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Title of Paper

General Equilibrium Pricing of Currency and Currency Options

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General Equilibrium Pricing of Currency and Currency Options[*]

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Abstract

This paper presents a consumption-based general equilibrium model for valuing foreign exchange contingent claims. The model identifies a novel economic mechanism by exploiting highly but imperfectly shared consumption disaster with variable intensities which are the concerns to the representative investor under recursive utility. When applied to the data, the model simultaneously replicates i) the moderate option-implied volatilities; ii) substantial variations in the risk-neutral skewness of currency returns; iii) the uncovered interest rate parity puzzle; and iv) the first two moments of carry trade returns. Furthermore, the model rationalizes salient features of the aggregate stock, government bonds, and equity index options.

1. Introduction

The foreign exchange market is the largest financial market in the world, yet the pricing of currencies and their derivatives poses challenges for modeling. In particular, several pricing regularities persist that defy easy explanations. First, the implied volatility of at-the-money (ATM) option contracts is fairly low, usually at around 8–12% (e.g., Carr and Wu, 2007). Second, the implied volatility of currency option quotes is, on average, a U-shaped function of moneyness, and the slope, which measures the risk-neutral skewness, exhibits substantial time variations suggesting that currency return skewness is strongly stochastic. Third, while the uncovered interest parity (UIP) implies that a regression of exchange rate changes on the interest rate differentials should produce a unit slope coefficient, empirical work since Fama (1984) consistently reveals negative slopes implying that high-interest currencies tend to appreciate. Fourth, the traditional carry trade strategy of buying high-interest currencies funded by selling low-interest currencies on average yields sizable, albeit significantly volatile, returns.

This paper proposes a consumption-based general equilibrium model for foreign exchange contingent claims. The central ingredients are the highly but imperfectly shared economic disasters with variable intensities, and the recursive preference (e.g., Epstein and Zin, 1989) that allows for a separation between elasticity of intertemporal substitution (EIS) and risk aversion. Following Barro (2006) and Wachter (2013), a disaster is modeled as a peso component in the consumption process. Under the recursive preference, investors care not only about the contemporaneous consumption shocks but also about the prospects of future consumption growth. When EIS is greater than one, they demand extra compensation for the increase of disaster rate out of fear of the substantial drop of consumption in the future. The results in this paper show that risks related to country-specific disaster components with variable intensities can simultaneously replicate many observed features in currency and currency option pricing in a quantitative manner.

The mechanism of the model is as follows. First, since disaster risks are highly shared across borders, the exchange rate needs to fluctuate less to prevent international arbitrage opportunities, which gives rise to the low ATM implied volatility. Second, with imperfect risk sharing, disaster rate at home can either rise above or fall below its foreign counterpart generating, respectively,

the negative and the positive skewness in currency returns. Stochastic skewness emerges when the two country-specific components evolve stochastically over time. Third, when disaster is more likely to strike at home than abroad such that the foreign currency pays the lower interest rate, the exchange rate is closely tied to the home-specific disaster intensity whose variations induce a negative correlation between the home pricing kernel and the exchange rate. Home investors thus demand a positive premium for holding the risky foreign currency, which drives up its valuation. Fourth, traditional carry trade strategy exploits the aforementioned risk compensation and at the same time is subject to the stochastic home- and foreign-specific disaster intensities, hence the sizable expected return coupled with the significant volatility.

In calibrating the model, I follow the literature (e.g., Backus, Foresi, and Telmer, 2001) by imposing complete symmetry in that all model parameters are identical across any two countries. Within a given country, I calibrate the variable disaster process according to i) the international evidence on its intensity and magnitude (e.g., Barro, 2006), ii) the match of some key moments for the aggregate stock. Across countries, I calibrate consumption correlation during normal times according to Brandt, Cochrane, and Santa-Clara (2006), and I choose a predominant global disaster component to match the observed high degree of risk sharing. The preference parameters are at levels deemed reasonable in the literature (e.g., Mehra and Prescott, 1985; Bansal and Yaron, 2004; Bansal, Gallant, and Tauchen, 2007). Finally, I estimate an inflation process to convert real variables into nominal ones.

To link the model to the data, I collect from Bloomberg option quotes written on three currency pairs that form a triangular relation: JPYUSD, GBPUSD, and GBPJPY. The option quotes are expressed as Garman and Kohlhagen (1983) implied volatilities at fixed times to maturity and fixed moneyness in terms of the Garman-Kohlhagen delta. In addition, I collect from Datastream the spot and one-month forward exchange rates of five major currencies, AUD, CAD, CHF, GBP, and JPY, quoted against the USD, which are used to study implications on carry trade.

The calibrated model delivers reasonable matches of the regularities in currency and currency option markets. First, it generates an average 9% volatility implied from one-month ATM options, as compared to the 10–11% volatilities implied from option quotes written on JPYUSD and GBPUSD. This result is in contrast to the traditional consumption-based model in which the implied volatility

is usually more than an order of magnitude higher than that in the data (e.g., Brandt, Cochrane, and Santa-Clara, 2006).

Second, the model generates substantial variations including frequent sign switches in the risk-neutral skewness of currency returns as measured by the slopes of the smile pattern for currency options. Quantitatively, the risk-neutral skewness is captured by risk reversal (RR)–the difference between the price of an out-of-the-money (OTM) call option and the price of an OTM put option with symmetric strikes. Taking the three-month ten-delta RR, for example, the model-implied RR standard deviation is 18.6%, as compared to its data counterparts of 19.0%, 13.4%, and 15.3% implied from JPYUSD, GBPUSD, and GBPJPY, respectively. Furthermore, I study the time variations of risk-neutral skewness based on i) RR series where the model implications are computed from the home- and the foreign-specific disaster rates backed out from the panel data of currency option quotes; ii) series of currency return skewness spanned from currency option prices in the spirit of Bakshi, Kapadia, and Madan (2003). The model again matches the data fairly well in both cases.

Third, the model replicates the UIP anomaly, i.e., high interest rate currencies tend to appreciate. The implied UIP slope coefficient is negative at -2 with a standard error of 1.2, which is consistent with their usually reported empirical values.

Fourth, the model matches the first two moments of carry trade returns. Under complete symmetry between the home and the foreign country, the expected carry trade returns are close to zero, and the implied volatility is 11.3%. By assuming a lower jump magnitude in the foreign country so that the foreign currency on average pays a higher interest than the home currency, the model is able to generate sizable expected returns of carry trade while keeping the implied volatility within 10.7–15.2%. Empirically, carry trades based on the fixed currency pairs yield low expected returns with volatilities ranging from 9% to 13%. I also consider dynamically rebalanced currency pairs based on their interest rate differentials, as embedded in the forward discount, whereby the lowest- (highest-) yielding currencies are selected to be sold (bought). This strategy is subject to around 11% volatility together with sizable expected returns that are replicated by the model as I vary the foreign disaster magnitudes from 20% to 15%.

In the model, the key to simultaneously matching the low level of ATM volatility and the

substantial cross-sectional variations for currency options lies in the highly but imperfectly shared economic disasters. To understand the source of the unshared disaster component to which exchange rates are subject, it is helpful to interpret country-specific disaster as a broader concept than "a disaster that strikes one country but not the other". For example, Table I in Barro (2006) reveals that the disaster associated with World War II struck major economies at different times. It is arguable, at least qualitatively, that such a disaster is not perfectly shared across borders. Since option prices are very sensitive to jumps, even a small fraction of the peso component that is unshared can produce substantial cross-sectional variations as observed in the data. Therefore, while it is difficult to distinguish econometrically the present model from that featuring 100% global disasters, the two setups have very different pricing implications.

The present paper shares with Carr and Wu (2007) and Bakshi, Carr, and Wu (2008; BCW) the objective of valuing currency options, but it differs in key aspects. First, both Carr and Wu (2007) and BCW impose constant interest rates, and thus are limited to addressing regularities in currency pricing.[1] In other words, they study currency option pricing while ignoring important pricing aspects of the underlying asset. Second, both Carr and Wu (2007) and BCW adopt a reduced-form setting in which the exchange rate process is subject to one Levy-type jump (e.g., Carr and Wu, 2004) exhibiting positive skewness, and another exhibiting negative skewness. As a result, their matches essentially reflect the successful projection of data features into the statistical characterization of the exchange rate dynamics. Given their work, the next step seems to explore a preference-based general equilibrium setup that maintains the internal consistency for pricing and in addition provides economic underpinnings to the observed currency and currency option pricing. This is precisely the goal of the present paper.

Linking to the literature on currency pricing, this paper also distinguishes itself along key dimensions. Different from Verdelhan (2010) and Bansal and Shaliastovich (2013) who, respectively, apply habit formation and long-run risk to explain the UIP anomaly, this paper exploits a variable disaster framework. Departing from Burnside, Eichenbaum, Kleshchelski, and Rebelo (2011) and Bakshi and Panayotov (2013), it provides a general equilibrium setup for studying carry trade.

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[1]Take the UIP anomaly, for example. Since the expected changes in exchange rate have no relation with the imposed constant interest rates, neither Carr and Wu (2007) nor BCW is able to study variations of exchange rates in response to interest rate differentials. In contrast, interest rate differentials and expected changes in exchange rates under my approach are intrinsically linked via the difference between the home- and the foreign-specific disaster rates.

Unlike Farhi and Gabaix (2011) who pursue the "production view" of exchange rates based on "expected resilience" incorporating both disaster intensity and magnitude, this paper emphasizes the imperfect sharing of disaster risks based on the no-arbitrage condition for exchange rate. Furthermore, whereas the results in Farhi, Fraiberger, Gabaix, Ranciere, and Verdelhan (2013) are derived from a constant disaster model, my objects of interest are the implications of the variability in disaster rates that are directly priced under the recursive utility.

The remainder of the paper is organized as follows. Sections 2 and 3 present the setup and summarize the theoretical implications, respectively. Section 4 describes the data and calibrates the model, and Section 5 provides the quantitative results. Finally, Section 6 concludes.

2. The setup

2.1. Imperfectly shared disasters

Following the peso problem literature (e.g., Barro, 2006), I assume that the aggregate home consumption evolves according to

$$\frac{dC_t}{C_t} = \mu dt + \sigma dB_{ct} + \left(e^Z - 1\right) dN_t, \tag{1}$$

where dB_{ct} is a standard Brownian; dN_t denotes the Poisson jumps with the arrival intensity $\lambda_t dt$, which models the rare economic disasters; $Z < 0$ denotes the log consumption jump size upon the occurrence of a disaster, which is assumed to be constant for simplicity without affecting the main economics. Following Wachter (2013), I introduce variability in disaster intensity λ_t as follows:

$$d\lambda_t = \kappa \left(\bar{\lambda} - \lambda_t\right) dt + \sigma_\lambda \sqrt{\lambda_t} dB_{\lambda t}, \tag{2}$$

where $dB_{\lambda t}$ is another standard Brownian independent of dB_{ct}. As plotted in Wachter (2013, Fig. 1), the square root term makes the stationary distribution of λ_t highly skewed. In particular, there are times when "rare" disasters can occur with high probability, but these times are themselves unlikely. On average, disasters strike at the intensity $\bar{\lambda}$.

Consistent with previous studies (e.g., Backus, Foresi, and Telmer, 2001; Verdelhan, 2011), I

assume parameters are identical beween any two countries, i.e., complete symmetry. The aggregate consumption and disaster intensity in the foreign country, super-indexed by "$*$", thus follow

$$\frac{dC_t^*}{C_t^*} = \mu dt + \sigma dB_{ct}^* + \left(e^Z - 1\right) dN_t^*, \tag{3}$$

$$d\lambda_t^* = \kappa\left(\bar{\lambda} - \lambda_t^*\right) dt + \sigma_\lambda \sqrt{\lambda_t^*} dB_{\lambda t}^*, \tag{4}$$

where N_t^* with intensity λ_t^* models the foreign economic disaster; B_{ct}^* and $B_{\lambda t}^*$ are mutually independent. Across borders, B_{ct}^* is correlated with B_{ct} by a constant ρ_C^{hf}; $dB_{\lambda t}$ and $dB_{\lambda t}^*$ are also correlated which will become clear very soon.

In each of the two countries, the Poisson process can be decomposed into the global component and the country-specific component. Mathematically,

$$\left\{\begin{array}{l} dN_t = dN_t^h + dN_t^g \\ dN_t^* = dN_t^f + dN_t^g \end{array}\right.,$$

where N_t^h and N_t^f model disasters specific to the home and the foreign country with intensities denoted by λ_t^h and λ_t^f, respectively; N_t^g models the global disasters shared across borders with the intensity λ_t^g; N_t^h, N_t^f, and N_t^g are mutually independent. I further specify that

$$d\lambda_t^h = \kappa\left(\bar{\lambda}^h - \lambda_t^h\right) dt + \sigma_\lambda \sqrt{\lambda_t^h} dB_{ht}, \tag{5}$$

$$d\lambda_t^f = \kappa\left(\bar{\lambda}^f - \lambda_t^f\right) dt + \sigma_\lambda \sqrt{\lambda_t^f} dB_{ft}, \tag{6}$$

$$d\lambda_t^g = \kappa\left(\bar{\lambda}^g - \lambda_t^g\right) dt + \sigma_\lambda \sqrt{\lambda_t^g} dB_{gt}, \tag{7}$$

where B_{ht}, B_{ft}, and B_{gt} are mutually independent. Under complete symmetry, $\bar{\lambda}^h = \bar{\lambda}^f$, but $\bar{\lambda}^g$ is allowed to be different. By combining (5)–(7), $\lambda_t \left(= \lambda_t^h + \lambda_t^g\right)$ and $\lambda_t^* \left(= \lambda_t^f + \lambda_t^g\right)$ indeed follow the processes of (2) and (4), respectively, if we define

$$dB_{\lambda t} \equiv \frac{1}{\sqrt{\lambda_t}} \left(\sqrt{\lambda_t^h} dB_{ht} + \sqrt{\lambda_t^g} dB_{gt}\right), \tag{8}$$

$$dB_{\lambda^* t} \equiv \frac{1}{\sqrt{\lambda_t^*}} \left(\sqrt{\lambda_t^f} dB_{ft} + \sqrt{\lambda_t^g} dB_{gt} \right). \tag{9}$$

Furthermore, (8)–(9) illustrate that $dB_{\lambda t}$ and $dB_{\lambda^* t}$ are correlated through dB_{gt}.

Decompositions of Poisson processes have been exploited previously (e.g., Duffie and Garleanu, 2001). Those studies tend to focus on processes with constant intensities. To the best of my knowledge, the above decomposition of Poisson process with stochastic intensity, also known as the Cox process, is new to the literature. It is built on the result, which I formally show in Appendix A.1, that the sum of two independent Cox processes is still a Cox process if their intensities are also independent of each other.

2.2. Recursive utility and pricing kernel

Assume the existence of a representative agent in the home country whose preference is described by the stochastic differential utility developed by Duffie and Epstein (1992), which is the continuous-time version of the recursive utility considered by Kreps and Porteus (1978) and Epstein and Zin (1989). Given the consumption process $\{C_s : s \geq 0\}$, the period-t utility of the agent, denoted by J_t, is defined recursively by

$$J_t = E_t \left[\int_t^\infty f(C_u, J_u) \, du \right], \tag{10}$$

where

$$f(C_t, J_t) = \frac{\beta}{1 - 1/\psi} \frac{C_t^{1-1/\psi} - [(1-\gamma) J_t]^{\frac{1-1/\psi}{1-\gamma}}}{[(1-\gamma) J_t]^{\frac{1-1/\psi}{1-\gamma} - 1}} \tag{11}$$

which is the normalized aggregator. In the above formula, β, γ, and ψ denote the subjective discount rate, the degree of risk aversion, and the elasticity of intertemporal substitution (EIS), respectively.

By homogeneity, J_t is separable in C_t and λ_t such that

$$J_t = \frac{C_t^{1-\gamma}}{1 - \gamma} [\beta I(\lambda_t)]^\theta, \tag{12}$$

where $\theta \equiv \frac{1-\gamma}{1-1/\psi}$. Applying Proposition 2 in Benzoni, Collin-Dufresne, and Goldstein (2011), $I(\lambda_t)$ in (12) denotes the wealth-consumption (W/C) ratio at the home country. By an accurate log-linear

approximation, $I(\lambda_t)$ can be written as

$$I(\lambda_t) = e^{a+b\lambda_t}, \tag{13}$$

where a and b are two constants. Alternatively, I express $I(\lambda_t)$ as a linear combination of general Chebyshev polynomials and solve the associated differential equation using the collocation method (e.g., Miranda and Fackler, 2002). The two approaches yield almost identical results with the average percentage deviations in the mean and standard deviation of the implied $I(.)$ being 0.19% and 0.53%, respectively. Furthermore, I find that the pricing implications based on these two approaches are also very similar. I therefore use (13) in the discussions that follow which greatly facilitates the exposition of model mechanism.

