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An application to sovereign bond markets in Europe

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Detecting contagion in a multivariate time series system: An application to sovereign bond markets in Europe[☆]

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Abstract

This paper proposes an original three-part sequential testing procedure (STP) with which to test for contagion using a multivariate model. First, conditional on breaks in the conditional mean, the procedure identifies distinct structural breaks in the volatility of a given set of countries. A further structural break test applied to the correlation matrix identifies and then dates the potential contagion mechanisms. As a third element, the STP tests for the distinctiveness of the break dates previously found. As a result of using multi-dimensional data, the STP has high testing power and is able to locate the dates of contagion more precisely. The application to European long-term interest rates shows that immediate contagion from Greece does not take place, but the dynamic spillovers are shown to increase after controlling for breaks in the different model parameters. For other countries we find evidence of both contagion and flight-to-quality mechanisms.

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1. Introduction

The recent European sovereign debt crisis has been characterized by a rapid diffusion across borders. However, this negative shock has been observed to diffuse differently across European countries; some countries, particularly Southern European countries such as Italy, Portugal and Spain, were immediately and negatively affected, whereas the direct impact on Northern and central European countries such as Germany and Austria has appeared much less severe. This fact is crucial for policy makers to establish efficient firewalls to stop the diffusion of such turmoil, and stress the problem of having a European global response to the crisis. This issue has been the starting point of a revival of empirical studies on the transmission of shocks and contagion in the euro area (**?**, **?**, **?**, **?** or **?**, to name but a few). These studies rely on distinct definitions of contagion and methodologies and lead to different conclusions. This paper proposes to extend these analyses by refining the empirical methodology to better detect the transmission of crises using time series data.

Since the seminal papers of **?**, **?** and **?**, (shift) contagion has often been considered a significant increase in the correlation between two countries' stock market indexes. A key methodological contribution is **?**, in which it is shown that contagion is over-accepted, if one ignores the changes that occur in the variance when testing for changes in correlation. Following **??**, another strand of literature prefers to focus on the transmission channels of a crisis and to investigate the stability of the spillovers between two countries' stock market indexes. Both approaches are particularly interesting because they adopt different timing perspectives, either distinguishing abrupt changes in contemporaneous dependence or changes in dynamic spillovers. We retain this distinction in the remainder of the paper. To be specific, we use the following terminology. *Interdependence* refers to existing linkages between markets. A *crisis* is turmoil in financial markets that occurs as a result of shocks, and it

materializes in the form of increased volatility. *Contagion* in general refers to the situation in which the degree of interdependence increases beyond its usual level during a crisis. In this paper, we distinguish between two forms of contagion. *Shift contagion* refers to breaks in contemporaneous correlation, i.e., an increase in the immediate shock transmission. *Spillover contagion* is associated with breaks in the dynamics of transmission channels; the latter form of contagion is directional, and typically requires a certain time lag to materialize. Finally, by *flight-to-quality* we mean the situation in which the level of interdependence decreases during a crisis, indicating a decoupling of safe haven countries that are believed to be unaffected by the crisis.[1]

Nevertheless, several issues should be taken into account when testing for contagion. First, many studies consider crisis dating as exogenous. In other words, the break date is not inferred from the data but imposed by the authors. Several procedures for endogenous break date determination have been proposed, e.g., in **??**, **?**, **?** or **?**.

Second, a typical assumption is the simultaneity of the structural breaks in volatility, typically representing increased turmoil and the outbreak of the crisis, and correlation breaks, representing the occurrence of contagion. **?** challenge this assumption and observe that during the Asian crisis, variance breaks preceded correlation shifts in most cases. The economic motivation behind this finding is that the intensification of interdependence is not immediate and takes place only when markets are already stressed. In such situations, assuming simultaneity between volatility and correlation shifts would lead to a combination of these two effects and hence to an underestimation of the presence of contagion.

Third, many existing papers such as **?**, **?**, or **?** exclusively analyze pairwise correlations. As noted by **?**, considering a multivariate approach is recommended to correctly apprehend contagion, whereas bivariate analysis may lead to biased conclusions. Indeed, a shock that

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[1]See inter alii **?** for a comparison between contagion and flight-to-quality in the case of stocks and bonds markets.

originates in country/market i does not necessarily impact country/market j directly but may indirectly transit via country/market k. Furthermore, from a purely econometric perspective, **?**, **?** and **?** show that the date of a structural break is estimated more precisely in a multivariate system than in a univariate regression.

The contribution of this paper is to propose and apply a novel sequential testing approach for contagion that addresses the abovementioned issues. Relying on the theory developed in **?**, we specify a vector autoregressive model (VAR) that is subject to multiple structural breaks. The key methodological innovation is to separate the structural breaks in the parameters for the conditional mean, the error variance and the correlation between the shocks. Furthermore, the procedure tests whether the inferred breaks are distinct from one another, in similar spirit to the test proposed in **?**. The sequential procedure is performed in a multivariate dynamic set-up (of dimension 10) to benefit from high testing power and more precise estimates of the break dates and thereby to better evaluate the presence of contagion.

The idea of decomposing mean, variance and correlation breaks is similar to the one proposed by **?**, who study structural breaks in cross-country inflation relations. Still, the Sequential Testing Procedure (hereafter STP) presents the advantage of decomposing the covariance matrix before testing and not after finding a break in the variance-covariance matrix. The procedure therefore requires breakpoint tests for fewer parameters, resulting in lower degrees of freedom. This increases the power and efficiency of the applied break tests.

Similarly to **?**, **?**, **?**, **?**, **?** and **?**, we analyze the 10-year bond yield spreads over Germany. Our analysis confirms previous conclusions regarding the absence of shift contagion between Greece and other countries. Furthermore, we are able to identify and date the sequence of shocks to the European sovereign bond markets. We observe that volatility breaks occur simultaneously in the whole system but are distinct from the structural breaks of the conditional mean parameters, which are again common across all equations. This finding stresses the coincidence of unrest hitting the euro area economies during the Sovereign Debt Crisis. The correlation breaks, on the other hand, occur over various dates and are not common

to all country pairs. Nevertheless, many of the breakpoints in correlations cluster around a small number of dates, and the distinctiveness tests enable us to identify a few 'contagion clubs', i.e., countries that have common correlation breaks. Furthermore, we find strong evidence of a flight-to-quality mechanism at various stages of the crisis that is associated with a decrease in correlation. The identified breakpoints and resulting parameter estimates are also very useful for analyzing the spillovers in a manner similar to that in **??** and **?**. Indeed, we demonstrate that the spillover indexes show significant time variation and that the distinct breakpoints translate into a well-interpretable evolution over time. In this case, the role of Greece appears different because there is evidence of spillover contagion at various times during the crisis.

The paper is structured as follows. Section 2 motivates and explains the model and the STP methodology. Section 3 describes the empirical application of our method to the case of the European crisis. Concluding remarks are made in Section 4. The appendix contains some technical details and complementary results, while Monte Carlo simulations on the method's properties can be found in an online appendix.

2. Methodology

2.1. The multivariate model

Our analysis of financial contagion builds on the following vector autoregressive model (VAR) for a vector of financial time series $\boldsymbol{y}_t = [y_{1,t}, \ldots, y_{n,t}]'$,

$$\boldsymbol{y}_t = \boldsymbol{B}_{0,t} + \sum_{i=1}^{p} \boldsymbol{B}_{i,t}\boldsymbol{y}_{t-1} + \boldsymbol{B}_{x,t}\boldsymbol{x}_t + \boldsymbol{\varepsilon}_t. \tag{1}$$

Here $t = 1, \ldots, T$, $\boldsymbol{x}_t$ is a vector of q exogenous variables, and the coefficient matrices $\boldsymbol{B}_{i,t}$ and $\boldsymbol{B}_{x,t}$ as well as the intercept vector $\boldsymbol{B}_{0,t}$ are potentially time-varying. The n-dimensional vector of error terms $\boldsymbol{\varepsilon}_t = [\varepsilon_{1,t}, \varepsilon_{2,t}, \ldots, \varepsilon_{n,t}]'$ follows some (unknown) distribution with co-variance matrix $\boldsymbol{\Sigma}_t$. Assumptions about the innovations are mild; they allow for the typical

features observed in financial returns, in particular conditional heteroscedasticity and auto-correlation, and can be found in Appendix A.1.

The parameters in equation (1) are allowed to be time-varying by being subject to structural breaks at unknown points in time. To be precise, below we explain how to test for and date structural breaks in the conditional mean, in the conditional variance and in conditional correlations in a manner similar to that described in **?**. The estimated breakpoints and the corresponding changes in parameter estimates in turn allow us to infer whether and to what extent contagion occurred.

2.2. Contagion in the multivariate model

For the moment, let us ignore any time variation in parameters of the conditional mean equation. Shift contagion is detected when the correlation between markets increases beyond its pre-crisis level. Because contemporaneous dependence is not part of the conditional mean model, it is captured by the covariance matrix $\boldsymbol{\Sigma}_t$ of the errors $\boldsymbol{\varepsilon}_t$. Thus, testing for contagion boils down to testing for an increase in the dependence among the residuals $\hat{\boldsymbol{\varepsilon}}_t$. However, as noted by **?**, a change in the covariance matrix $\boldsymbol{\Sigma}_t$ does not allow for the identification of contagion. The origin of a shift in a covariance term $\sigma_{ij} = \sigma_i \rho_{ij} \sigma_j$ would be unclear because it could result from an increase in the correlation or from a rise in the variance, where the latter is typically a sign of crisis outbreak or intensification. Therefore, we decompose the covariance matrix as follows:

$$\boldsymbol{\Sigma}_t = \boldsymbol{S}_t \boldsymbol{R}_t \boldsymbol{S}_t. \tag{2}$$

$\boldsymbol{R}_t$ is the matrix of $n(n-1)/2$ correlation coefficients $\rho_{ij,t}$ and $\boldsymbol{S}_t$ is a diagonal matrix containing n standard deviations $\sigma_{i,t}$, for $i, j = 1, \ldots, n$.

A test for (shift) contagion consists in detecting an increase in the elements of the correlation matrix $\boldsymbol{R}_t$, which measure contemporaneous interdependence only. However, during a financial crisis, some elements of the $\boldsymbol{S}_t$ matrix are likely to increase due to increased market risk. Moreover, there is no a priori reason to believe that the outbreak of a crisis in multiple

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countries occurs simultaneously but that contagion occurs in a sequential manner. In fact, it is likely that this contagious transmission may occur several periods after the initial outbreak of the crisis. Therefore, assuming concordance between shifts in volatility and dependence is overly restrictive and can lead to imprecise or even biased estimates of the unknown time of structural changes. In our approach, breaks in volatility and correlation are not assumed to be simultaneous, but we challenge this assumption and test whether these breakpoints are the same.

