac{2\gamma_1}{\gamma_1-\alpha}} \leq \frac{w_{1\ell}^{\frac{\gamma_1}{\gamma_1-\alpha}}}{w_{2\ell}^{\frac{\gamma_2}{\gamma_2-\alpha}}}\kappa,$$

which then proves part a. The proofs for parts b and c follow similarly. □

Lemma 2 also implies that teachers with $\lambda_j \in (\widetilde{\lambda}_r, q]$ choose the second school district. Thus, $L_{1r} = [0, \widetilde{\lambda}_r]$ and $L_{2r} = (\widetilde{\lambda}_r, q]$. The sorting of teachers across the districts is illustrated in Figure 1. Therefore, teachers are perfectly sorted across the districts in our model which is consistent with the findings of empirical studies such as Lankford et al. (2002), Clotfelter et al. (2011) and Kalogrides et al. (2012). This type of sorting pattern in our model also implies that average teacher quality in the second district is higher than the first district.

Now we can find the equilibrium wage rates in both districts using the budget constraints given by (1). Since teacher quality is uniformly distributed, then the budget constraints under each compensation regime can be reexpressed as follows after substituting the labor market clearing conditions:

$$R_1 = \int_0^{\widetilde{\lambda}_r} \frac{I_{1jr}}{q} d\lambda = \begin{cases} \int_0^{\widetilde{\lambda}_\ell} \frac{\lambda w_{1\ell}}{q} d\lambda = \frac{\widetilde{\lambda}_\ell^2 w_{1\ell}}{2q} & \text{if } r = \ell, \\ \int_0^{\widetilde{\lambda}_d} \frac{w_{1d}}{q} d\lambda = \frac{\widetilde{\lambda}_d w_{1d}}{q} & \text{if } r = d, \\ \int_0^{\widetilde{\lambda}_m} \frac{w_{1m}}{q} d\lambda = \frac{\widetilde{\lambda}_m w_{1m}}{q} & \text{if } r = m, \end{cases} \tag{5}$$

$$R_2 = \int_{\widetilde{\lambda}_r}^{q} \frac{I_{2jr}}{q} d\lambda = \begin{cases} \int_{\widetilde{\lambda}_\ell}^{q} \frac{\lambda w_{2\ell}}{q} d\lambda = \frac{\left(q^2-\widetilde{\lambda}_\ell^2\right) w_{2\ell}}{2q} & \text{if } r = \ell, \\ \int_{\widetilde{\lambda}_d}^{q} \frac{w_{2d}}{q} d\lambda = \frac{\left(q-\widetilde{\lambda}_d\right) w_{2d}}{q} & \text{if } r = d, \\ \int_{\widetilde{\lambda}_m}^{q} \frac{\lambda w_{2m}}{q} d\lambda = \frac{\left(q^2-\widetilde{\lambda}_m^2\right) w_{2m}}{2q} & \text{if } r = m, \end{cases} \tag{6}$$

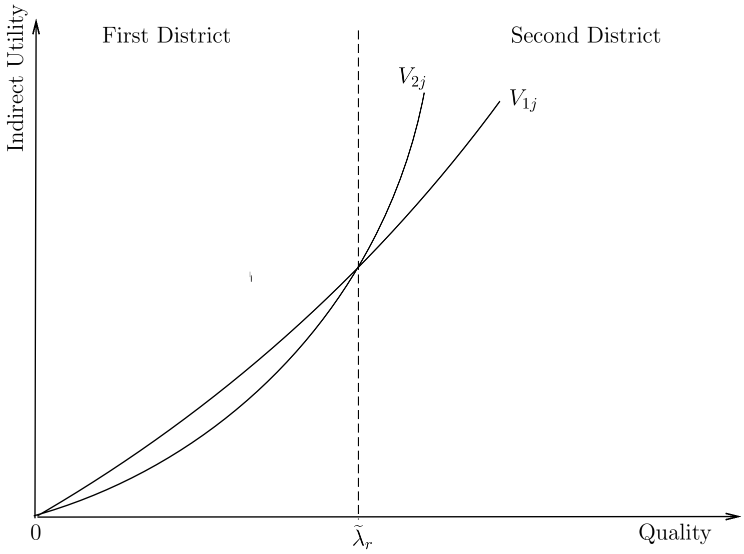

Figure 1: Teacher sorting under compensation regime r

Lemma 3. a) *The equilibrium wage rates under flexible compensation regime are given by:*

$$w_{1\ell} = \frac{2R_2 q + (2R_1 q)^{\frac{\gamma_1(\gamma_2-\alpha)}{\gamma_2(\gamma_1-\alpha)}}\,\Gamma_\ell^{\frac{2(\gamma_1-\gamma_2)\alpha}{\gamma_2(\gamma_1-\alpha)}}}{\left(\frac{\Gamma_\ell^2}{2R_1 q}\right)^{\frac{\alpha(\gamma_1-\gamma_2)}{\gamma_2(\gamma_1-\alpha)}} q^2},$$

$$w_{2\ell} = \frac{2R_2 q + (2R_1 q)^{\frac{\gamma_1(\gamma_2-\alpha)}{\gamma_2(\gamma_1-\alpha)}}\,\Gamma_\ell^{\frac{2(\gamma_1-\gamma_2)\alpha}{\gamma_2(\gamma_1-\alpha)}}}{q^2}.$$

b) *The equilibrium wages under fixed compensation regime are given by:*

$$w_{1d} = \frac{R_2 q + (R_1 q)^{\frac{\gamma_1(\gamma_2-\alpha)}{\gamma_2(\gamma_1-\alpha)}}\,\Gamma_d^{\frac{(\gamma_1-\gamma_2)\alpha}{\gamma_2(\gamma_1-\alpha)}}}{\left(\frac{\Gamma_d}{R_1 q}\right)^{\frac{\alpha(\gamma_1-\gamma_2)}{\gamma_2(\gamma_1-\alpha)}} q},$$

$$w_{2d} = \frac{R_2 q + (R_1 q)^{\frac{\gamma_1(\gamma_2-\alpha)}{\gamma_2(\gamma_1-\alpha)}}\,\Gamma_d^{\frac{(\gamma_1-\gamma_2)\alpha}{\gamma_2(\gamma_1-\alpha)}}}{q}.$$

