The Tragedy of Complexity*

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Abstract

This paper presents an equilibrium theory of product complexity. Complex products generate higher potential surplus, but require more attention from the consumer. Because consumer attention is a limited common resource, an *attention externality* arises: Sellers distort the complexity of their own products to *grab attention* from other products. This externality can lead to *too much* or *too little complexity* depending on product features and the attention constraint of the consumer. Products that are well understood in equilibrium are too complex, while products that are not well understood are too simple. We use our theory to analyze the complexity of retail financial products.

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1 Introduction

Products, real and financial, differ vastly in their complexity. Some products are exceedingly complicated: Never-ending options in retail financial products, such as insurance policies or retirement plans; endless features in smartphones and other personal technology items; overly complicated terms and conditions in online retail. Other products appear overly simplified: AAA-rated mortgage backed securities attempt to simplify a complex underlying security pool; the media and politicians often seem to oversimplify complex issues. But what is the right complexity of a good? And does the market deliver the efficient amount of complexity? This paper proposes an equilibrium theory of complexity to shed light on these issues.

The key premise of our analysis is that complex products generate higher potential surplus, but require more of the consumer's limited attention. This leads to an *attention externality*: When choosing the complexity of their goods, sellers do not take into account that attention is a common resource. In equilibrium, sellers therefore distort the complexity of their products in order to divert attention from the goods of other sellers.

Our analysis yields three main results. First, the presence of the attention externality means that equilibrium complexity is generally inefficient. The tragedy of the commons with respect to consumer attention can lead to *too much or too little complexity*, depending on the direction of the consumer's attention reallocation in response to changes in complexity. We refer to this generic inefficiency in complexity choice as the *tragedy of complexity*. Second, equilibrium complexity is more likely to be excessive when attention is abundant. This leads to a *complexity paradox*: Rather than helping consumers deal with complexity, increases in consumers' information processing capacity can make it more likely that complexity is excessive. In contrast, when many different sellers compete for a given amount of consumer attention, this can lead to inefficient dumbing down of products. Third, we characterize which types of goods end up being too complex and which too simple. Counterintuitively, products that are relatively well understood tend to be too complex, whereas products that are not well understood tend to be too simple. This result arises because the consumer's depth of understanding reveals her willingness to pay attention to a good, which drives the externality that leads to excessive complexity.

In our model, a consumer with limited attention purchases goods from a number of sellers of differentiated goods. We model limited attention by assuming that the consumer has a fixed time budget that she allocates across goods. Seller non-cooperatively choose the complexity of the good they are selling. The consumer's valuation of a good consists of two components. First, it directly depends on the good's complexity. What we have in mind is that, all else equal, a more complex good can be worth more to the consumer (e.g., because of additional features, functionality or customization). Second, the consumer's valuation is higher the more time she spends on understanding the good (e.g., a deeper understanding allows the consumer to make better use of the good's features, functionality, or customization). Therefore, as in Becker (1965), the consumer's time acts as an input to the value of consumption goods. Crucially, more complex goods require more attention to achieve the same depth of understanding. To capture this, we assume that when the complexity of a good doubles, it takes the consumer twice as long to reach the same depth of understanding. The buyer's understanding of a good therefore depends on the *effective attention* (time spent divided by complexity) paid to the good. Given this, an increase in product complexity therefore involves a tradeoff: A more complex good is potentially more valuable to the consumer, but the consumer also has to pay more attention to reach the same level of understanding.

When sellers choose the complexity of their good, they internalize that consumers respond by increasing or decreasing the attention allocated to their good. Sellers therefore have an incentive to distort the complexity of their good to increase the amount of attention paid to it by the buyer. More attention increases the good's value to the consumer, some of which is extracted by the seller. However, sellers do not internalize that attention is a common resource, such that an increase in attention paid to their good necessarily corresponds to a decrease in attention paid to other goods. These other goods decrease on value, resulting in an *attention externality*.

Because of the attention externality, equilibrium complexity and the resulting demand on the consumer's time are generally inefficient. The tragedy of the commons with respect to attention implies that, in equilibrium, there is more pressure on the consumer's time than under the planner's solution: The shadow price of attention is inefficiently high, such that the consumer constantly feels "out for time." However, equilibrium complexity can be either too high or too low, depending on the direction of the consumer's reaction to a change in complexity. In the language of consumer demand theory, this reaction can be characterized in terms of income and substitution effects. An increase in complexity is equivalent to an increase in the price of effective attention for the good in question. The consumer increases the time spent on this good if and only she reduces time spent on other goods, which occurs when the income effect for effective attention outweighs the substitution effect.

Our model allows us to explicitly characterize the direction of the complexity externality based on the characteristics of the good and the buyer's attention constraint. Paradoxically, goods tend to be overly complex precisely when the buyer has a relatively large attention budget. Therefore, rather than helping buyers deal with the complexities of everyday life, improvements in information processing capacity may therefore be a driver of excessive complexity—the *complexity paradox*. In contrast, when more goods compete for a fixed amount of buyer attention, goods can end up being inefficiently simple. Our model therefore provides an explanation for why the recent increase of online and social media outlets has gone hand in hand with a dumbing down of content.

When goods are heterogeneous, it is possible that some goods are too complex in equilibrium whereas others are too simple. Whether a goods ends up being too complex or too simple depends on its characteristics, such as how much the good benefits from increased complexity, how much consumer benefits from a better understanding of the good, and the good's relative importance in the buyer's consumption basket. Despite all of these factors, there is a simple way to characterize which goods are overly complex and which ones are dumbed down: Goods that, in equilibrium, are well understood (i.e., high effective attention) are too complex, whereas goods that are not well understood (low effective attention) are too simple. At first glance, this seems counterintuitive, as one might expect overly complex goods to not be well understood by the consumer. The reason is that if the buyer is willing to dedicate a lot of attention to understanding the good, then the seller has an incentive to make it more complex to draw even more attention from the consumer. Conversely, if for a given level of complexity the buyer does not pay much attention to a good, then the seller can make her pay more attention by simplifying the good.

Based on the above characterization, goods that are likely to be too complex include smartphones, checking accounts, and equity mutual funds. Most people invest significant time in using and understanding their smartphone. Producers seize on this through the development of new features and apps. In the case of checking accounts, our model predicts that banks have an incentive to add an inefficiently high number of contingent fees and promotional interest rates, which makes deposit contracts more complex than they should be. Similarly, actively managed mutual funds are generally much more complicated than simple index funds, which most researchers would consider the optimal choice for investors. In contrast, our model implies that intricate policy debates or financial products based on complicated pools of assets may end up being oversimplified. For example, despite the apparent complications, the question of whether the UK should leave the EU was often dumbed down to how much the UK contributes to the EU budget. Similarly, despite being based on a pool of complicated underlying assets, mortgage-backed securities are often oversimplified to the rating of the underlying asset pool.

By viewing time as an input to the value of consumption goods, our approach to modeling complexity builds on the classic work of Becker (1965). We extend this framework by introducing complexity choice. The choice of complexity affects the value of the good directly, but also how the consumer transforms her time into understanding the good. By assuming a limited time budget for the consumer, our framework captures that complexity is inherently a bounded rationality phenomenon (Brunnermeier and Oehmke, 2009). The constraint on the consumer's time serves a role similar to information processing constraints in information theory (e.g., Sims, 1998, 2003).

Our approach to complexity differs from the existing literature on this topic. In particular, in much of the existing literature, complexity is exploitative and mostly serves the purpose of obfuscation to increase market power or influence consumer choice (Carlin 2009, **?**, Piccione and Spiegler 2012, Spiegler 2016, and Asriyan, Foarta, and Vanasco 2018). Contrary to this literature, in our model complexity is potentially value-enhancing. Moreover, the cost of complexity is not an increase in market power or a distortion in the consumer's purchasing decision. Rather, it manifests itself an externality that the complexity of one good imposes on the equilibrium value of other goods.

A key aspect of our paper, competition for attention, is studied also by Bordalo, Gennaioli, and Shleifer (2016). In contract to our paper, their focus is on the salience of certain product attributes: Consumer attention can be drawn to either price or quality, resulting in equilibria that are price- or quality-salient. Despite the difference in focus, their analysis shares with ours an attention externality across goods. Finally, our work is related to the literature on providing default options, see Choi, Laibson, Madrian, and Metrick (2003). Specifically, privately optimal excess complexity may explain why sellers are often unwilling to provide default options that would make the product less time consuming to use.

2 Model

We consider an economy with N sellers (he) and a single buyer (she). Goods are differentiated and there is one seller per good. Each seller i is endowed with an indivisible unit of good i. Because goods are differentiated, sellers have some market power, which we capture in reduced form by assuming that seller i can extract a share $\theta_i \in (0, 1)$ of the surplus generated by good i.

The key decision for each seller i is to choose the complexity c_i of the good. While complexity has no direct cost (or benefit) for the seller, complexity matters because it affects the surplus generated by the good. For example, complexity can add value when it arises as a byproduct of customization that caters the buyer's individual needs. However, realizing the full value of a more complex good requires attention from the buyer. Returning to the previous example, the buyer may need to spend time to understand the precise nature of the more complex, customized good.[1] The total value of a complex good therefore arises from the combination of the characteristics of the good itself and the time that the buyer allocates to the good. In this respect, our paper builds on classic work on time as an input into the utility derived from market goods pioneered by Becker (1965).

To capture this more formally, we assume that the value to the buyer of consuming a unit of good i with complexity c_i, having allocated t_i units of time to good i, is given by

$$v_i\left(c_i, \frac{t_i}{c_i}\right), \tag{1}$$

which we assume is twice continuously differentiable in all arguments. The first argument of $v_i(\cdot,\cdot)$ captures that the value of the good depends directly on the complexity of the good. We assume that, for sufficiently low levels of complexity, $\frac{\partial v_i}{\partial c_i} > 0$ holds, such that *ceteris paribus* some complexity raises the value of the good. However, this benefit of complexity exhibits diminishing marginal returns, $\frac{\partial^2 v_i}{\partial c_i^2} < 0$ and, at some point, the marginal value of complexity could even turn negative.

The second argument of $v_i(\cdot,\cdot)$ reflects that the value of the good to the buyer increases with the attention (measured in units of time) that the buyer allocates to understand the good. However, we assume that a unit of attention is less valuable the more complicated the good. What matters, rather, is effective attention, which we define as time spent on the good divided by the good's complexity, t_i/c_i (to save space, we will sometimes simply denote effective attention by e_i). One can think of effective attention as the buyer's depth of understanding. More complex goods take more time to understand and make the best use of. In our particular specification, with effective attention equal to time spent divided by complexity, a good that is twice as complex takes twice as long to figure out. We make standard assumptions on the effect of effective attention and the value of the good: All

[1]This attention can be devoted to the good before the purchase (e.g. choosing specific features of a service contract) or during the use of the good (e.g. the use of the good is more time-consuming). Our model can accommodate both interpretations.

else equal, more effective attention increases the value of the good to the buyer, $\frac{\partial v_i}{\partial (t_i/c_i)} > 0$, but with diminishing marginal returns, $\frac{\partial^2 v_i}{\partial (t_i/c_i)^2} < 0$.