Table 1 illustrates a hypothetical crisis scenario that can be identified using our procedure. The crisis breaks out in market 1 first, which increases the standard deviation σ_1. Market 3 then enters a high-volatility crisis state, followed by market 2. The last standard deviation break occurs simultaneously with shift contagion between the first three markets, which means that the three correlation coefficients ρ_{12}, ρ_{13} and ρ_{23} shift to higher values. Finally, the fourth market is not affected by the crisis in any way; nevertheless, its covariance with the other markets changes. A test seeking instability in, for example, the covariance $\sigma_{13} = \sigma_1 \rho_{13} \sigma_3$ would produce a biased break date estimate in between the three distinct breaks. Instability in σ_{13} may be caused by changes in either the standard deviation or by a change in correlation. A direct test for a structural break in the covariance matrix would not be able to identify the source of instability. Furthermore, the number of parameters breaking is much smaller than the number of affected covariances. A test on the covariances would thus have more degrees of freedom, which would affect the power of the test.

To summarize, our test for shift contagion is a test for a structural break in the correlation matrix at an unknown point in time conditional on structural breaks at (possibly distinct) unknown dates in the series' volatility. However, the VAR coefficients in equation (1) are also likely to be subject to structural breaks; see **?** and **?**. Therefore, prior to testing for and estimating structural breaks in correlations and variances, we test for breakpoints in the VAR parameters $\boldsymbol{B}_{0,t}$, $\boldsymbol{B}_{i,t}$ and $\boldsymbol{B}_{x,t}$, which are collected in the coefficient matrix $\boldsymbol{B}_t$. A test for multiple structural breaks in the parameters of the conditional mean and covariance of

Table 1: Example of a sequence of crisis events

$$\begin{bmatrix} \sigma_1^{(1)} & 0 & 0 & 0 \\ 0 & \sigma_2^{(1)} & 0 & 0 \\ 0 & 0 & \sigma_3^{(1)} & 0 \\ 0 & 0 & 0 & \sigma_4 \end{bmatrix} \begin{bmatrix} 1 & \rho_{12}^{(1)} & \rho_{13}^{(1)} & \rho_{14} \\ \rho_{12}^{(1)} & 1 & \rho_{23}^{(1)} & \rho_{24} \\ \rho_{13}^{(1)} & \rho_{23}^{(1)} & 1 & \rho_{34} \\ \rho_{14} & \rho_{24} & \rho_{34} & 1 \end{bmatrix} \begin{bmatrix} \sigma_1^{(1)} & 0 & 0 & 0 \\ 0 & \sigma_2^{(1)} & 0 & 0 \\ 0 & 0 & \sigma_3^{(1)} & 0 \\ 0 & 0 & 0 & \sigma_4 \end{bmatrix} = \begin{bmatrix} \sigma_1^{2(1)} & \sigma_{12}^{(1)} & \sigma_{13}^{(1)} & \sigma_{14}^{(1)} \\ \sigma_{12}^{(1)} & \sigma_2^{2(1)} & \sigma_{23}^{(1)} & \sigma_{24}^{(1)} \\ \sigma_{13}^{(1)} & \sigma_{23}^{(1)} & \sigma_3^{2(1)} & \sigma_{34}^{(1)} \\ \sigma_{14}^{(1)} & \sigma_{24}^{(1)} & \sigma_{34}^{(1)} & \sigma_4^2 \end{bmatrix} = \Sigma^{(1)}$$

<center>Variance break in series 1</center>

$$\begin{bmatrix} \boldsymbol{\sigma_1^{(2)}} & 0 & 0 & 0 \\ 0 & \sigma_2^{(1)} & 0 & 0 \\ 0 & 0 & \sigma_3^{(1)} & 0 \\ 0 & 0 & 0 & \sigma_4 \end{bmatrix} \begin{bmatrix} 1 & \rho_{12}^{(1)} & \rho_{13}^{(1)} & \rho_{14} \\ \rho_{12}^{(1)} & 1 & \rho_{23}^{(1)} & \rho_{24} \\ \rho_{13}^{(1)} & \rho_{23}^{(1)} & 1 & \rho_{34} \\ \rho_{14} & \rho_{24} & \rho_{34} & 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{\sigma_1^{(2)}} & 0 & 0 & 0 \\ 0 & \sigma_2^{(1)} & 0 & 0 \\ 0 & 0 & \sigma_3^{(1)} & 0 \\ 0 & 0 & 0 & \sigma_4 \end{bmatrix} = \begin{bmatrix} \boldsymbol{\sigma_1^{2(2)}} & \boldsymbol{\sigma_{12}^{(2)}} & \boldsymbol{\sigma_{13}^{(2)}} & \boldsymbol{\sigma_{14}^{(2)}} \\ \boldsymbol{\sigma_{12}^{(2)}} & \sigma_2^{2(1)} & \sigma_{23}^{(1)} & \sigma_{24}^{(1)} \\ \boldsymbol{\sigma_{13}^{(2)}} & \sigma_{23}^{(1)} & \sigma_3^{2(1)} & \sigma_{34}^{(1)} \\ \boldsymbol{\sigma_{14}^{(2)}} & \sigma_{24}^{(1)} & \sigma_{34}^{(1)} & \sigma_4^2 \end{bmatrix} = \Sigma^{(2)}$$

<center>Variance break in series 3</center>

$$\begin{bmatrix} \sigma_1^{(2)} & 0 & 0 & 0 \\ 0 & \sigma_2^{(1)} & 0 & 0 \\ 0 & 0 & \boldsymbol{\sigma_3^{(2)}} & 0 \\ 0 & 0 & 0 & \sigma_4 \end{bmatrix} \begin{bmatrix} 1 & \rho_{12}^{(1)} & \rho_{13}^{(1)} & \rho_{14} \\ \rho_{12}^{(1)} & 1 & \rho_{23}^{(1)} & \rho_{24} \\ \rho_{13}^{(1)} & \rho_{23}^{(1)} & 1 & \rho_{34} \\ \rho_{14} & \rho_{24} & \rho_{34} & 1 \end{bmatrix} \begin{bmatrix} \sigma_1^{(2)} & 0 & 0 & 0 \\ 0 & \sigma_2^{(1)} & 0 & 0 \\ 0 & 0 & \boldsymbol{\sigma_3^{(2)}} & 0 \\ 0 & 0 & 0 & \sigma_4 \end{bmatrix} = \begin{bmatrix} \sigma_1^{2(2)} & \sigma_{12}^{(2)} & \boldsymbol{\sigma_{13}^{(3)}} & \sigma_{14}^{(2)} \\ \sigma_{12}^{(2)} & \sigma_2^{2(1)} & \boldsymbol{\sigma_{23}^{(2)}} & \sigma_{24}^{(1)} \\ \boldsymbol{\sigma_{13}^{(3)}} & \boldsymbol{\sigma_{23}^{(2)}} & \boldsymbol{\sigma_3^{2(2)}} & \boldsymbol{\sigma_{34}^{(2)}} \\ \sigma_{14}^{(2)} & \sigma_{24}^{(1)} & \boldsymbol{\sigma_{34}^{(2)}} & \sigma_4^2 \end{bmatrix} = \Sigma^{(3)}$$

<center>Variance break in series 2 and correlation breaks between series 1, 2 and 3</center>

$$\begin{bmatrix} \sigma_1^{(2)} & 0 & 0 & 0 \\ 0 & \boldsymbol{\sigma_2^{(2)}} & 0 & 0 \\ 0 & 0 & \sigma_3^{(2)} & 0 \\ 0 & 0 & 0 & \sigma_4 \end{bmatrix} \begin{bmatrix} 1 & \boldsymbol{\rho_{12}^{(2)}} & \boldsymbol{\rho_{13}^{(2)}} & \rho_{14} \\ \boldsymbol{\rho_{12}^{(2)}} & 1 & \boldsymbol{\rho_{23}^{(2)}} & \rho_{24} \\ \boldsymbol{\rho_{13}^{(2)}} & \boldsymbol{\rho_{23}^{(2)}} & 1 & \rho_{34} \\ \rho_{14} & \rho_{24} & \rho_{34} & 1 \end{bmatrix} \begin{bmatrix} \sigma_1^{(2)} & 0 & 0 & 0 \\ 0 & \boldsymbol{\sigma_2^{(2)}} & 0 & 0 \\ 0 & 0 & \sigma_3^{(2)} & 0 \\ 0 & 0 & 0 & \sigma_4 \end{bmatrix} = \begin{bmatrix} \sigma_1^{2(2)} & \boldsymbol{\sigma_{12}^{(3)}} & \boldsymbol{\sigma_{13}^{(4)}} & \sigma_{14}^{(2)} \\ \boldsymbol{\sigma_{12}^{(3)}} & \boldsymbol{\sigma_2^{2(2)}} & \boldsymbol{\sigma_{23}^{(3)}} & \boldsymbol{\sigma_{24}^{(2)}} \\ \boldsymbol{\sigma_{13}^{(4)}} & \boldsymbol{\sigma_{23}^{(3)}} & \sigma_3^{2(2)} & \boldsymbol{\sigma_{34}^{(2)}} \\ \sigma_{14}^{(2)} & \boldsymbol{\sigma_{24}^{(2)}} & \boldsymbol{\sigma_{34}^{(2)}} & \sigma_4^2 \end{bmatrix} = \Sigma^{(4)}$$

Note: Example of crisis and contagion events resulting in four covariance matrix regimes $\Sigma^{(d)}$, $d = 1, \ldots, 4$. The left side of the term indicates the decomposed covariance matrix, $SRS = \Sigma$. All parameter changes are highlighted in bold. Crisis breaks out in market 1 first, then in market 3 and reaches market 2 last, whereas contagion occurs at the same time as the crisis outbreak in market 2. Market 4 remains completely unaffected.

a multivariate system is proposed in **?**, and we adopt this approach. **?** propose an iterative procedure for separating breaks in the coefficients and the covariance matrix. We adapt this procedure to our setting, the difference being that we additionally allow for distinct breaks in correlations and volatilities. Our testing procedure operates as follows.

1. Determine the number of breakpoints in the VAR coefficients m_B, and estimate the break dates $\hat{T}_1^{(B)}, \ldots, \hat{T}_{m_B}^{(B)}$. Estimate the coefficients $\boldsymbol{\hat{B}}_t$ for the corresponding regimes.

2. Compute the residuals $\boldsymbol{\hat{\varepsilon}}_t$, and conditional on the breakpoints from Step 1, determine the number of breakpoints m_S in the standard deviations in $\boldsymbol{S}_t$ and the corresponding break dates $\hat{T}_1^{(S)}, \ldots, \hat{T}_{m_S}^{(S)}$. Estimate the regime-specific standard deviations $\hat{\sigma}_{it}$.

3. Compute the standardized residuals $\tilde{\varepsilon}_{it} = \hat{\varepsilon}_{it}/\hat{\sigma}_{it}$. Conditional on the breakpoints from Steps 1 and 2, determine the number of breakpoints m_R in the correlation matrix $\boldsymbol{R}_t$ and the corresponding break dates $\hat{T}_1^{(R)}, \ldots, \hat{T}_{m_R}^{(R)}$.

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4. Conditional on the breakpoints in $\boldsymbol{R}_t$, re-do Step 2.