c) *The equilibrium wages under mixed compensation regime are given by:*

$$w_{1m} = \left(\frac{2R_2 + R_1^2 q\left(\frac{\Gamma_m}{R_1 q}\right)^{\frac{\gamma_1\gamma_2+\gamma_1\alpha-2\gamma_2\alpha}{\gamma_2(\gamma_1-\alpha)}}}{\left(\frac{\Gamma_m}{R_1 q}\right)^{\frac{\gamma_1\gamma_2+\gamma_1\alpha-2\gamma_2\alpha}{\gamma_2(\gamma_1-\alpha)}} q}\right)^{\frac{1}{2}},$$

$$w_{2m} = \frac{2R_2 + R_1^2 q\left(\frac{\Gamma_m}{R_1 q}\right)^{\frac{\gamma_1\gamma_2+\gamma_1\alpha-2\gamma_2\alpha}{\gamma_2(\gamma_1-\alpha)}}}{q}.$$

Proof. **a)** Substituting $\widetilde{\lambda}_\ell = \frac{w_{1\ell}^{\frac{\gamma_1(\gamma_2-\alpha)}{2\alpha(\gamma_1-\gamma_2)}}}{w_{2\ell}^{\frac{\gamma_2(\gamma_1-\alpha)}{2\alpha(\gamma_1-\gamma_2)}}}\Gamma_\ell$ into (5) implies:

$$w_{2\ell} = \left(\frac{\Gamma_\ell^2}{2R_1 q}\right)^{\frac{\alpha(\gamma_1-\gamma_2)}{\gamma_2(\gamma_1-\alpha)}} w_{1\ell}.$$

Substituting this expression for $w_{2\ell}$ into (6) together with $\widetilde{\lambda}_\ell = \frac{w_{1\ell}^{\frac{\gamma_1(\gamma_2-\alpha)}{2\alpha(\gamma_1-\gamma_2)}}}{w_{2\ell}^{\frac{\gamma_2(\gamma_1-\alpha)}{2\alpha(\gamma_1-\gamma_2)}}}\Gamma_\ell$ implies the desired result.

b) Proof is similar to part a. Please see appendix.

c) Proof is similar to part a. Please see appendix.

□

Since teachers are perfectly sorted across the districts, then the average teacher quality in a district depends on the value of $\widetilde{\lambda}_r$. As $\widetilde{\lambda}_r$ rises, the average quality in each district rises and vice versa. Next lemma characterizes the equilibrium value of $\widetilde{\lambda}_r$ for each compensation regime.

Lemma 4. a) *The equilibrium value of $\widetilde{\lambda}_\ell$ under flexible compensation regime is given by:*

$$\widetilde{\lambda}_\ell = \frac{(2R_1 q)^{\frac{1}{2}}\left(\frac{\Gamma_\ell^2}{2R_1 q}\right)^{\frac{\alpha(\gamma_1-\gamma_2)}{2\gamma_2(\gamma_1-\alpha)}} q}{\left(2R_2 q + (2R_1 q)^{\frac{\gamma_1(\gamma_2-\alpha)}{\gamma_2(\gamma_1-\alpha)}} \Gamma_\ell^{\frac{2\alpha(\gamma_1-\gamma_2)}{\gamma_2(\gamma_1-\alpha)}}\right)^{\frac{1}{2}}}.$$

b) *The equilibrium value of $\widetilde{\lambda}_d$ under fixed compensation regime is given by:*

$$\widetilde{\lambda}_d = \frac{R_1 q^2 \left(\frac{\Gamma_d}{R_1 q}\right)^{\frac{\alpha(\gamma_1-\gamma_2)}{\gamma_2(\gamma_1-\alpha)}}}{R_2 q + (R_1 q)^{\frac{\gamma_1(\gamma_2-\alpha)}{\gamma_2(\gamma_1-\alpha)}} \Gamma_d^{\frac{\alpha(\gamma_1-\gamma_2)}{\gamma_2(\gamma_1-\alpha)}}}.$$

c) *The equilibrium value of $\widetilde{\lambda}_m$ under mixed compensation regime is given by:*

$$\widetilde{\lambda}_m = \frac{R_1 q^{\frac{3}{2}} \left(\frac{\Gamma_m}{R_1 q}\right)^{\frac{\gamma_1\gamma_2+\gamma_1\alpha-2\gamma_2\alpha}{2\gamma_2(\gamma_1-\alpha)}}}{\left(2R_2 + R_1^2 q \left(\frac{\Gamma_m}{R_1 q}\right)^{\frac{\gamma_1\gamma_2+\gamma_1\alpha-2\gamma_2\alpha}{\gamma_2(\gamma_1-\alpha)}}\right)^{\frac{1}{2}}}.$$

Proof. **a)** The budget constraint (5) implies $\widetilde{\lambda}_\ell = \left(\frac{2R_1 q}{w_{1\ell}}\right)^{\frac{1}{2}}$. Substituting into this the equilibrium value of $w_{1\ell}$ implies the above result.

b) Proof is similar to part a. Please see appendix.

c) Proof is similar to part a. Please see appendix.

□

4.1 Comparison of Compensation Regimes

In this section, we compare the equilibrium average teacher quality and teacher effort in a district across different compensation regimes. For that sake, the following lemma compares $\widetilde{\lambda}_\ell$, $\widetilde{\lambda}_d$, and $\widetilde{\lambda}_m$.

Lemma 5. *In equilibrium,* $\widetilde{\lambda}_d < \widetilde{\lambda}_m < \widetilde{\lambda}_\ell$.

Proof. See Appendix. □

Let us denote the average teacher quality in district i under compensation regime r with $E_{ir}(\lambda)$.

Proposition 1. *For any district* i*, in equilibrium* $E_{id}(\lambda) < E_{im}(\lambda) < E_{i\ell}(\lambda)$.

