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A Note on the Crowd-in Effect of Asset Bubbles in the Perpetual Youth Model

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Abstract

A perpetual youth overlapping generations model is presented in which the presence of financial frictions can create the *crowd-in* effect of asset bubbles that promotes capital accumulation. The existence of asset bubbles increases the equilibrium interest rate. Although the increased interest rate excludes less productive agents from production activity, these agents benefit from the liquidity of an intrinsically useless asset, rolling over the asset to the next period. As a result, allocative inefficiency is corrected, and capital accumulation is promoted under plausible parameter values.

Keywords: Asset bubbles; Crowd-in effect; Financial frictions; Capital accumulation; Heterogeneous agents.
JEL Classification Numbers: E44; O41.

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1 Introduction

An asset bubble is present when an asset's market price is over-valued compared to its fundamental value. Economic history has repeatedly witnessed output and asset prices growing synchronously at a high rate before a financial crisis, and serious depressions follow the collapse of asset bubbles. Despite these historical observations, the theoretical importance of the macroeconomic effects of asset bubbles has not been sufficiently acknowledged in mainstream macroeconomics. Many researchers, such as Tirole (1985), Weil (1987), Grossman and Yanagawa (1992), Futagami and Shibata (2000), and Mino (2008), have long discussed the macroeconomic effects of asset bubbles by applying the overlapping generations model. These studies in the traditional literature, however, derive only the *crowd-out* effect of asset bubbles that impedes capital accumulation, and regrettably, their theoretical prediction is not consistent with the historical facts of severe depressions following the collapse of asset bubbles. Given the failure of the traditional literature to capture realities, I present a dynamic general equilibrium model in which the *crowd-in* effect that promotes capital accumulation appears.

In contrast to the aforementioned studies, several researchers in the recently growing stream of literature on economic growth and asset bubbles have investigated not only the *crowd-out* effect but also the *crowd-in* effect that asset bubbles have on capital accumulation. Farhi and Tirole (2012) and Martin and Ventura (2012) apply the overlapping generations approach developed by Samuelson (1958) and Tirole (1985), whereas Kocherlakota (2009), Hirano and Yanagawa (2010), Miao and Wang (2011), Wang and Wen (2012), Kunieda and Shibata (2012), and Kiyotaki and Moore (2012) employ infinitely lived agent models of asset bubbles. The current paper belongs to this newly growing body of literature, but in contrast to the existing studies, I use the perpetual youth model developed by Blanchard (1985).

All of the studies in this new body of literature assume the presence of financial frictions, namely agents in an economy face credit constraints. As clarified by Martin and Ventura (2012), the presence of financial frictions can create the *crowd-in* effect of asset bubbles that promotes capital accumulation. The market interest rate is increased by the presence of asset bubbles, and the increased equilibrium interest rate excludes inefficient investment projects. As a result, production resources are used in more efficient projects and capital accumulation is promoted.

Although the presence of financial frictions is an important feature in the new body of literature that allows the *crowd-in* effect of asset bubbles to appear, there have been many studies that address asset bubbles in economies with financial frictions but that have not derived the *crowd-in* effect. Examples include the studies by Azariadis and Smith (1996), Boyd and Smith (1998), Kunieda (2008), Gokan (2011), and Matsuoka and Shibata (2012), all of which employ the two-period overlapping generations model. The model setting in the current paper is similar to that of Kunieda (2008) except that each agent foresees a longer lifetime horizon. Nevertheless, the *crowd-in* effect of asset bubbles can appear in the current model. Therefore, in many cases in which economies face financial frictions, the long-period lifetime setting should be one of the key factors to derive the *crowd-in* effect of asset bubbles.

2 Model

Although the basic structure of the current model is based on Kunieda's (2008) two-period overlapping generations model in which agents are heterogeneous regarding productivity in capital creation and face borrowing constraints, the model departs from Kunieda's model in that each agent foresees a long lifetime horizon but faces the probability of dying in each period. The economy consists of private agents and a representative firm. The economy continues from time $t = -\infty$ to $t = +\infty$ in discrete time. There is a probability ν that each agent's life ends unexpectedly in each period. According to the law of large numbers, a measure-$1 - \nu$ continuum of agents subsists and a measure-ν continuum of agents dies in each period. A measure-ν continuum of agents is newly born in each period, so that the total population is consistently equal to one.