5. Conditional on the breakpoints in $\boldsymbol{S}_t$ and $\boldsymbol{R}_t$ re-do Step 1.

6. Iterate between Steps 1 to 5 until the number of breakpoints and the estimated break dates do not change.

A few remarks must be made concerning the details of the algorithm. (i) All tests are based on (pseudo) likelihood ratio (LR) statistics relying on a multivariate normal distribution, as in **?**. Note that this does not mean that we assume a normal distribution or that the data are iid. Deviations from normality are accounted for by the asymptotic distribution of the resulting test statistics. Critical values are obtained by simulation. (ii) The maximum number of breakpoints m of each type must be set prior to the analysis. Furthermore, a minimum regime length between two breakpoints and at the boundaries of the sample must be chosen. In our application we allow for a maximum $m = 3$ breakpoints per model parameter and restrict each regime to contain at least 10% of all observations. (iii) The location of the multiple break locations can be estimated efficiently using the algorithms proposed in **?**. (iv) In Step 5, conditional on the breakpoints in the covariance matrix $\boldsymbol{\Sigma}_t$, a feasible generalized least squares estimator is applied to estimate the VAR; see **?** for details. (v) Confidence intervals for the break dates are obtained using a block bootstrap procedure. (vi) Although the break dates in $\boldsymbol{B}_t$, $\boldsymbol{S}_t$ and $\boldsymbol{R}_t$ are allowed to occur at distinct dates, all parameters within each type are assumed to share common breakpoints. This strong assumption results in favorable properties for the breakpoint test. However, when this assumption is violated the breakpoint test may not detect any structural change due to low power, or the break date estimates may be biased. In the next section, we discuss how to address this problem.

Until now we have described how to test for shift contagion, which is identified via an increase in correlations. Likewise, a decrease in correlation is a sign of a flight-to-quality behavior of investors. However, the combined structural breaks in mean, variance and correlations also help in studying the evolution of spillovers as measured by the dynamic spillover

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index of **??**, which we explain in Appendix A.2. An increase in the spillover index indicates the presence of what we termed spillover contagion. This concept identifies the direction of contagion and captures contagion that occurs with a time lag.

2.3. Testing for common breakpoints

One of the motivations for applying our test for contagion to multivariate data instead of relying on a bivariate analysis is that in the former case breakpoint tests have more power and produce more reliable estimates of the break dates (see **?, ?** and **?**). We illustrate these properties for the case of breakpoints in volatility and correlations in our Monte Carlo simulations in an online appendix of the paper. However, to exploit the advantages of having common breakpoints over a large set of parameters and equations, it is recommended that this assumption actually be questioned.

For each type of parameter we suggest testing whether the breakpoints in each equation can be assumed to occur at a common date.[2] A test for the coincidence of breakpoints in different model parameters is proposed by **?**. For the breakpoints in $\boldsymbol{B}_t$, $\boldsymbol{S}_t$ and $\boldsymbol{R}_t$, we test

$$H_0 : \text{All parameters share a common breakpoint}$$

against the alternative that the breakpoints are specific to each equation for $\boldsymbol{B}_t$ and $\boldsymbol{S}_t$ and for each variable pair in the case of the correlations in $\boldsymbol{R}_t$. This null hypothesis can be tested using a likelihood ratio test. The restricted likelihood assumes common breakpoints, whereas under the alternative, the number of breakpoints and their locations are determined for each equation/variable pair separately. Critical values are obtained using the following bootstrap algorithm.

1. Assuming common breakpoints, determine the number of structural breaks m and their

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[2]In the case of correlations, we test whether all breakpoints in pairwise correlations occur at the same date.

dates $\hat{T}_1, \ldots, \hat{T}_m$. Compute the log-likelihood under H_0, LL_0. Furthermore, determine the number of breakpoints, the break dates and the resulting log-likelihood LL_1 under the alternative. Compute the likelihood ratio statistic $LR = -2(LL_1 - LL_0)$.

2. Using the common breakpoints from Step 1, resample the multivariate observations using a block bootstrap scheme for each regime separately, i.e., T_1 observations from $t = 1, \ldots, T_1$, $T_2 - T_1$ observations from $t = T_1 + 1, \ldots, T_2$, etc.

3. Using the re-sampled data, determine the number of breaks, their locations and the log-likelihood under H_0 and H_1, and compute the bootstrap test statistic LR^*.

4. Repeat Steps 2 and 3 a large number of times to obtain the bootstrap distribution of the likelihood ratio statistic.

Note that in Step 2 the data are resampled either from the raw data, from $\hat{\varepsilon}_t$ or from $\tilde{\varepsilon}_t$, when testing the equality of the breaks in $\boldsymbol{B}_t$, $\boldsymbol{S}_t$ and $\boldsymbol{R}_t$, respectively.

If the null hypothesis of common breaks is rejected, we suggest studying the estimated breakpoints and their confidence intervals to determine subsets of the data that do share common breaks. Breakpoints in different parameters lying close to each other are an indication of common breaks in the corresponding subset of parameters. Although it is not possible to define what 'close' means in this situation, we suggest considering common breaks whenever the confidence intervals overlap. The presence of subsets with common breakpoints yields the same benefits in terms of the power and efficiency of the tests for structural breaks for the relevant subsets of parameters. Furthermore, the information pertaining to which variables/variable pairs can be grouped in the change point analysis is economically relevant information and may allow for interesting interpretations.

Although the search for subsets of the data that can be grouped is relevant from a statistical point of view, as well for the interpretation of results, the search does require the researcher to make a number of decisions. Most importantly, an initial guess for the grouping must be made. Furthermore, there may be mutually exclusive groupings, and the

researcher must chose one of them. The restrictions maximizing the overall likelihood should be considered in the case of conflicting groupings. Finally, overall, our procedure requires a large number of hypothesis tests, implying potential problems of multiple testing. Therefore, it is important to use conservative test sizes to control the overall size of the procedure. Because the number of hypothesis tests to be performed is not known prior to the analysis, we use a size of 1% in our application, but we note that in basically all cases we rejected a hypothesis the p-values were virtually equal to zero.

A complete Monte Carlo analysis that illustrates the small sample testing properties relevant to the STP is available in an online appendix of the article. It demonstrates some of the advantages of our approach. First, using multivariate systems when testing for contagion leads to higher power and a more reliable identification of the break dates. However, it is also observed that beyond a certain number of time series the rejection frequencies under an alternative hypothesis is decreasing; therefore, a certain saturation effect arises. Second, simulations illustrate that separating variance and correlation breaks is the recommended approach when the two do not break simultaneously.

3. Empirical Study

3.1. Contagion during the European debt crisis

We consider daily 10-year sovereign bond yield spreads of the 10 euro area countries Austria (AUS), Belgium (BEL), Finland (FIN), France (FRA), Greece (GRE), Ireland (IRE), Italy (ITA), the Netherlands (NET), Portugal (POR) and Spain (SPA) over the yield of Germany (GER). Data are extracted from Thomson Reuters datastream, and the sample covers the period Jan. 2, 2009 until Aug. 1, 2014. A preliminary analysis reveals that the unit root hypothesis cannot be rejected, and thus, the first differences in bond yield spreads are considered.

Concerning our model specification, $p = 1$ lag is chosen in the VAR considering the Bayesian Information Criterion (BIC). Because our sample covers the European debt crisis,

$q = 2$ exogenous variables are included to control for systemic risks. As in **?**, **?** and other studies, the Chicago Board Options Exchange Index (VIX) is included with one lag as a measure of global risk. Similarly to **?**, **?** and **?**, the lagged spread between the Euribor 3-month lending rate and the overnight reference rate EONIA is additionally included as a measure of European financial market stress.

Given our large sample length of $T = 1,456$, a moderate trimming $\kappa = 0.1$ has been considered, allowing for a minimal regime length of $h = \lceil \kappa T \rceil = 146$ days. As noted above, we allow for a maximum of $m = 3$ breaks in each type of parameter, which corresponds to the number of (common) breakpoints found in **?**. As previously described, we sequentially search for structural breaks assumed to be common within each class of parameters. Thus, in Step 1, we search for a maximum of three simultaneous co-breaks in $n(n + 1 + 2) = 130$ regression coefficients, Step 2 looks for a maximum of three simultaneous co-breaks in $n = 10$ standard deviations (conditional on the breaks found in Step 1), and Step 3 looks for a maximum of three simultaneous co-breaks in $\frac{n(n-1)}{2} = 45$ correlation coefficients (conditional on the breaks found in Steps 1 and 2).

Table 2 reports the results of the break tests in mean coefficients as well as in the variances. Note that the results presented are the final results obtained after iterating the breakpoint detection until convergence, as explained in Section 2. The STP detects *three* significant common breaks in both cases. Several remarks must be made: First, the three breaks in the conditional mean equations are common across all equations (at a p-value of 0.18), i.e., the assumption of common breakpoints in $\boldsymbol{B}$ cannot be rejected. Second, the breaks in the variances can also be assumed to be common across all equations (p-value of 0.32). As a result of having common breakpoints, the confidence intervals around the break dates tend to be relatively narrow, which is predicted by theory and by our simulation results. Third, the null hypothesis that the breaks in variances and means are located at common dates is rejected with a p-value of 0. This finding supports our intuition that the breaks in different types of parameters occur at distinct times. Therefore, assuming their synchronicity would

Table 2: Breakpoint analysis of conditional mean and variance

Mean regression	$m_B = 3$ breaks		
max LR	1010.63		
99% critical value	470.71		
p-value	0		
Break date estimates	15-Aug-2011	12-Mar-2012	02-Oct-2012
95% confidence intervals	[20-Apr-11,19-Aug-11]	[27-Feb-12,12-Mar-12]	[02-Oct-12,13-Jan-14]
Covariates	coefficient change	coefficient change	coefficient change
$Const$	-3.635	+3.709	-0.161
AUS_{t-1}	+1.757	-4.764	+5.308
BEL_{t-1}	+0.447	+5.577	-4.478
FIN_{t-1}	+8.131	-7.895	-0.838
FRA_{t-1}	-0.001	-1.139	+1.357
GRE_{t-1}	+0.646	-0.453	+ 0.297
IRE_{t-1}	-0.740	-1.776	+1.032
ITA_{t-1}	-2.741	+ 1.557	-0.493
NET_{t-1}	-13.218	+10.529	+2.320
POR_{t-1}	-0.063	+0.016	-0.279
SPA_{t-1}	+ 3.189	-2.425	-0.354
VIX_{t-1}	+ 0.066	-0.099	+ 0.027
$(Euribor - EONIA)_{t-1}$	2.538	-0.869	-1.100

Standard deviations	$m_S = 3$ breaks		
max LR	6877.00		
99% critical value	1222.76		
p-value	0		
Break date estimates	22-Jan-2010	05-Jul-2011	01-Oct-2012
95% confidence intervals	[30-Oct-09,11-Mar-10]	[22-Apr-11,01-Aug-11]	[03-Sep-12,25-Oct-12]
Series	standard deviation change	standard deviation change	standard deviation change
AUS	-0.001	+0.028	-0.034
BEL	+0.020	+0.036	-0.057
FIN	-0.002	+0.009	-0.011
FRA	+0.004	+0.040	-0.039
GRE	+0.242	+1.303	-1.421
IRE	+0.084	+0.012	-0.099
ITA	+0.022	+0.095	-0.088
NET	-0.002	+0.010	-0.013
POR	+0.116	+0.092	-0.140
SPA	+0.045	+0.068	-0.077

Note: Test results, estimated break dates and corresponding changes in coefficients for the conditional mean parameters (upper panel) and the standard deviations (lower panel). 95% confidence intervals we obtained via a block bootstrap. For breaks in the mean regression, changes in coefficients are summarized for each regressor listed in the first column: The shifts in the estimated regression coefficients are sums across all equations.

lead to substantial biases in the estimated break dates and the regime specific parameter estimates. However, it is noteworthy that two out of the three estimated breaks lie very close to each other, namely the breaks in July and August 2011 and the ones in October 2012, which differ only by one day. This suggests that the rejection of common breaks is only driven by one of the three break dates differing.