Under recursive preference, investors are concerned about the intertemporal consumption risk which is captured by $I(\lambda_t)$. In particular, $I(\lambda_t)$ loads negatively (positively) on λ_t if $\psi > 1$ $(\psi < 1)$. To understand it, notice that a positive λ_t shock implies a higher intensity of economic disasters which tends to substantially depress future consumption. On the one hand, the income effect makes the agent consume less today which raises the W/C ratio. On the other hand, the intertemporal substitution effect encourages the agent to borrow from the future which depresses the W/C ratio. The substitution effect dominates when $\psi > 1$. As a result, the W/C ratio is decreasing in λ_t which is captured by a negative b in (13). If in addition $\gamma > 1$ as in the usual calibration, the implied pricing kernel M_t loads positively on λ_t. Intuitively, the representative investor's marginal utility rises when she perceives a higher probability of disasters.

According to Duffie and Epstein (1992), the pricing kernel under the continuous-time recursive preference is given by

$$M_t = \exp\left(\int_0^t f_J ds\right) f_C,$$

where f_J and f_C denote the derivatives of the normalized aggregator with respect to its second and first argument, respectively. Evaluating derivatives in (11) and substituting for J_t using (12)–(13) gives

$$M_t = \exp\left(-\int_0^t \left[\beta\theta + \frac{1-\theta}{I(\lambda_s)}\right] ds\right) C_t^{-\gamma} I(\lambda_t)^{\theta-1}. \tag{14}$$

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Applying Ito's lemma with jumps (e.g., Duffie (2001, Appendix F)) to (14) gives

$$
\begin{aligned}
\frac{dM_t}{M_t} &= -r_t dt + (\theta - 1) b\sigma_\lambda \sqrt{\lambda_t^g} dB_{gt} + \left(e^{-\gamma Z} - 1\right)(dN_t^g - \lambda_t^g dt) \\
&\quad -\gamma\sigma dB_{ct} + (\theta - 1) b\sigma_\lambda \sqrt{\lambda_t^h} dB_{ht} + \left(e^{-\gamma Z} - 1\right)\left(dN_t^h - \lambda_t^h dt\right),
\end{aligned}
\tag{15}
$$

where r_t denotes the (real) risk-free rate at home with its formula given by (22). The pricing kernel process at the foreign country can be obtained via complete symmetry.

In (15), the first (excluding $-r_t dt$) and the second line capture the impacts of the global and the home-specific risk components, respectively. BCW also emphasize the differentiation between global and country-specific risks, which they find is the key to capturing the multidimensional structure of pricing kernels in international economies. Unlike this paper, they adopt the reduced-form approach by exogenously partitioning innovations of pricing kernels with fixed risk-free rates. In contrast, exchange rates and interest rates are jointly determined in the present setup (with details provided in Section 3.1), which maintains the internal consistency for currency pricing within a general equilibrium.

3. Currency and currency option pricing

This section presents the model's theoretical implications. After obtaining formulas of exchange rates and interest rates, I discuss the implications on currency option pricing and currency pricing, respectively.

3.1. Exchange rates and interest rates

No-arbitrage dictates that the ratio of the pricing kernels between two economies determines the exchange rate dynamics between them (e.g., Backus, Foresi, and Telmer, 2001; Brandt, Cochrane, and Santa-Clara, 2006; Bakshi, Carr, and Wu, 2008). Let S_t denote the time$-t$ home currency price of the foreign currency, and then

$$
S_t = M_t^*/M_t.
\tag{16}
$$

To introduce nominal variables, I take the home country as an example and assume the following process for the consumption price index (CPI) P_t:

$$\frac{dP_t}{P_t} = \pi_t dt + \sigma_P dB_{pt}, \tag{17}$$

where the expected inflation rate π_t is mean-reverting according to:

$$d\pi_t = \kappa_\pi \left(\bar{\pi} - \pi_t\right) dt + \sigma_\pi dB_{pt}. \tag{18}$$

The above specification implies that expected inflation and realized inflation are perfectly correlated which allows the former to be identified from inflation data alone (e.g., Wachter, 2006). By complete symmetry, the foreign CPI, P^*, follows the same dynamics as P, where B_{pt} is replaced with a new standard Brownian B_{pt}^*. I assume B_{pt}^* is independent of B_{pt} to avoid degeneration, and both B_{pt} and B_{pt}^* are independent of real shocks.

In its nominal (super-indexed by "\$") form, (16) becomes

$$S_t^{\$} = M_t^{\$,*}/M_t^{\$}, \tag{19}$$

where

$$M_t^{\$} = M_t/P_t; \quad M_t^{\$,*} = M_t^*/P_t^*. \tag{20}$$

By (15), (17)–(20), and the complete symmetry, $S_t^{\$}$ follows:

$$\frac{dS_t^{\$}}{S_t^{\$}} = \left[\begin{array}{c} \left(\lambda_t^h - \lambda_t^f\right) \left(\frac{\theta-1}{\theta} \left[e^{(1-\gamma)Z} - 1\right] + \frac{1}{2} \left(\theta - 1\right) \theta b^2 \sigma_\lambda^2\right) + \pi_t - \pi_t^* \\ + (\gamma\sigma)^2 \left(1 - \rho_C^{hf}\right) + \frac{1}{2} \left(\theta - 1\right)^2 b^2 \sigma_\lambda^2 \left(\lambda_t^h + \lambda_t^f\right) + \sigma_P^2 \end{array} \right] dt$$
$$+ \gamma\sigma \left(dB_{ct} - dB_{ct}^*\right) - \left(\theta - 1\right) b\sigma_\lambda \left(\sqrt{\lambda_t^h} dB_{ht} - \sqrt{\lambda_t^f} dB_{ft}\right)$$
$$+ \left(e^{\gamma Z} - 1\right) dN_t^h + \left(e^{-\gamma Z} - 1\right) dN_t^f + \sigma_P \left(dB_{pt} - dB_{pt}^*\right). \tag{21}$$

As expected, the dynamics of exchange rates are only driven by shocks that are not shared, which are captured by $\gamma\sigma \left(dB_{ct} - dB_{ct}^*\right)$ for consumption shocks, by $-\left(\theta - 1\right) b\sigma_\lambda(\sqrt{\lambda_t^h} dB_{ht} - \sqrt{\lambda_t^f} dB_{ft})$ for disaster intensity shocks, by $\left(e^{\gamma Z} - 1\right) dN_t^h$ and $\left(e^{-\gamma Z} - 1\right) dN_t^f$ for the arrival of disasters, and

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by $\sigma_P \left(dB_{pt} - dB_{pt}^* \right)$ for nominal shocks.

Closely related to the exchange rate dynamics is the determination of interest rates. Using (14) and the definition $r_t \equiv -E_t \left(\frac{dM_t}{M_t} \right)$ for the short-term real interest rate at home, it can be shown that:

$$r_t = \beta + \rho\mu - \frac{1}{2}\gamma \left(1 + \rho \right) \sigma^2 + \frac{1}{2} \left(\theta - 1 \right) b^2 \sigma_\lambda^2 \lambda_t - \lambda_t \left(e^{-\gamma Z} - 1 \right) - \frac{1 - \theta}{\theta} \lambda_t \left[e^{(1-\gamma)Z} - 1 \right]. \quad (22)$$

By (17), (20), and the definition $r_t^\$ = -E_t \left(\frac{dM_t^\$}{M_t^\$} \right)$, the nominal interest rate $r_t^\$ = r_t + \pi_t - \sigma_P^2$. The complete symmetry between the home and the foreign country thus implies:

$$r_t^\$ - r_t^{\$,*} = \left(\frac{1}{2} \left(\theta - 1 \right) b^2 \sigma_\lambda^2 - \left(e^{-\gamma Z} - 1 \right) - \frac{1 - \theta}{\theta} \left[e^{(1-\gamma)Z} - 1 \right] \right) \left(\lambda_t^h - \lambda_t^f \right) + \pi_t^* - \pi_t. \quad (23)$$

Under reasonable parameterization, $r_t^\$ - r_t^{\$,*}$ loads negatively on $\lambda_t^h - \lambda_t^f$. To see the intuition, note that the home agent saves more than the foreign agent out of the stronger motivations of precautionary saving when home-specific disaster rate λ_t^h is higher than its foreign counterpart. As a result, the real interest rate at home is depressed relative to its foreign counterpart. Taken together, when π_t and π_t^* are close as in the usual case for major economies, the lower λ_t^f is associated with the higher nominal rate paid by the foreign currency.

3.2. Option prices in relation to currency return dynamics

Options are written on nominal exchange rates, and their prices are denominated in the home currency. I first compute the spot characteristic function (e.g., Bakshi and Madan, 2000) of the currency return under the home currency risk-neutral measure Q^h,

$$\Psi \left(u; t, T \right) \equiv E_t^{Q^h} \left[\exp \left(-\int_t^T r_s^\$ ds \right) e^{iu \ln S_T^\$} \right], \quad (24)$$

where $i^2 = -1$; u is some complex number; and the dynamics of the log nominal exchange rate is

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given by (45). Within the present framework, I obtain the following closed-form:

$$\Psi(u; t, T) = \exp \left[\begin{array}{c} a(\tau) + b_h(\tau) \lambda_t^h + c_h(\tau) + b_f(\tau) \lambda_t^f + c_f(\tau) + b_g(\tau) \lambda_t^g \\ + c_g(\tau) + b_\pi(\tau) \pi_t + c_\pi(\tau) + b_{\pi^*}(\tau) \pi_t^* + c_{\pi^*}(\tau) \end{array} \right], \quad (25)$$

where $\tau \equiv T - t$; formulas for $a(\tau)$, $b(\tau)$, and $c(\tau)$ are provided in Appendix A.2. I then obtain option prices via fast Fourier inversion (Carr and Madan, 1999).

To examine the model implications on currency return dynamics and hence on the cross-sectional currency option pricing, I start with the special case where λs and πs are all constants. In this case, the variance (c_2), the third (c_3), and the fourth (c_4) cumulants for the currency return can be derived in closed-form as follows:[2]

$$\begin{aligned} c_2 &= (\gamma Z)^2 \left(\lambda^h + \lambda^f \right) + V_d, \\ c_3 &= (\gamma Z)^3 \left(\lambda^h - \lambda^f \right), \\ c_4 &= (\gamma Z)^4 \left(\lambda^h + \lambda^f \right), \end{aligned} \quad (26)$$

where $V_d \equiv (\theta - 1)^2 (b\sigma_\lambda)^2 \left(\lambda^h + \lambda^f \right) + 2 (\gamma \sigma)^2 (1 - \rho_C) + 2\sigma_P^2$ captures the variance contribution from the diffusion component. First, the diffusion component does not contribute to higher-order cumulants. Second, the currency returns show nonzero skewness or a nonzero third cumulant c_3 when $\lambda^h \neq \lambda^f$. Third, currency return kurtosis or the fourth cumulant c_4 is strictly positive as long as disasters are not prefectly shared across borders $(\lambda^h + \lambda^f > 0)$. In terms of magnitude, both skewness and kurtosis are higher when investors become more risk averse (higher γ) or when economic disaster becomes more severe (higher $|Z|$), which is intuitive.

Since all cumulants in (26) are constants, the model in this special case cannot capture the evidence from currency option markets that the currency return skewness is stochastic (e.g., Carr and Wu, 2007). In contrast, stochastic skewness arises naturally in the full model through the time variations in country-specific intensities λ_t^h and λ_t^f. First, currency options, like other derivatives, are sensitive to the jump component (e.g., Pan, 2002). Second, the pricing kernel loads positively

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[2] c_2, c_3, and c_4 are obtained by taking successive partial derivatives of the cumulant exponent (e.g., Wu, 2006) of the log (nominal) exchange rate which is defined by $k(u) \equiv \frac{1}{t} \ln E^{P^h} \left[e^{u \log(S_t^\$)} \right]$, where P^h is the actual measure at home; u is some complex number.

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on the disaster intensity under the usual calibration that γ and ψ are both greater than one (see discussion in Section 2.2). A positive shock on λ_t^h thus raises the home pricing kernel relative to its foreign counterpart, which, from (19), adds to the depreciation and hence the negative skewness of the foreign exchange. This result can also be seen from (26) with the note that Z is negative. The opposite logic holds for innovations of λ_t^f. Third, both λ_t^h and λ_t^f evolve stochastically and independently, hence the implied stochastic skewness.

In related research, BCW document several advantages of their setup relative to the previous literature of currency option pricing. The present paper differs from the work of BCW along several key dimensions. First, unlike BCW and as illustrated by (26), my approach links currency return dynamics, including the restrictions on the cross rate,[3] to both investors' preference (e.g., the risk aversion γ) and economic fundamentals (e.g., the log consumption jump size Z). Second, in contrast to BCW who impose constant interest rates, interest rates and exchange rates are jointly derived in my setup which facilitates the study of currency carry trade in relation to currency option pricing. Third, within a general equilibrium framework, the present model provides rich implications beyond currency and currency option pricing which are also consistent with the data.

3.3. The currency carry trade and the UIP anomaly

Closely related to exchange rate dynamics is the so-called currency carry trade. The typical carry trade strategy involves borrowing currencies with low interest rates and investing in currencies with high interest rates. A number of studies (e.g., Brunnermeier, Nagel, and Pedersen, 2009; Burnside, Eichenbaum, Kleshchelski, and Rebelo, 2011) show that carry trade is subject to crash risks. Indeed, a conventional saying among carry traders is that "exchange rates go up by the stairs and down by the elevator". It thus seems worthwhile to examine carry trade within a disaster framework.

To facilitate exposition, assume the home currency and the foreign currency offer the low and the high interest rate, respectively. Denote by r_t^e the (instantaneous) return of the following carry trade strategy: i) borrow one unit of the home currency; ii) convert it to the foreign currency and

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[3] For any three currency pairs that form a triangular relation (e.g., JPYUSD, AUDUSD, and AUDJPY), Appendix A.3 provides the mathematical results that restrictions on the cross rates (e.g., AUDJPY), which is essential to maintaining internal consistency among the three currency pairs, arises naturally from the present setup. Compared to BCW, my paper goes one step forward by attributing such restrictions to the economic primitives of the two countries involved (e.g., Japan and Australia).

lend it out at the foreign risk-free rate; iii) convert the earnings back to the home currency an instant later. Taken together,

$$r_t^e \equiv \left(r_t^{\$,*} - r_t^{\$}\right) + \frac{dS_t^{\$}/dt}{S_t^{\$}}, \tag{27}$$

where $r_t^{\$,*} - r_t^{\$} \, (> 0)$ accounts for the interest rate differential; $\frac{dS_t^{\$}/dt}{S_t^{\$}}$ accounts for the instantaneous return from holding the foreign currency.

To illustrate the model mechanism for carry trade, it is useful to replace $\frac{dS_t^{\$}/dt}{S_t^{\$}}$ in (27) with $d\ln\left(S_t^{\$}\right)/dt^4$ and define:

$$\hat{r}_t^e \equiv \left(r_t^{\$,*} - r_t^{\$}\right) + d\ln\left(S_t^{\$}\right)/dt. \tag{28}$$

In (28), taking conditional expectations on both sides and substituting (23) and (45) for $r_t^{\$,*} - r_t^{\$}$ and $E_t\left[d\ln\left(S_t^{\$}\right)/dt\right]$ yields:

$$E_t\left(\hat{r}_t^e\right) = \left[\frac{1}{2}\left(\theta - 1\right)^2 \left(b\sigma_\lambda\right)^2 + e^{-\gamma Z} - 1 + \gamma Z\right]\left(\lambda_t^h - \lambda_t^f\right). \tag{29}$$

$E_t\left(\hat{r}_t^e\right)$ is referred to as currency risk premium hereafter, and its expression provides a simple way to understand the profitability of carry trade. Since $e^x - 1 - x \geq 0$ for all x, $E_t\left(\hat{r}_t^e\right)$ loads positively on $\lambda_t^h - \lambda_t^f$ which is driven by the compensation to investors for bearing disaster intensity risks. To see the economics, assume $\lambda_t^h > \lambda_t^f$ so that the foreign currency is paying the higher interest rate due to less precautionary savings. First, a positive shock in λ_t^h raises the home pricing kernel which, according to (19), contributes to the depression of the foreign exchange rate. Second, since economic disasters are more probable at home than abroad, the effect of home-specific disaster intensity on the exchange rate determination dominates the effect of its foreign counterpart. Taken together, the home agent's marginal utility rises exactly when the foreign currency devalues conditional on the higher $r_t^{\$,*}$. As a result, she demands a positive compensation for holding the risky foreign currency.