Proof. For the first district under compensation regime r:

$$E_{1r}(\lambda) = \frac{\int\limits_0^{\widetilde{\lambda}_r} \lambda t(\lambda) d\lambda}{\int\limits_0^{\widetilde{\lambda}_r} t(\lambda) d\lambda} = \frac{\frac{\widetilde{\lambda}_r^2}{2q}}{\frac{\widetilde{\lambda}_r}{q}} = \frac{\widetilde{\lambda}_r}{2}.$$

For the second district under compensation regime r:

$$E_{2r}(\lambda) = \frac{\int\limits_{\widetilde{\lambda}_r}^{q} \lambda t(\lambda) d\lambda}{\int\limits_{\widetilde{\lambda}_r}^{q} t(\lambda) d\lambda} = \frac{\frac{q^2-(\widetilde{\lambda}_r)^2}{2q}}{\frac{q-\widetilde{\lambda}_r}{q}} = \frac{q+\widetilde{\lambda}_r}{2}.$$

Since $\widetilde{\lambda}_d < \widetilde{\lambda}_m < \widetilde{\lambda}_\ell$, then $E_{1d}(\lambda) < E_{1m}(\lambda) < E_{1\ell}(\lambda)$ and $E_{2d}(\lambda) < E_{2m}(\lambda) < E_{2\ell}(\lambda)$. □

Next, we compare teacher effort in a district under different compensation regimes.

Proposition 2. *In equilibrium in the first district,*

a) *teachers with* $\lambda_j < \min\{\widetilde{\lambda}_d, \overline{\lambda}_{1\ell d}\}$ *exert higher effort for each student under fixed compensation regime compared to flexible compensation regime where* $\overline{\lambda}_{1\ell d}$ *is defined as:*

$$\overline{\lambda}_{1\ell d} := \frac{R_2 q^2 + q\left(R_1 q\right)^{\frac{\gamma_1(\gamma_2-\alpha)}{\gamma_2(\gamma_1-\alpha)}} \kappa^{\frac{\gamma_2-\alpha}{\gamma_2}}}{2^{\frac{\alpha(\gamma_1-\gamma_2)}{\gamma_2(\gamma_1-\alpha)}}\left(2R_2 q + (2R_1 q)^{\frac{\gamma_1(\gamma_2-\alpha)}{\gamma_2(\gamma_1-\alpha)}} \kappa^{\frac{\gamma_2-\alpha}{\gamma_2}}\right)}.$$

b) *teachers with* $\lambda_j < \min\{\widetilde{\lambda}_m, \overline{\lambda}_{1\ell m}\}$ *exert higher effort for each student under mixed compensation regime compared to flexible compensation regime where* $\overline{\lambda}_{1\ell m}$ *is defined as:*

$$\overline{\lambda}_{1\ell m} := \frac{\left(\left(2R_2 + R_1\kappa^{\frac{\gamma_2-\alpha}{\gamma_2}}(R_1 q)^{\frac{(\gamma_2-\gamma_1)\alpha}{\gamma_2(\gamma_1-\alpha)}}\right)(R_1 q)^{\frac{\gamma_1(\gamma_2-\alpha)}{\gamma_2(\gamma_1-\alpha)}} q^3 \kappa^{\frac{\gamma_2-\alpha}{\gamma_2}}\right)^{\frac{1}{2}}}{2^{\frac{\alpha(\gamma_1-\gamma_2)}{\gamma_2(\gamma_1-\alpha)}}\left(2R_2 q + (2R_1 q)^{\frac{\gamma_1(\gamma_2-\alpha)}{\gamma_2(\gamma_1-\alpha)}}\kappa^{\frac{\gamma_2-\alpha}{\gamma_2}}\right)}.$$

c) *teachers with* $\lambda_j < \widetilde{\lambda}_d$ *exert higher effort for each student under fixed compensation regime compared to mixed compensation regime.*

Proof. **a)** Only those teachers with $\lambda_j < \widetilde{\lambda}_d$ live in the first district under both flexible and fixed compensation regimes since $\widetilde{\lambda}_d < \widetilde{\lambda}_\ell$ by Lemma 5. The optimal value of a teacher's effort is found in Lemma 1. Substituting into e_{1jsr}, the $w_{1\ell}$ and w_{1d} given by Lemma 3 implies the following equilibrium effort levels under flexible and fixed compensation regimes in district 1:

$$e_{1js\ell} = \left(\frac{\alpha\lambda_j^2 h(\upsilon_{1js})}{\mu(S_{1j})}\right)^{\frac{1}{\gamma_1-\alpha}}\left(\frac{2R_2 q + (2R_1 q)^{\frac{\gamma_1(\gamma_2-\alpha)}{\gamma_2(\gamma_1-\alpha)}}\kappa^{\frac{\gamma_2-\alpha}{\gamma_2}}}{\frac{\kappa^{\frac{\gamma_2-\alpha}{\gamma_2}} q^2}{(2R_1 q)^{\frac{\alpha(\gamma_1-\gamma_2)}{\gamma_2(\gamma_1-\alpha)}}}}\right)^{\frac{1}{\gamma_1-\alpha}},$$

$$e_{1jsd} = \left(\frac{\alpha\lambda_j h(\upsilon_{1js})}{\mu(S_{1j})}\right)^{\frac{1}{\gamma_1-\alpha}}\left(\frac{R_2 q + (R_1 q)^{\frac{\gamma_1(\gamma_2-\alpha)}{\gamma_2(\gamma_1-\alpha)}}\kappa^{\frac{\gamma_2-\alpha}{\gamma_2}}}{\frac{\kappa^{\frac{\gamma_2-\alpha}{\gamma_2}} q}{(R_1 q)^{\frac{\alpha(\gamma_1-\gamma_2)}{\gamma_2(\gamma_1-\alpha)}}}}\right)^{\frac{1}{\gamma_1-\alpha}},$$

where we also substituted $\Gamma_\ell = \kappa^{\frac{(\gamma_1-\alpha)(\gamma_2-\alpha)}{2\alpha(\gamma_1-\gamma_2)}}$ and $\Gamma_d = \kappa^{\frac{(\gamma_1-\alpha)(\gamma_2-\alpha)}{\alpha(\gamma_1-\gamma_2)}}$. Then $e_{1js\ell} < e_{1jsd}$ if and only if:

$$\frac{\lambda_j\left(2R_2 q + (2R_1 q)^{\frac{\gamma_1(\gamma_2-\alpha)}{\gamma_2(\gamma_1-\alpha)}}\kappa^{\frac{\gamma_2-\alpha}{\gamma_2}}\right)2^{\frac{\alpha(\gamma_1-\gamma_2)}{\gamma_2(\gamma_1-\alpha)}}}{q} < R_2 q + (R_1 q)^{\frac{\gamma_1(\gamma_2-\alpha)}{\gamma_2(\gamma_1-\alpha)}}\kappa^{\frac{\gamma_2-\alpha}{\gamma_2}}.$$

This implies $\lambda_j < \overline{\lambda}_{1\ell d}$. Therefore, those teachers with $\lambda_j < \min\{\widetilde{\lambda}_d, \overline{\lambda}_{1\ell d}\}$ both live in the first district under flexible and fixed compensation regimes and exert higher effort under fixed compensation regime.

b) Please see appendix.

c) Please see appendix.