The three breaks in conditional means (August 2011, March 2012 and October 2012) represent important systemic changes in Europe: the issuance of 4.6 billion euros to assist Ireland and Romania, as well as several meetings of the Ecofin and the first meeting of the European Systemic Board are associated with the August 2011 break. The March 2012 and October 2012 breaks can be associated with the different positive speeches of M. Draghi and O. Rehn and the first IMF/ECB/EC reports about the improvement of financial stability in the euro area. A change in the conditional mean parameters implies a change in the transmission mechanisms of a shock and can be associated with systemic risk deterioration or improvements.

The following findings reveal that all euro area countries faced three main breaks in volatility. The first one occurring in January 2010 was common to all European countries and was initiated by the Eurostat report questioning the Greek figures on public debt and deficit. Financial markets became concerned regarding the potential default of Greece, asking for a higher risk premium for holding Greek public bonds. Nevertheless, whereas this shock was associated with an increase in volatility for most of the countries, volatility actually decreased (by a small margin) for Austria, Finland and the Netherlands. This fact signals the heterogeneity across the different European countries: Volatility increases over the entire area except in these three countries, which are often considered the most virtuous ones in term of public debt. The next volatility break is found in July 2011, when the Eurogroup meeting stated a new financial plan to support Greece. Volatility in the bond markets was again exacerbated. Finally, in October 2012 we notice a return to a quieter regime and lower volatility following the IMF/EC reports signaling that Ireland and Portugal will satisfy the

objectives conditioning the safety plans. Both breaks are common in terms of timing and directions of volatility change among all European countries.

The third step of the STP consists in testing for the presence of breaks in the correlation coefficients conditional on those previously found in the mean and the variance. Ultimately, no common break is detected in the correlation matrix of all euro area countries. The p-value is equal to 0.36 testing the null of no breaks versus the alternative of the $m = 1$ single break hypothesis. This finding clearly indicates that Europe is heterogeneous, with some countries observing a decrease in the yield spread, indicating a lower risk premium, whereas others face an increase in refinancing capacities, and highlights that the diffusion (or contagion) of the shocks is not the same across the euro area. Therefore, we pursue our analysis by re-considering the nature of the correlation breaks. To be specific, in a first step, we analyze the pairwise correlations and study the different breaks corresponding to all country pairs. Not only is this information interesting in its own right; the procedure used to obtain the information is also a preliminary stage required to begin with the multivariate investigation of contagion. Using the results of the first step helps us to identify suitable groups of countries to test for the presence of contagion clubs, i.e., subsets of countries that have common correlation breaks. Therefore, we try to identify subsets of countries that may in fact be characterized by common correlation breaks. We consider several clusters of countries as candidates based on the estimated break dates of pairwise correlations[3] and perform the test for common breakpoints in correlation against the alternative of distinct breaks in pairwise correlations introduced in Section 2.3. Due to the high computational burden, the test results are based on 100 bootstrap replications. Several clusters of countries emerge. First, common correlation breaks cannot be rejected for AUS–POR–SPA at a p-value of 0.35. The correlations move in the same direction for the first two breaks between these

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[3]The candidates for the clusters are BEL–FIN–FRA–ITA–NET, AUS–POR–SPA, BEL–FRA–NET, BEL–FIN–ITA, BEL–ITA–NET, FIN–FRA–NET, BEL–FIN–FRA, and BEL–FIN–NET

Table 3: Breakpoints in correlations part 1

	AUS	BEL	FIN	FRA	GRE
AUS		(0.646) +0.231 −0.052 −0.320 (0.505)	0.497	0.644 +0.182 −0.475 +0.461 0.811	0.278
BEL	09-Jan-2012 [26-May-12,13-Jan-12] 07-Aug-2012 [31-Jul-12,07-Aug-12] 27-Feb-2013 [27-Feb-13,09-Jan-14]		0.287 +0.471 −0.365 +0.039 0.432	0.725 −0.020 +0.222 −0.332 0.595	0.335
FIN	-	03-Aug-2009 [24-Jul-09,05-Aug-09] 25-Feb-2010 [23-Feb-10,28-Mar-12] 07-Nov-2012 [02-Dec-10,18-Oct-13]		0.356 +0.163 +0.340 −0.413 0.446	0.175
FRA	04-Nov-2010 [20-Oct-09,05-Mar-12] 21-May-2013 [27-May-11,21-May-13] 11-Dec-2013 [11-Dec-13,09-Jan-14]	26-Feb-2010 [06-Aug-09,22-Feb-11] 02-Mar-2012 [20-Sep-10,16-Mar-12] 08-Oct-2012 [24-Sep-12,03-Jan-14]	11-Aug-2009 [24-Jul-09,14-Sep-09] 06-Apr-2010 [03-Mar-10,09-Apr-10] 01-Nov-2013 [27-Oct-10,13-Nov-13]		0.293
GRE	-	-	-	-	-
IRE	31-Mar-2010 [31-Aug-09,16-Jun-11] 13-Jun-2013 [04-Nov-10,29-May-13] 08-Jan-2014 [03-Jan-14,09-Jan-14]	-	-	28-May-2010 [26-Oct-09,16-Sep-11] 10-Apr-2013 [21-Dec-10,06-May-13] 29-Nov-2013 [31-Oct-13,09-Jan-14]	-
ITA	-	03-Aug-2009 [24-Jul-09,05-Aug-09] 25-Feb-2010 [23-Feb-10,28-Mar-12] 07-Nov-2012 [02-Dec-10,18-Oct-13]	03-Aug-2009 [24-Jul-09,05-Aug-09] 25-Feb-2010 [23-Feb-10,28-Mar-12] 07-Nov-2012 [02-Dec-10,18-Oct-13]	-	-
NET	-	26-Feb-2010 [06-Aug-09,22-Feb-11] 02-Mar-2012 [20-Sep-10,16-Mar-12] 08-Oct-2012 [24-Sep-12,03-Jan-14]	28-Jul-2009 [24-Jul-09,04-Aug-09] 03-Mar-2010 [17-Feb-10,16-Aug-12] 01-Apr-2013 [28-Sep-10,26-Dec-13]	26-Feb-2010 [06-Aug-09,22-Feb-11] 02-Mar-2012 [20-Sep-10,16-Mar-12] 08-Oct-2012 [24-Sep-12,03-Jan-14]	-
POR	13-Aug-2009 [24-Jul-09,03-Sep-09] 31-Mar-2010 05-Mar-10,09-Apr-13] 30-Dec-2013 [02-Nov-10,09-Jan-14]	-	11-Aug-2009 [24-Jul-09,21-Aug-09] 02-Apr-2010 [04-Mar-10,04-Dec-13]	03-Mar-2010 [04-Aug-09,31-Aug-10] 29-Apr-2011 [28-Sep-10,10-Aug-11] 01-Mar-2012 [08-Dec-11,30-Jul-13]	-
SPA	13-Aug-2009 [24-Jul-09,03-Sep-09] 31-Mar-2010 05-Mar-10,09-Apr-13] 30-Dec-2013 [02-Nov-10,09-Jan-14]	12-Jan-2011 [24-Jul-09,12-Jan-11] 04-Aug-2011 [04-Aug-11,09-Nov-11] 31-May-2012 [24-Feb-12,01-Jan-14]	27-Jul-2009 [24-Jul-09,25-Aug-09] 02-Apr-2010 [16-Feb-10,05-Aug-10] 28-Feb-2011 [25-Oct-10,06-Dec-13]	-	-

Note: This table should be read jointly with Table 4. Correlation breaks obtained by applying sequential procedure from Section 2. Break date point estimates (95% confidence intervals obtained by a block bootstrap in parentheses) for correlation breaks are reported in the lower triangle, whereas the upper triangle reports the correlation before the first break, the changes in correlations corresponding to the breakpoints and the correlation after the last break.

Table 4: Breakpoints in correlations part 2

	IRE	ITA	NET	POR	SPA
AUS	0.547 −0.244 +0.191 −0.276 0.217	0.511	0.561	0.611 +0.064 −0.375 −0.068 0.232	0.464 +0.294 −0.281 −0.154 0.323
BEL	0.436	0.641 +0.157 −0.053 −0.282 0.463	0.741 −0.229 +0.088 −0.154 0.446	0.434	0.643 +0.217 −0.176 −0.264 0.421
FIN	0.262	0.244 +0.392 −0.352 −0.019 0.265	0.294 +0.535 −0.224 −0.256 0.348	0.194 +0.388 −0.401 0.181	0.206 +0.443 −0.247 −0.196 0.206
FRA	0.557 −0.251 +0.234 −0.291 0.249	0.555	0.708 −0.036 −0.041 −0.159 0.472	0.621 −0.165 −0.335 +0.215 0.337	0.509
GRE	0.375	0.410	0.205	0.466	0.389
IRE		0.553	0.465 −0.319 +0.340 −0.289 0.197	0.543	0.544
ITA	-		0.521 +0.227 −0.259 −0.191 0.298	0.759 −0.313 +0.286 0.732	0.515 +0.359 −0.058 +0.077 0.893
NET	15-Oct-2010 [13-Oct-09,28-Jun-11] 10-Apr-2013 [08-Jun-11,19-Jun-13] 09-Jan-2014 [31-Oct-13,09-Jan-14]	05-Aug-2009 [24-Jul-09,05-Aug-09] 25-Feb-2010 [25-Feb-10,08-Sep-10] 31-Mar-2011 [20-Sep-10,27-Aug-13]		0.504 +0.153 −0.278 −0.205 0.174	0.743 −0.363 −0.255 +0.282 0.376
POR	-	25-Nov-2010 [24-Aug-10,22-Feb-11] 30-Dec-2013 [22-Jun-11,09-Jan-14]	11-Aug-2009 [24-Jul-09,11-Aug-09] 03-Mar-2010 [03-Mar-10,23-Aug-10] 15-Mar-2011 [27-Sep-10,29-Jun-12]		0.528 +0.242 −0.259 +0.258 0.769
SPA	-	01-Oct-2009 [13-Aug-09,10-Dec-09] 03-Nov-2011 [23-Apr-10,14-Mar-13] 28-Oct-2013 [21-Jan-13,25-Dec-13]	03-Mar-2010 [18-Sep-09,23-May-10] 05-Nov-2012 [23-Sep-10,16-Nov-12] 10-Jun-2013 [29-May-13,09-Jan-14]	13-Aug-2009 [24-Jul-09,03-Sep-09] 31-Mar-2010 [05-Mar-10,09-Apr-13] 30-Dec-2013 [02-Nov-10,09-Jan-14]	

Note: This table should be read jointly with Table 3. Correlation breaks obtained by applying sequential procedure from Section 2. Break date point estimates (95% confidence intervals obtained by a block bootstrap in parentheses) for correlation breaks are reported in the lower triangle, whereas the upper triangle reports the correlation before the first break, the changes in correlations corresponding to the breakpoints and the correlation after the last break.

countries, while the last break on Dec. 30, 2013 is a sign of shift contagion between Portugal and Spain but of flight-to-quality towards Austria. The second cluster found is BEL–FRA–NET at a p-value of 0.85. This second cluster stresses the potential contagion between this core of European countries, which share strong cross-border banking activities (Dexia, ING, etc.). Finally, we observe a third cluster composed of BEL–FIN–ITA at a p-value of 0.4. For all the remaining candidate clusters, the null hypothesis of common correlation breaks is rejected with a p-value of 0.