2.1 Production

A Cobb-Douglas production function produces general goods Y_t using capital Z_t and labor L_t as follows:

$$Y_t = Z_t^{\alpha} L_t^{1-\alpha},$$

where $\alpha \in (0,1)$ is a capital share of output. Capital Z_t depreciates entirely in one period.[1] It follows that $L_t = \nu$ because each agent supplies one unit of labor *only* in the initial period of her lifetime and thus the population of workers in each period is equal to ν. General goods can be used interchangeably for consumption and investment. Define $y_t := Y_t/\nu$. It then follows that $y_t = z_t^{\alpha}$, where $z_t = Z_t/\nu$ is the capital-to-labor ratio. The capital and labor markets are perfectly competitive and the production factors are paid their marginal products:

$$q_t = \alpha z_t^{\alpha-1} \tag{1}$$

$$w_t = (1-\alpha) z_t^{\alpha}, \tag{2}$$

where q_t is the price of capital and w_t is the wage rate.

2.2 Agents

2.2.1 Two Saving Methods

An agent born at time τ (called agent i_τ) is endowed with one unit of labor *only* in the initial period of her lifetime.[2] She grows up to be a potential capital producer and acquires her income only from savings from her second period onward. There are two types of saving methods: agent i_τ can begin to produce capital or buy an intrinsically useless asset. However, there is no financial market where she can borrow resources to invest in a project or purchase the intrinsically useless asset, namely she faces borrow-

[1] It is implicitly assumed that the general goods are also perishable in one period, implying that there is no technology available to preserve the general goods for the next period.

[2] The current model differs from Blanchard's model in that agents are endowed with labor only in the initial period of their lifetimes.

ing constraints. One may observe that capital created by agents broadly includes both physical and human capital. Because potential capital producers receive uninsured idiosyncratic productivity shocks in each period, agent i_τ's return on savings $R_t(i_\tau)$ from time $t = \tau + 1$ onward is individually specific. As mentioned, however, her life will end with a probability ν at the beginning of each period before consuming her income from savings.

The total nominal supply of the intrinsically useless asset is constant, given by M. The price of the intrinsically useless asset at time t is denoted by p_t. Then, $B_t := p_t M$ is its real value at time t. The intrinsically useless asset is freely disposable, and thus B_t is non-negative. A bubble on the intrinsically useless asset is present if B_t is strictly positive.[3] If B_t is strictly positive, we have:

$$B_t = r_t B_{t-1}, \tag{3}$$

where $r_t = p_t/p_{t-1}$ is the return on holding the intrinsically useless asset from time $t-1$ to time t.

2.2.2 Utility Maximization

As assumed in the model of Blanchard (1985), competitive insurance companies provide an insurance program to agents such that for all $t > \tau$, agent i_τ receives $x_t(i_\tau) := R_t(i_\tau)a_{t-1}(i_\tau)\nu/(1-\nu)$ if she subsists through time t, but she returns $R_t(i_\tau)a_{t-1}(i_\tau)$ to the insurance company if she dies at the beginning of time t, where $a_{t-1}(i_\tau)$ represents savings at time $t-1$.[4] Because the insurance market is competitive, a representative insurance company obtains zero expected profits.

The whole set of potential capital producers at time t (i.e., all agents who are alive at minimum until the beginning of time $t+1$) is denoted by Ω_t. Agent $i_\tau \in \Omega_t$ $(\tau \leq t)$

[3] One can define an asset bubble as the difference between an asset's fundamental and market values.

[4] Given that the possibility of dying and the idiosyncratic productivity shocks are independent across both time and agents, the law of large numbers with respect to agents enables a representative insurance company to offer this insurance contract to agents. There is a mass of agents whose potential income is identical to $R_t(i_\tau)a_{t-1}(i_\tau)$, which sustains this insurance program.

with a subjective discount factor $\beta \in (0,1)$ maximizes her expected lifetime utility as follows:

$$\max \ E_t \left[\sum_{s=t}^{\infty} [\beta(1-\nu)]^{s-t} \ln c_s(i_\tau) \right],$$

subject to

$$c_s(i_\tau) + a_s(i_\tau) = I_s(i_\tau), \tag{4}$$

for $s \geq t \geq \tau$, where agent i_τ's income $I_s(i_\tau)$ at time s is defined as:

$$I_s(i_\tau) = \begin{cases} w_s & \text{if } s = \tau \\ R_s(i_\tau)a_{s-1}(i_\tau) + x_s(i_\tau) = \frac{R_s(i_\tau)}{1-\nu} a_{s-1}(i_\tau) & \text{if } s \geq \tau + 1, \end{cases} \tag{5}$$