The implied positive risk compensation hinges on $\lambda_t^h > \lambda_t^f$ which is reflected by the higher foreign interest rate. Since $\bar{\lambda}^h = \bar{\lambda}^f$ tends to hold empirically for a long enough time period, (29) yields zero currency risk premium, on average. By examining carry trade strategies based on six major

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[4]Theoretically, $E_t d\ln\left(S_t^{\$}\right)/dt$ differs from $\frac{dS_t^{\$}/dt}{S_t^{\$}}$ by a convex adjustment component. Empirically, $\ln S_{t+\Delta} - \ln S_t$ is close to $\frac{S_{t+\Delta}}{S_t} - 1$ when the time interval Δ is small.

currency pairs that are fixed, Bakshi and Panayotov (2013) find the expected returns are generally not statistically different from zero. Eq. (29) provides a simple explanation for their findings.

Closely related to its implication on the expected return of carry trade, the model also generates the UIP anomaly that currencies with higher than average interest rates tend to appreciate. To see the details, I follow the literature (e.g., Backus, Foresi, and Telmer, 2001) and examine the expected changes of the log exchange rate in response to the interest rate differential:

$$E_t \left(d \ln S_t^\$ \right) / dt = E_t \left(\hat{r}_t^e \right) + \left(r_t^\$ - r_t^{\$,*} \right), \tag{30}$$

where $\hat{r}_t^e$ is defined in (28). Previous analyses indicate that $E_t \left(\hat{r}_t^e \right)$ and $r_t^\$ - r_t^{\$,*}$ load in opposite signs on $\lambda_t^h - \lambda_t^f$. Ignoring nominal shocks, $E_t \left(d \ln S_t^\$ \right) / dt$ moves in the opposite direction to $r_t^\$ - r_t^{\$,*}$ when the impact of $E_t \left(\hat{r}_t^e \right)$ dominates, which is the case when the investors are not too risk averse. Consequently, the foreign currency which pays the higher interest (assuming $\lambda_t^h > \lambda_t^f$) tends to appreciate generating the UIP anomaly. While nominal shocks induce the comovement of $E_t \left(d \ln S_t^\$ \right) / dt$ and $r_t^\$ - r_t^{\$,*}$ by the purchasing power parity relationship, the impact is dominated in the model by that of $E_t \left(\hat{r}_t^e \right)$ driven by the disaster intensity shocks.[5]

4. Data and calibration

4.1. Currency and currency option data

I obtain over-the-counter quotes of currency options from Bloomberg for three currency pairs that form a triangular relation: JPYUSD (the dollar price of one yen), GBPUSD (the dollar price of one pound), and GBPJPY (the yen price of one pound), where the sample period is from October 1, 2003 to May 20, 2011. Options on each currency pair have seven fixed maturities at one, two, three, six, nine, 12, and 18 months, and I ignore the one-week quotes out of concern that short-dated contracts are more subject to liquidity or supply/demand premium (e.g., Pan and Singleton, 2008) not modeled in the present paper. At each maturity, quotes are available at five deltas in the form of delta-neutral straddle implied volatilities, 10- and 25-delta risk reversals (RRs), and 10-

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[5]This result is consistent with Hollifield and Yaron's (2003) finding that risks from the real side of the economy are vital to capturing the UIP anomaly.

and 25-delta butterfly spreads (BFs). The data are available at the daily frequency, and each time series contains 1,993 observations.

By industry convention, currency options are quoted in the form of Garman-Kohlhagen (1983) implied volatility (G/K-vol). The G/K deltas are computed by

$$\left\{ \begin{array}{l} \Delta^c = e^{-r^*\tau} N\left(d_1\right) \ \text{ for call options} \\ \Delta^p = -e^{-r^*\tau} N\left(-d_1\right) \ \text{ for put options} \end{array} \right., \tag{31}$$

where $N\left(.\right)$ denotes the cumulative normal function;

$$d_1 = \frac{\ln\left(e_t/K\right) + \left(r - r^*\right)\tau}{IV\sqrt{\tau}} + \frac{1}{2}IV\sqrt{\tau}, \tag{32}$$

where K, τ, and IV denote, respectively, the strike, the times to maturity, and the volatility input. G/K-vol are directly available for the delta-neutral straddle defined as the sum of a call option and a put option with the same strike satisfying $\Delta^c + \Delta^p = 0$. Since the delta-neutral restriction implies $d_1 = 0$, the implicit strike is very close to the spot price. For this reason, this quote is referred to as at-the-money implied volatility (ATMV) hereafter.

Besides ATMV, the Bloomberg data also provide quotes for RRs and BFs which record variations of G/K-vol across different option deltas. RR is defined as the volatility difference between an out-of-the-money (OTM) call option and an OTM put option with the same times to maturity, and it captures the skewness of the risk-neutral currency return distribution. Mathematically, RR at the given option delta, Δ, is computed by

$$RR\left[\Delta\right] \equiv IV^c\left[\Delta\right] - IV^p\left[\Delta\right]. \tag{33}$$

On the other hand, option traders use the BF, defined as the difference between the average OTM implied volatilities and ATMV, to quantity the kurtosis of risk-neutral currency return distribution. At the given option delta, Δ, the implied BF is computed by

$$BF\left[\Delta\right] \equiv \left(IV^c\left[\Delta\right] + IV^p\left[\Delta\right]\right)/2 - ATMV. \tag{34}$$

From the empirical quotes, we can derive the implied volatilities at the five levels of delta using (33)–(34) as follows: i) $IV^c[25] = BF[25] + ATMV + RR[25]/2$, ii) $IV^p[25] = BF[25] + ATMV - RR[25]/2$, iii) $IV^c[10] = BF[10] + ATMV + RR[10]/2$, and iv) $IV^p[10] = BF[25] + ATMV - RR[10]/2$. To back out the time-series variations of disaster rates implied from the model, we need to convert the implied volatilities into option prices and the deltas into strike prices using the domestic and foreign interest rates. The maturity-matched interest rates are stripped using LIBOR and swap rates from Bloomberg for all the three currencies involved, assuming piece-wise constant forward rates.

Quotes of exchange rates are from Datastream. To study carry trade, I collect data on spot and one-month forward contracts of five major currencies (excluding EUR due to its shorter history) which are quoted against the USD. The five currencies are: AUD, CAD, CHF, GBP, and JPY. One advantage of using forward contracts is that they facilitate the incorporation of transaction costs (e.g., Bessembinder, 1994). To study the impact of disaster state on currency return skewness and kurtosis, I also examine countries which have experienced financial/economic disasters since 1985. In particular, I choose Indonesia, Korea, Philippines, Mexico, Thailand, Malaysia, Argentina, and Russia, with their currencies being IDR, KRW, PHP, MXN, THB, MYR, ARS, and RUB, respectively, which are quoted against the USD. For all currency pairs involved, the sample period is from January 1985 to January 2012 at the monthly frequency.

4.2. Calibration

Table 1 reports the base case calibration of the model at the annual frequency. Panel A describes the preference parameters. Consistent with the macro finance literature, I set the subjective time discount rate β to 0.02. The degree of risk aversion γ is set at 6 which is roughly the midpoint of the range deemed reasonable (e.g., Mehra and Prescott, 1985). I use $\psi = 2$ which is close to the estimation by Attanasio and Weber (1989) and Bansal, Gallant, and Tauchen (2007).

In the model, μ and σ denote the average and the volatility of the aggregate consumption growth conditional on no disasters. I thus calibrate their annualized values to the quarterly real consumption divided by the total population data in the U.S. for the period between 1952Q2 and 2006Q4, during which no economic disasters are documented. The correlation between the home

and the foreign consumption growths, ρ_C^{hf}, is set to 0.3, which is roughly the midpoint of its empirical range from 0.17 to 0.14 documented by Brandt, Cochrane, and Santa-Clara (2006, Table 4). These numbers are reported in Panel B of Table 1.

Parameters related to disasters are presented in Panel C of the same table. I set the mean-reversion parameter κ and the volatility parameter σ_λ in the λ_t−process at 0.142 and 0.09, respectively.[6] Using the postwar data in G7 countries, Barro (2006) reports an average disaster probability of 0.017 per annum, to which the long-run disaster intensity $\bar{\lambda}$ is calibrated. From the U.S. experience, the largest annual consumption drop which occurred in 1932 is around 10%. Using the S&P 500 Index option data for 1988–2008, Du (2011) estimates a 15.8% consumption jump that is factored into option pricing. Consumption jump sizes tend to be much higher from international experiences.[7] In view of this evidence, I choose, on balance, a consumption jump size of 20%, i.e., $e^Z - 1 = -0.2$, which is still conservative compared to that used by Barro (2006). In an unreported exercise, I have verified that moments (mean, volatility, skewness, kurtosis) of model-implied consumption growth under the above calibration are broadly consistent with the empirical evidence.[8]

The last disaster parameter, $\bar{\lambda}^g/\bar{\lambda}$, is calibrated to the empirical levels of the risk-sharing index (RSI) developed by Brandt, Cochrane, and Santa-Clara (2006). Specifically, I set $\bar{\lambda}^g/\bar{\lambda} = 0.98$ at which the model-implied RSI (0.96) matches its empirical counterpart (0.95–0.98). Overall, the 98% risk-sharing implies that economic disasters are very likely to simultaneously strike different countries, which is consistent with the historical evidence of the Great Depression. Given the increasing economic ties among countries, this implication seems more valid nowadays than ever

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[6]Since λ_t is the only state in the one-country version of the model, the autocorrelation of the price-dividend ratio for the aggregate equity (discussed in Appendix B) is approxmiately that of λ_t. I thus calibrate κ so that the implied first-order autocorrelation of the price-dividend ratio equals its data value of 0.87 (e.g., Lettau and Wachter, 2011). Next, I choose σ_λ so that the model generates reasonable return volatility for the aggregate equity. I have conducted an extensive comparative analysis and found that the model implications on currency and currency option pricing are largely robust to the choices of κ and σ_λ.

[7]For example, Barro (2006) reports that both Germany and Japan experienced declines of more than 50% in real GDP per capita over the two-year period towards the end of World War II. Using a novel data set, Barro and Ursua (2008) show that the results on consumption are similar. Heston and Summers (1991) report that several countries have experienced a one-year decline in real GDP or consumption of more than 20% since 1950, which include Algeria, Angola, Chad, Iran, Iraq, Namibia, Nicaragua, Niger, Nigeria, Sierra Leone, and Uganda.

[8]Since my paper studies an endowment economy under which consumption equals output (e.g., Lucas, 1978; Mehra and Prescott, 1985; Campbell and Cochrane, 1999; Barro, 2006), I calculate the empirical moments based on the GDP data for G7 countries during 1890–2008. Another reason for this choice is that both the availability and quality of international consumption data are limited. For example, the available non-US consumption data typically do not distinguish between expenditures on non-durables plus service and expenditures on durables.

before. To understand the source of the unshared components, it is helpful to interpret country-specific disaster as a broader concept than "a disaster that strikes one country but not the other". For example, Table I in Barro (2006) reveals that the disaster associated with World War II struck major economies at different times. It is thus arguable, at least qualitatively, that this disaster is not perfectly shared across borders. My work demonstrates that the imperfect risk-sharing is vital to understanding currency and currency option pricing.

Finally, Panel D of Table 1 reports parameterization on the nominal side which is estimated using the Kalman filter based on US inflation data from 1952Q1 to 2008Q4. These values are close to those used by Lettau and Wachter (2011) and Bansal and Shaliastovich (2013). To summarize, none of the model parameters is directly calibrated to the interest rate, exchange rate, or the currency option data. As a result, quantitative implications on currency and currency option pricing to be presented in next section are largely out of the sample, and the matches to be reported in the following are mainly driven by the model structure and the underlying economics.

5. Quantitative implications

Sections 5.1 and 5.2 discuss quantitative implications on currency option pricing and currency pricing, respectively. Since derivative valuations are insensitive to variations in expected inflation, I fix π and π^* at their long-run averages when computing the model-implied currency option prices.

5.1. Currency option pricing

I start with ATMV which determines the currency option prices. I then discuss how the model matches key features of the cross-sectional currency option data.

5.1.1. ATM implied volatility

Columns 2–5 in Table 2 report the average ATMV at four maturities: one/three/six/twelve months, implied from both the model (Panel A) and the data (Panel B), where the model values are computed as averages over the simulated stationary distributions of $\left(\lambda_t^h, \lambda_t^f, \lambda_t^g\right)$. Taking the one-month ATMV, for example, the model average is 9%, as compared to 11.0% for JPYUSD, 10.0% for GBPUSD, and 13.7% for GBPJPY. Across maturities, the ATMVs exhibit a flat term

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structure in both the model and the data. The low ATMVs are attributed to the smooth exchange rate volatility which on average equals 9.5% in the model, as compared to 10.6%, 9.11%, and 13.2% in the data implied from the above three currency pairs.

In terms of the magnitude, previous general equilibrium models tend to generete a highly volatile exchange rate process. Indeed, Brandt, Cochrane, and Santa-Clara (2006) report that the exchange rate volatility implied from a traditional consumption-based asset pricing model is at least 50% per year. The smooth exchange rate process and hence the low ATMV generated in the present setup are thus desirable, which lays the foundation for exploring the cross-sectional variations in currency option prices.

To understand the model mechanism for the low volatility, I calculate from (45) the instantaneous variance of the changes in the log nominal exchange rate as follows:

$$Var_t\left(d\ln\left(S_t^\$\right)\right)/dt = 2\left(\gamma\sigma\right)^2\left(1 - \rho_C^{hf}\right) + \left[(\theta - 1)^2 (b\sigma_\lambda)^2 + (\gamma Z)^2\right]\left(\lambda_t^h + \lambda_t^f\right) + 2\sigma_P^2. \quad (35)$$

Eq. (35) consists of three terms attributed to the imperfectly shared consumption shocks, the imperfectly shared economic disasters, and nominal shocks. In both the model and the data, volatilities of real and nominal exchange rates are very close suggesting that impacts from nominal shocks are small. Due to compensations to investors for bearing the risks of variable disaster rates, the present setup can generate the observed high equity premium under low risk aversion. This result, combined with the low σ, implies the low volatility attributed to consumption diffusive shocks. If economic disasters are highly shared across borders, i.e., $\lambda_t^h + \lambda_t^f$ is close to zero, the second term's contribution would also be small, hence the low exchange rate volatility.

5.1.2. Cross-sectional option pricing: unconditional values

Starting from ATMVs, option-implied volatilities vary as we change moneyness, or equivalently, option deltas. In particular, variations in price differences between OTM puts and OTM calls are captured by changes in RR defined by (33), which measures the skewness of the risk-neutral exchange rate distribution. Specifically, the positive (negative) RR captures the positive (negative) risk-neutral skewness.

Panel A of Table 3 reports the model- and data-implied standard deviation for the 10- and 25-delta risk reversals, denoted by RR10 and RR25, respectively, over various times to maturity, where the RRs are normalized as percentages of the maturity-matched ATMV. To obtain the model values, I first search for the strike, using (31)–(32) and conditional on the given states $\left(\lambda_t^h, \lambda_t^f, \lambda_t^g\right)$, to match the option deltas, where the home and foreign interest rates are at their equilibrium levels. I then compute option prices at the backed-out strikes from which I obtain RR10 and RR25. Finally, I compute the standard deviations of the RRs over the stationary distribution of $\left(\lambda_t^h, \lambda_t^f, \lambda_t^g\right)$. To present the matches of RR fluctuations from another perspective, Panel B of the same table reports the ranges of RR variations implied from both the model and the data, where the model values are again based on the λ-stationary distribution. Due to space limitation, I report RR ranges only for JPYUSD and GBPUSD.

The model does a good job in capturing the substantial variations of risk-neutral skewness observed in the data. To illustrate it, I use the three-month RRs for example. The model-implied standard deviations of RR10 and RR25 are 18.6% and 8.43%, respectively, as compared to 19.0% and 9.82% from JPYUSD, 13.4% and 7.40% from GBPUSD, and 15.3% and 8.49% from GBPJPY. Across maturities, the model predicts a rising term structure for the RR standard deviations which is also consistent with the data. Turning to the RR ranges, the model implications are again close/ comparable to the data in terms of the sizes of the ranges measured as the maximum RRs minus the minimum RRs. For example, the sizes of the model-implied RR ranges are 91.1% and 43.0% for the three-month RR10 and RR25, respectively, as compared to their data counterparts of 118% and 59.9% from JPYUSD, and 47.2% and 32.6% from GBPUSD. Furthermore, by breaking the perfect symmetry in calibration, I find the model is also able to replicate the average RRs in the data[9] while leaving the matches of RR standard deviation largely unaffected (see footnote 14 for the details).