Tables 3 and 4 gather the results of the breakpoint analysis in the correlations. The restrictions of common breakpoints from the identification of clusters have been imposed on these results. The tables should be read jointly, forming a $10{\times}10$ matrix with estimated break dates in the lower triangular part and the estimated correlations, as well as their changes, in the upper triangular part. In Table 5, we summarize the timeline of the estimated structural breaks in all parameters, which contains the same qualitative information as the previous three tables but likely makes it easier for the reader to extract the pertinent information.

To interpret the results of these tables, it is worth recalling that the presence of breaks among the correlations can either be associated with a decrease or with an increase in correlation. The first case suggests that an increase in the yield spread of a particular country coincides with a cheaper refinancing rate in another one. This virtuous transmission is usually labeled as flight-to-quality. In the opposite case of a synchronous upward movement in the yield spread of the other country, contagious transmission is thus supported.

We have already observed several breaks in correlation for 13 country pairs in and around August 2009, before the first estimated volatility break of January 2010. All changes in correlation are positive, providing preliminary evidence of the danger of contagion. Interestingly, the correlation between Portugal and Spain rises by 0.318 to a high 0.841 on July 29, 2009, and the correlation between Italy and Spain rises by 0.359 to a remarkable level of 0.874 on Oct. 1, 2009. It thus appears that the European sovereign debt crisis had already begun in 2009 before the occurrence of the first variance break, as a consequence of government bailouts

Table 5: Timeline of estimated structural breaks

Date			Break	Countries	Date			Break	Countries
2009	Jul	27	↑ R	FIN–SPA	2012	Jan	09	↑ R	AUS–BEL
		28	↑ R	FIN–NET		Mar	01	↑ R	FRA–POR
	Aug	03	↑ R	BEL–FIN–ITA			02	↑ R	BEL–FRA, BEL–NET
	Aug	05	↑ R	ITA–NET				↓ R	FRA–NET
		11	↑ R	FIN–FRA, FIN–POR, NET–POR			12	B	euro area
		13	↑ R	AUS–POR–SPA		May	31	↓ R	BEL–SPA
	Oct	01	↑ R	ITA–SPA		Aug	07	↓ R	AUS–BEL
2010	Jan	22	↑ S	BEL, FRA, GRE, IRE, ITA, POR, SPA		Oct	01	↓ S	euro area
			↓ S	AUS, FIN, NET			02	B	euro area
	Feb	25	↓ R	BEL–FIN–ITA, ITA–NET			08	↓ R	BEL–FRA–NET
		26	↓ R	BEL–FRA–NET		Nov	05	↓ R	NET–SPA
	Mar	03	↓ R	FIN–NET, FRA–POR, NET–POR, NET–SPA			07	↓ R	BEL–ITA, FIN–ITA
		31	↓ R	AUS–IRE, AUS–POR–SPA				↑ R	BEL–FIN
	Apr	02	↓ R	FIN–POR, FIN–SPA	2013	Feb	27	↑ R	AUS–BEL
		06	↑ R	FIN–FRA		Apr	01	↓ R	FIN–NET
	May	28	↓ R	FRA–IRE			10	↑ R	FRA–IRE, IRE–NET
	Oct	15	↓ R	IRE–NET		May	21	↓ R	AUS–FRA
	Nov	04	↑ R	AUS–FRA		Jun	10	↑ R	NET–SPA
		25	↓ R	ITA–POR			13	↑ R	AUS–IRE
2011	Jan	12	↑ R	BEL–SPA		Oct	28	↑ R	ITA–SPA
	Feb	28	↓ R	FIN–SPA		Nov	01	↓ R	FIN–FRA
	Mar	15	↓ R	NET–POR			29	↓ R	FRA–IRE
		31	↓ R	ITA–NET		Dec	11	↑ R	AUS–FRA
	Apr	29	↓ R	FRA–POR			30	↑ R	ITA–POR, POR–SPA
	Jul	05	↑ S	euro area				↓ R	AUS–POR, AUS–SPA
	Aug	04	↓ R	BEL–SPA	2014	Jan	08	↓ R	AUS–IRE
		15	B	euro area			09	↓ R	IRE–NET
	Nov	03	↓ R	ITA–SPA					

Note: This table summarizes the results from Tables 2 to 4. B, S and R represent breaks in parameters for the conditional mean, the variance and the correlation, respectively. The arrows indicate whether the parameters have increased or decreased at the corresponding breakpoints.

during the Global Financial Crisis and growing concerns about accumulated high debt levels, following the financial distress of countries such as Latvia or Hungary. This interpretation is supported by several reports, such as the one published by the European Commission on January 2009, which gave rise to concerns about debt sustainability (**?** and **?**). Following the first volatility break in January 2010, in early 2010 we observe correlation breaks in a total of 19 country pairs. Almost all of those correlation breaks are associated with a decrease in correlations. The breaks clearly indicate the presence of a flight-to-quality mechanism, in favor of Austria, Finland and the Netherlands, which benefit from a decrease in their yield spread and thus more interesting refinancing conditions.[4] As previously mentioned, this result highlights a high level of heterogeneity among euro area countries, supporting the conclusions of **?** and **?**, who find that the EMU periphery mainly spread to other periphery countries as well as to France and Belgium, but not to Austria, Finland or the Netherlands. In the second half of 2010 and the first half of 2011, several further breaks in correlations occur; thus, the flight-to-quality mechanisms appears to persist.

After the second volatility break in July 2011, which signals the entry into a higher volatility period, the evolution of the correlation becomes ambiguous. The STP detects only a few breaks in bilateral correlations, both positive and negative ones. It can be envisioned that market participants realize that the euro area is not particularly homogeneous and begin to make arbitrage between countries, leading to various effects on correlations. Similar conclusions can be drawn after the third variance shock in October 2012, after which relatively few breaks in correlation are observed. In late 2013 and early 2014, again several breaks are observed over a short period, involving 10 country pairs. Correlations among periphery countries are rising (POR–SPA or POR–ITA), but there is also evidence of an amplification of the flight-to-quality mechanism involving mostly Austria and the Netherlands.

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[4]It is important to note that we are considering yield spreads over the German 10-year bond, and therefore, Germany itself is not considered in this study.

A further noteworthy observation are the correlation breaks involving France beginning in 2012 and lasting until late 2013. Most breaks were negative, but there were also positive ones. Interestingly, France had two breaks with each the Netherlands and Belgium in 2012 and two breaks with each Austria and Ireland in 2013. These findings highlight the uncertainty concerning the role of France in the crisis and specific problems associated with the French economy.

Another interesting and striking result is that STP does not detect any correlation break involving Greece and only very few ones involving Ireland (only with Austria, France and the Netherlands), implying that we reject contagion from Greece/Ireland to the rest of the European countries. It appears that the contemporaneous shock transmission to the rest of Europe is not intensified, i.e., that no shift contagion is observed. Formally, this corroborates our idea of separating the dates of breaks in conditional means, variances and correlations. Furthermore, the important de-leveraging of the German and Dutch banks of their Greek assets (see **?**), as well as the regulation measures taken by the Irish government, may explain such an independence of the Greek/Irish yield spread. The consequence of such a finding is that a Greek default[5] will most likely not be characterized by contagion to the rest of Europe. This result has radical policy consequences and before being accepted should be assessed by further studies relying on complementary contagion approaches.

3.2. Spillovers vs shift contagion

To better understand the situation of Greece, but also to gain additional insight concerning other problematic countries, we rely on the spillover approach developed by **??** and implemented in the same context as ours by **?**. The authors calculate indexes based on the analysis of the forecast error variance decomposition and conclude that contagion is supported when abrupt changes (i.e., breaks) in spillovers are observed. This approach is de-

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[5]The situation in Ireland has sharply improved in recent years, prompting us not to consider the default of Ireland as a credible case.
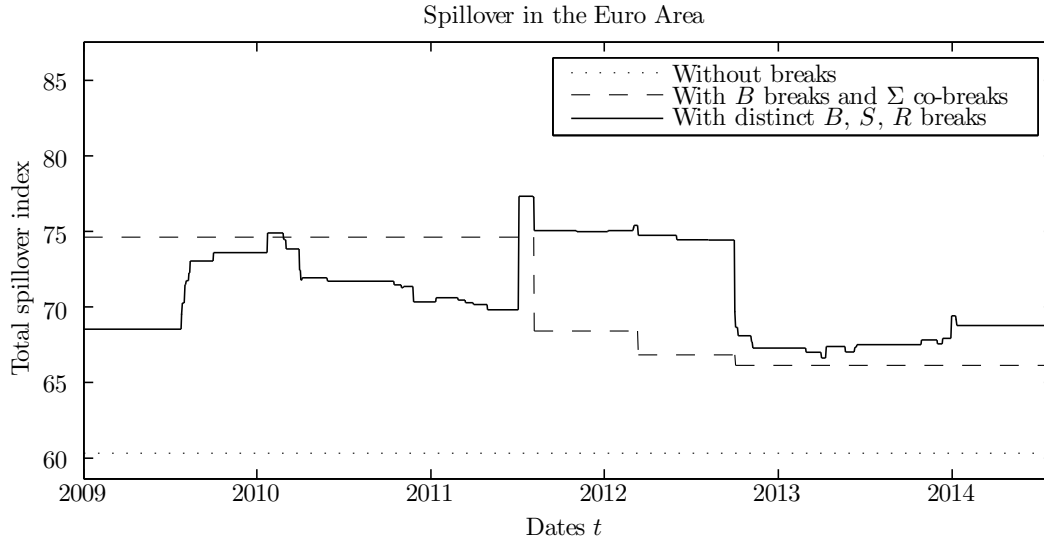
Figure 1: Total spillover index, 20-step-ahead forecast, between 02 January 2009 and 01 August 2014. Either following a model without breaks, with breaks in the mean parameters B as estimated by the STP and allowing covariance breaks at the same dates, or with distinct breaks in the mean parameters B, the variances S and the correlation coefficients R according to the STP.

tailed in Appendix A.2. We consider a 20-step-ahead forecast error variance decomposition[6], corresponding to approximately one trading month, which is calculated for each country (transmitting or receiving) and the euro area as a whole.