where $c_s(i_\tau)$ and $a_s(i_\tau)$ are entrepreneur i_τ's consumption and net worth (savings) at the end of time s, respectively. Denote $\tilde{R}_t(i_\tau) := R_t(i_\tau)/(1-\nu)$. Given information until time t, the expectation operator $E_t[\cdot]$ is associated with uncertainty about the individually specific return. Note that uncertainty regarding sudden death is already taken into consideration in the expected lifetime utility. The Euler equation associated with agent i_τ is obtained as follows:

$$\frac{1}{c_t(i_\tau)} = E_t \left[\frac{\beta(1-\nu)\tilde{R}_{t+1}(i_\tau)}{c_{t+1}(i_\tau)} \right].$$

From the Euler equation, the budget constraints (4), and the transversality condition, it is straightforward to demonstrate that the log-linear lifetime utility yields a simple linear relationship between net worth $a_t(i_\tau)$ and income $I_t(i_\tau)$ as follows:

$$a_t(i_\tau) = \beta(1-\nu)I_t(i_\tau). \tag{6}$$

Next, we determine $\tilde{R}_{t+1}(i_\tau)$. Agents optimally allocate net worth through one of the aforementioned saving methods in each period. More concretely, given net worth $a_t(i_\tau)$ and information on productivity $\Phi_t(i_\tau)$, agent $i_\tau \in \Omega_t$ chooses k_t and b_t to maximize her income at time $t+1$. It is important to note that each agent knows her productivity

$\Phi_t(i_\tau)$ when solving the income maximization problem. In other words, for $t+1 \geq \tau$, we have:

$$I_{t+1}(i_\tau) = \max_{k_t, b_t} \frac{q_{t+1}\Phi_t(i_\tau)k_t + r_{t+1}b_t}{1-\nu}, \tag{7}$$

subject to:

$$k_t + b_t = a_t(i_\tau), \tag{8}$$

$$b_t \geq 0, \tag{9}$$

$$k_t \geq 0, \tag{10}$$

where k_t is investment and b_t is the purchase of an intrinsically useless asset. The investment project at time t transforms one unit of general goods into Φ_t units of capital goods that are sold to the representative firm at a price q_{t+1} for use as input. $\Phi_t(i_\tau)$ is an idiosyncratic productivity shock at time t for capital creation, which is independent and identically distributed across both time and agents (the i.i.d. assumption). $\Phi_t(i_\tau)$ has support over $[0, h]$ where $h > 0$ and the cumulative distribution function is given by $G(\Phi_t(i_\tau))$, which is continuous, differentiable, and strictly increasing over the support. Eq. (9) is a non-negativity constraint of the purchase of the intrinsically useless asset, which reflects the free disposal of the asset. Eq. (10) is the non-negativity constraint of investment.

From the income maximization problem, it follows that if agent i_τ receives an idiosyncratic shock $\Phi_t(i_\tau)$ greater than r_{t+1}/q_{t+1}, she initiates an investment project, but if she receives an idiosyncratic shock $\Phi_t(i_\tau)$ less than r_{t+1}/q_{t+1}, she purchases the intrinsically useless asset by using her total net worth. In other words, $\phi_t := r_{t+1}/q_{t+1}$ is a cutoff for productivity shocks that divides agents into producers and asset holders at time t. The solution to the income maximization problem of agent $i_\tau \in \Omega_t$ is given as follows:

$$k_t(i_\tau) = \begin{cases} 0 & \text{if } \Phi_t(i_\tau) \leq \phi_t \\ a_t(i_\tau) & \text{if } \Phi_t(i_\tau) > \phi_t, \end{cases} \tag{11}$$

and

$$b_t(i_\tau) = \begin{cases} a_t(i_\tau) & \text{if } \Phi_t(i_\tau) \le \phi_t \\ 0 & \text{if } \Phi_t(i_\tau) > \phi_t. \end{cases} \tag{12}$$

From Eqs.(5), (7), (11) and (12), one can obtain:

$$a_t(i_\tau) = \begin{cases} \beta(1-\nu)w_t & \text{if } t = \tau \\ \beta(1-\nu)\tilde{R}_t(i_\tau)a_{t-1}(i_\tau) & \text{if } t \ge \tau+1, \end{cases} \tag{13}$$

where

$$\tilde{R}_{t+1}(i_\tau) = \frac{\max\{q_{t+1}\Phi_t, r_{t+1}\}}{1-\nu}.$$

2.3 Equilibrium

A competitive equilibrium is expressed by sequences of the interest rate $\{r_t\}$, capital price $\{q_t\}$, wage rate $\{w_t\}$, and allocation $\{(c_t(i_\tau)), (a_t(i_\tau)), (k_t(i_\tau)), (b_t(i_\tau))\}$, $\{Z_t, L_t\}$ and $\{B_t\}$ for all $t \in (-\infty, \infty)$ and $\tau \le t$ so that all agents' and the representative firm's optimization conditions hold, and the asset, capital, and labor markets clear.