Why is the present setup able to generate substantial variations in currency return skewness as captured by RR? First, investors are concerned about the prospects of future consumption growth

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[9]Under the base case calibration assuming complete symmetry, the model-implied RR variations are largely symmetric. On the other hand, the data exhibit asymmetry towards the negative RRs which is attributed to the recent financial crisis originated from the US generating, from the lens of the present model, the imbalances between the home- and the foreign-specific disaster rates. Indeed, data in the earlier period from January 1996 to January 2004, as studied by Carr and Wu (2007), exhibit more symmetry. For example, Carr and Wu (2007) report that the average three-month RR10 and RR25 implied from GBPUSD are -0.37% and -0.10%, respectively, during their sample period.

under the recursive utility. Variable disaster intensities are therefore directly priced through which they have impacts on exchange rate dynamics. Second, under the imperfectly shared disaster risks, the home-specific disaster rate can either rise above or fall below its foreign counterpart generating, respectively, the negative and the positive skewness in exchange rates. Third, since the two country-specific components evolve independently, their relative impacts on skewness change over time leading to the substantial RR variations.

Fig. 1 illustrates the above mechanism. Fixing the global intensity λ_t^g at its long-run average $\bar{\lambda}^g$, I plot the implied three-month 10-delta RR as a function of the home- $\left(\lambda_t^h\right)$ and the foreign-specific $\left(\lambda_t^f\right)$ disaster intensities, where λ_t^h and λ_t^f are both quoted as multiples of $\bar{\lambda}^h \left(= \bar{\lambda}^f\right)$ and vary between zero and five. RR is virtually zero when there are no jumps in the exchange rate process at $\lambda_t^h = \lambda_t^f = 0$ (the rightmost point). Starting from $\lambda_t^h = \lambda_t^f = 0$, RR drops monotonically with the increase in λ_t^h, since higher λ_t^h adds to the negative skewness of currency returns by raising the likelihood of potential downward jumps. Conversely, RR rises monotonically with the increase of λ_t^f. By comparison, RR changes little along the line $\lambda_t^h = \lambda_t^f$ and is still close to zero at $\lambda_t^h = \lambda_t^f = 5\bar{\lambda}^h$ (the leftmost point). This exercise shows that λ_t^h and λ_t^f both have substantial and nearly symmetric impacts on the implied RR.

While RR captures cross-sectional differences between OTM puts and OTM calls, BF defined in (34) captures differences between OTM and ATM options. Table 4 reports the model- and data-implied standard deviations and averages for the 10- and 25-delta BFs, denoted by BF10 and BF25, respectively, over various maturities, where the BFs are also normalized as percentages of the maturity-matched ATMV. Overall, model values are close/comparable to their data counterparts. Taking the six-month BF10 for example, the model-implied standard deviation (Panel A) and average (Panel B) are 5.34% and 9.39%, as compared to 4.45% and 14.3% from JPYUSD, 4.04% and 10.4% from GBPUSD, and 3.70% and 12.6% from GBPJPY. Simultaneously, the model generates a rising term structure for both the standard deviations and the averages of BFs which is also consistent with the data.

To corroborate the above matches, I pursue an alternative method that allows the risk-neutral volatility, skewness, and kurtosis of asset returns to be spanned by option prices (e.g., Bakshi, Kapadia, and Madan, 2003; Jiang and Tian, 2005; Bakshi and Madan, 2006). In the same spirit as

that described by Bakshi, Kapadia, and Madan (2003), I first express the risk-neutral skewness of the $\tau-$period currency returns $\ln S_\tau^\$$ as functions of the fair valuations of volatility contract, cubic contract, and quartic contract, denoted respectively by U, V, and W. I then use prices of $\tau-$period option contracts across a collection of strikes to span valuations of the three contracts.

Currency option prices in the data are deduced from quotes of ATMV, RR, and BF which are directly available at only five strikes (deltas). To minimize the discretization error, I apply the curve-fitting method (e.g., Jiang and Tian, 2005) to obtain option prices at any strike within the empirical ranges of strikes.[10] The unconditional skewness (sk) and kurtosis (kt) thus spanned are computed based on the time series from Octobor 2003 to May 2011. Correspondingly in the model, I obtain the unconditional sk and kt based on the stationary distribution of the states. I find the model implications match their data counterparts reasonably well. Take the standard deviation of sk and kt, for example. Their model values are 0.316 and 0.322, respectively, compared with 0.299 and 0.284 for JPYUSD, 0.221 and 0.260 for GBPUSD, and 0.341 and 0.422 for GBPJPY. To summarize, the model is able to explain the data in terms of the unconditional mean and standard deviation of currency return skewness and kurtosis under the risk-neutral measure.

5.1.3. Cross-sectional option pricing: time variations

Turning to time variations of the implied cross-sectional option pricing, I use currency option prices to back out the time series of λ_t^h and λ_t^f with λ^g fixed at its long-run average.[11] More specifically, let N_t be the number of option prices on period t, and $O_n^{(d)}(t, \tau_n, K_n)$ and $O_n^{(m)}(t, \tau_n, K_n, \hat{\theta}; \lambda_t^h, \lambda_t^f, \bar{\lambda}^g)$ be, respectively, the observed and the model price of the nth option $(n = 1, 2, ...N_t)$, where τ_n and K_n denote the time to maturity and strike price; $\hat{\theta}$ denote the calibrated parameters reported in Table 1. The strikes K_n, common to both data and model prices, are determined by matches to option deltas through the Garman-Kohlhagen formula, where the interest rates and the implied volatilities used are from data quotes. Given the currency pair, period t's

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[10]While option prices outside of the empirical ranges can also be deduced by extrapolation, I only use interpolation for two reasons. First, the implied strikes cover a wide range. Taking the three-month contracts written on GBPUSD, for example, the implied strikes quoted as the ratio of the underlying exchange rate range from 0.83 to 1.17. Second, I use similar ranges of strikes when computing the model implications, and the implied matches are robust to marginal variations of the strike ranges.

[11]Alternatively, I back out $\left\{ \hat{\lambda}_t^h, \hat{\lambda}_t^f, \hat{\lambda}_t^g \right\}$ simultaneously from the option data, and the implied time-series matches are largely unaffected.

disaster rates are identified by minimizing the sum of the squared pricing errors as follows:

$$\left(\hat{\lambda}_t^h, \hat{\lambda}_t^f\right) = \underset{\lambda_t^h, \lambda_t^f}{\arg\min} \sum_{n=1}^{N_t} \left[O_n^{(d)}(t, \tau_n, M_n) - O_n^{(m)}(t, \tau_n, M_n, \hat{\Theta}; \lambda_t^h, \lambda_t^f, \bar{\lambda}^g)\right]^2, \qquad (36)$$

where $\hat{\lambda}_t^h$ and $\hat{\lambda}_t^f$ are restricted to be non-negative. To avoid putting too much weight on high price quotes, I scale all option prices by their Garman-Kohlhagen vega. Although data are available daily, I back out the weekly series of $\left\{\hat{\lambda}_t^h, \hat{\lambda}_t^f\right\}$ which illustrates the point. In particular, Eq. (36) is repeated for JPYUSD and GBPUSD every Wednesday between October 1, 2003 and May 20, 2011. Due to the missing data, series for GBPJPY start from May 17, 2006 instead.

Using the backed-out $\{\hat{\lambda}_t^h, \hat{\lambda}_t^f\}$ as inputs, I compute the model-implied RR series which are then compared to the data. Take RR10 at the three-month horizon, for example. The model series correlate their data counterparts by 95.6%, 96.4%, and 95.6% for JPYUSD, GBPUSD, and GBPJPY, respectively, and the results are plotted in the left three panels of Fig. 2. I furthermore pursue the method of option spanning described in Section 5.1.2 and compute the time variations of risk-neutral currency return skewness, where the model and the data values are computed at the same discretization of strikes for the given trading day. The right three panels of Fig. 2 plot the results at the three-month horizon which again exhibit reasonable matches: the correlations between the model- and the data-implied sk series are 91.6% for JPYUSD, 93.5% for GBPUSD, and 93.7% for GBPJPY.

In the present setup, time variations of RR are mainly driven by fluctuations of country-specific disaster rates that investors with intertemporal preferences are concerned about. Indeed, by setting $\lambda_t^h = \lambda_t^f = 0$, the left panels of Fig. 3 show that the implied RR series fail miserably at capturing the observed stochastic skewness. The above mechanism advocates that it is the concern about the (unshared) disaster risks, or "crash-o-phobia", that matters the most to the observed pricing. The same economics also explain the regularities of carry trade returns (Section 5.2.1), the UIP anomaly (Section 5.2.2), the time variations of volatility smirk in equity index option pricing (Appendix B.2), and is consistent with the findings by Bollerslev and Todorov (2011) that investors' fear of jump tail risks plays a crucial role for explaining risk premia embedded in aggregate equity.

Linking to the literature on currency option pricing, the regularity of stochastic skewness is

first addressed by Carr and Wu (2007) using a class of reduced-form models built on the general framework of time-changed Levy processes developed in their earlier work. Unlike the present setup in which stochastic skewness arises endogenously from imperfectly shared disaster risks, Carr and Wu (2007) start with an exogenous exchange rate process subject to two Levy-jump components that exhibit positive (upward jumps) and negative (downward jumps) skewness. Stochastic skewness emerges when the two jump components, and hence their relative weights, vary stochastically over time. Despite success, Carr and Wu point out in their conclusion: "for future research, it is important to understand the economic underpinnings of the stochastic skewness suggested by currency option prices". Partly motivated by their work, this paper pursues a preference-based general equilibrium setup which enhances the understanding of stochastic skewness based on investors' intertemporal preference and the impefectly shared disaster risks.

Moving to the fourth moments, I show previously in Section 5.1.2 that the present model is able to match the unconditional mean and standard deviation of BF and the risk-neutral currency return kurtosis (kt). Perhaps not surprisingly, matching time variations of BF and kt turns out to be more challenging. To accomodate matches along this dimension without affecting the main economics, I generalize the country-specific disaster rate process by allowing the average disaster rates to be stochastic in a persistent way.[12] I find the extended model is able to deliver reasonable matches of BF and sk time variations. Take BF10 at the three-month horizon, for example. The correlations between the model and the data series are 93.9%, 94.7%, and 90.7% for JPYUSD, GBPUSD, and GBPJPY, respectively, and the results are plotted in the right three panels of Fig. 3. Simultaneously, I have verified that the extended model maintains time-series matches of lower moments as well as the matches of unconditional values that are implied from the base model before the extension.

---

[12]Specifically, for country j $(j = h, f)$, the generalized country-specific disaster rate follows: $d\lambda_t^j = \kappa \left(\bar{\lambda}^j \eta_t^j - \lambda_t^j \right) dt + \sigma_\lambda \sqrt{\lambda_t^j} dB_{jt}$. In this specification, η_t^j is referred to as the stochastic central tendency factor for country j which follows $d\eta_t^j = \kappa_\eta \left(1 - \eta_t^j \right) dt + \sigma_\eta \sqrt{\eta_t^j} dB_{\eta t}^j$, where $B_{\eta t}^h$ and $B_{\eta t}^f$ are mutually independent which are also independent of the other Brownians. I maintain the original λ^g−process with the verification that adding stochastic central tendency to λ_t^g has little impact on the implied matches. In calibration, I set $\kappa_\eta = \frac{1}{4}\kappa$ and $\sigma_\eta = \frac{1}{4}$ so that i) central tendency factors are more persistent than disaster rates; ii) central tendency factors are subject to significant amounts of time variations. Fixing λ_t^g at its long-run average, I back out from option prices the time series of $\left\{ \hat{\lambda}_t^h, \hat{\lambda}_t^f, \hat{\eta}_t^h, \hat{\eta}_t^f \right\}$ which I then use to compute BFs implied from the extended model.

5.2. Currency pricing

5.2.1. The first two moments of carry trade returns

Pursuing the strategy described by (27), I first present formulas of instantaneous expectation and variance of model-implied carry trade returns. Next, I discuss the empirical implementation of (27). Finally, I report quantitative implications both under complete symmetry and an asymmetric generalization that allows the foreign disaster magnitude to be different from its domestic counterpart.

In (27), taking expectation on both sides and substituting (23) and (21) for $r_t^{\$,*} - r_t^{\$}$ and $E_t\left(\frac{dS_t^{\$}/dt}{S_t^{\$}}\right)$, I obtain the instantaneous expected return of carry trade as follows:

$$
\begin{aligned}
E_t\left(r_t^e\right) &= \left[\frac{1}{2}(\theta-1)^2(b\sigma_\lambda)^2 + e^{-\gamma Z} - 1\right]\left(\lambda_t^h - \lambda_t^f\right) \\
&\quad + (\gamma\sigma)^2\left(1 - \rho_C^{hf}\right) + \frac{1}{2}(\theta-1)^2 b^2\sigma_\lambda^2\left(\lambda_t^h + \lambda_t^f\right) \\
&\quad + \left(e^{\gamma Z} - 1 - \gamma Z\right)\lambda_t^h + \left(e^{-\gamma Z} - 1 + \gamma Z\right)\lambda_t^f + \sigma_P^2.
\end{aligned}
\tag{37}
$$

To compute the second moment, note that both $r_t^{\$,*}$ and $r_t^{\$}$ are short-term interest rates which can be treated as constants over an instant of time. The instantaneous variance of carry trade is thus completely determined by that of $\frac{dS_t^{\$}/dt}{S_t^{\$}}$ which equals

$$
\begin{aligned}
Var_t\left(r_t^e\right) &= 2(\gamma\sigma)^2(1 - \rho_C) + \left[(\theta-1)^2(b\sigma_\lambda)^2 + \left(e^{\gamma Z} - 1\right)^2\right]\lambda_t^h \\
&\quad + \left[(\theta-1)^2(b\sigma_\lambda)^2 + \left(e^{-\gamma Z} - 1\right)^2\right]\lambda_t^f + 2\sigma_P^2,
\end{aligned}
\tag{38}
$$

where I've used the independences between dB_{ht} and dB_{ft} and that between dN_t^h and dN_t^f.

Eqs. (37)–(38) formalize that carry trade is subject to crash risks. The same result holds for the aggregate stock, as indicated by (61)–(62) which give the model-implied equity premium and the stock return volatility. Empirically, Burnside (2012) concludes that "a unifying explanation of stock market and carry trade returns based on observed fluctuations in measures of risk remains elusive". He conjectures that an alternative explanation should pursue investors' concerns about out-of-sample events, which is formalized in this paper.

To implement (27) empirically, consider the following strategy: i) borrow one unit of the home

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currency; ii) convert it into the foreign currency and lend it out at the foreign risk-free rate; iii) convert the earnings back to the home currency one period later. The implied payoff is given by:

$$\left(1 + r_t^{\$,*}\right) \frac{S_{t+1}^{\$}}{S_t^{\$}} - \left(1 + r_t^{\$}\right), \tag{39}$$

which is the empirical counterpart of (27). Based on the covered interest rate parity,

$$\frac{1 + r_t^{\$}}{1 + r_t^{\$,*}} = \frac{F_t^{\$}}{S_t^{\$}}, \tag{40}$$

where $F_t^{\$}$ denotes the (nominal) forward exchange rate. Using (40), Eq. (39) can be equivalently rewritten as:

$$\left(1 + r_t^{\$}\right) \left(\frac{S_{t+1}^{\$}}{F_t^{\$}} - 1\right). \tag{41}$$

Empirically, I focus on the one-month carry trade returns which are computed using the practical version of (41) that accounts for the transaction costs.[13]

I compute the unconditional mean and volatility of model-implied carry trade returns by exploiting (37)–(38) over the stationary distributions of $\left(\lambda_t^h, \lambda_t^f\right)$. From discussions in Section 3.3, $E_t\left(r_t^e\right) = E_t\left(\hat{r}_t^e\right)$ +convex adjustment, where $E_t\left(\hat{r}_t^e\right)$ denotes the currency risk premium whose (unconditional) average is zero under complete symmetry. Accounting for the convex adjustment component, the model generates an average carry trade return of 0.485%. From (38), variations of carry trade returns arise from i) the unshared home- and foreign-specific disaster rates which evolve stochastically and independently; ii) the unshared consumption diffusions and nominal shocks. On average, the model produces a return volatility of 11.3%.

Empirically, I perform computations with respect to fixed country pairs for which the average levels of λ_t^h and λ_t^f tend to converge. Specifically, I choose five currency pairs, AUDUSD, USDJPY, AUDCHF, GBPJPY, and GBPCHF, that are usually used for carry trade. The data are monthly from January 1985 to January 2012. I find that the implied average returns are 3.42%, -0.493%,

---

[13]Taking into account the transaction costs, (41) becomes $\left(1 + r_t^{\$}\right) \left(\frac{S_{t+1}^{\$,b}}{F_t^{\$,a}} - 1\right)$, where the superscripts "b" and "a" denote the bid rate and the ask rate, respectively. Interest rate quotes are computed as averages of the bid rates and ask rates. In terms of magnitude, accounting for bid/ask differentials for interest rate has a very small impact on the implied carry trade returns.