In addition to the relatively standard assumptions on positive but diminishing returns to effective attention, we make two additional assumption on how effective attention affects the value of the good.

Assumption 1. *The marginal value of effective attention t_i/c_i is independent of the level of complexity:* $\frac{\partial^2 v_i}{\partial c_i \partial (t_i/c_i)} = 0$.

Assumption 2. *The value of good i is bounded above and below in effective attention:* $v_i\left(c_i, 0\right) > 0$ *and* $v\left(c_i, \infty\right) < \infty$.

Assumption 1 states that the marginal value effective attention is independent of the complexity of the good. Therefore, when a good is twice as complicated, the buyer has to spend twice as much time on the good to gain the same level of understanding. Assumption 2 implies that good i is valuable even when buyers pay no attention to it, $v_i\left(c_i, 0\right) > 0$, and that the value of the good remains bounded even when effective attention is infinite, $v_i\left(c_i, \infty\right) < \infty$. The first part of this assumption guarantees that all goods are consumed in equilibrium. The second part ensures that very simple goods ($c_i \to 0$) have finite value.

2.1 The Buyer's Problem

The buyer takes the complexity of each good as given. The buyer's maximization problem is then to choose which goods to consume and how much attention t_i to allocate to these goods, taking into account that she receives a share $1-\theta_i$ of the surplus generated by good i. We assume that the buyer's utility is quasi-linear in the benefits derived from the N goods and wealth, and that the buyer is deep pocketed. This assumption implies that our results are driven by the buyer's attention constraint (introduced in more detail below) rather than a standard budget constraint. By Assumption 2, the buyer receives positive utility from consuming good i even when paying no attention to it. It is therefore optimal for the buyer to purchase all N goods. The buyer's maximization problem therefore reduces to choosing the amount of attention she allocates to each good, taking as given complexity c_i:

$$\max_{t_1,..t_N} \sum_{i=1}^{N} (1-\theta_i) \cdot v_i \left(c_i, \frac{t_i}{c_i} \right). \tag{2}$$

The key constraint faced by the buyer is that her attention is limited. Specifically, the buyer has a fixed amount of time T (the attention budget) that can be allocated across the N goods. Therefore, the buyer maximizes (2) subject to the attention constraint

$$\sum_{i=1}^{N} t_i \leq T. \tag{3}$$

By rewriting the this attention constraint as $\sum_{i=1}^{N} \frac{t_i}{c_i} \cdot c_i \leq T$ (i.e., multiplying and dividing by c_i), we see that one can think of the attention constraint as a standard budget constraint, where the good purchased by the buyer is effective attention t_i/c_i, the price of effective attention for good i is the complexity of that good, c_i, and the buyer's wealth is her endowment of time, T. As we will see below, this interpretation can be useful because it allows us to draw parallels to classic results from consumer demand theory.

The buyer's first-order condition, which pins down the optimal attention choice $t_i(c_1, \ldots, c_N)$ is then given by

$$(1-\theta_i) \cdot \frac{\partial v_i \left(c_i, \frac{t_i(c_1,\ldots,c_N)}{c_i} \right)}{\partial \left(\frac{t_i}{c_i} \right)} \cdot \frac{1}{c_i} \leq \lambda, \tag{4}$$

where λ denotes the Lagrange multiplier on the attention constraint. This first-order condition holds with equality when $t_i > 0$. Intuitively, the first-order condition states that the marginal value to the buyer of an additional unit of attention to good i (if she pays attention to the good at all) equals the shadow price of attention λ. Because the buyer can only extract a fraction $1-\theta_i$ of the surplus generated by good i, all else equal it is optimal to allocate more time to goods for which this fraction is large.

2.2 Equilibrium Complexity: The Seller's Problem

Seller i's objective is to maximize profits, given by a fraction θ_i of the surplus generated by good i. The seller's only choice variable is the complexity c_i of his good. However, in choosing c_i, the seller internalizes that his complexity choice affects the amount of attention allocated to the good by the buyer. In other words, like a Stackelberg leader the seller internalizes that the attention the buyer pays to his good, $t_i(c_1, \ldots, c_N)$, is function of c_i. The seller's objective function is therefore

$$\max_{c_i} \theta_i \cdot v_i \left(c_i, \frac{t_i(c_1, \ldots, c_N)}{c_i} \right), \tag{5}$$

with an associated first-order condition of

$$\theta_i \cdot \frac{d}{dc_i} v_i \left(c_i, \frac{t_i(c_1, \ldots, c_N)}{c_i} \right) \leq 0. \tag{6}$$

As usual, this first order condition holds with equality whenever $c_i > 0$. Assuming that c_i is indeed an internal solution[2] and taking the total derivative, the first-order condition (6) can be rewritten as

$$\frac{\partial v_i \left(c_i, \frac{t_i}{c_i} \right)}{\partial c_i} = \frac{\partial v_i \left(c_i, \frac{t_i}{c_i} \right)}{\partial \left(\frac{t_i}{c_i} \right)} \cdot \frac{t_i}{{c_i}^2} - \frac{\partial v_i \left(c_i, \frac{t_i}{c_i} \right)}{\partial \left(\frac{t_i}{c_i} \right)} \cdot \frac{1}{c_i} \cdot \frac{\partial t_i}{\partial c_i}. \tag{7}$$

This first-order condition states that, from the seller's perspective, the optimal level of complexity equates the marginal increase in value from additional complexity to the value reduction that arises from lower levels of effective attention (holding the buyer's attention to the good constant), net of the change in the good's value that arises from the buyer's change in attention paid to good i in response to an increase of the complexity of that good. In equilibrium, this first-order condition must hold for each seller i.

The key observation from the seller's optimization problem is that sellers take into account that a change in the complexity of their good changes the amount of attention that the buyer will allocate to

[2] A sufficient condition for $c_i > 0$ is that a standard Inada condition holds with respect to complexity.

their good, as indicated by $\frac{\partial t_i}{\partial c_i}$. Sellers perceive the additional attention paid to their good in response to a change in complexity as a net gain, even though in aggregate this is merely a reallocation—any additional attention paid to good i would otherwise be allocated to goods of other sellers. Because the seller of good i is essentially diverting attention away from other goods, we refer to this as the *attention grabbing* effect.

Using the buyer's first-order condition (4), we can rewrite the seller's optimality condition (7) in terms of the shadow price of attention λ, which for $c_i > 0$ and $t_i > 0$ yields

$$\frac{\partial v_i\left(c_i, \frac{t_i}{c_i}\right)}{\partial c_i} = \frac{\lambda}{1-\theta_i}\left(\frac{t_i}{c_i} - \frac{\partial t_i}{\partial c_i}\right). \tag{8}$$

Rewriting the first-order condition in this more concise way is useful when comparing the seller's first-order condition to the planner's optimality condition derived in the next section.

2.3 Optimal Complexity: The Planner's Problem

We now turn to the planner's choice of product complexity. The key difference compared to the seller's profit-maximization problem described above is that the planner takes into account that the buyer optimally allocates attention across all goods. Therefore, the planner takes into account the effect of a change in the complexity of good i not only on the value of good i but also, via the buyer's attention reallocation, on all other goods $j \neq i$.

Mathematically, the planner chooses the product complexities of all N goods to maximize total surplus,

$$\max_{c_1,\ldots c_N} \sum_{i=1}^{N} v_i\left(c_i, \frac{t_i(c_1,\ldots,c_N)}{c_i}\right). \tag{9}$$

Following the same steps as in the derivation for the seller's first-order condition (including the assumption that c_i^* is an internal solution), the optimality condition for the planner's complexity choice

c_i^* for good i is given by

$$\frac{\partial v_i\left(c_i, \frac{t_i}{c_i}\right)}{\partial c_i} = \frac{\partial v_i\left(c_i, \frac{t_i}{c_i}\right)}{\partial\left(\frac{t_i}{c_i}\right)} \cdot \frac{t_i}{c_i^2} - \sum_{j=1}^{N} \frac{\partial v_j\left(c_j, \frac{t_j}{c_j}\right)}{\partial\left(\frac{t_j}{c_j}\right)} \cdot \frac{1}{c_j} \cdot \frac{\partial t_j}{\partial c_i}. \tag{10}$$

The planner's optimality condition highlights the difference between the sellers' complexity choice (8) and the planner's solution. In particular, whereas the seller of good i only takes into account the change in the valuation of good i that results from the reallocation of attention to or from good i, the planner takes into account the changes in valuation that results from the reallocation of attention across all goods, resulting in $N-1$ additional terms on the right hand side. In general, the privately optimal complexity choice therefore differs from the planner's solution—reallocation of attention from other goods to good i results in an externality that is not taken into account by the seller of good i.

As before, using the buyer's first-order condition (4), we can rewrite the planner's optimality condition (10) in terms of the shadow price of attention λ, which for internal solutions yields

$$\frac{\partial v_i\left(c_i, \frac{t_i}{c_i}\right)}{\partial c_i} = \frac{\lambda}{1-\theta_i} \cdot \left(\frac{t_i}{c_i} - \frac{\partial t_i}{\partial c_i}\right) - \sum_{j \neq i} \frac{\lambda}{1-\theta_j} \cdot \frac{\partial t_j}{\partial c_i}, \tag{11}$$

where the second term on the right hand side captures the externality that is neglected by the seller.

A particularly simple case arises when all sellers have equal market power, such that $\theta_i = \theta$. In this case, the planner's optimality condition reduces to

$$\frac{\partial v_i\left(c_i, \frac{t_i}{c_i}\right)}{\partial c_i} = \frac{\lambda}{1-\theta} \cdot \frac{t_i}{c_i}. \tag{12}$$

This simplified condition results from the fact that, when viewed across all goods, attention is merely reallocated (i.e., $\sum_{j=1}^{N} t_j = T$ implies that $\sum_{j=1}^{N} \frac{\partial t_j}{\partial c_j} = 0$).

2.4 The Complexity Externality

To better understand the externality that arises in complexity choice, denote the planner's complexity choices by $(c_1^*, \ldots, c_N^*)$. The seller of good i has an incentive to deviate from the socially optimal complexity c_i^* whenever there is a private benefit from doing so. Under equal market power of sellers ($\theta_i = \theta$), a simple comparison of the first-order conditions (8) and (12) shows that this is the case whenever at the optimal complexity c_i^* the attention grabbing effect is nonzero, $\frac{\partial t_i}{\partial c_i} \neq 0$. When $\frac{\partial t_i}{\partial c_i} > 0$ the seller of good i has an incentive to increase the complexity of his good beyond the socially optimal level, whereas when $\frac{\partial t_i}{\partial c_i} < 0$ the seller if good i wants to decrease complexity below the socially optimal level. In both cases, the direction of the externality is therefore driven by the desire to divert the buyer's attention away from other goods. The above result is true also when market power differs across sellers, as shown formally in the following proposition.