2.3.1 Aggregation

The i.i.d. assumption for the realization of the stochastic productivity shocks renders the derivations of aggregate variables tractable. Define $\Psi_t := \{i_\tau \in \Omega_t : \Phi_t(i_\tau) > \phi_t\}$. Agents in Ψ_t become capital producers, and agents in Ω_t/Ψ_t become asset holders. Because the total demand for the intrinsically useless asset at time t is given by $\int_{i_\tau \in \Omega_t/\Psi_t} a_t(i_\tau)di_\tau$, the asset-market clearing condition is written as:

$$B_t = \int_{i_\tau \in \Omega_t/\Psi_t} a_t(i_\tau)di_\tau = \beta(1-\nu)\left(\nu z_t^\alpha + r_t B_{t-1}\right) G(\phi_t). \tag{14}$$

From Eq. (11), each capital producer invests $a_t(i_\tau)$, and thus, the total capital Z_{t+1} used for the next period of production is given by:

$$Z_{t+1} = \int_{i_\tau \in \Psi_t} \Phi_t(i_\tau) a_t(i_\tau) di_\tau = \beta(1-\nu)\left(\nu z_t^\alpha + r_t B_{t-1}\right) F(\phi_t), \tag{15}$$

where $F(\phi_t) := \int_{\phi_t}^{\infty} \Phi_t(i_\tau) dG(\Phi_t(i_\tau))$. In both Eqs. (14) and (15), the second equalities hold because of the i.i.d. assumption for the realization of the stochastic productivity shocks.

2.3.2 Dynamical Systems and the Bubbly Steady State

From Eqs. (3), (14), and (15) with $r_t = \alpha\phi_{t-1} z_t^{\alpha-1}$, we obtain the laws of motions of the cutoff and capital in equilibrium as follows:

$$\frac{\beta(1-\nu)G(\phi_t)}{1-\beta(1-\nu)G(\phi_t)} = \frac{\alpha\phi_{t-1}G(\phi_{t-1})}{F(\phi_{t-1})} \tag{16}$$

$$z_{t+1} = \frac{\beta(1-\nu)F(\phi_t)}{1-\beta(1-\nu)G(\phi_t)} z_t^\alpha. \tag{17}$$

Eq. (16) is a difference equation with respect to a single variable ϕ_t. It has two steady states ϕ^* and ϕ^{**} such that:

$$G(\phi^*) = 0 \iff \phi^* = 0$$

and

$$\frac{\beta(1-\nu)F(\phi^{**})}{1-\beta(1-\nu)G(\phi^{**})} = \alpha\phi^{**}. \tag{18}$$

It is clearly shown that ϕ^{**} is uniquely determined in $(0, h)$. One notes from Eq. (14) that ϕ^* gives a bubbleless state and ϕ^{**} gives a bubbly state in the dynamical system that consists of Eqs. (16) and (17). Note that I do not reference a bubbleless or bubbly “steady" state here. This is because even if the economy stays in ϕ^* and ϕ^{**}, which are the steady states in Eq. (16), the economy may be on a transitional path in the dynamical system that consists of Eqs. (16) and (17). It is straightforward to demonstrate that ϕ^* is locally stable and that ϕ^{**} is unstable in Eq. (16), and the phase diagram associated with the dynamic behavior of ϕ_t is provided in Figure 1. Because ϕ_t is not a predetermined variable, the equilibrium is locally determinate in the neighborhood of ϕ^{**}; if one focuses on the small neighborhood of ϕ^{**}, $\phi_t = \phi^{**}$ for all t is only the rational expectations equilibrium. However, if we investigate the global dynamic behavior, equilibrium is indeterminate.[5]

Define a function $\Lambda(\phi)$ such that:

$$\Lambda(\phi) := \frac{\beta(1-\nu)F(\phi)}{1-\beta(1-\nu)G(\phi)},$$

which is identical to the coefficient of z_t^α in Eq.(17) and the left-hand side of Eq. (18).