0.432%, 0.840%, and -0.765%, while the implied volatilities are 11.7%, 11.5%, 12.9%, 12.7%, and 8.77%. To summarize, my model under complete symmetry generates the key features of empirical carry trade returns based on fixed currency pairs: low expected returns and volatilities ranging from 9% to 13%.

As reported by Bakshi and Panayotov (2013), carry trade is more profitable when implemented on a dynamic rebalancing basis, under which investors always short the low-interest currency and long the high-interest currency. Correspondingly, on the model side, I need to break the complete symmetry so as to produce positive expected returns for carry trade. One simple way to do that is to maintain the base case jump size (20%) for the home country, but assume a lower jump magnitude for the foreign country. In this case (with all other parameter values remaining at their base levels), the foreign country pays the higher interest rate due to less precautionary saving, and the strategy of shorting the home currency and longing the foreign currency yields positive returns, on average.

To illustrate the above mechanism analytically, again I resort to the currency risk premium and expand Eq. (29) into

$$
E_t^{asy}(\hat{r}_t^e) = \left[\frac{1}{2}(\theta-1)^2(b\sigma_\lambda)^2 + e^{-\gamma Z} - 1 + \gamma Z\right]\lambda_t - \left[\frac{1}{2}(\theta-1)^2(b^*\sigma_\lambda)^2 + e^{-\gamma Z^*} - 1 + \gamma Z^*\right]\lambda_t^*,
$$
(42)

where b^* is the foreign counterpart of b controlling the sensitivity of the log W/C ratio to the disaster intensity at the home country (see Eq. (13)). Although $E(\lambda_t) = E(\lambda_t^*) = \bar{\lambda}$, the average $E_t^{asy}(\hat{r}_t^e)$ is positive for two reasons. First, $|b| > |b^*|$ when $|Z| > |Z^*|$. Intuitively, at the lower disaster magnitudes, W/C ratio reacts less to the changes of their arrival probabilities. Second, $f(x) = e^x - 1 - x$ is increasing in x so that $e^{-\gamma Z} - 1 + \gamma Z > e^{-\gamma Z^*} - 1 + \gamma Z^*$. The convex adjustment component further raises the average carry trade return relative to the average currency risk premium.

Quantitatively, I combine data on spot and forward contracts of five major currencies (excluding the euro for its shorter history) under independent floating schemes, which are quoted against the USD. The currencies involved are: AUD, CAD, CHF, GBP, and JPY. I then design two strategies that short and long the five currencies which are rebalanced at the end of each month between January 1985 and January 2011. Currencies are included according to their interest rate differentials

inferred from the forward discount. The first strategy buys the highest-yielding currency and sells the lowest-yielding counterpart, while the second strategy buys the second highest-yielding currency and sells the second lowest-yielding counterpart. Based on these two strategies, the implied average carry returns are 4.64% and 4.43% while the implied return volatilities are 11.5% and 10.7%.

On the model side, I choose various foreign disaster magnitudes $\left|e^{Z^*} - 1\right|$ ranging from 15% and 19.5%, which are reported in column 1 of Table 5. Correspondingly, columns 2–3 of the same table report the unconditional mean and volatility of carry trade returns. Consistent with intuition, the implied average return increases as $\left|e^{Z^*} - 1\right|$ decreases signaling the greater degree of asymmetry. In terms of levels, the average carry trade returns range from 0.816% to 5.77%. Simultaneoulsy, the model generates volatilities ranging from 10.7% to 15.2%. In conclusion, by introducing asymmetry in disaster magnitude, the present model is able to replicate the first two moments of empirical carry trade returns based on dynamically rebalanced currency pairs.[14]

5.2.2. The UIP anomaly

I next turn to the UIP anomaly. If the currency risk premium $E_t\left(\hat{r}_t^e\right)$ is a constant, Eq. (30) implies a one-to-one relation between $r_t^\$ - r_t^{\$,*}$ and $E_t\left(d\ln S_t^\$\right)/dt$ which is sometimes referred to as the uncovered interest parity (UIP). If UIP holds, the slope coefficient in the regression

$$\ln S_{t+1}^\$ - \ln S_t^\$ = a_1 + a_2\left(r_t^\$ - r_t^{\$,*}\right) + \text{residual}$$

should be close to one. Since Fama (1984), however, people have found that a_2, referred to as the UIP coefficient, is consistently less than one and usually negative: high interest-paying currencies tend to appreciate instead of depreciating, which indicates that $E_t\left(\hat{r}_t^e\right)$ is time-varying.

In my setup, the time-varying $E_t\left(\hat{r}_t^e\right)$ arises from the unshared disaster risks as discussed in

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[14]The introduction of asymmetry in disaster magnitude leaves the model matches of currency option pricing largely intact. Take the three-month 10-delta RR, for example. The model-implied RR standard deviation is 11.2% at $\left|e^{Z^*} - 1\right| = 15\%$ which is still comparable to its data counterpart (e.g., 13.4% from GBPUSD). The lower standard deviation relative to that under complete symmetry (18.6%) is attributed to the higher ATMV against which RR is normalized. Indeed, the implied RR standard deviation at $\left|e^{Z^*} - 1\right| = 15\%$ would be 19.7% if we still use the base case level of ATMV for normalization. Furthermore, allowing for smaller foreign disaster magnitudes facilitates the matches of average skewness observed during my sample period. Taking implications at the three-month horizon, for example, the average RR10 and RR25 at $\left|e^{Z^*} - 1\right| = 15\%$ are -30.4% and -15.2%, respectively, as compared to -33.2% and -17.8% from JPYUSD, -9.75% and -5.36% from GBPUSD, and -35.1% and -17.6% from GBPJPY.

Section 3.3. To explore the quantitative implications, I run the UIP regressions at the monthly frequency based on the simulated data of nominal exchange rates and the one-month nominal bond yields, where I have used the closed-form bond pricing formula given by (64). As reported in Panel A of Table 6, the model-implied UIP coefficient is -2.0 with the standard deviation of 1.2. Both numbers are in line with their empirical values documented in previous studies (e.g., Bansal and Shaliastovich, 2009; Verdelhan, 2010). The basic intuition is that the home investor requires a positive premium for holding the foreign currency when disaster is more likely at home than abroad. When the risk aversion γ is not too low (at a level greater than 2.2 conditional on the base case calibration of the other parameters), the required premium will be able to offset the effect of interest rate differentials and in addition drives up the appreciation of the foreign currency.

Building on the work of Campbell and Cochrane (1999) and case II of Bansal and Yaron (2004), respectively, Verdelhan (2010) and Bansal and Shaliastovich (2013) show that habit formation and long-run risk models are also able to resolve the forward premium anomaly. Time-varying currency risk premiums arise from variable risk aversions induced by habit formation in Verdelhan (2010), and from the short-term fluctuations of consumption volatility in Bansal and Shaliastovich (2013). In this paper, I resort to the disaster story instead because I also aim at pricing currency derivatives: as implied from studies by Benzoni, Collin-Dufresne, and Goldstein (2011) and Du (2011), neither habit formation nor long-run risk prices derivative assets well if the underlying asset returns are not subject to jumps. More evidence on the importance of a jump component to currency option pricing is provided in Appendix A.4.

5.2.3. Various other regularities on currency pricing

Panel B of Table 6 reports the data-implied currency return skewness and kurtosis under the physical measure which are based on countries that have experienced financial/economic disasters from January 1985 to January 2012. Specifically, I choose Indonesia, Korea, Philippines, Mexico, Thailand, Malaysia, Argentina, and Russia, with their currencies being IDR, KRW, PHP, MXN, THB, MYR, ARS, and RUB, respectively, which are quoted against the USD. To accomodate the negative skewness as suggested by the data, I use the similar asymmetry as discussed in Section 5.2.1. At the foreign disaster magnitude of 15%, the model-implied currency return skewness and

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kurtosis are -1.01 and 8.20, respectively. These numbers are close/comparable to their empirical counterparts as reported in Panel B of Table 6. For example, THB in my sample period has skewness and kurtosis of -1.08 and 10.7, respectively. Furthermore, the model-implied currency return volatility equals 15.6% under the same asymmetry, which is comparable/close to 18.9%, 15.3%, 12.4%, 19.0%, 13.5%, 9.39%, 17.1%, and 23.0% implied from IDR, KRW, PHP, MXN, THB, MYR, ARS, and RUB, respectively.

Panel C of Table 6 reports some additional regularities related to currency pricing, where the data values are average implications based on multiple currency pairs considered in this paper. Wherever RRs are involved, I report the numbers for the six-month 10-delta contracts, for example. The model produces a stark contrast between the low persistence of exchange rates and the high persistence of interest rates, a positive correlation between RRs and (nominal) interest rate differentials, as well as a positive correlation between changes in RR and the contemporaneous (nominal) currency returns.[15] All these implications are consistent with the data.

6. Conclusions

This paper presents a consumption-based general equilibrium model featuring i) variable economic disasters that are imperfectly shared across borders, and ii) recursive utility under which disaster risks are directly priced. The model proves useful in capturing i) the moderate exchange rate volatility and hence the moderate option-implied volatilities when disaster risks are highly shared; ii) the stochastic skewness when the unshared disaster components evolve stochastically and independently across time; iii) the sizable expected return of carry trade as compensation for bearing disaster risks; and iv) the UIP anomaly when risk aversion is not too low at which the demanded risk premium offsets the interest rate differential and in addition drives up the valuation of the risky foreign currency.

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[15] The positive correlation between changes in RR and the contemporaneous (nominal) currency returns can be seen analytically from the lens of my setup. From previous analysis with respect to Fig. 1, RR can be proxied by $\lambda_t^f - \lambda_t^h$ whose (instantaneous) change at period t equals $d\left(\lambda_t^f - \lambda_t^h\right) = -\kappa\left(\lambda_t^f - \lambda_t^h\right)dt + \sigma_\lambda\left(\sqrt{\lambda_t^f}dB_{ft} - \sqrt{\lambda_t^h}dB_{ht}\right)$, where I have used (5)–(6). Comparing this equation with (21), the positive correlation follows from $\theta < 0$ and $b < 0$ under the usual calibration that γ and ψ are both greater than one. Intuitively, the decrease of λ_t^f and the increase of λ_t^h both add to the potential downward jumps of the foreign currency exchange rate, which leads to its contemporaneous devaluation.

Linking back to the literature, previous studies on foreign exchange contingent claims tend to adopt the reduced-form setting and examine pricing regularities separately. A match between data and reduced-form models unfortunately only reflects the successful projection of data features onto the statistical characterization of exchange rates or pricing kernels, and examining regularities in isolation leaves unanswered an important question on the connections among different pricing phenomena. A general equilibrium study is thus desirable which provides an economic story for various regularities of currency and currency option pricing that are determined in a uniform and internally consistent manner.

The present setup also has rich implications beyond the valuations of foreign exchange contingent claims. It replicates the aggregate stock market behaviors such as the high equity premium and return volatility; it matches the first two moments of short-term real bond yields; it also captures the salient pricing feature of volatility smirk for equity index options in terms of both levels and time-series variations. The details are provided in Appendix B. Taken together, the present model has the potential to unify pricing regularities in various financial markets with a coherent disaster story.

Appendix A. Proofs, formulas, and further discussions

A.1. Sum of two independent Cox processes

This subsection shows that the sum of two independent Cox processes is still a Cox process if their intensities are independent of each other. Given a Cox process N_t with the stochastic intensity λ_t, define the $\sigma-$field

$$\Im_t = \sigma\left\{\lambda_s : 0 \leq s \leq t\right\},$$

which is equivalent to knowing the evolution of λ up to period t. The moment generating function (mgf) for N_t can be computed by

$$\psi_N(m) \equiv Ee^{mN_t} = E\left[E\left(e^{mN_t}|\Im_t\right)\right] = E\left[\sum_{n=0}^{\infty} e^{mn}\frac{\left(\int_0^t \lambda_u du\right)^n}{n!}e^{-\int_0^t \lambda_u du}\right],$$

where the third equality follows that a Cox process is an inhomogeneous Poisson process conditional on a particular realization of its stochastic intensity. Evaluating the above formula yields

$$\psi_N(m) = E\left\{\exp\left[\left(\int_0^t \lambda_u du\right)(e^m - 1)\right]\right\}. \tag{43}$$

Next consider two independent Cox processes N_t^h and N_t^g whose arrival intensities are λ_t^h and λ_t^g, respectively. The mgf for $N_t^g + N_t^h$ is thus

$$
\begin{aligned}
\psi_{N^g + N^h}(m) &\equiv Ee^{m\left(N_t^h + N_t^g\right)} = \left(Ee^{mN_t^h}\right)\left(Ee^{mN_t^g}\right) \\
&= E\left\{\exp\left[\left(\int_0^t \lambda_u^h du\right)(e^m - 1)\right]\right\} \cdot E\left\{\exp\left[\left(\int_0^t \lambda_u^g du\right)(e^m - 1)\right]\right\},
\end{aligned}
$$

where I have used the independence and (43) for the second and the third equality, respectively. In the present model, λ_t^h and λ_t^g as characterized by (5) and (7) are also independent, which is due to the independences between dB_{ht} and dB_{gt}. Therefore,

$$
\begin{aligned}
\psi_{N^g + N^h}(m) &= E\left\{\exp\left[\left(\int_0^t \lambda_u^h du\right)(e^m - 1)\right] \cdot \exp\left[\left(\int_0^t \lambda_u^g du\right)(e^m - 1)\right]\right\} \\
&= E\left\{\exp\left[\left(\int_0^t \left[\lambda_u^h + \lambda_u^g\right] du\right)(e^m - 1)\right]\right\}.
\end{aligned}
\tag{44}
$$

It is well known that knowing the mgf is equivalent to knowing the distribution itself. By comparing (44) with (43), I conclude that $N_t^g + N_t^h$ is also a Cox process with intensity equaling $\lambda_t^h + \lambda_t^g$.

A.2. Closed-form valuation of $\Psi(.)$

I first obtain the dynamics of log nominal exchange rate which is used to obtain the closed-form spot characteristic function $\Psi(.)$ defined by (24). Applying Ito's lemma with jumps to (21) yields:

$$
\begin{aligned}
d\ln\left(S_t^\$\right) &= \left[\left(\lambda_t^h - \lambda_t^f\right)\left(\frac{\theta - 1}{\theta}\left[e^{(1-\gamma)Z} - 1\right] + \frac{1}{2}(\theta - 1)\theta b^2 \sigma_\lambda^2\right) + \pi_t - \pi_t^*\right] dt \\
&\quad + \gamma\sigma\left(dB_{ct} - dB_{ct}^*\right) - (\theta - 1)b\sigma_\lambda\left(\sqrt{\lambda_t^h}dB_{ht} - \sqrt{\lambda_t^f}dB_{ft}\right) \\
&\quad + \gamma Z\left(dN_t^h - dN_t^f\right) + \sigma_P\left(dB_{pt} - dB_{pt}^*\right).
\end{aligned}
\tag{45}
$$

The closed-form valuation of $\Psi(.)$ is given by (25), where

$$
a(\tau) = -\beta - \rho\mu + \frac{1}{2}\gamma(1+\rho)\sigma^2 + \sigma_P^2 - \left[(\gamma\sigma)^2(1-\rho_C) + \sigma_P^2\right](iu + u^2);
\tag{46}
$$

$$
b_c(\tau) = \frac{2\psi_c(1 - e^{-\eta_c\tau})}{2\eta_c - (\eta_c - \xi_c)(1 - e^{-\eta_c\tau})}
\tag{47}
$$

$$
c_c(\tau) = -\frac{\kappa\bar{\lambda}^c}{\sigma_\lambda^2}\left[(\eta_c - \xi_c)\tau + 2\ln\left(1 - \frac{\eta_c - \xi_c}{2\eta_c}(1 - e^{-\eta_c\tau})\right)\right]
\tag{48}
$$

with $\eta_c = \sqrt{\xi_c^2 + 2\omega_c^2\psi_c}$ for $c = h, f, g$;

$$
b_d(\tau) = \frac{\psi_c}{\kappa_\pi}\left(1 - e^{-\kappa_\pi\tau}\right)
\tag{49}
$$

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$$c_d(\tau) = \left[\frac{\xi_c \psi_c}{\kappa_\pi} + \frac{1}{2}\sigma_\pi^2 \left(\frac{\psi_c}{\kappa_\pi}\right)^2\right]\tau + \frac{\psi_c}{\kappa_\pi^2}\left(\xi_c + \sigma_\pi^2 \frac{\psi_c}{\kappa_\pi}\right)\left(e^{-\kappa_\pi \tau} - 1\right) - \frac{\sigma_\pi^2}{4\kappa_\pi}\left(\frac{\psi_c}{\kappa_\pi}\right)^2 \left(e^{-2\kappa_\pi \tau} - 1\right) \quad (50)$$

for $d = \pi, \pi^*$. In (47)–(48),

$$\psi_h = \left[\begin{array}{c} -(1-iu)\Theta + \frac{1}{2}(1-iu)^2(\theta-1)^2(b\sigma_\lambda)^2 \\ +e^{-(1-iu)\gamma Z} - 1 \end{array}\right], \quad \xi_h = \kappa - (1-iu)(\theta-1)b\sigma_\lambda^2,$$

$$\psi_f = -iu\Theta - \frac{1}{2}u^2(\theta-1)^2(b\sigma_\lambda)^2 + e^{-iu\gamma Z} - 1, \qquad \xi_f = \kappa - iu(\theta-1)b\sigma_\lambda^2,$$

$$\psi_g = -\Theta + \frac{1}{2}(\theta-1)^2(b\sigma_\lambda)^2 + e^{-\gamma Z} - 1, \qquad \xi_g = \kappa - (\theta-1)b\sigma_\lambda^2,$$

where

$$\Theta \equiv \frac{1}{2}\theta(\theta-1)(b\sigma_\lambda)^2 + \frac{\theta-1}{\theta}\left[e^{(1-\gamma)Z} - 1\right].$$

In (49)–(50),

$$\psi_\pi = iu - 1, \quad \xi_\pi = \kappa_\pi \bar{\pi} + (-1+iu)\sigma_P \sigma_\pi$$

$$\psi_{\pi^*} = -iu, \qquad \xi_{\pi^*} = \kappa_\pi \bar{\pi} - iu\sigma_P \sigma_\pi.$$

A.3. Cross-currency restrictions

In the literature, many papers on currency pricing often start by exogenously specifying the exchange rate dynamics for an underlying currency pair. These papers essentially treat the specified dynamics on a standalone basis, and thus have ignored restrictions on the cross exchange rate. Below, I use JPYUSD, AUDUSD, and AUDJPY which form a triangular relation as an example to show that the cross rate restrictions are automatically satisfied in my setup. The analysis is based on (45) which gives the exchange rate dynamics for a generic currency pair.