Three VAR models are considered to compute spillover indexes: The first one is the VAR model without any structural breaks. The second one follows the approach of **?**. In particular, the model assumes common breaks in the conditional mean and the covariance matrix of the errors. The third model is the most complete one, which is obtained by applying the STP and assuming distinct breaks in the conditional means, variances and correlations. The breaks within the correlation matrix are also distinct, and common correlation breaks are only used for the groups of countries identified by our tests for the abovementioned common correlation breaks. As a consequence, a large number of regimes are observed, which leads to notable time variation in the spillover indexes. This number of regimes guarantees a more refined interpretation of the results because distinct events can be separated. Figure 1 represents the total spillover index, calculated as the sum of either all transmitted spillover indexes or

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[6]The results for other forecast horizons such as 10 steps are virtually identical to the ones reported here.

all received spillover indexes, excluding each own variation share. The figure reveals that spillover from other countries explains more than 60% of the interest rate dynamics, which is extremely high. In addition, this figure illustrates the additional information gained by allowing for distinct breaks in variance and correlation as done by the STP. When considering the VAR model without breaks, we obtain a spillover index of approximately 60%, whereas the complete model with break leads to spillovers explaining more than 70% of the forecast error variance. Furthermore, separating the break dates in conditional means, variances and correlations (solid line in the graph) yields notably different results from the model that assumes common breakpoints in all parameters (dashed line).[7] This result unambiguously suggests that not considering the presence of breaks leads to an underestimation of the importance of spillovers, whereas setting unjustified restrictions on the break dates may severely affect the resulting dynamics in the spillover index. Considering the VAR model with distinct breaks, we observe a time-varying path of the spillover index. The changes in the index correspond to the breakpoints identified in the previous section and allow us to assess their importance in terms of shock transmissions. The index appears to be particularly high between mid-2011 and mid-2012 (with a value around 75%), a period during which the crisis was particularly intense, but then decreases to stabilize at approximately 70%. The break observed in mid-2011 supports the idea of contagion. Still, we must be careful in comparing with the concept of contagion developed in the previous subsection. In this case, the concept corresponds to an abrupt increase in dynamic spillovers, i.e., the spillover contagion, whereas previously it corresponded to an increase in contemporaneous correlation, which is typically labeled shift contagion. The increase in spillover associated with no break in correlation implies that contagion is not immediate but takes time before occurring. Again, this difference in results supports our intuition of separating the different types of breaks to

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[7]These differences also appear when looking at the directional spillovers for individual countries; detailed results are available from the authors upon request.
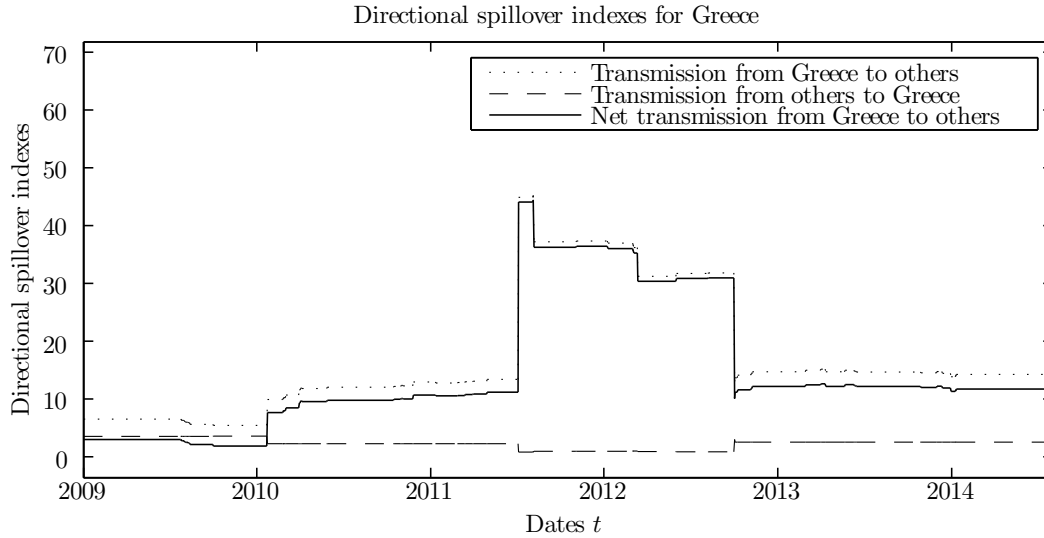
Directional spillover indexes for Greece

Figure 2: Directional spillovers of Greece, 20-step-ahead forecast, between 02 January 2009 and 01 August 2014, following a model with distinct breaks in the mean parameters B, the variances S and the correlation coefficients R according to the STP. Spillover transmitted from Greece to other countries, spillover transmitted from other countries to Greece, and net spillover transmitted from Greece to other countries.

obtain a more refined picture of the situation.

Now, focusing on the specific case of Greece[8], Figure 2 reports the directional spillover indexes obtained for this country. We observe a positive net transmission from Greece to the other countries, indicating that the linkages to Greece are structural sources of spillovers among European countries. Beginning in 2010, several of the structural breaks show that the transmission from Greece has increased severely, before decreasing sharply in late 2012. This result confirms the previous findings of **?** and **?**. However, this finding also illustrates that even if we have not obtained any evidence for an increase in contemporaneous correlation between Greece and the rest of Europe most recently, the spillover index remains extremely high (around 10), even after having decreased in 2012. From a political perspective, the message is thus more pessimistic than the one gathered from the previous subsection. A default of Greece may not immediately affect the other countries because contemporaneous correlations are not subject to breaks, but still the transmission should take place soon after. The positive result is that even if the negative wave were to hit the European countries,

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[8]Results for all the other countries can be found in Appendix A.3.

authorities would have some time left to set up firewalls.

A positive net transmission is also observed for three other periphery countries (Italy, Spain and Portugal), which decreased in the middle of the sample followed by an increase in late 2012. Although the directional transmission indexes of these countries are lower than the Greek index (5 for Italy and Portugal and 2.5 for Spain), we observe that the indexes have increased since mid-2012, and according to the contagion analysis they belong to clusters involving European core countries. This result suggests that a default, or to a lesser extent a higher tension of repayment of the sovereign debt, could immediately impact European core countries (via a break in correlation) but would also transmit dynamically. This result thus suggests that European authorities should implement specific policies to limit this potential transmission.

Finally, we remark on the situation of Ireland, for which the net transmission has declined sharply in mid-2011, confirming that the financial markets have recognized the successful crisis management in this country.

4. Conclusion

This paper proposes a new approach to testing for contagion that has three distinct features. First, the approach distinguishes breaks in conditional mean, volatility, and correlation using a sequence of tests, allowing for distinct dates of breaks. Second, the approach is implemented within a multivariate system, instead of relying on bivariate systems as in many other studies. The tests rely on the techniques developed in **?**. Third, although structural breaks are generally assumed to occur at distinct dates, we test whether some parameters share a common breakpoint using an approach similar to that proposed in **?**. This procedure can lead to a simpler model and to more reliable testing results but also allows the researcher to challenge restrictive assumptions concerning the equality of break dates. The setup of the STP is supported by the analysis of small sample properties, which favor multivariate break testing as long as the breaks in fact occur simultaneously.

The application of our approach to the recent European debt crisis offers new insight into the way the crisis has spread over the region. The implementation of STP indicates that mean breaks are common over the euro area countries. The same is true for the volatility breaks. Still, these two types of breaks are observed to occur at distinct dates. Conditional on these two types of breaks, a large number of distinct breaks in bivariate correlations occur. Many of these breaks cluster around a few dates and common correlation breaks can be assumed to occur within a few small groups of countries. We observe some contagion clubs but notice that Greece and Ireland belong to none of them. This finding is particularly interesting for Greece, which constitutes a case study for European authorities. Such findings support the idea that it is not recommendable to assume all structural breaks are coincident, as it is currently assumed in most of the literature. Crisis periods begin with mean and volatility breaks, and shocks are transmitted to other markets with some delay. However, there is also evidence of sudden increases in correlation, indicating the presence of immediate (shift) contagion. To refine our conclusions, we consider the spillover index proposed by **??**. We notice that global spillovers vary significantly over time due to the break dates estimated by our algorithm. A high level of spillover is observed between mid-2011 and mid-2012 during the peak of the turmoil. More specifically, periphery countries transmit an enormous part of their shock to the rest of Europe, the most important country being Greece, with a net spillover index of up to 40. This finding demonstrates that even if Greek distress may not necessarily be contagious contemporaneously, it does transmit to the rest of Europe, hence confirming the findings of **?**.

Some guidelines for European policy can be derived from the results of this paper. First, a default or a higher risk premium on the Greek debt will not immediately affect the yield spreads of other euro area countries. Such shocks will, however, spill over eventually, which leaves authorities some time to set up firewalls or a bail-out plan. In the case of Italy, Spain and Portugal, the spillover index is lower than that for Greece, suggesting a lower trans- mission. Nevertheless, these countries are potentially contagious for Europe, as correlations

between these countries have abruptly increased during the crisis and stayed on high levels. This highlights a potential threat in the event of a debt shock originating from these countries. In conclusion, the European sovereign bond market is heterogeneous, and credible threats are still present. Specific regulations and policy actions are thus required to reduce systemic risks.

A. Appendix

A.1. Assumptions

We make the following assumptions about the error term, adapted from assumptions A4 and A5 in **?**. They are formulated with respect to regimes, as $\varepsilon_t \sim (0, \Sigma_j)$, for $[\lambda_{j-1}T] + 1 \leq t \leq [\lambda_j T]$, with break fractions $\lambda_j = t_j/T$, $\lambda_0 = 0$, $\lambda_{m+1} = 1$. The assumptions cover the typical features observed in financial returns. In particular, the presence of conditional heteroscedasticity is allowed. Assumption A3 guarantees asymptotic distinctiveness of stability regimes.

- Assumption A1: Let $\mathscr{F}_t = \sigma - \text{field}\{\ldots, \varepsilon_{t-2}, \varepsilon_{t-1}\}$. ε_t is weakly stationary within each segment and (a) $\{\varepsilon_t, \mathscr{F}_t\}$ forms a strongly mixing (α-mixing) sequence with size $-4r/(r-2)$ for some $8 > r > 2$, (b) $E(\varepsilon_t) = 0$ and $\sup_t ||\varepsilon_t||_{2r+\delta} < M < \infty$ for some $\delta > 0$ and $M > 0$, where $||X||_r = (\sum_i \sum_j E|X_{i,j}|^r)^{1/r}$, for $r \geq 1$, is the L_r-norm of a random matrix X, (c) let $S_{k,j}(l) = \sum_{T_{j-1}^0 + l + 1}^{T_{j-1}^0 + l + k}(\varepsilon_t \varepsilon_t')$, $j = 1, \ldots, m$, for each $e \in \mathbb{R}^n$ of length 1, $var(\langle e, S_{k,l}(0)\rangle) \geq v(k)$ for some function $v(k) \to \infty$ as $k \to \infty$ (with $\langle \cdot \rangle$, the usual inner product).

- Assumption A2: $\{\varepsilon_t \varepsilon_t' - \Sigma_j^0\}$ satisfies the conditions stated for ε_t in Assumption A1.

- Assumption A3: $0 < \lambda_1^0 < \ldots < \lambda_m^0 < 1$, where $T_j^0 = [T\lambda_j^0]$, $j = 1, \ldots, m+1$.

A.2. The spillover index

Our procedure is based on tests for detecting contagion using the structural breaks in volatility and correlations. Recently, **??** provide an additional tool for analyzing the relationships between financial markets via spillovers. They construct spillover indexes that build on the structural, dynamic model linkages between countries. A VAR model framework such as ours enables the computation of the indexes; see **?** for an applications closely related to ours. An index increase indicates spillover contagion. Also, shift contagion analysis can be enhanced, as each previously detected data regime delimited by structural breaks can be

associated with a spillover index value. Then, directional spillover values provide information about the source and destination of dynamic shock transmission at the time.