Lemma 1. *For $\phi \in [0, h]$, the maximum of $\Lambda(\phi)$ is given at $\hat{\phi} \in (0, h)$ where $\hat{\phi}$ is given by $\Lambda(\hat{\phi}) = \hat{\phi}$.*

Proof. See the appendix.

Figure 2 illustrates the positions of ϕ^{**} and $\hat{\phi}$. It is noted that $\hat{\phi} < \phi^{**}$ and as α increases, ϕ^{**} approaches $\hat{\phi}$. The dynamical system that consists of Eqs. (16) and (17) has two steady states (ϕ^*, z^*) and (ϕ^{**}, z^{**}) where

$$z^* = [\Lambda(\phi^*)]^{\frac{1}{1-\alpha}} = [\beta(1-\nu)F(0)]^{\frac{1}{1-\alpha}}$$

and

$$z^{**} = [\Lambda(\phi^{**})]^{\frac{1}{1-\alpha}} = [\alpha\phi^{**}]^{\frac{1}{1-\alpha}}.$$

Clearly, (ϕ^*, z^*) is the bubbleless steady state, and (ϕ^{**}, z^{**}) is the bubbly steady state. From the determinations of z^* and z^{**}, it follows that $z^{**} > z^*$ if $\beta(1-\nu)F(0) < \alpha\phi^{**}$. Figure 3 illustrates the dynamic behaviors of z_t that follow the difference equation (17) when the economy stays in the bubbly and bubbleless states. As observed in Figure 3, the presence of asset bubbles promotes capital accumulation if $\beta(1-\nu)F(0) < \alpha\phi^{**}$.

[5] In other words, an uncountably infinite number of equilibrium trajectories exist that converge to ϕ^*, originating from the left-side neighborhood of ϕ^{**}. See Figure 1.

Formally, Proposition 1 is obtained.

Proposition 1. *Suppose that $\beta(1-\nu)F(0) < \alpha\phi^{**}$. Then, the presence of asset bubbles promotes capital accumulation, particularly for the same initial capital stock z_0, the economy from time $t = 1$ onward accumulates more capital stock when it stays in the bubbly state ϕ^{**} than in the bubbleless state ϕ^*.*

Proof. The claim follows from the determinations of z^* and z^{**} and the difference equation (17). $\square$.

Example

Suppose that $\Phi_t(i_\tau)$ follows a uniform distribution in $[0, 1]$. I set $\beta = 0.98$ and $\nu = 0.02$. Because capital in this model broadly includes both physical and human capital, I set $\alpha = 0.66$ following the estimation of Mankiw et al. (1992). When $\alpha > 1/2$, I obtain:

$$\phi^{**} = \frac{\alpha - \sqrt{\alpha^2 - (2\alpha - 1)\beta^2(1-\nu)^2}}{(2\alpha - 1)\beta(1-\nu)},$$

which yields $\alpha\phi^{**} = 0.6126$. Meanwhile, because $F(0)$ is the mean of $\Phi_t(i_\tau)$, I have $\beta(1-\nu)F(0) = \beta(1-\nu)/2 = 0.4802$. These parameter values provide a case in which the presence of asset bubbles promotes capital accumulation.

3 Concluding Remarks

One can create a rational expectations equilibrium with stochastic bubbles from the current model in which the crash of asset bubbles is self-fulfilling and caused by extrinsic uncertainty. In the self-fulfilling equilibrium, when asset bubbles collapse because of agents' expectations, a depression follows the crash of asset bubbles under the condition given in Proposition 1. This outcome is consistent with historical observations and has not been investigated by the traditional literature that only considers the crowd-out effect of asset bubbles impeding capital accumulation.

The long-period lifetime setting creates the liquidity effect of an intrinsically useless asset. The presence of asset bubbles increases the equilibrium interest rate, which excludes less productive agents from production activity; however, these agents benefit from the liquidity of the intrinsically useless asset, rolling it over to the next period. As a result, allocative inefficiency is corrected, and the presence of asset bubbles promotes capital accumulation under plausible parameter values.