Proposition 1. Complexity Externality. *Starting from the planner's solution $(c_1^*, ...c_N^*)$, keeping the complexity of all other $i \neq j$ goods fixed at c_j^*, the seller of good i*

(i) has an incentive to increase complexity c_i above its optimum c_i^* if $\frac{\partial t_i^*}{\partial c_i} > 0$;

(ii) has an incentive to decrease complexity c_i below its optimum c_i^* if $\frac{\partial t_i^*}{\partial c_i} < 0$;

(iii) has no incentive to change complexity c_i from its optimum c_i^* if $\frac{\partial t_i^*}{\partial c_i}$=0.

Proposition 1 states that the complexity externality has the same sign as the attention grabbing effect: Sellers have a (local) incentive to increase the complexity of their good beyond its optimal level if buyers respond by increasing the amount of attention paid to the good. In contrast, if buyers respond by decreasing the amount of attention allocated to the good when its complexity increases, sellers have a (local) incentive to decrease the complexity of their product below the socially optimal level.

A key implication of Proposition 1 is that the equilibrium complexity choices $(c_1^e, \ldots, c_N^e)$, which are given by a fixed point at which the first-order condition (8) holds for all sellers, generally differ from the planner's solution. In general the equilibrium distortion is not necessarily in the same direction as sign partial derivative $\frac{\partial t_j}{\partial c_i}$ at the optimal complexity levels because of "wealth effects" through the equilibrium shadow price of attention λ. However, in a symmetric equilibrium when $\theta_i = \theta$ and $v_i = v$, the partial derivative is sufficient to sign the equilibrium distortion.

Proposition 2. Equilibrium Complexity Distortion under Symmetry. *Assume that market power θ_i and value functions $v_i(\cdot, \cdot)$ are the same across sellers. Compared to the planner's complexity choice c_i^*, the complexity c_i^e chosen by sellers in a symmetric equilibrium is*

(i) too high if $\left.\frac{\partial t_i}{\partial c_i}\right|_{c_i=c_i^*} > 0$ (i.e., the goods are too complex in equilibrium);

(ii) too low if $\left.\frac{\partial t_i}{\partial c_i}\right|_{c_i=c_i^*} < 0$ (i.e., the goods are too simple in equilibrium);

(iii) exactly right if $\left.\frac{\partial t_i}{\partial c_i}\right|_{c_i=c^*} = 0$.

Therefore, in a symmetric setting, the sellers' local incentive to increase complexity beyond the socially optimal amount in order to attract more attention for its good results in an equilibrium in which all goods are too complex. In contrast, if the seller can attract more attention by making the good simpler than the socially optimal amount of complexity, the equilibrium features "dumbed down" goods that are too simple compared to the planner's choice.

Whereas Propositions 1 and 2 described the externality in complexity choice in terms of the attention grabbing effect $\frac{\partial t_i}{\partial c_i}$, there are a number other summary statistics than can equally be used as sufficient statistics to characterize the direction of the externality. As stated in Lemma 1 below, one can equivalently look at (1) the effect of a change in complexity on the shadow cost of attention, (2) the attention grabbing effect given a fixed shadow cost of attention, and (3) a simple complementarity condition between complexity and buyer attention. To state these results concisely, it is useful to introduce some additional notation. First, using (4) it will sometimes be useful to write attention as a function of the good's own complexity and the shadow cost of attention, $\tilde{t}_i(c_i, \lambda)$. Second, for the last equivalence result we will rewrite the value of good i in terms of attention instead of effective attention (i.e., we define $\tilde{v}(c, t) = v(t, t/c)$).

Lemma 1. Attention Grabbing: Equivalence Results. *For any given* $(c_1, \ldots, c_N)$ *the following have the same sign:*

(i) the attention grabbing effect for good i, $\frac{\partial t_i(c_1,..c_N)}{\partial c_i}$*;*

(ii) the effect of good i's complexity on the shadow cost of attention, $\frac{\partial \lambda(c_1,..c_N)}{\partial c_i}$*;*

(iii) the attention grabbing effect for good i, keeping the shadow cost of complexity fixed, $\frac{\partial \tilde{t}_i(c_i,\lambda)}{\partial c_i}\big|_\lambda$*;*

(iv) the complementarity (substitutability) of attention and complexity, $\frac{\partial^2 \tilde{v}(c_i,t_i)}{\partial c_i \partial t_i}$.

Statement (ii) in Lemma 1 provides intuition for why attention grabbing is an externality: It drives up the shadow price of attention for other goods which then get less attention. This also highlights the importance of the assumption of limited attention. If attention could be bought or sold at a fixed cost (i.e., λ is independent of the sellers' complexity choices), there would be no externality, since increasing the amount of attention allocated to one good would not mean that the attention paid to other goods has to diminish. Statement (iv) in Lemma 1 provides a microeconomic interpretation of the complexity externality: There is an incentive for sellers to increase complexity beyond the optimal level when attention and complexity are complements. In contrast, when attention and complexity are substitutes, sellers have an incentive to decrease complexity below the optimal level.

2.5 The Tragedy of Complexity as a Tragedy of Commons

The difference between the equilibrium complexity choice and the planner's solution has parallels with the classic tragedy of the commons. Like grass on a common grazing meadow, attention is a shared resource that is used by all goods. Similar to the farmer who is deciding whether to send an additional cow onto the grazing ground, the seller of good i does not internalize that an increase in the complexity of his good affects all other sellers. However, there is an important subtlety in how this translates into the equilibrium complexity choice. Following the logic of the tragedy of the commons, there is an overuse of the common resource (attention) in equilibrium, because sellers divert attention from other products to their own. However, as we saw above, attention grabbing can manifest itself in *too* *much or too little complexity*, depending on whether "overcomplicating" or "dumbing down" leads to an increase in buyer attention paid to a particular product. As a result, equilibrium complexity can be above or below what a planner would choose. One possibility is that buyers react to an increase in the complexity of good i by allocating more attention to good i (i.e., $\partial t_i / \partial c_i > 0$). In this case, the amount of the common resource available to other sellers decreases, leading to the classic tragedy-of-the-commons result: there is too much complexity in equilibrium. However, another possibility is that buyers react to an increase in the complexity of good i by diverting attention away from good i (i.e., $\partial t_i / \partial c_i < 0$). In this case, the seller of good i has an incentive to "dumb down" his product in order to attract the buyer's attention, resulting in too little complexity in equilibrium.

Whereas the complexity externality can go either way, the scarce resource of attention is always overused irrespective of the direction of the externality. Unchecked competition for the buyer's attention implies that the shadow price of attention is higher in equilibrium than it would be under the planner's solution, $\lambda^e \geq \lambda^*$, with strict inequality whenever $c_i^e \neq c_i^*$ for at least one good. In words, the buyer constantly feels short of time when sellers compete for her attention.

Proposition 3. The Buyer is Short of Time. *Suppose that equilibrium and optimal complexity differ for at least one good. Then the equilibrium shadow price of attention strictly exceeds the shadow price of attention under the planner's solution, $\lambda^e > \lambda^*$.*

Thus the conventional *tragedy of commons* intuition does hold for the fixed supply common resource used by all goods, attention. As mentioned before, the non-trivial contribution of our paper is to show that this maps into complexity choice in a non-trivial way: there might be too much or too little complexity, this is what we call the *the tragedy of complexity*.

What type of policy intervention would solve the tragedy of complexity? According to the above analysis, a regulation that simply aims to reduce complexity is not the right policy. After all, equilibrium complexity can be too high or too low. Rather, the optimal regulation would have to induce sellers to internalize the shadow cost of attention. In principle, this could be achieved via tradable permits. However, it is not clear how tradable permits on consumer attention could be implemented.

2.6 The Direction of the Externality

We now characterize the direction of the externality: Which goods are dumbed down and which are too complex? The key to signing the externality is to understand how attention allocated to good i changes when this seller changes the complexity of his good c_i, keeping the complexity of all other goods unchanged. Accordingly, we are interested in the shape of the function $t_i(c_i)$.

In Proposition 4 we show that $t_i(c_i)$ is hump shaped: It is increasing at low levels of c_i and decreasing at higher levels of c_i. In fact, in Section 3 we show that, using an explicit functional form for $v_i(c_i, e_i)$, the resulting $t_i(c_i)$ function is single peaked (see Figure 1).

Proposition 4. The Direction of the Complexity Externality. *For any complexity levels c_{-i} of other sellers (with at least one strictly positive), there exists a $\bar{c}_i > 0$ and $\bar{\bar{c}}_i < \infty$ such that*

(i) $\frac{\partial t_i}{\partial c_i} > 0$ *if* $c_i < \bar{c}_i$,

(ii) $\frac{\partial t_i}{\partial c_i} < 0$ *if* $c_i > \bar{\bar{c}}_i$ *and* $t_i > 0$.

The following observations help in understanding the shape of the $t_i(c_i)$ function: First, observe that if the seller chooses to make a simple good (c_i close to zero), then the buyer pays little attention to the good (t_i is also close to zero). This is the case because even with very little attention the consumer can understand the good very well (i.e., even a small amount of time spent on the good leads to high effective attention). Second, if the good has an intermediate level of complexity, the buyer pays attention to it. Third, if the good becomes very complex, the buyer pays very little attention to the good. For very complex goods, the return from allocating an additional unit of time to understand them becomes vanishingly small, so that the buyer focuses her attention on other goods.

Based on the hump shape of the $t_i(c_i)$ function, sellers of relatively complex goods have an incentive to dumb down the good they sell, whereas sellers of relatively simple goods have an incentive to make their goods more complex. Which goods are relatively simple in equilibrium and which are relatively complex depends on all the parameters of the model, which means that solving the model explicitly can be complicated. However, note that simple goods with little attention allocated to them are well understood (high effective attention) in equilibrium, while complex goods that the buyer does not pay much attention to are not well understood (low effective attention). In Section 3 we formalize the above insight using an explicit functional form for $v_i(c_i, e_i)$ (see Proposition 8).

If the seller increases c_i above a certain point, the buyer may even choose not to pay any attention to the good at all. At some point paying attention to a very complex good becomes a waste of time: Even if the buyer were to allocate all of her attention to the good, she would not understand it well (i.e., effective attention would still be low). Because the good is valuable to the buyer with zero attention paid to it, the buyer may simply give up on understanding this good, focusing her attention on other goods. We call this result the *giving up effect* and formalize it in the following proposition.

Proposition 5. Giving Up on Very Complex Goods. *For any complexity levels c_{-i} of other sellers (with at least one strictly positive), there exists a $\hat{c}_i > 0$ such that $t_i(c_i) = 0$ for $c_i > \hat{c}_i$.*

Real world examples, where buyers often choose not to pay any attention include the terms and conditions associated with software or online purchases. This is a classic instance of information overload in the context of complexity (see Brunnermeier and Oehmke (2009)). It follows from Proposition 1 that, if the buyer buys pays no attention to a good, there is no complexity externality for this good, such that the equilibrium level of complexity coincides with the social optimum.