□

It should be noted that we compare in Proposition 2 the efforts of those teachers who work in the first school district under the two compensation regimes of interest. For instance, in part a of the proposition, we concentrate on those teachers that work in the first district under both fixed and flexible compensation regimes. Based on the proposition, if fixed compensation regime initially prevails in the first district, then moving to flexible compensation regime decreases the efforts of teachers with $\lambda_j < \overline{\lambda}_{1\ell d}$ and moving to mixed compensation regime decreases the efforts of all teachers. Also, if mixed compensation regime initially prevails in the first district, then moving to flexible compensation regime decreases the efforts of teachers with $\lambda_j < \overline{\lambda}_{1\ell m}$.

Parts a and c of Proposition 2 implies that teachers with $\lambda_j < \min\{\widetilde{\lambda}_d, \overline{\lambda}_{1\ell d}\}$ exert the highest effort under fixed compensation regime. Moreover, parts a and b of the proposition implies that teachers with $\lambda_j < \min\{\widetilde{\lambda}_d, \overline{\lambda}_{1\ell d}, \overline{\lambda}_{1\ell m}\}$ exert the lowest effort under flexible compensation regime. In sum, low quality teachers exert highest effort under fixed compensation regime and exert lowest effort under flexible compensation regime.[6]

We next analyze the teachers' efforts in the second district under alternative compensation regimes. Again, we concentrate on those teachers that work in the second district under the two compensation regimes of interest.

Proposition 3. *In equilibrium in the second district,*

a) *teachers with $\lambda_j > \max\{\widetilde{\lambda}_\ell, \overline{\lambda}_{2\ell d}\}$ exert higher effort for each student under flexible compensation regime compared to fixed compensation regime where $\overline{\lambda}_{2\ell d}$ is defined as:*

$$\overline{\lambda}_{2\ell d} := \frac{R_2 q^2 + q\left(R_1 q\right)^{\frac{\gamma_1(\gamma_2-\alpha)}{\gamma_2(\gamma_1-\alpha)}} \kappa^{\frac{\gamma_2-\alpha}{\gamma_2}}}{2R_2 q + \left(2R_1 q\right)^{\frac{\gamma_1(\gamma_2-\alpha)}{\gamma_2(\gamma_1-\alpha)}} \kappa^{\frac{\gamma_2-\alpha}{\gamma_2}}}.$$

b) *teachers with $\lambda_j > \widetilde{\lambda}_\ell$ exert higher effort for each student under flexible compensation regime compared to mixed compensation regime.*

c) *teachers with $\lambda_j > \max\{\widetilde{\lambda}_m, \overline{\lambda}_{2dm}\}$ exert higher effort for each student under mixed compensation regime compared to fixed compensation regime where $\overline{\lambda}_{2dm}$ is defined as:*

$$\overline{\lambda}_{2dm} := \frac{R_2 q + \left(R_1 q\right)^{\frac{\gamma_1(\gamma_2-\alpha)}{\gamma_2(\gamma_1-\alpha)}} \kappa^{\frac{\gamma_2-\alpha}{\gamma_2}}}{2R_2 + R_1\left(R_1 q\right)^{\frac{\alpha(\gamma_2-\gamma_1)}{\gamma_2(\gamma_1-\alpha)}} \kappa^{\frac{\gamma_2-\alpha}{\gamma_2}}}.$$

Proof. **a)** Only those teachers with $\lambda_j > \widetilde{\lambda}_\ell$ live in the second district under both flexible and fixed compensation regimes since $\widetilde{\lambda}_d < \widetilde{\lambda}_\ell$ by Lemma 5. The optimal value of a teacher's effort is found in Lemma 1. Substituting into e_{2jsr}, the $w_{2\ell}$ and w_{2d} given by Lemma 3 implies the following equilibrium effort levels under flexible and fixed compensation regimes in district 2:

$$e_{2js\ell} = \left(\frac{\alpha\lambda_j^2 h(\upsilon_{2js})}{\mu(S_{2j})}\right)^{\frac{1}{\gamma_2-\alpha}} \left(\frac{2R_2 q + \left(2R_1 q\right)^{\frac{\gamma_1(\gamma_2-\alpha)}{\gamma_2(\gamma_1-\alpha)}} \kappa^{\frac{\gamma_2-\alpha}{\gamma_2}}}{q^2}\right)^{\frac{1}{\gamma_2-\alpha}},$$

[6]We define a low quality teacher with $\lambda_j < \min\{\widetilde{\lambda}_d, \overline{\lambda}_{1\ell d}, \overline{\lambda}_{1\ell m}\}$.

$$e_{2jsd} = \left(\frac{\alpha\lambda_j h(\upsilon_{2js})}{\mu(S_{2j})}\right)^{\frac{1}{\gamma_2-\alpha}} \left(\frac{R_2 q + (R_1 q)^{\frac{\gamma_1(\gamma_2-\alpha)}{\gamma_2(\gamma_1-\alpha)}} \kappa^{\frac{\gamma_2-\alpha}{\gamma_2}}}{q}\right)^{\frac{1}{\gamma_2-\alpha}},$$

where we also substituted $\Gamma_\ell = \kappa^{\frac{(\gamma_1-\alpha)(\gamma_2-\alpha)}{2\alpha(\gamma_1-\gamma_2)}}$ and $\Gamma_d = \kappa^{\frac{(\gamma_1-\alpha)(\gamma_2-\alpha)}{\alpha(\gamma_1-\gamma_2)}}$. Then $e_{2js\ell} > e_{2jsd}$ if and only if:

$$\lambda_j \left(2R_2 q + (2R_1 q)^{\frac{\gamma_1(\gamma_2-\alpha)}{\gamma_2(\gamma_1-\alpha)}} \kappa^{\frac{\gamma_2-\alpha}{\gamma_2}}\right) > R_2 q^2 + q(R_1 q)^{\frac{\gamma_1(\gamma_2-\alpha)}{\gamma_2(\gamma_1-\alpha)}} \kappa^{\frac{\gamma_2-\alpha}{\gamma_2}}.$$