Let λ_{ij}^h be the h-step-ahead forecast error variance decomposition, i.e., the fraction of the forecast error variance for forecasting variable i that is due to shocks to variable j. Variance decomposition requires orthogonal innovations, and **?** suggest using the generalized impulse response framework of **?** and **?**, which is invariant to the ordering of the variables. Furthermore, the λ_{ij}^h are normalized to satisfy $\sum_{j=1}^n \lambda_{ij}^h = 1$ and $\sum_{i=1}^n \sum_{j=1}^n \lambda_{ij}^h = n$. The total spillover index is defined as the fraction of overall forecast error variance that is due to shocks to other markets. Formally,

$$TS^h = 100 \cdot \frac{\sum_{i=1}^n \sum_{j=1, j \neq i}^n \lambda_{ij}^h}{\sum_{i=1}^n \sum_{j=1}^n \lambda_{ij}^h}. \tag{3}$$

Furthermore, directional spillover indexes are defined as the spillover transmitted from i to all other markets,

$$DS_{\leftarrow i}^h = 100 \cdot \frac{\sum_{j=1, j \neq i}^n \lambda_{ji}^h}{n}, \tag{4}$$

and as the spillover received by i from the other markets,

$$DS_{\rightarrow i}^h = 100 \cdot \frac{\sum_{j=1, j \neq i}^n \lambda_{ij}^h}{n}. \tag{5}$$

Finally, the net spillover by market i is defined as

$$NS_i^h = DS_{\leftarrow i}^h - DS_{\rightarrow i}^h. \tag{6}$$

The spillover index depends on the VAR coefficients and on the covariance matrix of the residuals. We compute (and plot) these indexes by taking into account the structural breaks of each parameter type identified using our procedure described above. This yields an easily interpretable summary of the effects the various parameter changes of the model have and allows the researcher to assess whether spillover contagion has occurred.

A.3. Spillover in the euro area
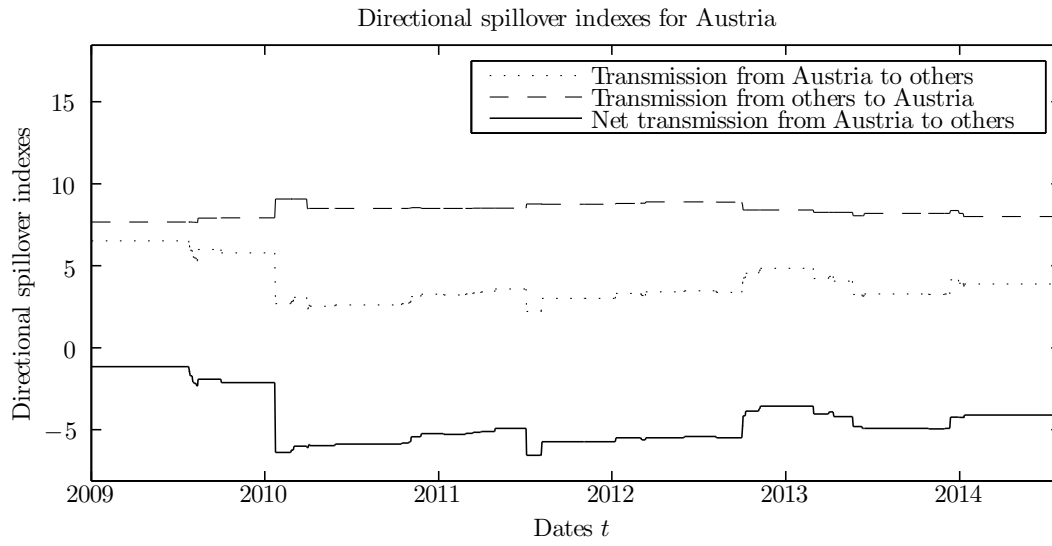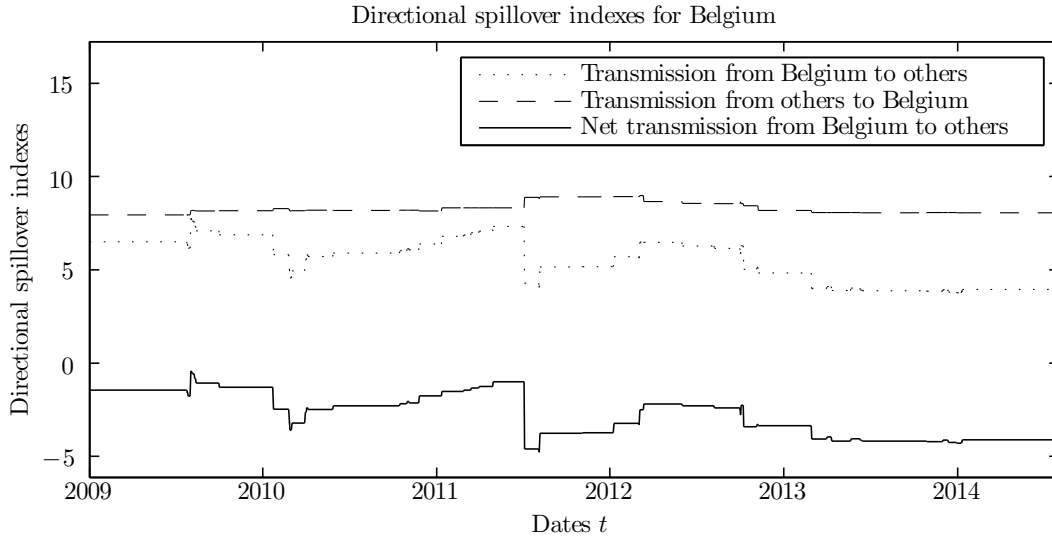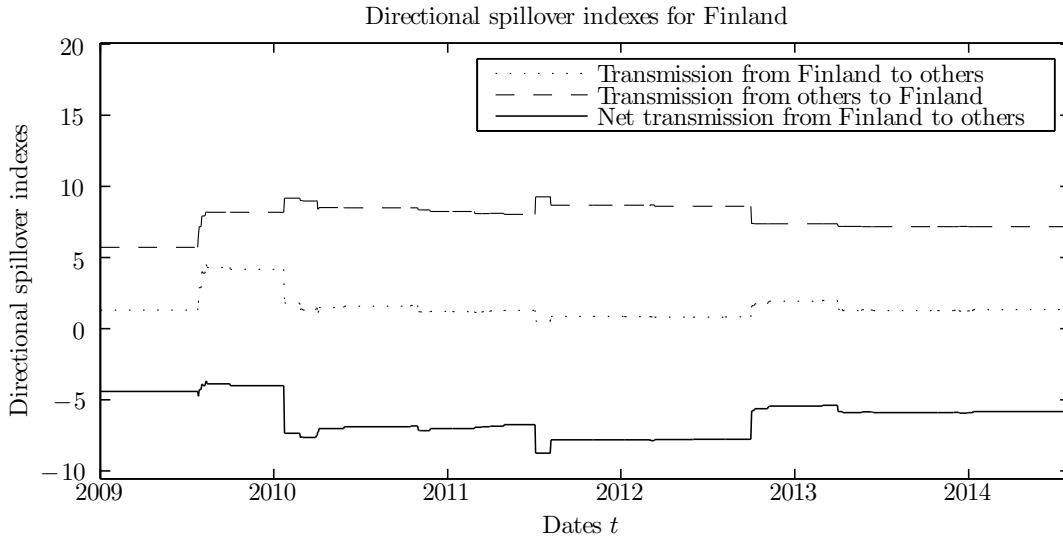
Directional spillover indexes for Austria

Figure 3: Directional spillovers of Austria, 20-step-ahead forecast, between 02 January 2009 and 01 August 2014, following a model with distinct breaks in the mean parameters B, the variances S and the correlation coefficients R according to the STP. Spillover transmitted from Austria to other countries, spillover transmitted from other countries to Austria, and net spillover transmitted from Austria to other countries.

Directional spillover indexes for Belgium

Figure 4: Directional spillovers of Belgium, 20-step-ahead forecast, between 02 January 2009 and 01 August 2014, following a model with distinct breaks in the mean parameters B, the variances S and the correlation coefficients R according to the STP. Spillover transmitted from Belgium to other countries, spillover transmitted from other countries to Belgium, and net spillover transmitted from Belgium to other countries.

Directional spillover indexes for Finland

Figure 5: Directional spillovers of Finland, 20-step-ahead forecast, between 02 January 2009 and 01 August 2014, following a model with distinct breaks in the mean parameters B, the variances S and the correlation coefficients R according to the STP. Spillover transmitted from Finland to other countries, spillover transmitted from other countries to Finland, and net spillover transmitted from Finland to other countries.
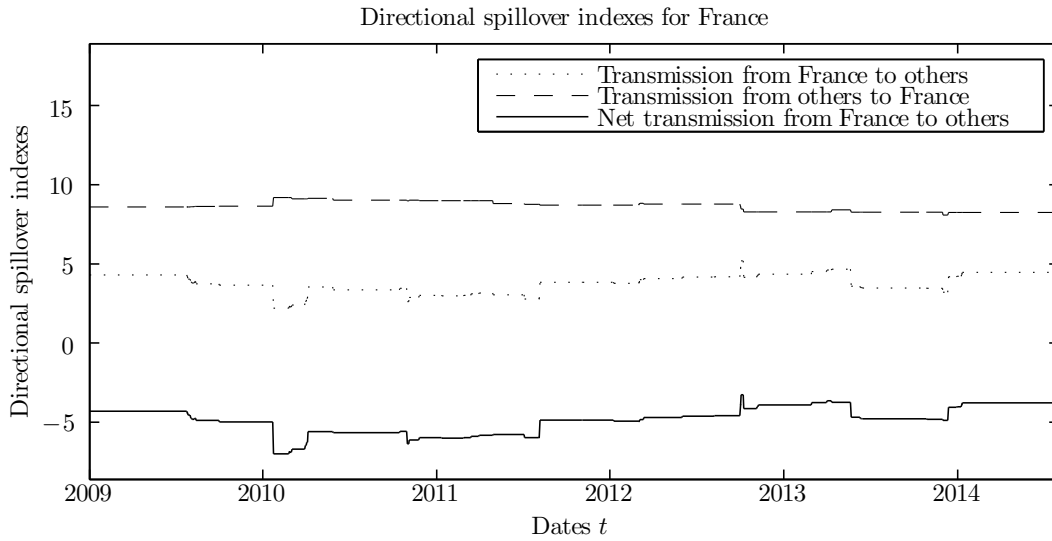
Figure 6: Directional spillovers of France, 20-step-ahead forecast, between 02 January 2009 and 01 August 2014, following a model with distinct breaks in the mean parameters B, the variances S and the correlation coefficients R according to the STP. Spillover transmitted from France to other countries, spillover transmitted from other countries to France, and net spillover transmitted from France to other countries.
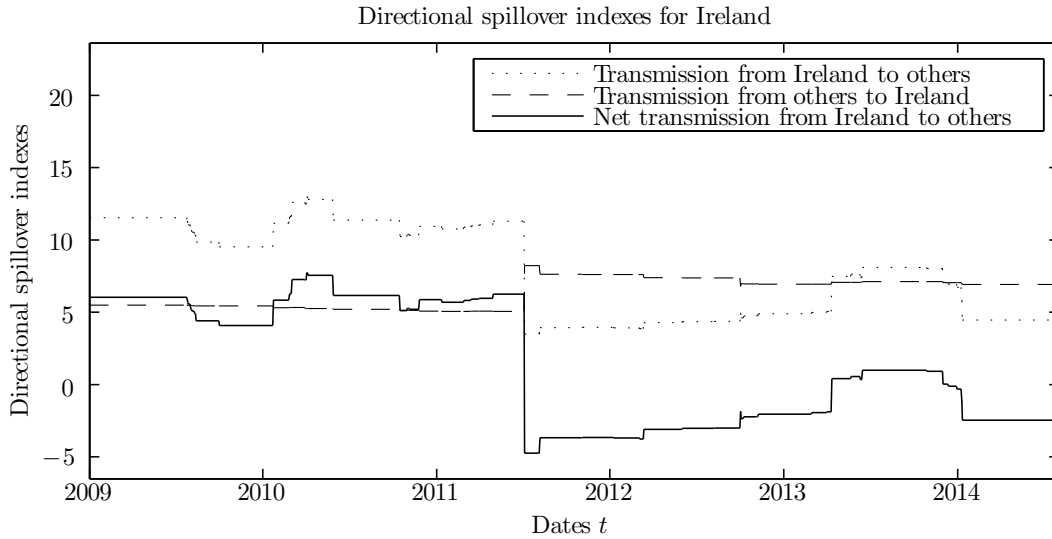