The seller's incentive to attract additional attention from the consumer explains a familiar phenomenon: Often a new or updated product initially appears worse than its predecessor. Consider the release of an update to a software one uses regularly. Without investing more time, it often seems as if the product has become worse than it was before. For example, following the release of a new version of Excel a user complained: *"I hate the new product I bought. It has far too many features that I will never use and cannot get rid of. [...] Why do u have to make things so difficult?"* Another user replied: *"That's normal. Many people found that the new interface in Excel 2007 was a nightmare [...*

However,] there are so much cool functions and features added. Just take some time to adapt to the new interface."[3]

Our model reconciles these seemingly contrasting views: Without investing more time it often seems as if the product has become worse than it was before. As stated in Proposition 6, these are exactly the instances when the complexity chosen by the seller is excessive. Moreover, our model rationalizes why, despite the apparent reduction in value that arises when attention is held constant, the seller engages in this type of behavior. Once we account for the extra attention allocated to the good by the consumer in response to the change in complexity, the valuation of the good increases. Some of this additional surplus is then extracted by the seller. The flip side, not internalized by the seller, is that the extra time allocated to the good is taken away from other goods, so that the valuation of those goods decreases. Overall, after the reallocation of attention, the good in question is worth more to both the buyer and the seller, but the value of all other goods suffers to an extent that the buyer is worse off overall. In line with the example above, we refer to this result as the software update effect.

Proposition 6. The Software Update Effect. *The seller has an incentive to increase complexity above the efficient level (i.e., $\frac{\partial t_i}{\partial c_i} > 0$ at $c_i = c_i^e$) if and only if the value of good i to the buyer decreases when time allocated to the good constant, i.e., $\frac{\partial \tilde{v}_i(c_i,t_i)}{\partial c_i} < 0$.*

2.7 Complexity through the lens of demand theory

The conditions that lead to excess complexity can be cast in the language of consumer demand theory. For simplicity, we demonstrate this in a two-good setting. Rewriting the attention constraint in terms of effective attention, $e_i = \frac{t_i}{c_i}$, we obtain

$$c_1 e_1 + c_2 e_2 = T. \tag{13}$$

[3] https://answers.microsoft.com/en-us/msoffice/forum/all/why-is-excel-so-complicated/a2fc9495-1fb6-4bf0-965a-07c2b037606b (August 14, 2015), last accessed November 17, 2018.

This version of the attention constraint shows that we can think of product complexity as the price of a unit of effective attention. Under this interpretation, we can then express the buyer's choice of effective attention for good $i = 1, 2$ as

$$e_i(c_1, c_2, T), \tag{14}$$

the Marshallian demand for effective attention as a function of the complexity of the two goods, c_1 and c_2, and the attention budget, T.

We can now use standard concepts from consumer demand theory to characterize when excess complexity emerges in equilibrium. Suppose seller 1 increases the complexity of his good. Now consider a standard Slutsky decomposition that divides the change in effective attention the buyer allocates to good 2, $\frac{\partial e_2(c_1,c_2,T)}{\partial c_1}$, into a substitution effect and an income effect. The substitution effect results in a reallocation of effective attention from good 1 to good 2. When the price of effective attention for good 1 is increased, the buyer optimally increases the effective attention paid to good 2. The income effect, on the other hand, results in a decrease in effective attention paid to both goods. When the income effect outweighs the substitution effect, then the increase in the complexity of good 1 leads to reduction in the effective attention paid to good 2. Because c_2 is unchanged, this implies that t_2 decreases and t_1 increases (because $t_1 + t_2 = T$). Therefore, excess complexity arises ($\frac{\partial t_1}{\partial c_1} > 0$) if and only if the income effect for effective attention outweighs the substitution effect.[4]

[4] Alternatively, one can show that complexity is excessive when the demand for effective inattention is inelastic. Under the interpretation of complexity as the price of a unit of effective attention, the time spent on good i is equal to the buyer's expenditure on that good (i.e., $t_i = c_i e_i$). A standard result from consumer demand theory is that an increase in the price of good i leads to an increase in the total expenditure on that good if and only if the own-price demand elasticity of that good is smaller than one, $\eta_i = \left| \frac{\partial e_i}{\partial c_i} \frac{c_i}{e_i} \right| < 1$.

3 An Explicit Characterization of the Equilibrium

In this section, we analytically characterize equilibrium complexity choice using an explicit functional form for the value of good. Specifically, we assume that the value of good i is given by:

$$v_i\left(c_i, \frac{t_i}{c_i}\right) = w_i \cdot \left(f_i(c_i) + \delta_i - \delta_i \cdot \frac{1}{1 + \frac{t_i}{c_i}}\right). \tag{15}$$

Given this functional form, the direct benefit from complexity (assuming zero time spent on the good) is given by $f_i(c_i)$. w_i is the utility weight (or size) of good i, while δ_i captures the benefit of understanding the good. The additional value from understanding the good is an increasing function of effective attention, t_i/c_i, paid to the good. This functional form satisfies all assumptions we made before, including Assumptions 1 and 2.

We use this specific functional form because, in conjunction with a quadratic benefit function,

$$f_i(c_i) = \alpha_i \cdot c_i - c_i^2, \tag{16}$$

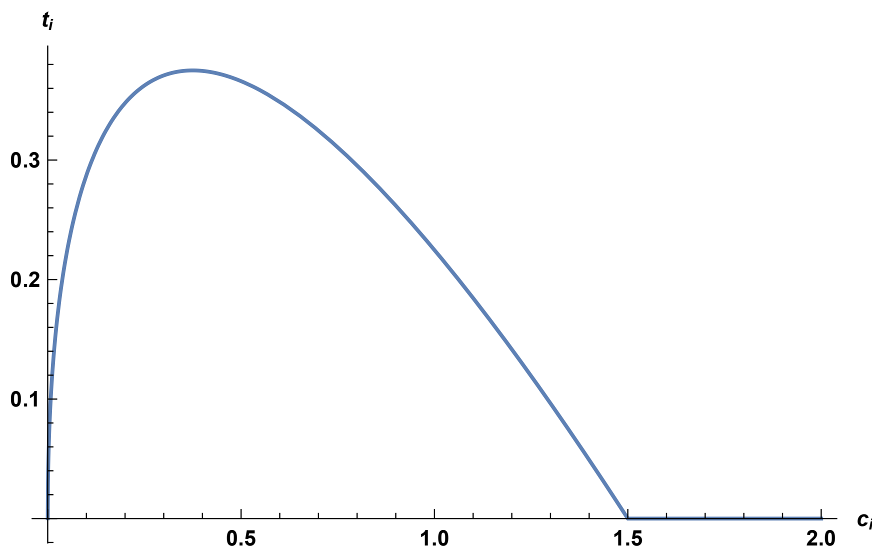
it allows us to derive closed-form solutions. In the quadratic benefit function, α_i parametrizes the benefit of added complexity for good i. Note that, given the quadratic nature of the benefit function, increasing complexity beyond some level reduces buyer utility. Therefore, even without a constraint on buyer attention, the optimal level of complexity of good i is finite and given by $\frac{\alpha_i}{2}$.

Given these assumptions, we can explicitly derive how much attention is paid to good i. Substituting the functional form v_i into (4) and rearranging yields

$$\tilde{t}_i(c_i, \lambda) = \sqrt{\frac{c_i \cdot \delta_i \cdot (1 - \theta_i) \cdot w_i}{\lambda}} - c_i. \tag{17}$$

Figure 1 illustrates the attention allocated to good i as a function of the complexity of good i, holding the shadow cost of attention λ fixed. Attention follows the hump shape already suggested by Propositions 4 and 5. Recall from Lemma 1 that holding λ fixed does not influence the sign of the slope of the $t_i(c_i)$ curve. We can therefore read the direction of the externality directly off the figure. For low levels of c_i, $\frac{\partial t_i}{\partial c_i} > 0$, such that an increase in the complexity of good i leads to an increase in the attention paid to the good. For higher levels of c_i, the direction of the externality reverses and an increase in complexity of good i leads to a reduction in attention to good i, $\frac{\partial t_i}{\partial c_i} > 0$. Finally, above some critical level of complexity, the buyer completely gives up on learning and pays no attention to good i at all (even though she still buys the good).

Figure 1: **Attention as a function of complexity with fixed cost of attention**



Homogenous goods, parameters: $N = 2$, $\delta = 0.9$, $w = 1$, $\alpha = 2$, $\theta = 0.5$, $\lambda = 0.3$.

3.1 Homogeneous Goods

Using the functional form of (15) and assuming homogeneous goods, the equilibrium first order condition (8) and the planner's first-order condition (11) can be written as stated in the following lemma.

Lemma 2. Symmetric First Order Conditions. *In a symmetric equilibrium with homogenous goods using the functional form (15), the equilibrium first-order condition is*

$$f'(c) = \left[\frac{T}{N \cdot c} - \frac{N-1}{2 \cdot N} \cdot \left(\frac{T}{N \cdot c} - 1\right)\right] \cdot \frac{\delta}{c \cdot \left(1 + \frac{T}{N \cdot c}\right)^2}, \tag{18}$$

whereas the planner's first order condition is

$$f'(c) = \frac{T}{N \cdot c} \cdot \frac{\delta}{c \cdot \left(1 + \frac{T}{N \cdot c}\right)^2}. \tag{19}$$

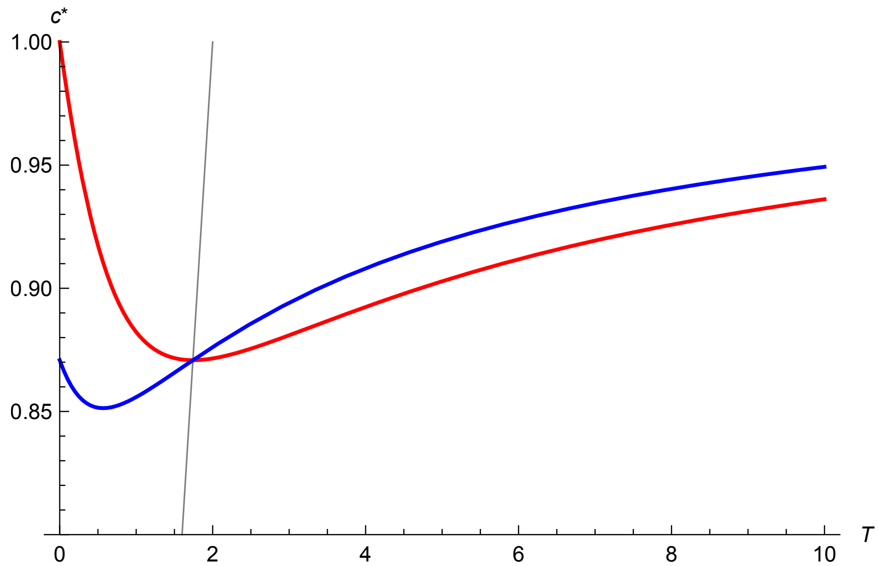
Under the quadratic benefit function, the marginal benefit of complexity is $f'(c) = \alpha - 2c$, so that solving for the equilibrium level of complexity requires solving a third-order polynomial. The solution can be expressed in closed form, but it is relatively complicated. Nevertheless, the key features of the equilibrium can be learnt by simply comparing the two first-order conditions (18) and (19). In particular, equilibrium complexity is larger than planner's choice of complexity if and only if

$$c^e < \frac{T}{N}. \tag{20}$$

Condition (20) defines a separating hyperplane in the parameter space. On one side ($c^e < \frac{T}{N}$) there is too much complexity, on the other side ($c^e > \frac{T}{N}$) there is too little complexity. Note, however, that c^e is an endogenous quantity. We therefore illustrate equilibrium complexity c^e and planner's optimal complexity c^* in Figure 2, plotted as a function of the buyer's attention capacity T. The blue line illustrates the equilibrium complexity of the two goods, whereas the red line corresponds to the planner's solution. The grey line illustrates the separating hyperplane implied by (20). To the right of this line, equilibrium complexity is higher than the complexity that a planner would choose. To the left of the grey line, equilibrium complexity is lower than the planner's solution.