Applying (45) to the case where the US and Japan are treated as the home and the foreign country, respectively, we have

$$d\ln\left(S_t^{\$,JPYUSD}\right) = \left[(\lambda_t^{US} - \lambda_t^{JP})\Theta + \pi_t^{US} - \pi_t^{JP}\right]dt + \gamma\sigma\left(dB_{ct}^{US} - dB_{ct}^{JP}\right) \quad (51)$$

$$- (\theta-1)b\sigma_\lambda\left(\sqrt{\lambda_t^{US}}dB_t^{US} - \sqrt{\lambda_t^{JP}}dB_t^{JP}\right) + \gamma Z\left(dN_t^{US} - dN_t^{JP}\right) + \sigma_P\left(dB_{pt}^{US} - dB_{pt}^{JP}\right),$$

where $\Theta \equiv \frac{\theta-1}{\theta}\left[e^{(1-\gamma)Z} - 1\right] + \frac{1}{2}(\theta-1)\theta b^2\sigma_\lambda^2$; $S^{\$}$ denotes the nominal exchange rate. If instead the US and Australia are treated as the home and the foreign country, respectively, we have

$$d\ln\left(S_t^{\$,AUDUSD}\right) = \left[(\lambda_t^{US} - \lambda_t^{AU})\Theta + \pi_t^{US} - \pi_t^{AU}\right]dt + \gamma\sigma\left(dB_{ct}^{US} - dB_{ct}^{AU}\right) \quad (52)$$

$$- (\theta-1)b\sigma_\lambda\left(\sqrt{\lambda_t^{US}}dB_t^{US} - \sqrt{\lambda_t^{AU}}dB_t^{AU}\right) + \gamma Z\left(dN_t^{US} - dN_t^{AU}\right) + \sigma_P\left(dB_{pt}^{US} - dB_{pt}^{AU}\right).$$

The cross rate AUDJPY has to satisfy the triangular arbitrage relation which implies:

$$d\ln\left(S_t^{\$,AUDJPY}\right) = d\ln\left(S_t^{\$,JPYUSD}\right) - d\ln\left(S_t^{\$,AUDUSD}\right). \tag{53}$$

Substituting (51)–(52) into (53) gives

$$d\ln\left(S_t^{\$,AUDJPY}\right) = \left[(\lambda_t^{AU} - \lambda_t^{JP})\Theta + \pi_t^{AU} - \pi_t^{JP}\right]dt + \gamma\sigma\left(dB_{ct}^{AU} - dB_{ct}^{JP}\right) \tag{54}$$

$$- (\theta - 1)b\sigma_\lambda\left(\sqrt{\lambda_t^{AU}}dB_t^{AU} - \sqrt{\lambda_t^{JP}}dB_t^{JP}\right) + \gamma Z\left(dN_t^{AU} - dN_t^{JP}\right) + \sigma_P\left(dB_{pt}^{AU} - dB_{pt}^{JP}\right).$$

It is easy to see that (54) is consistent with (45) when Australia and Japan are treated as the home and the foreign country, respectively. In other words, my setup guarantees, for any three countries forming a triangular relation, that the cross rate is redundant and completely determined by the other two primary currency pairs.

In the literature, BCW are among the earliest to address the issue of triangular arbitrage relation. They start with the exogenous pricing kernel processes, from which exchange rates are determined as the ratio of the pricing kernels between any two economies. This way, for any three currency pairs that form a triangular relation, the cross rate (e.g., AUDJPY) arises naturally as the ratios of the other two exchange rates (e.g., JPYUSD and AUDUSD). The same logic applies in my setup if we treat the implied pricing kernels as exogenously given. Due to the general equilibrium that endogenizes the implied pricing kernels, my approach goes one step forward relative to BCW by attributing exchange rate dynamics, including the cross rate restriction, to investors' preference and economic fundamentals.

A.4. Can stochastic volatility price currency and its derivatives well?

To further illustrate the importance of a disaster component for pricing currency options, I investigate in this subsection an alternative model that pursues the stochasticity of the (diffusive) volatility of consumption growth. Using the home country as an example, aggregate consumption now follows:

$$\frac{dC_t}{C_t} = (\mu + x_t)\,dt + \sqrt{\Omega_t}dB_{ct},$$

where

$$dx_t = -\kappa_x x_t dt + \sigma_x dB_{xt};$$

$$d\Omega_t = \kappa_\Omega\left(\bar{\Omega} - \Omega_t\right)dt + \sigma_\Omega\sqrt{\Omega_t}dB_{\Omega t};$$

B_{ct}, $B_{\Omega t}$, and B_{xt} are mutually independent. Under the recursive preference of (10)–(11), x_t and Ω_t, which capture, respectively, the long-run risk and the stochastic volatility, are both directly priced. As a result, the home pricing kernel M_t takes the form

$$M_t = \exp\left(-\int_0^t \left[\beta\theta + \frac{1-\theta}{I(\Omega_s, x_s)}\right]ds\right)C_t^{-\gamma}I(\Omega_t, x_t)^{\theta-1}. \tag{55}$$

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Similar models have been studied by Bansal and Yaron (2004), and Bansal and Shaliastovich (2013), but neither touches on the subject of currency option pricing or carry trade.

Using the log-linear approximation

$$I\left(\Omega_t, x_t\right) = e^{a + b\Omega_t + cx_t},$$

it follows that M_t

$$\frac{dM_t}{M_t} = -r_t dt - \gamma \sqrt{\Omega_t} dB_{ct} + \left(\theta - 1\right) b\sigma_\Omega \sqrt{\Omega_t} dB_{\Omega t} + \left(\theta - 1\right) c\sigma_x dB_{xt},$$

where r_t denotes the short-term interest rate at home. On the nominal side, the consumption price index P_t still follows (17)–(18). To facilitate comparison, again I assume complete symmetry between the home and the foreign country. Furthermore, I follow Bansal and Shaliastovich (2013) by assuming that i) the long-run growth prospects across two countries are identical in that x_t is perfectly shared; and ii) there are short-run differences captured by the independent evolutions of consumption volatility between countries. Again, B_{ct} and B_{ct}^* are correlated by ρ_C^{hf}.

By Ito's lemma with jumps, the implied nominal exchange rate follows:

$$\frac{dS_t^\$}{S_t^\$} = \left[\begin{array}{c} \left(\Omega_t - \Omega_t^*\right)\left(\frac{1}{2}\left(\theta - 1\right)\theta\left(b\sigma_\Omega\right)^2 + \frac{1}{2}\gamma\left(\gamma - 1 - \rho\right)\right) + \pi_t - \pi_t^* \\ +\frac{1}{2}\gamma^2\left(\Omega_t + \Omega_t^* - 2\rho_C^{hf}\sqrt{\Omega_t \Omega_t^*}\right) + \frac{1}{2}\left(\theta - 1\right)^2\left(b\sigma_\Omega\right)^2\left(\Omega_t + \Omega_t^*\right) + \sigma_P^2 \end{array} \right] dt$$
$$+\gamma\left(\sqrt{\Omega_t} dB_{ct} - \sqrt{\Omega_t^*} dB_{ct}^*\right) - \left(\theta - 1\right) b\sigma_\Omega\left(\sqrt{\Omega_t} dB_{\Omega t} - \sqrt{\Omega_t^*} dB_{\Omega t}^*\right) + \sigma_P\left(dB_{pt} - dB_{pt}^*\right), \quad (56)$$

where innovations due to imperfectly shared risks are driven by $\gamma\left(\sqrt{\Omega_t} dB_{ct} - \sqrt{\Omega_t^*} dB_{ct}^*\right)$ for consumption shocks, by $-\left(\theta - 1\right) b\sigma_\Omega\left(\sqrt{\Omega_t} dB_{\Omega t} - \sqrt{\Omega_t^*} dB_{\Omega t}^*\right)$ for stochastic volatility, and by $\sigma_P\left(dB_{pt} - dB_{pt}^*\right)$ for nominal shocks. The log nominal exchange rate follows a similar process except that the convex adjustment component in the second line of (56) is missing. Next, I compute the implied currency option prices using the same procedure as that for the variable disaster model which exploits the Fourier inversion with respect to the spot characteristic function. Finally, I derive formulas of the return moments associated with the carry trade strategy of (27). Specifically, the instantaneous expectation and variance of carry trade returns under the stochastic volatility model are given by:

$$E_t\left(r_t^e\right) = \left(\Omega_t - \Omega_t^*\right)\left(\frac{1}{2}\left(\theta - 1\right)^2\left(b\sigma_\Omega\right)^2 + \frac{1}{2}\gamma^2\right) \quad (57)$$
$$+\frac{1}{2}\gamma^2\left(\Omega_t + \Omega_t^* - 2\rho_C^{hf}\sqrt{\Omega_t \Omega_t^*}\right) + \frac{1}{2}\left(\theta - 1\right)^2\left(b\sigma_\Omega\right)^2\left(\Omega_t + \Omega_t^*\right) + \sigma_P^2.$$

$$Var_t\left(r_t^e\right) = \gamma^2\left(\Omega_t + \Omega_t^* - 2\rho_C^{hf}\sqrt{\Omega_t \Omega_t^*}\right) + \left(\theta - 1\right)^2\left(b\sigma_\Omega\right)^2\left(\Omega_t + \Omega_t^*\right) + 2\sigma_P^2. \quad (58)$$

I keep the model calibration as close to those of Bansal and Yaron (2004) and Bansal and Shaliastovich (2013) as possible and summarize it in Panels A–D of Table A.1. Panel E presents

model implications on currency option pricing, where I use the six-month contracts as an example. The implied standard deviations of RR and BF are no more than 0.01%, far below their empirical levels. This is the case even though the diffusive risks as captured by Ω_t and Ω_t^* are not shared at all. On the other hand, the lack of sharing in consumption volatility risks implies an unconditional carry trade volatility of 44.1%, as reported in Panel F of Table A.1, which appears too high compared with its data values (9–13%). These results provide direct evidence that stochastic volatility alone cannot replicate key features in currency and currency option pricing.

Appendix B. Pricing of other assets

B.1. Aggregate equity and government bonds

Restricting our analysis to the home country, the aggregate dividend follows:

$$\frac{dD_t}{D_t} = \mu_D dt + \sigma_D \left(\rho_D dB_{ct} + \sqrt{1 - \rho_D^2} dB_{dt} \right) + \left(e^{Z_D} - 1 \right) dN_t, \tag{59}$$

where B_{dt} is a standard Brownian independent of consumption diffusion (driven by B_{ct}). Consistent with the US experience during the Great Depression, dividend and consumption jump simultaneously upon the economic disaster. To obtain quantitative implications, I set $(\mu_D, \sigma_D, \rho_D, Z_D) = (3\%, 9\%, 0.1, -0.4)$. In an unreported exercise, I have verified that (59) under this calibration is able to explain the US dividend growth dynamics in terms of the first four moments (mean, volatility, skewness, kurtosis) and the correlation with the consumption growth.

Denote by P_t^S the period-t price of the aggregate equity as the claim to all future dividends. In the spirit of the derivation of the W/C ratio, I express the stock price-dividend ratio by the log-linear approximation as follows:

$$I^S \left(\lambda_t \right) \equiv \frac{P_t^S}{D_t} = e^{a_D + b_D \lambda_t}, \tag{60}$$

where a_D and b_D are constants. Using (60), the equity premium and the stock return volatility are

$$EP_t = \gamma \rho_D \sigma \sigma_D - (\theta - 1) b b_D \sigma_\lambda^2 \lambda_t + \lambda_t \left(1 - e^{-\gamma Z} \right) \left(e^{Z_D} - 1 \right), \tag{61}$$

$$volR_t = \sqrt{\sigma_D^2 + b_D^2 \sigma_\lambda^2 \lambda_t + \lambda_t \left(e^{Z_D} - 1 \right)^2}, \tag{62}$$

respectively. In (61), the first and third terms are the usual compensations for bearing risks of consumption diffusions and consumption jumps; the second term is unique to variable disaster models which compensates the home investor for bearing the risks of shocks to the disaster intensity. Similar interpretations apply to (62). By combining (61)–(62) with (37)–(38), my setup provides a unified framework for carry trade and the pricing of aggregate stock.

By its definition, the price of a nominal (zero-coupon) government bond is given by

$$B_{t,T} = E_t^{Q^h} \left[\exp \left(- \int_t^T r_s^\$ ds \right) \right], \tag{63}$$

which is a special case of (24) with u set to zero. $B_{t,T}$ in (63) can be derived in the following closed-form:

$$B_{t,T} = e^{a_B(\tau) + b_B(\tau)\lambda_t + c_B(\tau)\pi_t}, \tag{64}$$

where $\tau = T - t$; $a_B(\tau)$, $b_B(\tau.)$, and $c_B(\tau)$ satisfy a series of ordinary differential equations (ODEs). The price of a real government bond can be calculated in a similar way.

Panels A–B of Table A.2 present the key moments characterizing behaviors of the aggregate equity and the three-month real government bonds. Both the model implications and their usual empirical value ranges are reported. The model-implied average price-dividend ratio, average equity premium, and volatility of the aggregate equity returns are 26, 6.0%, and 16.0%, respectively, which are all in line with their usual data estimates. These results essentially replicate those of Wachter (2013) who is the first to show that variability in disaster intensity combined with recursive preference provides another way to understand behaviors of the aggregate stock. Turning to government bonds, the implied mean and the standard deviation of bond yields are low and the first autocorrelation of yields is high in both the model and the data.

Model implications on bond pricing are further examined in Fig. A.1. As plotted in the top left panel and consistent with empirical findings (e.g., Ang, Bekaert, and Wei, 2008; Verdelhan, 2010), the model generates a fairly flat and slightly downward-sloping yield curve for real bonds. Turning to nominal yields, the top right panel plots the data- and model-implied weekly time series of the three-month bond yields from January 1996 to May 2011. The data values are proxied by LIBOR rates for the USD. I keep the expected inflation π_t fixed when generating the model values. Driven only by disaster rate shocks $\left\{ \hat{\lambda}_t \right\}$ backed out from the panel data of equity index options (see descriptions in Appendix B.2), the model-implied series captures 32% of the variations in the data. For example, the estimated $\hat{\lambda}_t$ decreases persistently during 2005–2006. From the lens of my model, bond yields should gradually increase during the same period due to less precautionary savings, which is supported by the data.

B.2. Equity index options

Index option prices written on the aggregate equity are obtained in the same spirit as currency options. Specifically, I first compute the spot characteristic function of the stock return under the home risk-neutral measure Q^h,

$$\Psi(u; t, T) \equiv E_t^{Q^h} \left[\exp \left(- \int_t^T r_s^\$ ds \right) e^{iu \ln P_T^S} \right], \tag{65}$$

which can be derived in the following closed-form:

$$\Psi\left(u;t,T\right) = \exp\left[a\left(\tau\right) + b_\lambda\left(\tau\right)\lambda_t + b_\pi\left(\tau\right)\pi_t + c_\lambda\left(\tau\right) + c_\pi\left(\tau\right)\right],\tag{66}$$

where $a\left(\tau\right)$, $b\left(\tau\right)$, and $c\left(\tau\right)$ satisfy a series of ODEs. I then obtain option prices via fast Fourier inversion (Carr and Madan, 1999). When calculating Black-Scholes volatility (B/S-vol) from both the model and the data, I fix the interest rate and the payout ratio as the two arguments for the inversion of the B/S formula at 5% and 3%, respectively.