This implies $\lambda_j > \overline{\lambda}_{2\ell d}$. Therefore, those teachers with $\lambda_j > \max\{\widetilde{\lambda}_\ell, \overline{\lambda}_{2\ell d}\}$ both live in the second district under flexible and fixed compensation regimes and exert higher effort under flexible compensation regime.

b) Please see appendix.

c) Please see appendix.

□

In words, Proposition 3 implies that if flexible compensation regime initially prevails in the second district, then moving to fixed compensation regime decreases efforts of teachers with $\lambda_j > \overline{\lambda}_{2\ell d}$ and moving to mixed compensation regime decreases efforts of all teachers. Also, if mixed compensation regime initially prevails in the second district, then moving to a fixed compensation regime decreases the efforts of teachers with $\lambda_j > \overline{\lambda}_{2dm}$.

Parts a and b of Proposition 3 implies that teachers with $\lambda_j > \max\{\widetilde{\lambda}_\ell, \overline{\lambda}_{2\ell d}\}$ exert the highest effort under flexible compensation regime. Moreover, parts a and c of the proposition implies that teachers with $\lambda_j > \max\{\widetilde{\lambda}_\ell, \overline{\lambda}_{2\ell d}, \overline{\lambda}_{2dm}\}$ exert the lowest effort under fixed compensation regime. In sum, high quality teachers exert highest effort under flexible compensation regime and exert lowest effort under fixed compensation regime.[7]

It should be noted that, the mean achievement rises in a teacher's class as effort for each student rises. Therefore, based on the Propositions 2 and 3, we can compare the mean achievement in a teacher's class under different compensation regimes. Unfortunately, it is not possible in our model to compare economy-wide mean achievement under different compensation regimes.

5 Conclusion

Since 2011, several states in U.S. decreased the power the teacher unions in bargaining over teacher wages. This led school districts to switch to more flexible teacher compensation regimes at which wage rises with quality. In this paper, we compared different teacher compensation regimes with regards to their effects on teachers' efforts and average teacher qualities in each district. For that

[7]We define a high quality teacher with $\lambda_j > \max\{\widetilde{\lambda}_\ell, \overline{\lambda}_{2\ell d}, \overline{\lambda}_{2dm}\}$.

sake, we set up a fully tractable general equilibrium model of local teacher labor markets with two school districts. The supply of teacher quality in each district is determined from the spatial sorting of teachers who maximize their utilities by optimally choosing effort for each student in their classrooms. The districts' demand for teacher quality is determined from the districts' budgets. We solved for the equilibrium wages in both districts, teachers' efforts, and the cutoff teacher quality separating the school districts under each compensation regime.

In our model, there are only two districts. We believe our model can be extended to an arbitrary number of districts which is left for future research. Our model also lacks household choices which is another fruitful direction for future research. By extending our model in these directions, one can quantitatively compare mean achievement, variance of achievement, household welfare, and teacher welfare under different compensation regimes.

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Appendix

Proof of Lemma 3. **b)** Substituting $\widetilde{\lambda}_d = \frac{w_{1d}^{\frac{\gamma_1(\gamma_2-\alpha)}{\alpha(\gamma_1-\gamma_2)}}}{w_{2d}^{\frac{\gamma_2(\gamma_1-\alpha)}{\alpha(\gamma_1-\gamma_2)}}}\Gamma_d$ into (5) implies:

$$w_{2d} = \left(\frac{\Gamma_d}{R_1 q}\right)^{\frac{\alpha(\gamma_1-\gamma_2)}{\gamma_2(\gamma_1-\alpha)}} w_{1d}.$$

Substituting this expression for w_{2d} into (6) together with $\widetilde{\lambda}_d = \frac{w_{1d}^{\frac{\gamma_1(\gamma_2-\alpha)}{\alpha(\gamma_1-\gamma_2)}}}{w_{2d}^{\frac{\gamma_2(\gamma_1-\alpha)}{\alpha(\gamma_1-\gamma_2)}}}\Gamma_d$ implies the desired result.

c) Substituting $\widetilde{\lambda}_m = \frac{w_{1m}^{\frac{\gamma_1(\gamma_2-\alpha)}{\gamma_1\gamma_2+\gamma_1\alpha-2\gamma_2\alpha}}}{w_{2m}^{\frac{\gamma_2(\gamma_1-\alpha)}{\gamma_1\gamma_2+\gamma_1\alpha-2\gamma_2\alpha}}}\Gamma_m$ into (5) implies:

$$w_{2m} = \left(\frac{\Gamma_m}{R_1 q}\right)^{\frac{\gamma_1\gamma_2+\gamma_1\alpha-2\gamma_2\alpha}{\gamma_2(\gamma_1-\alpha)}} w_{1m}^2.$$

Substituting this expression for w_{2m} into (6) together with $\widetilde{\lambda}_m = \frac{w_{1m}^{\frac{\gamma_1(\gamma_2-\alpha)}{\gamma_1\gamma_2+\gamma_1\alpha-2\gamma_2\alpha}}}{w_{2m}^{\frac{\gamma_2(\gamma_1-\alpha)}{\gamma_1\gamma_2+\gamma_1\alpha-2\gamma_2\alpha}}}\Gamma_m$ implies the desired result.

$\square$

Proof of Lemma 4. **b)** The budget constraint (5) implies $\widetilde{\lambda}_d = \frac{R_1 q}{w_{1d}}$. Substituting into this the equilibrium value of w_{1d} implies the desired result.

c) The budget constraint (5) implies $\widetilde{\lambda}_m = \frac{R_1 q}{w_{1m}}$. Substituting into this the equilibrium value of w_{1m} implies the desired result.