The figure illustrates the *complexity paradox*: As attention capacity increases (i.e., larger T), the equilibrium level of complexity (blue line) lies above that chosen by the planner (red line). Therefore, complexity rises to inefficiently high levels precisely when information processing capacity grows. Rather than helping the buyer deal with complexity, increased information processing capacity can therefore be a source of excessive complexity in the economy. This confirms the intuition gained from equation (20): The first order effect of raising T is that it is more likely that equilibrium complexity c^e lies below the separating hyperplane. The adjustment of c^e in response to raising T does not overturn

Figure 2: **Equilibrium and planner's optimal complexity as a function of attention capacity**



Homogenous goods, parameters: $N = 2$, $\delta = 0.9$, $w = 1$, $\alpha = 2$, $\theta = 0.5$. The blue line is the equilibrium choice, the red line the planner's choice. Both converge to the unconstrained optimal complexity of 1 as $T \to \infty$. The thin grey line is the separating hyperplane (here a line) between too low and too high complexity.

this first order effect. The following proposition formalizes this insight and establishes a number of additional comparative statics.

Proposition 7. The Complexity Paradox and Other Comparative Statics. *In the homogenous goods case, if benefit of understanding a good δ is not too high, all goods get the same amount of attention. Equilibrium complexity of goods is too high compared to the panner's optimum if*

- *δ is high (i.e., it is the important to of pay attention);*
- *T is high (i.e., attention is abundant);*
- *α is low (i.e., complexity is not very valuable);*
- *N is low (i.e., there are fewer goods);*

but it does not depend on θ, the bargaining power of the seller.

In addition to the complexity paradox, another interesting prediction of Proposition 7 is that increasing the number of goods that the buyer consumes (which one may argue is a natural byproduct of economic development) leads to goods that are overly simple. The reason is that in symmetric equilibrium all goods get the same $\frac{T}{N}$ amount of attention. Therefore, more goods necessarily lead to less attention paid to each good (similar to decreasing the overall attention capacity T). Therefore, using the same logic by which lower T leads to overly simplistic goods, so does increasing the number of goods N.

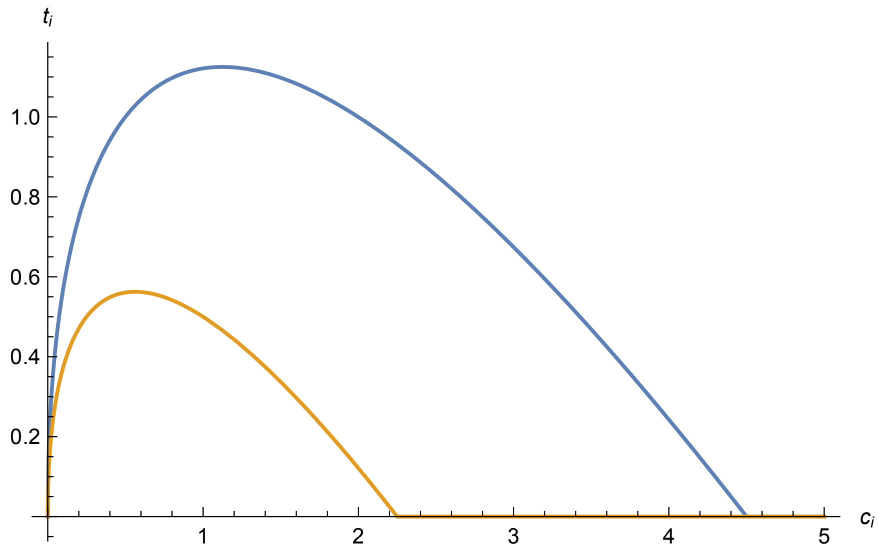
Note, however, that we are only stating that higher N leads to oversimplified goods relative to the planner's solution. It is not necessarily the case that goods become simpler in an absolute sense. In fact, Figure 2 illustrates that decreasing the attention budget T can lead to an increase in complexity. The reason is that when attention is severely limited, goods are not understood very well no matter what, so that it becomes optimal to increase their complexity to gain some value in that dimension. Using a similar argument, a lower benefit from complexity α and a higher cost of inattention δ lead to too much complexity by lowering the equilibrium complexity c_i^e, making condition (20) more likely to hold.

Recall that we stated Proposition 7 assuming the benefit of understanding the good δ is not too large. In this case, the equilibrium is symmetric. When δ is high, there is potentially also an asymmetric equilibrium for small T. For high δ and small T it is optimal (both for seller and the planner) to allocate complexity asymmetrically across otherwise identical goods: One good is very simple ($c_i = 0$), whereas the other good is complex ($c_j > 0$). Therefore, fundamentally similar products can have very different levels of complexity. As usual, the equilibrium does not pin down which good ends up being the complex one.

3.2 (Ex Ante) Heterogeneous Goods

In this section, we extend our analysis to consider ex ante heterogeneous goods. Ex ante heterogeneity in the characteristics of goods translates into ex post heterogeneity in their complexity and attention paid to the goods. Using Proposition 2, the seller of good i has an incentive to choose higher complexity than the planner if $\frac{\partial t_i(c_1,..c_N)}{\partial c_i} > 0$. Note that from Lemma 1 we know that $\frac{\partial t_i(c_1,..c_N)}{\partial c_i}$ has the same

Figure 3: **Attention to heterogenous goods as a function of complexity**



Heterogenous goods, parameters: $N = 2$, $T = 1$, $\delta = 0.9$, $w_1 = 1$, $w_2 = 2$, $\alpha = 2$, $\theta = 0.5$, shadow cost of attention fixed at $\lambda = 0.2$.

sign as $\frac{\partial t_i(c_i,\lambda)}{\partial c_i}\big|_\lambda$. Using (17), we can rewrite the condition $\frac{\partial t_i(c_i,\lambda)}{\partial c_i}\big|_\lambda > 0$ as

$$\frac{(1-\theta_i)\cdot\delta_i\cdot w_i}{c_i\cdot 4\cdot\lambda} > 1 \tag{21}$$

Comparing (17) and (21) it is clear that the goods that are too complex are the ones that get a lot of attention for a given level of complexity. More formally, we can rewrite (17) as:

$$\frac{t_i}{c_i} = 2\cdot\sqrt{\frac{(1-\theta_i)\cdot\delta_i\cdot w_i}{4\cdot c_i\cdot\lambda}} - 1. \tag{22}$$

Therefore, $\frac{t_i}{c_i} > 1$ if and only if (21), holds, (i.e., when $\frac{\partial t_i(c_i,\lambda)}{\partial c_i}\big|_\lambda > 0$). The following proposition formalizes this insight.

Proposition 8. Complexity and the Depth of Understanding. *When goods are ex ante heterogeneous, the seller*

- *has an incentive to overly complicate goods that are relatively well understood in the planner's solution,* $\frac{t_i^*}{c_i^*} > 1$*;*

- *has an incentive to oversimplify goods that are not well understood in the planner's solution,* $\frac{t_i^*}{c_i^*} < 1$

Proposition 8 provides a simple characterization of the distortion of product complexity. While the proposition is stated in terms of depth of understanding in the planner's solution, the depth of understanding usually does change drastically when we move from the planner's solution to the competitive equilibrium. Therefore, goods that in the planner's solution are relatively well understood end up being too complex. In contrast, goods for which the planner chooses a relatively shallow level of understanding end up being dumbed down even further.

Initially, this result may seem surprising. In particular, one may think that the hallmark of the goods that are too complex is that the buyer does not understand them well. However, contrary to this intuition, the goods that are understood well are the ones that the buyer is willing to invest her time in. Therefore, from the seller's perspective, it pays to make these goods more complex, in order to draw more attention from the consumer.

Based on the above characterization, goods that are likely to be too complex include smartphones, checking accounts, and equity mutual funds. Most people invest significant time in using and understanding their smartphone. Producers seize on this through the development of new features and apps. In the case of checking accounts, our model predicts that banks have an incentive to add an inefficiently high number of contingent fees and promotional interest rates, which makes deposit contracts more complex than they should be. Similarly, actively managed mutual funds are generally much more complicated than simple index funds, which most researchers would consider the optimal choice for investors. In contrast, our model implies that intricate policy debates or financial products based on complicated pools of assets may end up being oversimplified. For example, despite the apparent complications, the question of whether the UK should leave the EU was often dumbed down to how much the UK contributes to the EU budget. Similarly, despite being based on a pool of complicated underlying assets, mortgage-backed securities are often oversimplified to the rating of the underlying asset pool.

While Proposition 8 is stated in terms of depth of understanding in the planner's solution, the depth of understanding usually does not change drastically when we move from the planner's solution to the competitive equilibrium. Therefore, loosely speaking, goods that are well understood in equilibrium are made exceedingly complex, whereas goods not deeply understood by the buyer are dumbed down.

Equation (21) provides intuition for which goods tend to be well understood and therefore too complex. This is likely to be the case for goods that have a large utility weight (high w_i), goods for which the buyer gets to keep more of the surplus (low θ_i), and goods for which not paying attention is very costly (high δ_i). For such goods, the equilibrium level of complexity is likely to be on the upward sloping part of the $\tilde{t}_i(c_i, \lambda)$ function. However, simply reading these results from equation (21) is not quite correct because equilibrium complexity c_i^e is not unchanged if the above parameters change. In Proposition 9, we therefore show that the above intuition is not overturned by these adjustments in the level of equilibrium complexity. We can therefore characterize the effect of heterogeneity in the case of two goods as follows.

Proposition 9. Heterogenous goods. *Assume that there are two goods ($N = 2$) that are identical in all parameters $\{w, \delta, \alpha, \theta\}$, and that at these parameters the equilibrium and planner's level of complexity coincide. If we allow one of the above good-specific parameters $\pi \in \{w, \delta, \alpha, \theta\}$ to differ across the two goods, s.t. the first good has $\pi_1 = \pi - \epsilon$ and the second has $\pi_2 = \pi + \epsilon$, then for small enough $\epsilon > 0$ the following holds:*

(i) Importance ($\pi = w$). The less important good (smaller w_i) is simpler and too simple. The more important good (larger w_i) is more complex and too complex.