Figure 7: Directional spillovers of Ireland, 20-step-ahead forecast, between 02 January 2009 and 01 August 2014, following a model with distinct breaks in the mean parameters B, the variances S and the correlation coefficients R according to the STP. Spillover transmitted from Ireland to other countries, spillover transmitted from other countries to Ireland, and net spillover transmitted from Ireland to other countries.
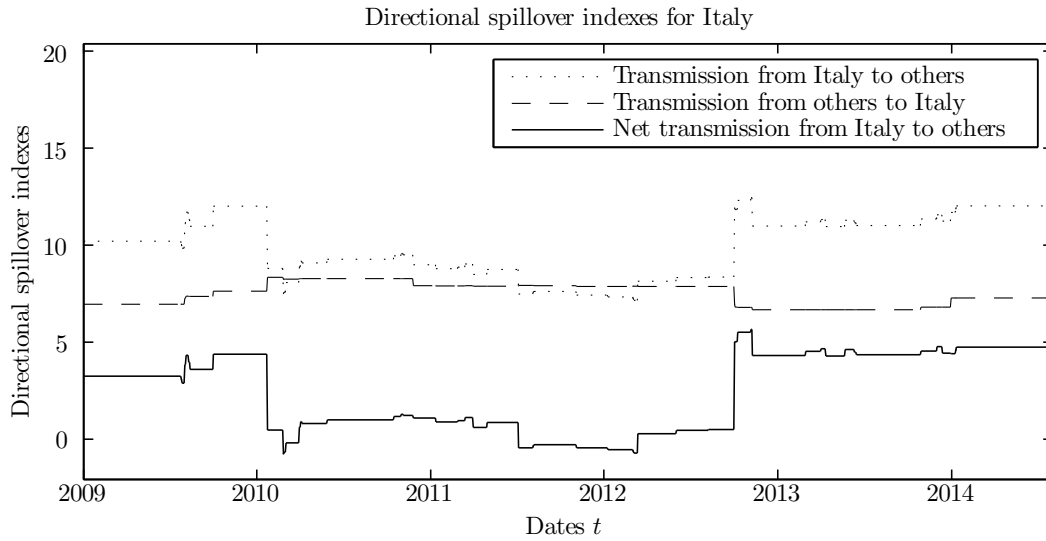
Directional spillover indexes for Italy

Figure 8: Directional spillovers of Italy, 20-step-ahead forecast, between 02 January 2009 and 01 August 2014, following a model with distinct breaks in the mean parameters B, the variances S and the correlation coefficients R according to the STP. Spillover transmitted from Italy to other countries, spillover transmitted from other countries to Italy, and net spillover transmitted from Italy to other countries.
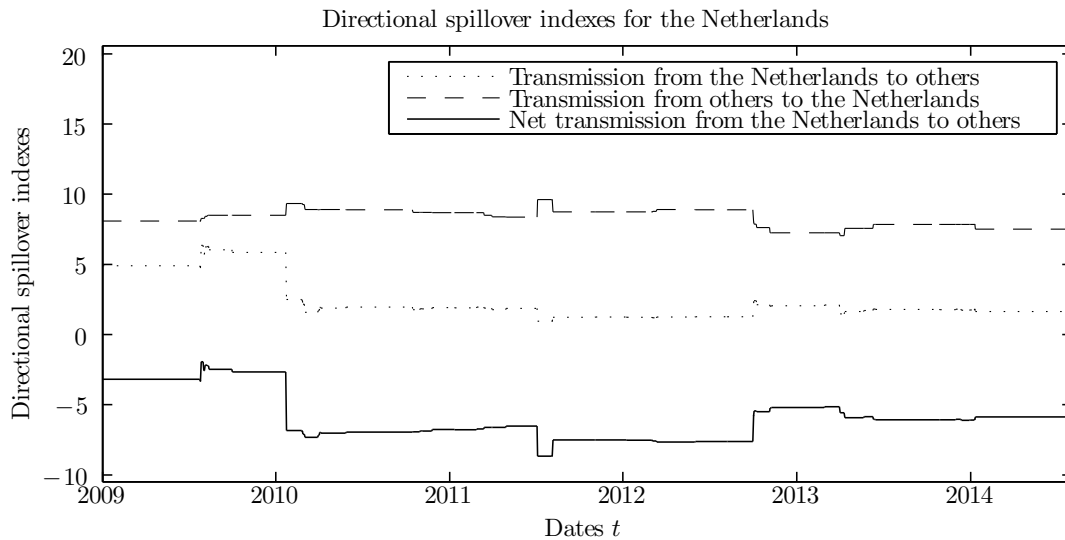
Directional spillover indexes for the Netherlands

Figure 9: Directional spillovers of the Netherlands, 20-step-ahead forecast, between 02 January 2009 and 01 August 2014, following a model with distinct breaks in the mean parameters B, the variances S and the correlation coefficients R according to the STP. Spillover transmitted from the Netherlands to other countries, spillover transmitted from other countries to the Netherlands, and net spillover transmitted from the Netherlands to other countries.
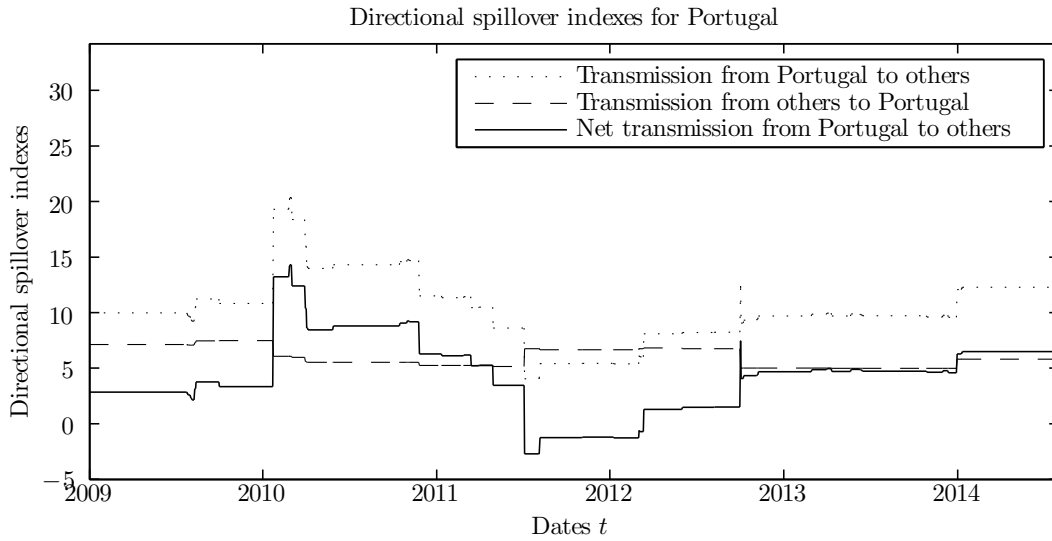
Directional spillover indexes for Portugal

Figure 10: Directional spillovers of Portugal, 20-step-ahead forecast, between 02 January 2009 and 01 August 2014, following a model with distinct breaks in the mean parameters B, the variances S and the correlation coefficients R according to the STP. Spillover transmitted from Portugal to other countries, spillover transmitted from other countries to Portugal, and net spillover transmitted from Portugal to other countries.
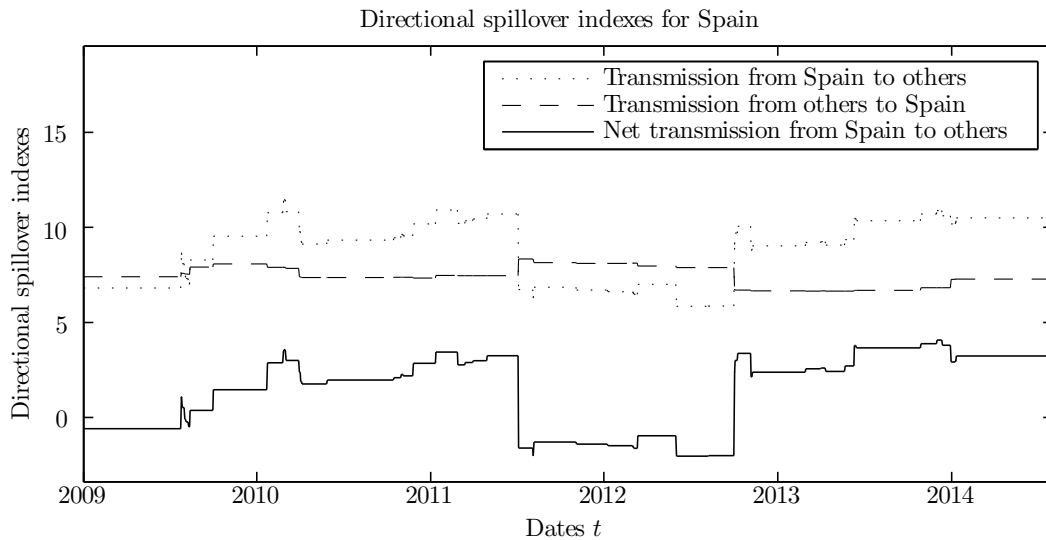

Directional spillover indexes for Spain

Figure 11: Directional spillovers of Spain, 20-step-ahead forecast, between 02 January 2009 and 01 August 2014, following a model with distinct breaks in the mean parameters B, the variances S and the correlation coefficients R according to the STP. Spillover transmitted from Spain to other countries, spillover transmitted from other countries to Spain, and net spillover transmitted from Spain to other countries.

References