$\square$

Proof of Lemma 5. Let us first show $\widetilde{\lambda}_d < \widetilde{\lambda}_m$. Since $R_2^2 q^2 > 0$, then:

$$\frac{1}{R_2^2 q^2 + \Gamma_d^{\frac{\alpha(\gamma_1-\gamma_2)}{\gamma_2(\gamma_1-\alpha)}} (R_1 q)^{\frac{\gamma_1(\gamma_2-\alpha)}{\gamma_2(\gamma_1-\alpha)}} \left(2R_2 q + \Gamma_d^{\frac{\alpha(\gamma_1-\gamma_2)}{\gamma_2(\gamma_1-\alpha)}} (R_1 q)^{\frac{\gamma_1(\gamma_2-\alpha)}{\gamma_2(\gamma_1-\alpha)}}\right)}$$
$$< \frac{1}{\Gamma_d^{\frac{\alpha(\gamma_1-\gamma_2)}{\gamma_2(\gamma_1-\alpha)}} (R_1 q)^{\frac{\gamma_1(\gamma_2-\alpha)}{\gamma_2(\gamma_1-\alpha)}} \left(2R_2 q + \Gamma_d^{\frac{\alpha(\gamma_1-\gamma_2)}{\gamma_2(\gamma_1-\alpha)}} (R_1 q)^{\frac{\gamma_1(\gamma_2-\alpha)}{\gamma_2(\gamma_1-\alpha)}}\right)}.$$

Taking square root of both sides implies:

$$\frac{1}{R_2 q+\Gamma_d^{\frac{\alpha(\gamma_1-\gamma_2)}{\gamma_2(\gamma_1-\alpha)}}(R_1 q)^{\frac{\gamma_1(\gamma_2-\alpha)}{\gamma_2(\gamma_1-\alpha)}}} < \frac{1}{\left(\Gamma_d^{\frac{\alpha(\gamma_1-\gamma_2)}{\gamma_2(\gamma_1-\alpha)}}(R_1 q)^{\frac{\gamma_1(\gamma_2-\alpha)}{\gamma_2(\gamma_1-\alpha)}}\left(2R_2 q+\Gamma_d^{\frac{\alpha(\gamma_1-\gamma_2)}{\gamma_2(\gamma_1-\alpha)}}(R_1 q)^{\frac{\gamma_1(\gamma_2-\alpha)}{\gamma_2(\gamma_1-\alpha)}}\right)\right)^{\frac{1}{2}}}.$$

Rearranging this inequality implies:

$$\frac{q^{\frac{1}{2}}\Gamma_d^{\frac{\alpha(\gamma_1-\gamma_2)}{2\gamma_2(\gamma_1-\alpha)}}(R_1 q)^{\frac{\gamma_1(\gamma_2-\alpha)}{2\gamma_2(\gamma_1-\alpha)}}}{R_2 q+\Gamma_d^{\frac{\alpha(\gamma_1-\gamma_2)}{\gamma_2(\gamma_1-\alpha)}}(R_1 q)^{\frac{\gamma_1(\gamma_2-\alpha)}{\gamma_2(\gamma_1-\alpha)}}} < \frac{1}{\left(2R_2+\Gamma_d^{\frac{\alpha(\gamma_1-\gamma_2)}{\gamma_2(\gamma_1-\alpha)}}(R_1 q)^{\frac{\alpha(\gamma_2-\gamma_1)}{\gamma_2(\gamma_1-\alpha)}}R_1\right)^{\frac{1}{2}}}.$$

By definition, $\Gamma_d^{\frac{\alpha(\gamma_1-\gamma_2)}{\gamma_2(\gamma_1-\alpha)}}=\Gamma_m^{\frac{\gamma_1\gamma_2+\gamma_1\alpha-2\gamma_2\alpha}{\gamma_2(\gamma_1-\alpha)}}$ which can be used to reexpress the above inequality as:

$$\frac{q\left(\frac{\Gamma_d}{R_1 q}\right)^{\frac{\alpha(\gamma_1-\gamma_2)}{\gamma_2(\gamma_1-\alpha)}}}{R_2 q+(R_1 q)^{\frac{\gamma_1(\gamma_2-\alpha)}{\gamma_2(\gamma_1-\alpha)}}\Gamma_d^{\frac{\alpha(\gamma_1-\gamma_2)}{\gamma_2(\gamma_1-\alpha)}}} < \frac{q^{\frac{1}{2}}\left(\frac{\Gamma_m}{R_1 q}\right)^{\frac{\gamma_1\gamma_2+\gamma_1\alpha-2\gamma_2\alpha}{2\gamma_2(\gamma_1-\alpha)}}}{\left(2R_2+R_1^2 q\left(\frac{\Gamma_m}{R_1 q}\right)^{\frac{\gamma_1\gamma_2+\gamma_1\alpha-2\gamma_2\alpha}{\gamma_2(\gamma_1-\alpha)}}\right)^{\frac{1}{2}}}.$$

Multiplying both sides of this inequality with $R_1 q$ implies that $\widetilde{\lambda}_d<\widetilde{\lambda}_m$. Now let us show $\widetilde{\lambda}_m<\widetilde{\lambda}_\ell$. Since $\frac{\alpha(\gamma_1-\gamma_2)}{\gamma_2(\gamma_1-\alpha)}<1$, then $2^{\frac{\alpha(\gamma_1-\gamma_2)}{\gamma_2(\gamma_1-\alpha)}}<2$. Using this implies:

$$\frac{1}{2R_2 q+\Gamma_\ell^{\frac{2\alpha(\gamma_1-\gamma_2)}{\gamma_2(\gamma_1-\alpha)}}(R_1 q)^{\frac{\gamma_1(\gamma_2-\alpha)}{\gamma_2(\gamma_1-\alpha)}}} < \frac{1}{2^{\frac{\alpha(\gamma_1-\gamma_2)}{\gamma_2(\gamma_1-\alpha)}}R_2 q+\Gamma_\ell^{\frac{2\alpha(\gamma_1-\gamma_2)}{\gamma_2(\gamma_1-\alpha)}}(R_1 q)^{\frac{\gamma_1(\gamma_2-\alpha)}{\gamma_2(\gamma_1-\alpha)}}}.$$