(ii) Bargaining power ($\pi = \theta$). The good for which the seller has more bargaining power (higher θ_i) is simpler in equilibrium and too simple. The good for which sellers have less bargaining power (lower θ_i) is more complex and too complex.

(iii) Importance of attention ($\pi = \delta$). The good for which attention is less important (low δ_i) is more complex but too simple. The good for which attention is more important (higher δ_i) is simpler but too complex.

(iv) Value of complexity ($\pi = \alpha$). The good for which complexity is more valuable (higher α_i) is more complex but too simple. The good for which complexity is less valuable (lower α_i) is simpler but too complex.

We summarize the characterization of goods from Proposition 9 in the complexity matrix illustrated in Table 1. The complexity matrix categorizes goods based on (i) whether they are simple or complex in an absolute sense and (ii) whether they are too simple or too complex relative to the planner's solution. Depending on their characteristics, goods can be complex (or simple) but not complex (or simple) enough, or they can be complex (or simple) and too complex (or simple) relative to the panner's optimum. While Proposition 8 provides a way of telling which goods are too complex or too simple based on the observed depth of understanding, Proposition 9 helps us understand why this is likely to be the case.

Table 1: **The Complexity Matrix**

	too simple	too complex
simple	low w_i $\sim$ less important good high θ_i $\sim$ seller bargaining power	high δ_i $\sim$ attention important low α_i $\sim$ complexity not beneficial
complex	low δ_i $\sim$ attention not important high α_i $\sim$ complexity beneficial	high w_i $\sim$ more important good low θ_i $\sim$ buyer bargaining power

Based on the observation that smartphones are typically well understood by their users, we argued that are likely too complex. But what drives this result? Using Table 1, we can identify a number of potential reasons. One the one hand, it could be the case that smartphones are too complex because attention an important component of they value they generate. However, according to Table 1, in this case it must also be the case that smartphones be simple in an absolute sense. On the other hand, one it could be the case that smartphones are too complex because either their utility weight w_i in the buyer's utility is high or because the buyer gets to keep most of the surplus (high $1-\theta_i$). In this case, Table 1 indicates that smartphones should also be complex in an absolute sense. Absolute complexity is the amount of time needed to understand a good up to a certain level. With the assumed functional form for v_i, complexity c_i is equal to the amount of time t_i that has to be invested by the buyer to gain exactly half of the utility increase that one could get if one moved from not understanding the good at all ($t_i = 0$) to understanding the good perfectly ($t_i = \infty$). If one is willing to argue that it is relatively time consuming to learn to use a smartphone reasonably well, then we would be in the latter case: smartphones are complex in an absolute sense and also too complex relative to what a planner would choose.

4 Conclusion

Complexity is an important choice variable for the sellers of goods and financial products. By developing an equilibrium model of product complexity, this paper shows that the complexity decision has important economic repercussions. In particular, in a world where buyers have a limited amount of time that they can devote to the goods they consume, equilibrium complexity choices generally involve an *attention externality*: When choosing the complexity of their goods, sellers do not take into account that attention is a common resource that is shared with other products. In equilibrium, sellers therefore distort the complexity of their products in order to steal attention from the goods of other sellers. However,in contrast to the classic tragedy of the commons, this can lead to *too much or too little complexity* in equilibrium.

Paradoxically, the equilibrium is more likely to feature excessive complexity when attention is abundant. Therefore, rather than helping buyers deal with complexity, increases in information processing capacity make it more likely that complexity is excessive—the *complexity paradox*. When goods are heterogeneous, our model shows that some products end up being be too complex in equilibrium, while other are too simple. Our model allows us to characterize the factors that determine whether goods are simple or complex, both in an absolute sense and relative to the efficient level of complexity. Counterintuitively, it is the goods that are well understood that are too complex in equilibrium, regardless of whether they are simple or complex in an absolute sense.

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A Proofs

Proof of Proposition 1.

What the incentives of seller i to change the level of complexity c_i of good i from the planner's optimum $(c_1^*, ...c_N^*)$ depends on the difference between the seller's first-order condition (8) and that of the planner (11), both evaluated at the planner's choice $(c_1^*, ...c_N^*)$. We rewrite the difference between the right-hand side of the two first-order conditions as

$$\sum_{j \neq i} \frac{\lambda^*}{1-\theta_j} \cdot \frac{\partial t_j(c_1^*, ..., c_N^*)}{\partial c_i} = \sum_{j \neq i} \frac{\lambda^*}{1-\theta_j} \cdot \frac{d\tilde{t}_j(c_j^*, \lambda(c_1^*, ..., c_N^*))}{dc_i} = \frac{\partial \lambda(c_1^*, ..., c_N^*)}{\partial c_i} \cdot \sum_{j \neq i} \frac{\lambda^*}{1-\theta_j} \cdot \frac{\partial \tilde{t}_j(c_j^*, \lambda^*)}{\partial \lambda}, \quad \text{(A1)}$$

where we use t_i^* as a shorthand for $t_i(c_1^*, ..c_N^*)$ and λ^* denotes the shadow cost of attention at the planner's solution, $\lambda(c_1^*, ..c_N^*)$. The first step in equation (A1) uses the fact that we can rewrite the buyers attention choice $t_j(c_1^*, ..c_N^*)$ as a function of the complexity of good j and the shadow cost of attention λ as $\tilde{t}_j(c_j^*, \lambda(c_1^*, ..c_N^*))$. The second step applies the chain rule, $\frac{d\tilde{t}_j(c_j^*, \lambda(c_1^*, ..c_N^*))}{dc_i} = \frac{\partial \lambda(c_1^*, ..c_N^*)}{\partial c_i} \frac{\partial \tilde{t}_j(c_j^*, \lambda^*)}{\partial \lambda}$ and moves the first (constant) term outside of the summation sign.

Note that raising the shadow cost of attention leads to less attention being paid to all goods, because the inverse function theorem implies

$$\frac{\partial \tilde{t}_i\left(c_i, \lambda\right)}{\partial \lambda} = \frac{1}{\frac{\partial \tilde{v}_i^2\left(c_i, \tilde{t}_i(c_i, \lambda)\right)}{\partial \tilde{t}_i^2}} < 0, \quad \text{(A2)}$$

where we used $\tilde{v}_j(c_j, \tilde{t}_j) = v_j(c_j, \frac{t_j}{c_j})$ and $\frac{\partial \tilde{v}_j^2\left(c_j, \tilde{t}_j(c_j, \lambda)\right)}{\partial \tilde{t}_j^2} < 0$ by assumption.

Thus from (A1), the externality is negative if $\frac{\partial t_i(c_1^*, ..c_N^*)}{\partial c_i} > 0$, meaning that the panner's optimum must entail a lower level of c_i, which in turn increases the left hand side of (11) (due to the decreasing benefits of complexity we have assumed). □

Proof of Proposition 2.

[TBA] □

Proof of Lemma 1.

Attention t_i allocated to good i can be implicitly expressed from (4) using $\tilde{t}_i\left(c_i, \lambda\right)$. Attention grabbing $\frac{\partial t_i(c_1, ..c_N)}{\partial c_i}$ can be written as:

$$\frac{\partial t_i(c_1, ..c_N)}{\partial c_i} = \frac{d\tilde{t}_i(c_i, \lambda(c_1, ..c_N))}{dc_i} = \frac{\partial \tilde{t}_i(c_i, \lambda)}{\partial c_i} + \frac{\partial \tilde{t}_i(c_i, \lambda)}{\partial \lambda} \cdot \frac{\partial \lambda(c_1, ..c_N)}{\partial c_i} \quad \text{(A3)}$$

where the first term is the effect of c_i on t_i keeping λ fixed, while the second the indirect effect through λ.

The defining implicit equation for equilibrium $\lambda(c_1, ..c_N)$ is:

$$T = \sum_j t_j = \sum_j \tilde{t}_j \left(c_j, \lambda\right) \tag{A4}$$

Without a specific functional form for v we cannot express $\lambda(c_1, ..c_N)$ explicitly in the heterogenous setup but we can take the derivative of interest $\frac{d.}{dc_i}$ of (A4):

$$0 = \frac{\partial \tilde{t}_i \left(c_i, \lambda\right)}{\partial c_i} + \sum_{j=1}^{N} \frac{\partial \tilde{t}_j \left(c_j, \lambda\right)}{\partial \lambda} \cdot \frac{\partial \lambda}{\partial c_i}, \tag{A5}$$

from which one can express

$$\frac{\partial \lambda}{\partial c_i} = \frac{\frac{\partial \tilde{t}_i(c_i, \lambda)}{\partial c_i}}{\sum_{j=1}^{N} - \frac{\partial \tilde{t}_j(c_j, \lambda)}{\partial \lambda}}. \tag{A6}$$

Plugging this into (A3) yields

$$\frac{\partial t_i(c_1, ..c_N)}{\partial c_i} = \frac{\partial \tilde{t}_i(c_i, \lambda)}{\partial c_i} + \frac{\partial \tilde{t}_i(c_i, \lambda)}{\partial \lambda} \cdot \frac{\frac{\partial \tilde{t}_i(c_i, \lambda)}{\partial c_i}}{\sum_{j=1}^{N} - \frac{\partial \tilde{t}_j(c_j, \lambda)}{\partial \lambda}} = \frac{\partial \tilde{t}_i(c_i, \lambda)}{\partial c_i} \cdot \left[1 - \frac{-\frac{\partial \tilde{t}_i(c_i, \lambda)}{\partial \lambda}}{\sum_{j=1}^{N} - \frac{\partial \tilde{t}_j(c_j, \lambda)}{\partial \lambda}}\right] \tag{A7}$$

The second term is positive as $\frac{\partial \tilde{t}_i(c_i, \lambda)}{\partial \lambda} < 0$ for all j (see (A2)). Thus $\frac{\partial \tilde{t}_i(c_i, \lambda)}{\partial c_i}$ has the same sign as $\frac{\partial t_i(c_1, ..c_N)}{\partial c_i}$. By the same argument, it is obvious from (A6) that $\frac{\partial \lambda}{\partial c_i}$ also has the same sign as $\frac{\partial t_i(c_1, ..c_N)}{\partial c_i}$.