Moments for the equity index options are reported in Panel C of Table A.2. Consistent with the data, the model-implied ATMV is slightly higher than the stock return volatility. The smirk premium, measured as the difference in B/S-vol between 10% OTM put and ATM, is on average 8.2% in the model, which is in line with the 8–10% data values documented since the 1987 market crash (e.g., Benzoni, Collin-Dufresne, and Goldstein, 2011).

In the present setup, variations of index option prices are exclusively driven by the home disaster rate λ_t at the given maturity and moneyness. To explore this prediction, I back out the model-implied weekly time series of λ_t using the index option data over the sample period from January 10, 1996 to May 18, 2011. Using the backed-out $\{\hat{\lambda}_t\}$ to compute model values, the bottom two panels of Fig. A.1 plot together the model- and data-implied time series of ATMV and smirk premium, respectively, for the one-month option contracts.

The model captures a large fraction of variations in both ATMV and smirk premium with correlations between the model- and the data-implied series being 0.95 and 0.45, respectively. In the model, as in the data, ATMV rises monotonically in λ_t. This monotone relationship is intuitive since empirically, ATMV for index options has been called the "investor fear gauge". From the lens of the present model, the degree of fear is captured by investors' perception of the probability of economic disasters. In contrast, a higher disaster rate has two offsetting effects on the implied smirk premium. First, it reflects investors' greater concern about the potential disasters which gives them stronger incentives to hedge, hence the higher option premium for the given diffusive stock volatility σ_{Pt}^s. Second, it raises σ_{Pt}^s which diminishes the relative importance of jumps in the total stock return variations. As a result, investors have less incentives to use OTM puts as insurance against disasters which drives down the smirk premium for the given λ_t. The second effect dominates leading to the decreasing smirk premium in λ_t. Taken together, the model predicts a rising smirk premium as ATMV decreases. The data largely support this prediction (e.g., Du, 2011).

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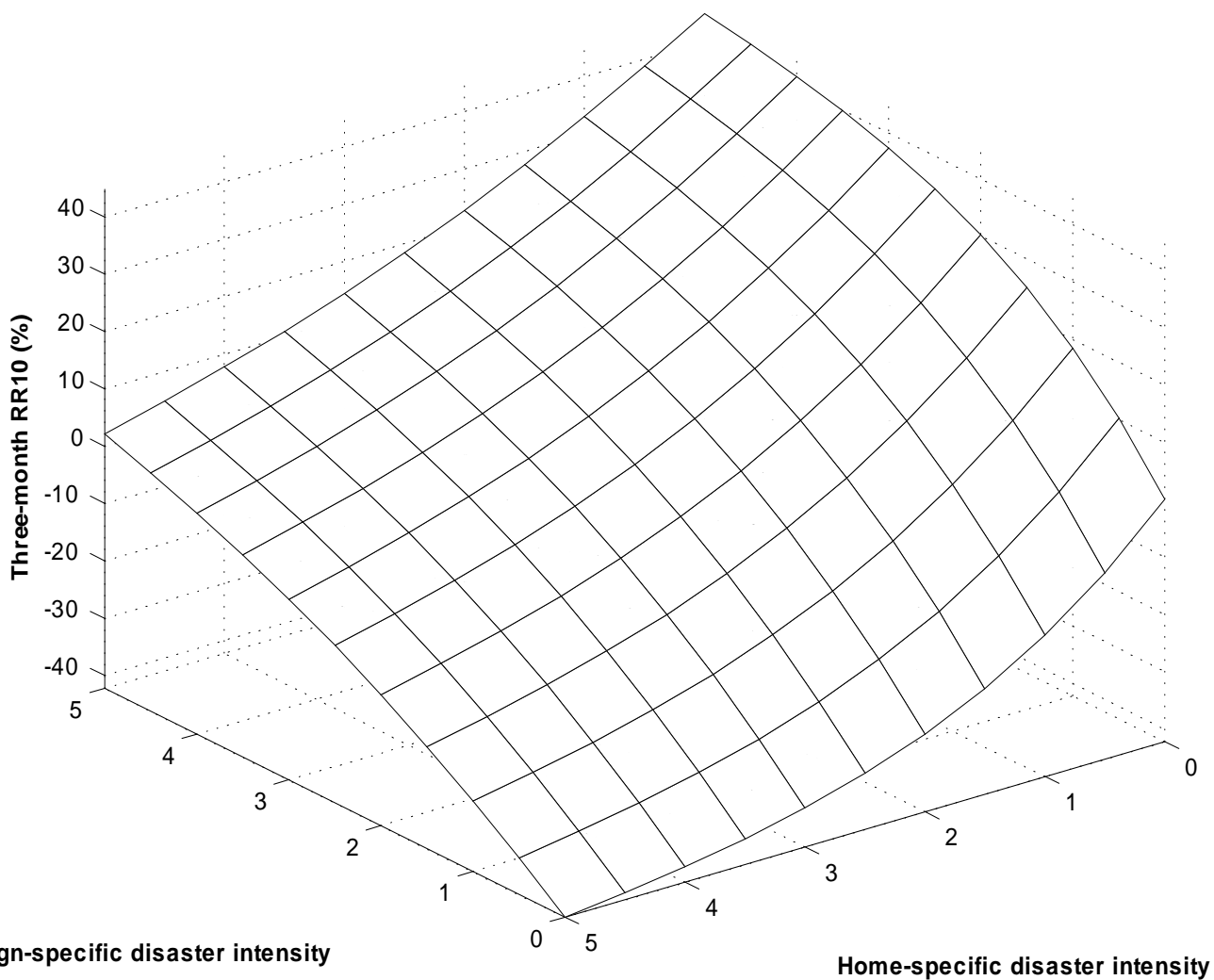
Fig. 1. Risk reversal as the function of country-specific disaster intensities. Fig. 1 plots the model-implied three-month 10-delta risk reversal as a function of the home- $\left(\lambda_t^h\right)$ and the foreign-specific $\left(\lambda_t^f\right)$ disaster intensities, where the global intensity λ_t^g is fixed at its long-run average $\bar{\lambda}^g$. Both λ_t^h and λ_t^f are quoted as multiples of $\bar{\lambda}^h \left(= \bar{\lambda}^f\right)$ and vary between zero to five.
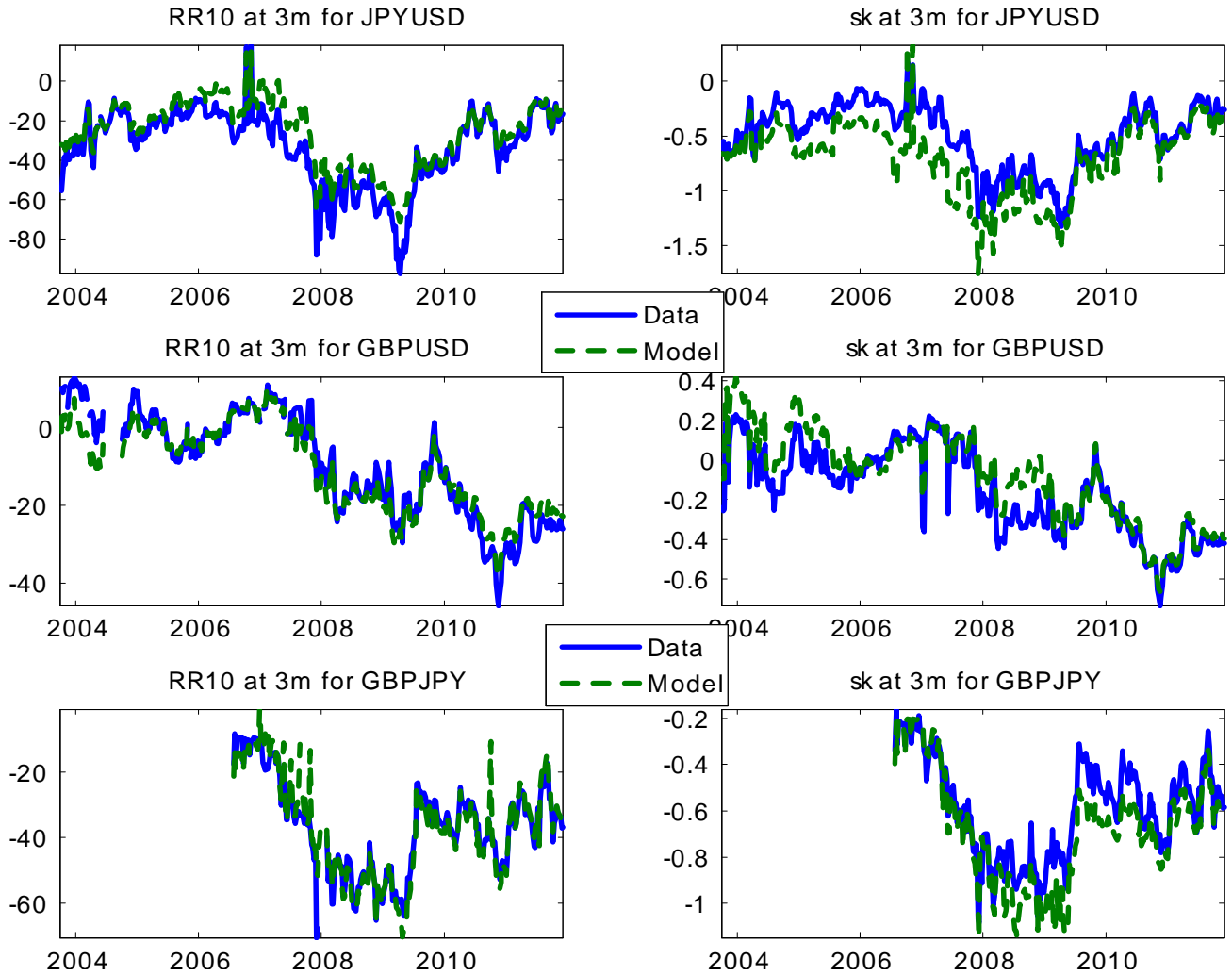
Fig. 2. Time variations of risk reversals (RR) and risk-neutral currency return skewness. The left three panels plot the model- and the data-implied time series of the three-month 10-delta RRs for JPYUSD, GBPUSD, and GBPJPY. To compute model values, I use as inputs the time series of country-specific disaster rates, λ_t^h and λ_t^f, which are backed out by minimizing the sum of the squared option pricing errors for the given trading day. The right three panels plot time variations of risk-neutral skewness (sk) at the three-month horizon that are spanned from option prices (e.g., Bakshi, Kapadia, and Madan, 2003) for the same currency pairs, where the model and the data prices are obtained at the same discretization of strikes within the empirical ranges of strike prices. In all panels, the period is from October 1, 2003 to May 18, 2011 at the weekly frequency.
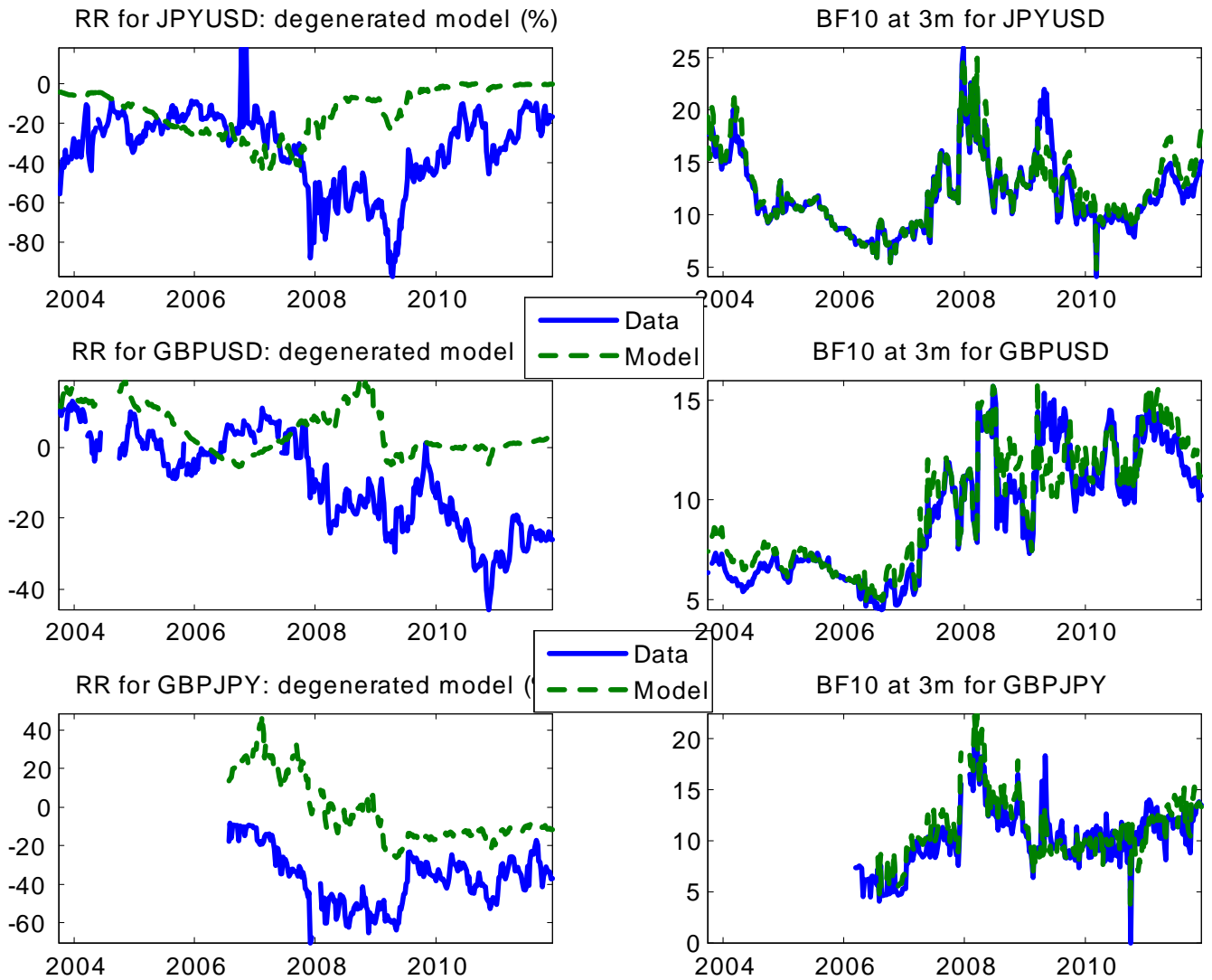
Fig. 3. The left three panels of Fig. 3 plot the time variations of three-month 10-delta risk reversals for JPYUSD, GBPUSD, and GBPJPY implied from both data and a degenerated model in which the country-specific disaster rates, λ_t^h and λ_t^f, are set to zero. The right three panels of Fig. 3 plot the time variations of three-month 10-delta butterfly spreads for the same currency pairs implied from the data and an extended version of variable disaster model. Relative to the base model of the variable disasters, the extended model generalizes the disaster rate processes by allowing the mean of both the home- and foreign-specific disaster rates to be stochastic. The details of the extension are provided in footnote 12.

Table 1

Base case calibration

Table 1 reports the base case calibration for the variable disaster model. Panel A describes the calibration of preference parameters, where β, γ, and ψ denote the subjective discount rate, the degree of risk aversion, and the elasticity of intertemporal substitution, respectively. Panel B calibrates non-disaster parameters related to the consumption process, where μ, σ, and ρ_C^{hf} denote, respectively, the average and the volatility of consumption growth rate conditional on no disasters, and the correlation between the home and the foreign consumption growths. Panel C reports disaster-related parameters, where κ, σ_λ, and $\bar{\lambda}$ control, respectively, the degree of mean-reversion, the volatility, and the average of the variable disaster intensity. The last two parameters, Z and $\bar{\lambda}^g/\bar{\lambda}$ denote, respectively, the log consumption jump size and the fraction of the global component in potential disasters. Panel D reports the estimated inflation process using the Kalman filter based on US inflation data, where $\bar{\pi}$ and κ_π control, respectively, the long-run average and the mean reversion of the expected inflation; and σ_P and σ_π denote standard deviations of unexpected and expected inflation, respectively. All parameter values are reported at the annual frequency.