This inequality can be reexpressed as:

$$\frac{R_1(R_1 q)^{\frac{-\gamma_1\gamma_2-\gamma_1\alpha+2\gamma_2\alpha+\alpha(\gamma_1-\gamma_2)}{\gamma_2(\gamma_1-\alpha)}}}{2R_2+R_1\Gamma_\ell^{\frac{2\alpha(\gamma_1-\gamma_2)}{\gamma_2(\gamma_1-\alpha)}}(R_1 q)^{\frac{\alpha(\gamma_2-\gamma_1)}{\gamma_2(\gamma_1-\alpha)}}} < \frac{2^{\frac{\gamma_1(\gamma_2-\alpha)}{\gamma_2(\gamma_1-\alpha)}}}{2R_2 q+2^{\frac{\gamma_1(\gamma_2-\alpha)}{\gamma_2(\gamma_1-\alpha)}}\Gamma_\ell^{\frac{2\alpha(\gamma_1-\gamma_2)}{\gamma_2(\gamma_1-\alpha)}}(R_1 q)^{\frac{\gamma_1(\gamma_2-\alpha)}{\gamma_2(\gamma_1-\alpha)}}},$$

which can be reexpressed as:

$$\frac{R_1(R_1 q)^{\frac{-\gamma_1\gamma_2-\gamma_1\alpha+2\gamma_2\alpha}{\gamma_2(\gamma_1-\alpha)}}}{2R_2+R_1\Gamma_\ell^{\frac{2\alpha(\gamma_1-\gamma_2)}{\gamma_2(\gamma_1-\alpha)}}(R_1 q)^{1-\frac{\gamma_1\gamma_2+\gamma_1\alpha-2\gamma_2\alpha}{\gamma_2(\gamma_1-\alpha)}}} < \frac{2^{1-\frac{\alpha(\gamma_1-\gamma_2)}{\gamma_2(\gamma_1-\alpha)}}(R_1 q)^{-\frac{\alpha(\gamma_1-\gamma_2)}{\gamma_2(\gamma_1-\alpha)}}}{2R_2 q+2^{\frac{\gamma_1(\gamma_2-\alpha)}{\gamma_2(\gamma_1-\alpha)}}\Gamma_\ell^{\frac{2\alpha(\gamma_1-\gamma_2)}{\gamma_2(\gamma_1-\alpha)}}(R_1 q)^{\frac{\gamma_1(\gamma_2-\alpha)}{\gamma_2(\gamma_1-\alpha)}}}.$$

By definition, $\Gamma_\ell^{\frac{2\alpha(\gamma_1-\gamma_2)}{\gamma_2(\gamma_1-\alpha)}}=\Gamma_m^{\frac{\gamma_1\gamma_2+\gamma_1\alpha-2\gamma_2\alpha}{\gamma_2(\gamma_1-\alpha)}}$ which can be used to reexpress the above inequality as:

$$\frac{R_1^2 q^3\left(\frac{\Gamma_m}{R_1 q}\right)^{\frac{\gamma_1\gamma_2+\gamma_1\alpha-2\gamma_2\alpha}{\gamma_2(\gamma_1-\alpha)}}}{2R_2+R_1^2 q\left(\frac{\Gamma_m}{R_1 q}\right)^{\frac{\gamma_1\gamma_2+\gamma_1\alpha-2\gamma_2\alpha}{\gamma_2(\gamma_1-\alpha)}}} < \frac{2R_1 q^3\left(\frac{\Gamma_\ell^2}{2R_1 q}\right)^{\frac{\alpha(\gamma_1-\gamma_2)}{\gamma_2(\gamma_1-\alpha)}}}{2R_2 q+(2R_1 q)^{\frac{\gamma_1(\gamma_2-\alpha)}{\gamma_2(\gamma_1-\alpha)}}\Gamma_\ell^{\frac{2\alpha(\gamma_1-\gamma_2)}{\gamma_2(\gamma_1-\alpha)}}}.$$

Taking square root of both sides implies $\widetilde{\lambda}_m < \widetilde{\lambda}_\ell$. □

Proof of Proposition 2. **b)** Only those teachers with $\lambda_j < \widetilde{\lambda}_m$ live in the first district under both flexible and mixed compensation regimes since $\widetilde{\lambda}_m < \widetilde{\lambda}_\ell$ by Lemma 5. The optimal value of a teacher's effort is found in Lemma 1. Substituting into e_{1jsm}, the w_{1m} given by Lemma 3 implies the following equilibrium effort level under mixed compensation regime in district 1:

$$e_{1jsm} = \left(\frac{\alpha\lambda_j h(\upsilon_{1js})}{\mu(S_{1j})}\right)^{\frac{1}{\gamma_1-\alpha}} \left(\frac{2R_2 + R_1^2 q \left(\frac{\Gamma_m}{R_1 q}\right)^{\frac{\gamma_1\gamma_2+\gamma_1\alpha-2\gamma_2\alpha}{\gamma_2(\gamma_1-\alpha)}}}{\left(\frac{\Gamma_m}{R_1 q}\right)^{\frac{\gamma_1\gamma_2+\gamma_1\alpha-2\gamma_2\alpha}{\gamma_2(\gamma_1-\alpha)}} q}\right)^{\frac{1}{2(\gamma_1-\alpha)}},$$

where $\Gamma_m = \kappa^{\frac{(\gamma_1-\alpha)(\gamma_2-\alpha)}{\gamma_1\gamma_2+\gamma_1\alpha-2\gamma_2\alpha}}$. The equilibrium value of $e_{1js\ell}$ is provided in the proof of part a above. Then $e_{1js\ell} < e_{1jsm}$ if and only if $\lambda_j < \overline{\lambda}_{1\ell m}$. Therefore, those teachers with $\lambda_j < \min\{\widetilde{\lambda}_m, \overline{\lambda}_{1\ell m}\}$ both live in the first district under flexible and mixed compensation regimes and exert higher effort under mixed compensation regime.