Appendix

Proof of Lemma 1

From $\Lambda(\phi) = \beta(1-\nu)F(\phi)/[1-\beta(1-\nu)G(\phi)]$, one can obtain:

$$\Lambda'(\phi)[1-\beta(1-\nu)G(\phi)]^2 = \beta(1-\nu)G'(\phi)H(\phi)$$

where

$$H(\phi) = [1-\beta(1-\nu)G(\phi)][\Lambda(\phi)-\phi].$$

Because $H'(\phi) = -1+\beta(1-\nu)G(\phi) < 0$, $H(\phi)$ is a strictly decreasing function where $H(0) = \beta(1-\nu)F(0) > 0$ and $H(h) = -h[1-\beta(1-\nu)] < 0$. Therefore, $H(\phi) = 0$ has a unique solution, which is $\hat{\phi}$. Moreover, if $\phi \in [0, \hat{\phi})$, $\Lambda(\phi)$ is increasing and if $\phi \in (\hat{\phi}, h]$, $\Lambda(\phi)$ is decreasing, which implies that $\hat{\phi}$ gives the maximum of $\Lambda(\phi)$. $\square$

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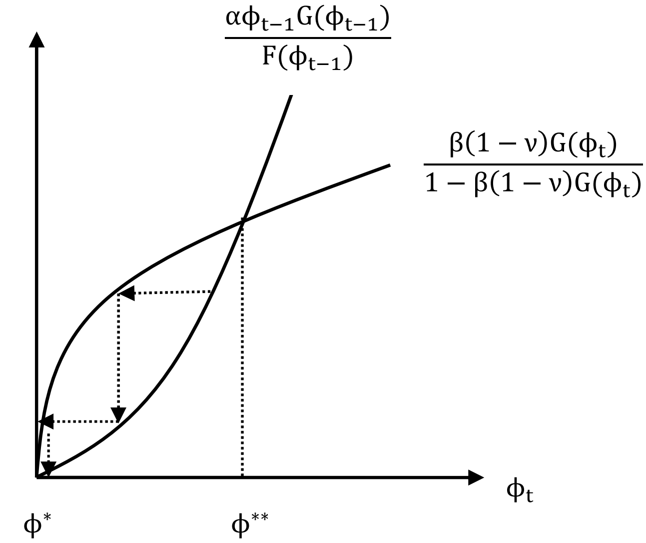

Fig. 1: Dynamic Behavior of ϕ_t

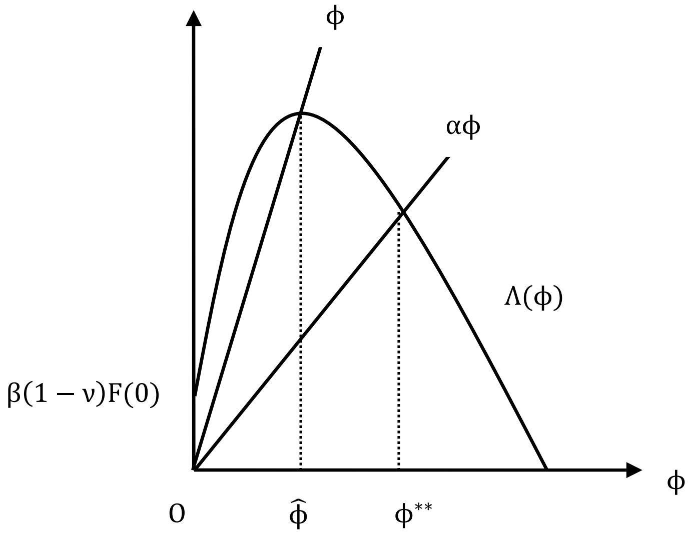

Fig. 2: Positions of $\hat{\phi}$ and ϕ^{**}

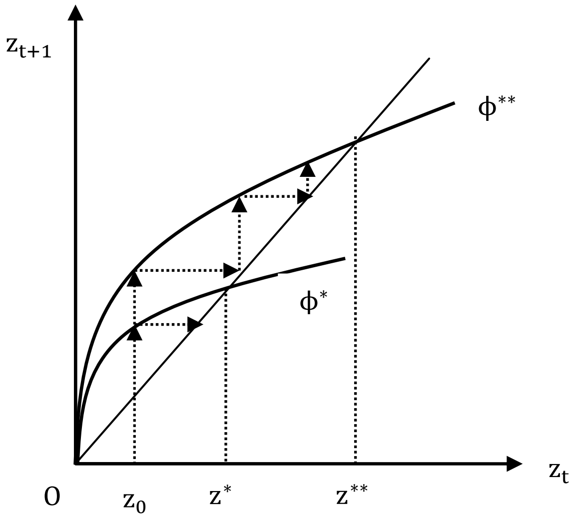

Fig. 3: Dynamic behavior of z_t (when $\beta(1-\nu)F(0) < \alpha\phi^{**}$)