Now we turn to $\frac{\partial^2 \tilde{v}(c_i, t_i)}{\partial c_i \partial t_i}$. One can rewrite the buyers problem (2) in terms of $\tilde{v}$:

$$\max_{t_1, ..t_N} \sum_{i=1}^{N} (1 - \theta_i) \cdot \tilde{v}_i \left(c_i, t_i\right). \tag{A8}$$

which is maximized subject to (3) and we get the counterpart of (4) wich for interior solution of t_i can be written as:

$$\frac{1}{(1 - \theta_i)} \cdot \lambda = \frac{\partial \tilde{v}_i \left(c_i, t_i\right)}{\partial t_i} \tag{A9}$$

Here the seller treats c_i as given. What happens if we allow this predetermined c_i to vary? Taking the partial with respect to c_i while keeping all other c's as fixed:

$$\frac{1}{1 - \theta_i} \cdot \frac{\partial \lambda}{\partial c_i} = \frac{\partial^2 \tilde{v}_i \left(c_i, t_i\right)}{\partial c_i \partial t_i} + \frac{\partial^2 \tilde{v}_i \left(c_i, t_i\right)}{\partial t_i^2} \cdot \frac{\partial \tilde{t}_i(c_i, \lambda)}{\partial c_i} + \frac{\partial^2 \tilde{v}_i \left(c_i, t_i\right)}{\partial t_i^2} \cdot \frac{\partial \tilde{t}_i(c_i, \lambda)}{\partial \lambda} \cdot \frac{\partial \lambda}{\partial c_i} \tag{A10}$$

where we took into account that t_i is a function of c_i and λ, thus $\tilde{t}_i(c_i,\lambda)$ and at the same time λ is the function of all other c's. Using (A2) that $\frac{\partial^2 \tilde{v}_i(c_i,t_i)}{\partial t_i^2} \cdot \frac{\partial \tilde{t}_i(c_i,\lambda)}{\partial \lambda} = 1$ and rearranging:

$$\frac{\theta_i}{1-\theta_i} \cdot \frac{\partial \lambda}{\partial c_i} = \frac{\partial^2 \tilde{v}_i\left(c_i,t_i\right)}{\partial c_i \partial t_i} + \frac{1}{\frac{\partial \tilde{t}_i(c_i,\lambda)}{\partial \lambda}} \cdot \frac{\partial \tilde{t}_i(c_i,\lambda)}{\partial c_i}. \tag{A11}$$

Using (A6) to substitute $\frac{\partial \lambda}{\partial c_i}$ one arrives at

$$\frac{\theta_i}{1-\theta_i} \cdot \frac{\frac{\partial \tilde{t}_i(c_i,\lambda)}{\partial c_i}}{\sum_{j=1}^{N} - \frac{\partial \tilde{t}_j(c_j,\lambda)}{\partial \lambda}} = \frac{\partial^2 \tilde{v}_i\left(c_i,t_i\right)}{\partial c_i \partial t_i} + \frac{1}{\frac{\partial \tilde{t}_i(c_i,\lambda)}{\partial \lambda}} \cdot \frac{\partial \tilde{t}_i(c_i,\lambda)}{\partial c_i}. \tag{A12}$$

Rearranging again yields:

$$\frac{\frac{\theta_i}{1-\theta_i} \cdot \frac{\partial \tilde{t}_i(c_i,\lambda)}{\partial c_i} + \sum_{j=1}^{N} \frac{\partial \tilde{t}_j(c_j,\lambda)}{\partial \lambda}}{\frac{\partial \tilde{t}_i(c_i,\lambda)}{\partial c_i} \cdot \sum_{j=1}^{N} \frac{\partial \tilde{t}_j(c_j,\lambda)}{\partial \lambda}} \cdot \frac{\partial \tilde{t}_i(c_i,\lambda)}{\partial c_i} = \frac{\partial^2 \tilde{v}_i\left(c_i,t_i\right)}{\partial c_i \partial t_i}, \tag{A13}$$

since $\frac{\partial \tilde{t}_i(c_i,\lambda)}{\partial \lambda} < 0$ for all $i \in \{1,..N\}$, it follows that $\frac{\partial \tilde{t}_i(c_i,\lambda)}{\partial c_i}$ and $\frac{\partial^2 \tilde{v}_i(c_i,t_i)}{\partial c_i \partial t_i}$ have the same sign. □

Proof of Proposition 3.

[TBA] □

Proof of Proposition 4.

Let us first concentrate on the case of $c_i \to 0$. We first prove that $\lim_{c_i \to 0} \frac{\partial t_i}{\partial c_i} > 0$ and $\lim_{c_i \to 0} t_i = 0$. Since $v(c,e)$ is bounded from above and strictly increasing in e, the following Inada condition holds:

$$\lim_{e_i \to \infty} \frac{\partial v\left(\cdot, e_i\right)}{\partial e_i} = 0 \tag{A14}$$

Note that $t_i = 0$ cannot be a solution if $c_i \to 0$ because $\lim_{c_i \to 0} \frac{\partial v(c_i,e_i)}{\partial e_i}\big|_{t_i=0} = \lim_{c_i \to 0} \frac{\partial v(c_i,0)}{\partial e_i} > 0$ by assumption and thus first-order condition of the buyer (4) would have to hold with inequality $(1-\theta_i)\frac{\partial v(c_i,e_i)}{\partial e_i}\frac{1}{c_i} < \lambda$. However this cannot hold for $t_i = 0$ as the left hand side would diverge to ∞. Thus $t_i > 0$, such that t_i satisfies the first-order condition $(1-\theta_i)\frac{\partial v(c_i,e_i)}{\partial e_i}\frac{1}{c_i} = \lambda$ with equality, where λ is a finite positive number.

Then the only way that the first-order condition can hold in the limit as $c_i \to 0$ is if $\frac{\partial v(c_i,e_i)}{\partial e_i}$ converges to zero, which requires that e_i converges to infinity. But then by l'Hôpital's rule

$$\lim_{c_i \to 0} e_i = \lim_{c_i \to 0} \frac{t_i}{c_i} = \lim_{c_i \to 0} \frac{\frac{\partial t_i}{\partial c_i}}{1} = \lim_{c_i \to 0} \frac{\partial t_i}{\partial c_i} = \infty > 0, \tag{A15}$$

thus indeed there exists a $\bar{c}_i > 0$, such that $\frac{\partial t_i}{\partial c_i} > 0$ for any $c_i < \bar{c}_i$.

Let us turn to the case of $c_i \to \infty$. This also implies $e_i \to 0$ since $t_i \leq T$. First we show that $\lim_{c_i \to \infty} t_i = 0$. Recall that we assumed $v(c,e)$ is bounded from both above and below, thus $\lim_{e \to 0} e \cdot v(c,e) = 0$. Write the following:

$$\lim_{e \to 0} v(c,e) = \lim_{e \to 0} \frac{e \cdot v(c,e)}{e} = \lim_{e \to 0} \frac{v(c,e) + e \cdot \frac{\partial v(c,e)}{\partial e}}{1} = \lim_{e \to 0} v(c,e) + \lim_{e \to 0} \frac{\frac{\partial v(c,e)}{\partial e}}{1/e} \tag{A16}$$

where the second equality follows from l'Hôpital's rule. Thus (A16) implies:

$$\lim_{e_i \to 0} \frac{\frac{\partial v(c_i,e_i)}{\partial e_i}}{1/e_i} = 0. \tag{A17}$$

By the first-order condition (4), for any $e > 0$ and $c > 0$ we have:

$$\frac{\frac{\partial v(c_i,e_i)}{\partial e_i}}{1/e_i} = t_i \cdot \frac{\frac{\partial v(c_i,e_i)}{\partial e_i}}{c_i} = t_i \cdot \frac{\lambda}{1-\theta_i}. \tag{A18}$$

We now take the limit as $c_i \to \infty$, which also implies $e_i \to 0$. Because by assumption $\frac{\partial^2 v(c_i,e_i)}{\partial c_i \partial e_i} = 0$, we have

$$\lim_{c_i \to \infty} \frac{\frac{\partial v(c_i,e_i)}{\partial e_i}}{1/e_i} = \lim_{e_i \to 0} \frac{\frac{\partial v(c_i,e_i)}{\partial e_i}}{1/e_i} = 0, \tag{A19}$$

which, by equation (A18), implies that

$$\lim_{c_i \to \infty} t_i \cdot \lambda = 0. \tag{A20}$$

Now notice that in equilibrium we cannot have $\lambda = 0$ since then buyers would allocate infinite time to a good, contradicting the assumption that the attention constraint is no binding. Thus λ is bounded from below, which implies $\lim_{c_i \to \infty} t_i = 0$. [TBA: partial derivative at higher c]

Proof of Proposition 5.

If $t_i > 0$ then the first-order condition of the buyer (4) would have to hold with equality $(1-\theta_i)\frac{\partial v(c_i,e_i)}{\partial e_i}\frac{1}{c_i} = \lambda$. However, if $c_i \to \infty$ then this cannot hold unless $\frac{\frac{\partial v(c_i,e_i)}{\partial e_i}}{c_i}$ or λ diverges to ∞. We show that neither can be the case. [TBA]

$\square$

Proof of Proposition 6.

The partial derivative of interest can be written in terms of effective attention $\frac{t_i}{c_i}$

$$\frac{\partial \tilde{v}_i(c_i,t_i)}{\partial c_i} = \left.\frac{dv_i\left(c_i,\frac{t_i}{c_i}\right)}{dc_i}\right|_{t_i} = \frac{\partial v_i\left(c_i,\frac{t_i}{c_i}\right)}{\partial c_i} - \frac{t_i}{c_i^2}\cdot\frac{\partial v_i\left(c_i,\frac{t_i}{c_i}\right)}{\partial\left(\frac{t_i}{c_i}\right)}. \tag{A21}$$

On the right hand side, we can substitute $\frac{\partial v_i\left(c_i,\frac{t_i}{c_i}\right)}{\partial c_i}$ from the first-order condition (7) of the seller i and arrive at

$$\frac{\partial \tilde{v}_i(c_i,t_i)}{\partial c_i} = -\frac{1}{c_i}\cdot\frac{\partial v_i\left(c_i,\frac{t_i}{c_i}\right)}{\partial\left(\frac{t_i}{c_i}\right)}\cdot\frac{\partial t_i}{\partial c_i}. \tag{A22}$$

Since $\frac{\partial v_i\left(c_i,\frac{t_i}{c_i}\right)}{\partial\left(\frac{t_i}{c_i}\right)}$ is strictly positive by assumption, we have shown that $\frac{\partial \tilde{v}_i(c_i,t_i)}{\partial c_i}$ and $\frac{\partial t_i}{\partial c_i}$ have opposite signs.

□

Proof of Lemma 2.