c) Only those teachers with $\lambda_j < \widetilde{\lambda}_d$ live in the first district under both fixed and mixed compensation regimes since $\widetilde{\lambda}_d < \widetilde{\lambda}_m$ by Lemma 5. The equilibrium effort levels e_{1jsd} and e_{1jsm} are provided in the proofs of parts a and b above. Now we would like to prove that $e_{1jsd} > e_{1jsm}$ for any teacher living in the first district under both regimes. For that sake, let us start with the following inequality:

$$\left(R_2 q (R_1 q)^{\frac{\alpha(\gamma_1-\gamma_2)}{\gamma_2(\gamma_1-\alpha)}} + R_1 q \kappa^{\frac{\gamma_2-\alpha}{\gamma_2}}\right)^2 > 2R_2 q (R_1 q)^{\frac{\gamma_1\gamma_2+\gamma_1\alpha-2\gamma_2\alpha}{\gamma_2(\gamma_1-\alpha)}} \kappa^{\frac{\gamma_2-\alpha}{\gamma_2}} + \left(R_1 q \kappa^{\frac{\gamma_2-\alpha}{\gamma_2}}\right)^2,$$

which implies:

$$\left(\frac{R_2 q + (R_1 q)^{\frac{\gamma_1(\gamma_2-\alpha)}{\gamma_2(\gamma_1-\alpha)}} \kappa^{\frac{\gamma_2-\alpha}{\gamma_2}}}{\frac{\kappa^{\frac{\gamma_2-\alpha}{\gamma_2}} q}{(R_1 q)^{\frac{\alpha(\gamma_1-\gamma_2)}{\gamma_2(\gamma_1-\alpha)}}}}\right)^2 > \frac{2R_2 + R_1^2 q \kappa^{\frac{\gamma_2-\alpha}{\gamma_2}} (R_1 q)^{-\frac{\gamma_1\gamma_2+\gamma_1\alpha-2\gamma_2\alpha}{\gamma_2(\gamma_1-\alpha)}}}{(R_1 q)^{-\frac{\gamma_1\gamma_2+\gamma_1\alpha-2\gamma_2\alpha}{\gamma_2(\gamma_1-\alpha)}} \kappa^{\frac{\gamma_2-\alpha}{\gamma_2}} q}.$$

Taking $\frac{1}{2(\gamma_1-\alpha)}$ power of both sides and then multiplying both sides of the last inequality with $\left(\frac{\alpha\lambda_j h(\upsilon_{1js})}{\mu(S_{1j})}\right)^{\frac{1}{\gamma_1-\alpha}}$ implies that $e_{1jsd} > e_{1jsm}$.

□

Proof of Proposition 3. **b)** Only those teachers with $\lambda_j > \widetilde{\lambda}_\ell$ live in the second district under both flexible and mixed compensation regimes since $\widetilde{\lambda}_m < \widetilde{\lambda}_\ell$ by Lemma 5. The optimal value of a teacher's effort is found in Lemma 1. Substituting into e_{2jsm}, the w_{2m} given by Lemma 3 implies the following equilibrium effort level under mixed compensation regime in district 2:

$$e_{2jsm} = \left(\frac{\alpha\lambda_j^2 h(\upsilon_{2js})}{\mu(S_{2j})}\right)^{\frac{1}{\gamma_2-\alpha}} \left(\frac{2R_2 + R_1^2 q\left(\frac{\Gamma_m}{R_1 q}\right)^{\frac{\gamma_1\gamma_2+\gamma_1\alpha-2\gamma_2\alpha}{\gamma_2(\gamma_1-\alpha)}}}{q}\right)^{\frac{1}{\gamma_2-\alpha}},$$

where $\Gamma_m = \kappa^{\frac{(\gamma_1-\alpha)(\gamma_2-\alpha)}{\gamma_1\gamma_2+\gamma_1\alpha-2\gamma_2\alpha}}$. The equilibrium effort levels $e_{2js\ell}$ is provided in the proof of part a above. Now we would like to prove that $e_{2js\ell} > e_{2jsm}$ for any teacher living in the second district under both regimes. Since $2^{\frac{\gamma_1(\gamma_2-\alpha)}{\gamma_2(\gamma_1-\alpha)}} > 1$, then:

$$2R_2 q + (2R_1 q)^{\frac{\gamma_1(\gamma_2-\alpha)}{\gamma_2(\gamma_1-\alpha)}} \kappa^{\frac{\gamma_2-\alpha}{\gamma_2}} > 2R_2 q + (R_1 q)^{2-\frac{\gamma_1\gamma_2+\gamma_1\alpha-2\gamma_2\alpha}{\gamma_2(\gamma_1-\alpha)}} \kappa^{\frac{\gamma_2-\alpha}{\gamma_2}},$$

which implies:

$$\frac{2R_2 q + (2R_1 q)^{\frac{\gamma_1(\gamma_2-\alpha)}{\gamma_2(\gamma_1-\alpha)}} \kappa^{\frac{\gamma_2-\alpha}{\gamma_2}}}{q^2} > \frac{2R_2 + R_1^2 q (R_1 q)^{-\frac{\gamma_1\gamma_2+\gamma_1\alpha-2\gamma_2\alpha}{\gamma_2(\gamma_1-\alpha)}} \kappa^{\frac{\gamma_2-\alpha}{\gamma_2}}}{q},$$

Taking $\frac{1}{\gamma_2-\alpha}$ power of both sides and then multiplying both sides of the last inequality with $\left(\frac{\alpha\lambda_j^2 h(\upsilon_{2js})}{\mu(S_{2j})}\right)^{\frac{1}{\gamma_2-\alpha}}$ implies that $e_{2js\ell} > e_{2jsm}$.

c) Only those teachers with $\lambda_j > \widetilde{\lambda}_m$ live in the second district under both fixed and mixed compensation regimes since $\widetilde{\lambda}_d < \widetilde{\lambda}_m$ by Lemma 5. The equilibrium effort levels e_{2jsd} and e_{2jsm} are provided in the proofs of parts a and b above. Then $e_{2jsd} < e_{2jsm}$ if and only if $\lambda_j > \overline{\lambda}_{2dm}$. Therefore, those teachers with $\lambda_j > \max\{\widetilde{\lambda}_m, \overline{\lambda}_{2dm}\}$ both live in the second district under fixed and mixed compensation regimes and exert higher effort under mixed compensation regime.

□