Panel A: Preferences		
$\beta = 0.02$	$\gamma = 6$	$\psi = 2$

Panel B: Consumption not related to disasters		
$\mu = 0.0243$	$\sigma = 0.0095$	$\rho_C^{hf} = 0.3$

Panel C: Disasters				
$\kappa = 0.142$	$\sigma_\lambda = 0.09$	$\bar{\lambda} = 0.017$	$e^Z - 1 = -0.2$	$\bar{\lambda}^g/\bar{\lambda} = 0.98$

Panel D: Inflation			
$\bar{\pi} = 0.0354$	$\kappa_\pi = 0.411$	$\sigma_\pi = 0.0055$	$\sigma_P = 0.013$

Table 2

ATM implied volatility and exchange rate volatility

Panel A reports model implications on the average ATM implied volatility (ATMV) at four maturities: one/three/six/twelve months, as well as the average exchange rate volatility reported in the last column, where the model values are computed based on the simulated stationary distributions of variable disaster rates. Panel B reports the same implications from the data which are based on option prices and exchange rates for three currency pairs that form a triangular relation: JPYUSD, GBPUSD, and GBPJPY, where the sample period is from October 1, 2003 to May 20, 2011.

Panel A: Model-implied ATMV and exchange rate volatility				
ATMV (%)				Exchange rate volatility (%)
1m	3m	6m	12m	
Base case 8.95	8.80	8.68	8.60	9.51

Panel B: Data-implied ATMV and exchange rate volatility				
ATMV (%)				Exchange rate volatility (%)
1m	3m	6m	12m	
JPYUSD 11.0	10.8	10.7	10.7	10.6
GBPUSD 10.0	10.1	10.3	10.4	9.11
GBPJPY 13.7	13.5	13.4	13.5	13.2

Table 3

Moment implications about risk reversals

Panel A reports the model- and data-implied standard deviation for the 10- and the 25-delta risk reversals which are denoted by RR10 and RR25, respectively. The data values are from option quotes written on three currency pairs, JPYUSD (JU in short), GBPUSD (GU), and GBPJPY (GJ), which form a triangular relation. The model values are based on the simulated stationary distribution of the disaster rates, and the results are reported at various maturities ranging from one month to 18 months. Panel B reports the ranges of risk reversal variations at the same deltas and times to maturity as that in Panel A, where the model values are again based on the stationary distribution of the disaster rates. Due to the space limitation, I report the implied RR ranges only for JPYUSD and GBPUSD. In both panels, the data sample period is from October 1, 2003 to May 20, 2011.

Panel A: Standard deviations of risk reversals							
	RR10 (%)				RR25 (%)		
	Model	JU	GU	GJ	Model	JU	GU	GJ
1m	11.2	16.3	10.6	13.8	5.41	8.68	6.03	6.85
2m	15.5	17.5	12.0	14.0	7.25	9.31	6.69	7.72
3m	18.6	19.0	13.4	15.3	8.43	9.82	7.40	8.49
6m	24.7	22.2	14.9	18.4	10.3	11.4	8.04	10.0
9m	28.5	26.0	15.0	19.8	11.1	13.2	8.16	10.1
12m	31.3	26.0	15.9	20.9	11.4	13.0	8.52	11.3
18m	34.8	29.7	14.8	21.0	11.4	14.8	7.93	11.0

Panel B: Ranges of risk reversal variations					
	RR10 (%)			RR25 (%)		
	Model	JU	GU	Model	JU	GU
1m	-26.3~26.8	-82.1~18.5	-41.9~12.3	-12.7~12.8	-44.1~3.93	-23.1~6.92
2m	-36.7~38.1	-92.0~16.6	-45.1~12.4	-17.7~18.0	-48.4~9.20	-24.4~6.87
3m	-44.3~46.8	-100~18.2	-46.9~13.0	-21.2~21.8	-51.8~-4.68	-25.1~7.49
6m	-59.7~66.3	-112~23.9	-50.5~13.1	-27.5~29.6	-57.3~-1.47	-26.5~10.4
9m	-69.5~72.9	-120~-14.0	-51.3~10.5	-30.9~34.5	-61.0~-7.31	-26.8~6.88
12m	-76.5~69.8	-127~29.8	-52.1~12.8	-32.7~37.9	-65.1~-8.62	-27.3~6.98
18m	-76.8~63.3	-136~-17.5	-49.7~10.3	-34.2~42.3	-69.3~-9.27	-25.9~6.19

Table 4

Moment implications about butterfly spreads

Panel A reports the model- and data-implied standard deviation for the 10- and the 25-delta butterfly spreads which are denoted by BF10 and BF25, respectively. The data values are from option quotes written on three currency pairs, JPYUSD (JU in short), GBPUSD (GU), and GBPJPY (GJ), which form a triangular relation. The model values are based on the simulated stationary distribution of the disaster rates serving as the states. Results are reported at various maturities ranging from one to 18 months. Panel B reports the average butterfly spreads at the same deltas and maturities as those in Panel A for the same currency pairs, where the model values are again based on the stationary distribution of the disaster rates. In both panels, the data sample period is from October 1, 2003 to May 20, 2011.

Panel A: Standard deviations of butterfly spreads

	BF10 (%)				BF25 (%)			
	Model	JU	GU	GJ	Model	JU	GU	GJ
1m	1.28	2.69	1.97	2.48	0.315	0.602	0.578	1.45
2m	2.27	2.95	2.56	2.58	0.565	0.572	0.715	0.691
3m	3.15	3.43	3.09	2.94	0.785	0.586	0.914	0.751
6m	5.34	4.45	4.04	3.70	1.29	0.672	1.09	0.939
9m	7.02	5.79	3.98	4.15	1.61	0.741	1.10	1.06
12m	8.33	6.06	4.59	4.83	1.82	0.743	1.29	1.69
18m	10.2	7.17	7.28	4.38	2.04	1.18	1.16	3.00

Panel B: Averages of butterfly spreads

	BF10 (%)				BF25 (%)			
	Model	JU	GU	GJ	Model	JU	GU	GJ
1m	2.83	9.78	7.04	7.64	0.688	2.59	2.31	2.03
2m	4.71	10.8	8.03	9.05	1.13	2.71	2.55	2.12
3m	6.20	12.0	9.24	10.2	1.45	2.86	2.76	2.22
6m	9.39	14.3	10.4	12.6	2.05	3.06	3.07	2.46
9m	11.5	16.2	13.0	13.7	2.34	3.05	3.71	2.39
12m	12.9	17.2	11.8	15.1	2.48	3.34	3.36	2.64
18m	14.7	18.7	14.0	16.4	2.61	3.58	3.94	3.17

Table 5

The first two moments of model-implied carry trade returns under asymmetry in disaster magnitude

I consider the following carry trade strategy: i) borrow one unit of the home currency; ii) convert it into the foreign currency and lend it out at the foreign risk-free rate; iii) convert the earnings back to the home currency one period later. I break the complete symmetry between the home and the foreign country as follows: I maintain the base case jump size (20%) for the home country, but assume a lower jump size (in absolute value) for the foreign country. All other parameter values remain at their base levels. This way, the foreign currency pays the higher interest rate due to less precautionary savings, and the carry trade strategy yields positive expected return. Column 1 of Table 5 reports the assumed foreign disaster magnitude $\left|e^{Z^*} - 1\right|$. Correspondingly, columns 2–3 report the first two moments, i.e., the unconditional mean and volatility, of the model-implied one-month carry trade returns.

$\left|e^{Z^*} - 1\right|$	Mean (%)	Volatility (%)
-0.195	0.816	11.1
-0.19	1.11	11.0
-0.185	1.40	10.9
-0.18	1.67	10.8
-0.175	1.96	10.7
-0.17	2.51	10.9
-0.165	3.53	11.9
-0.16	4.51	13.2
-0.155	5.20	14.3
-0.15	5.77	15.2

Table 6

UIP anomaly and various other regularities related to currency pricing

  Panel A reports the UIP coefficient and its standard deviation based on the regression from the monthly changes of the log nominal exchange rates onto the one-month nominal bond yield differential between the home and the foreign country, where the data values are the usual ones reported in the literature. Panel B reports the data-implied currency return skewness and kurtosis under the physical measure which are based on eight countries that have experienced financial/ economic disasters since 1985. Specifically, I choose Indonesia, Korea, Philippines, Mexico, Thailand, Malaysia, Argentina, and Russia, with their currencies being IDR, KRW, PHP, MXN, THB, MYR, ARS, and RUB, respectively, which are quoted against the USD. Panel C reports some additional regularities related to currency pricing, where $\Delta \ln S^{\$}$, $r^{\$}$, $r^{\$,*}$, RR, and ΔRR denote, respectively, changes in the log nominal exchange rate, the nominal interest rates at home and abroad, risk reversal, and changes in risk reversal. The autocorrelations are reported on an annual basis, and the data values are average implications based on multiple currency pairs considered in this paper. For computations that involve risk reversals, I use the six-month 10-delta contracts for an example.

Panel A: UIP anomaly

	Model	Data
UIP coefficient	-2.0	-2.8~-0.9
Standard deviation for UIP coef.	1.2	0.6~1.0

Panel B: Skewness and kurtosis of "disaster currencies"

Currency	IDR	KRW	PHP	MXN	THB	MYR	ARS	RUB
Skewness	-2.24	-0.808	-1.08	-1.30	-1.08	-3.09	-1.48	-1.63
Kurtosis	9.32	6.03	4.53	3.93	10.7	14.5	4.05	4.89

Panel C: Other regularities related to currency pricing

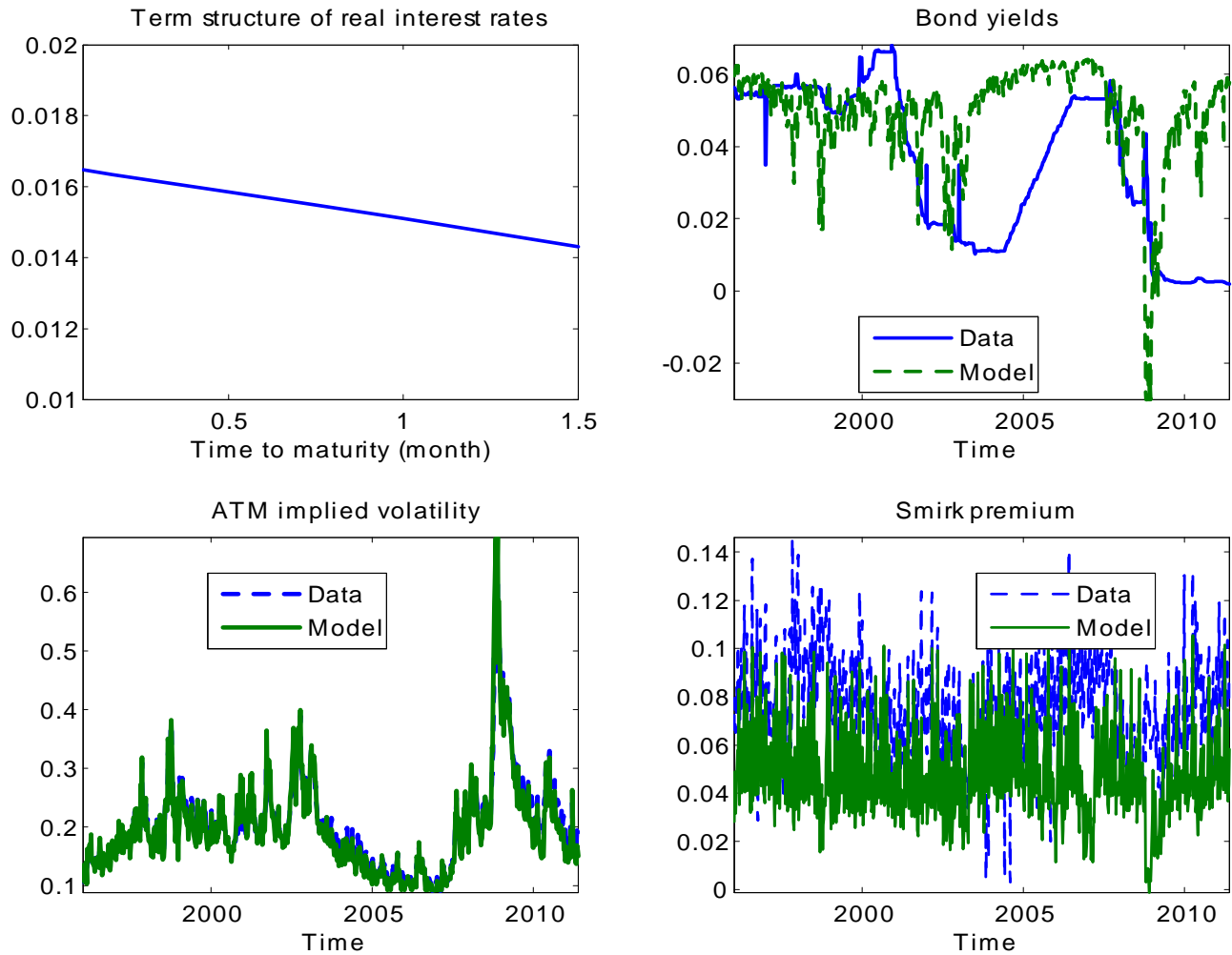|		
Autocorrelation of $\Delta \ln S^{\$}$	-0.07	-0.08
Autocorrelation of $r^{\$}$	0.75	0.73
Correlation between RR and $r^{\$} - r^{\$,*}$	0.35	0.17
Correlation between ΔRR and $\Delta \ln S^{\$}$	0.44	0.26

Fig. A.1. Bond pricing and equity index option pricing. The top left panel plots the model-implied yield curve for real bonds. The top right panel plots the weekly time series of the three-month nominal bond yields implied from both the model and the data, where the model values are calculated at the disaster rates backed out from the panel data of equity index options. The bottom two panels plot the time series of ATM implied volatility (ATMV) and the time series of smirk premium for equity index options implied from both the model and the data. The smirk premium is calculated as the volatility difference between 10% OTM options and ATMs. The model values are calculated at the disaster rates backed out from the panel data of equity index options by minimizing the sum of the squared pricing errors for each trading day. Except for the top left panel, the time period in the other three panels is from January 10, 1996 to May 18, 2011 at the weekly frequency.

Table A.1

Calibration and implications of a stochastic volatility model

I investigate the implications of a stochastic volatility model, where the consumption process is subject to both the long-run risk component in its expected growth and stochastic volatility. Panels A–D report the model calibration, where I keep the numbers as close to those in Bansal and Yaron (2004) and Bansal and Shaliastovich (2013) as possible. Panel E reports implications on currency option pricing. Specifically, columns 1–2 report the standard deviation of risk reversals (RRs); column 3 reports the standard deviation of butterfly spread (BF). I use six-month contracts as an example, and the numbers following RR and BF denote the option delta. Panel F reports the unconditional mean and volatility of one-month carry trade returns implied from the model.

Panel A: Preferences			
$\beta = 0.0264$	$\gamma = 10$	$\psi = 1.5$	
Panel B: Consumption volatility			
$\sqrt{\overline{\Omega}} = 0.0312$	$\kappa_\Omega = 0.156$	$\sigma_\Omega = 0.8 \times 10^{-5}$	
Panel C: Consumption growth			
$\mu = 0.018$	$\kappa_x = 0.252$	$\sigma_x = 0.0012$	$\rho_C^{hf} = 0$
Panel D: Inflation			
$\bar{\pi} = 0.033$	$\kappa_\pi = 0.812$	$\sigma_\pi = 0.0004$	$\sigma_P = 0.0107$

Panel E: Implications on currency option pricing		
Std of RR10 (%)	Std of RR25 (%)	Std of BF10 (%)
<0.01	<0.01	<0.01

Panel F: Implications on carry trade	
Mean (%)	Volatility (%)
9.75	44.1

Table A.2

Moments of aggregate equity, government bonds, and equity index options

    Table A.2 presents moment implications for various other assets including the aggregate equity (Panel A), government bonds (Panel B), and equity index options (Panel C). More specifically, Panel A reports the average price-dividend ratio, the equity premium, and the return volatility; Panel B reports the average, the standard deviation, and the first-order autocorrelation of short-term yields from real bonds. Panel C reports the average volatilities implied from ATM and deep OTM equity index options, as well as the average smirk premium calculated as their difference. The data values in the last column are the usual ones reported in the literature.

	Model	Data
Panel A: Aggregate equity		
Average price-dividend ratio	26	25~35
Average equity premium (%)	6.0	6~8
Volatility of the aggregate equity returns (%)	16.0	14~17
Panel B: Real yields of short-term government bonds		
Average (%)	1.8	1~1.5
Standard deviation (%)	1.7	1~1.5
First order autocorrelation	0.77	0.7~0.8
Panel C: Equity index options		
ATM implied volatility (%)	16.7	15~18
Implied volatility at moneyness equaling 0.9 (%)	24.9	24~27
Smirk premium (%)	8.2	8–10