We use the heterogenous setup until noted otherwise. The first-order condition of the buyer for a given λ is:

$$\tilde{t}_i(c_i,\lambda) = \sqrt{\frac{c_i\cdot\delta_i\cdot(1-\theta_i)\cdot w_i}{\lambda}} - c_i. \tag{A23}$$

Plugging the above into (3) that time adds up to T one can express λ:

$$\lambda = \frac{\left(\sum_{k=1}^N \sqrt{c_k\delta_k\left(1-\theta_k\right)w_k}\right)^2}{\left(\sum_{j=1}^N c_j + T\right)^2}, \tag{A24}$$

substituting this back into (A23) we arrive at:

$$t_i(c_1,..c_N) = \frac{\sqrt{c_i\delta_i\left(1-\theta_i\right)w_i}}{\sum_{k=1}^N \sqrt{c_k\delta_k\left(1-\theta_k\right)w_k}}\cdot\left(\sum_{j=1}^N c_j + T\right) - c_i. \tag{A25}$$

Taking the derivative of interest that captures the attention grabbing:

$$\frac{\partial t_i(c_1,..c_N)}{\partial c_i} = \frac{\sum_{j\neq i}\sqrt{c_j\delta_j\left(1-\theta_j\right)w_j}}{\left(\sum_{k=1}^N \sqrt{c_k\delta_k\left(1-\theta_k\right)w_k}\right)^2}\cdot\frac{1}{2}\cdot\frac{1}{\sqrt{c_i}}\cdot\sqrt{\delta_i\left(1-\theta_i\right)w_i}\cdot\left(\sum_{j=1}^N c_j + T\right) + \frac{\sqrt{c_i\delta_i\left(1-\theta_i\right)w_i}}{\sum_{k=1}^N \sqrt{c_k\delta_k\left(1-\theta_k\right)w_k}} - 1. \tag{A26}$$

Imposing symmetry one gets:

$$\frac{\partial t}{\partial c} = \frac{N-1}{2\cdot N}\cdot\left(\frac{T}{N\cdot c} - 1\right) \tag{A27}$$

and

$$\lambda = \frac{w \cdot \delta \cdot (1-\theta)}{c \cdot \left(1 + \frac{T}{N \cdot c}\right)^2}. \tag{A28}$$

Plugging these into (8) and (12) and using that the for the symmetric case with the given functional form $\frac{\partial v_i\left(c_i, \frac{t_i}{c_i}\right)}{\partial c_i} = w \cdot f'(c)$ and that in a symmetric equilibrium all goods get the same amount of attention $t = \frac{T}{N}$ we arrive at the equations stated in the lemma. □

Proof of Proposition 7.

The statement is made for low enough $\delta < \delta^*$, because if δ is too high then the planner wants to differentiate goods. In the following we calculate a lower bound for δ^*. First note that the maximum v that can be derived from a good is when the complexity is chosen to be its first best (unconstrained) optimum of $c_i = \frac{\alpha_i}{2}$ and effective attention is chosen to be infinite $\frac{t_i}{c_i} = \infty$. Thus:

$$v_i\left(c_i, \frac{t_i}{c_i}\right) < v_i\left(\frac{\alpha_i}{2}, \infty\right) = w_i \cdot \left(\delta_i + \frac{\alpha_i^2}{4}\right) \tag{A29}$$

Also with N ex ante symmetric goods if all goods have the same amount of complexity and thus get the same amount of attention from the buyer, then v_i can be bounded from below because if $c = \frac{\alpha}{2}$ is not optimal, then v would be higher with the optimal choice:

$$v_i\left(c_i, \frac{t_i}{c_i}\right) > v\left(\frac{\alpha}{2}, \frac{T/N}{\alpha/2}\right) = \frac{w\left(\alpha^3 N + 2T\left(\alpha^2 + 4\delta\right)\right)}{4\alpha N + 8T} \tag{A30}$$

We need to show that

$$N \cdot v\left(c_s^*, \frac{t_{s,s}^*}{c_s^*}\right) > (N-1) \cdot v\left(c_a^*, \frac{t_{s,a}^*}{c_a^*}\right) + 1 \cdot v\left(0, \infty\right) \tag{A31}$$

where c_s^* is the panner's optimum in the symmetric case and c_a^* in the asymmetric one (in which one of the goods has zero complexity but the others have the same). A sufficient condition for the symmetric solution is that

$$\frac{w\left(\alpha^3 N + 2T\left(\alpha^2 + 4\delta\right)\right)}{4\alpha N + 8T} > (N-1) \cdot w \cdot \left(\delta + \frac{\alpha^2}{4}\right) + w \cdot \delta \tag{A32}$$

which simplifies to

$$\delta < \frac{\alpha(\alpha N + 2T)}{4N^2} \tag{A33}$$

and holds for small enough δ and is more likely to be violated for small attention capacity T. It holds for all T (including $T = 0$) if $\delta < \frac{\alpha^2}{4 \cdot N}$: this is the sufficient (but not necessary) condition for the panner's optimum to be symmetric.

[TBA: proof that if the planner's optimum is symmetric, so is the competitive equilibrium]

The separating hyperplane for which the optimal and equilibrium levels of complexity are the same happens at critical attention level T at which all goods have complexity $c = \frac{T}{N}$ (see Eq. 20). Plugging this into Eq. 11 yields a quadratic equation for T, for which the only solution that corresponds to a maximum of the social welfare function is (this can be checked by signing the second order condition):

$$T^{crit} = \frac{N}{4}\left(\alpha + \sqrt{\alpha^2 - 2\delta}\right) \tag{A34}$$

This defines a separating hyperplane in the parameter space, so by continuity of equilibrium complexity in the parameters, all we have to check is whether there is too much complexity on one side of the hyperplane arbitrarily close to the hyperplane itself.

Taking the total derivative of the two first order conditions Eq. 8 and 11 with respect to δ and substituting $c = \frac{T}{N}$ (at $c^e = c^*$), and expressing $c^{e\prime}(\delta)$ and $c^{*\prime}(\delta)$ the two FOCs yield:

$$c^{e\prime}(\delta) = \frac{2NT}{\delta N(N+1) - 16T^2} \tag{A35}$$

$$c^{*\prime}(\delta) = \frac{NT}{\delta N^2 - 8T^2} \tag{A36}$$

We want to show that if δ is a bit higher than on the hyperplane, then equilibrium complexity is higher than the panner's optimum. Thus we have to show $c^{e\prime}(\delta) > c^{*\prime}(\delta)$ at $T = T^{crit}$ which holds if,

$$\frac{\sqrt{\alpha^2 - 2\delta} + \alpha}{\left(\alpha\sqrt{\alpha^2 - 2\delta} + \alpha^2 - 2\delta\right)\left(N\left(2\alpha\sqrt{\alpha^2 - 2\delta} + 2\alpha^2 - 3\delta\right) - \delta\right)} \geq 0, \tag{A37}$$

which holds if $\delta < \frac{\alpha^2}{2}$.

Taking the total derivative of the two first order conditions Eq. 8 and 11 with respect to T and substituting $c = \frac{T}{N}$ (at $c^e = c^*$), and expressing $c^{e\prime}(T)$ and $c^{*\prime}(T)$ the two FOCs yield:

$$c^{e\prime}(T) = \frac{\delta - \delta N}{\delta N(N+1) - 16T^2} \tag{A38}$$

$$c^{*\prime}(T) = 0 \tag{A39}$$

$c^{e\prime}(T) > c^{*\prime}(T)$ holds at $T = T^{crit}$ if $\delta < \frac{\alpha^2}{2}$.

Taking the total derivative of the two first order conditions Eq. 8 and 11 with respect to α and substituting $c = \frac{T}{N}$ (at $c^e = c^*$), and expressing $c^{e\prime}(\alpha)$ and $c^{*\prime}(\alpha)$ the two FOCs yield:

$$c^{e\prime}(\alpha) = \frac{8T^2}{16T^2 - \delta N(N+1)} \tag{A40}$$

$$c^{*\prime}(\alpha) = \frac{4T^2}{8T^2 - \delta N^2} \tag{A41}$$

$c^{e\prime}(\alpha) < c^{*\prime}(\alpha)$ holds at $T = T^{crit}$ if $\delta < \frac{\alpha^2}{2}$.

Taking the total derivative of the two first order conditions Eq. 8 and 11 with respect to N and substituting $c = \frac{T}{N}$ (at $c^e = c^*$), and expressing $c^{e\prime}(N)$ and $c^{*\prime}(N)$ the two FOCs yield:

$$c^{e\prime}(N) = \frac{\delta(N-1)T}{N\left(\delta N(N+1) - 16T^2\right)} \tag{A42}$$

$$c^{*\prime}(N) = 0 \tag{A43}$$

$c^{e\prime}(N) < c^{*\prime}(N)$ holds at $T = T^{crit}$ if $\delta < \frac{\alpha^2}{2}$. □

Proof of Proposition 8.

See text. □

Proof of Proposition 9.

Heterogeneity in δ: $\delta_1 = \delta - \epsilon$ for good 1 and $\delta_2 = \delta + \epsilon$ for good 2, where δ is the level at which the equilibrium and planner's levels of complexity coincide for both goods. We know from (20) that the level of complexity for both goods at $\epsilon = 0$ is very simple

$$c_1^s = c_2^s = c_1^p = c_2^p = \frac{T}{2}. \tag{A44}$$

The strategy of the proof is to look at the derivative of equilibrium and planner's optimal complexity in ϵ. This way we can conclude which complexity is higher if ϵ is small enough. We can express these derivatives by taking total differential of the first-order conditions for the two goods and then setting $\epsilon = 0$: Equation 7 in the equilibrium case and Equation 10 in the planner's case. In the equilibrium, this gives us two equations and we can solve for the two unknowns: $\left.\frac{\partial c_1^p}{\partial \epsilon}\right|_{\epsilon=0} = \left.\frac{\partial c_1^p}{\partial \delta_1}\right|_{\delta_1=\delta}$ and $\left.\frac{\partial c_2^p}{\partial \epsilon}\right|_{\epsilon=0} = \left.\frac{\partial c_2^p}{\partial \delta_2}\right|_{\delta_2=\delta}$. Similarly we have two equations and two unknowns for the derivatives of the planner's optimal complexity $\left.\frac{\partial c_1^s}{\partial \epsilon}\right|_{\epsilon=0} = \left.\frac{\partial c_1^s}{\partial \delta_1}\right|_{\delta_1=\delta}$ and $\left.\frac{\partial c_2^s}{\partial \epsilon}\right|_{\epsilon=0} = \left.\frac{\partial c_2^s}{\partial \delta_2}\right|_{\delta_2=\delta}$.

First note that in all cases it must be the case that the parameters are chosen s.t. the level of T is equal to (A34) because this is the combination of parameters at equilibrium and planner's levels of complexity coincide.

The derivatives of interest in the panner's optimum are

$$\left.\frac{\partial c_1^s}{\partial \theta_1}\right|_{\theta_1=\theta} = -\left.\frac{\partial c_2^s}{\partial \theta_2}\right|_{\theta_2=\theta} = \frac{T}{2\left(2T^2-\delta\right)}, \tag{A45}$$

for the equilibrium

$$\left.\frac{\partial c_1^p}{\partial \theta_1}\right|_{\theta_1=\theta} = -\left.\frac{\partial c_2^p}{\partial \theta_2}\right|_{\theta_2=\theta} = \frac{3T}{2\left(8T^2-3\delta\right)}. \tag{A46}$$

If $T > \sqrt{\frac{3}{8}\delta}$ then it follows that

$$\left.\frac{\partial c_2^p}{\partial \theta_2}\right|_{\theta_2=\theta} < \left.\frac{\partial c_1^p}{\partial \theta_1}\right|_{\theta_1=\theta} \tag{A47}$$

thus if ϵ is small enough then good 2 (with the higher δ) is simpler than good 1 (with the lower δ). $T > \sqrt{\frac{3}{8}\delta}$ holds since T is defined by (A34) (with $N = 2$) and $\alpha^2 - 2\delta > 0$ follows from the existence (assumed in the statement of the theorem) of a critical T where the planner's and equilibrium complexity coincide.

If furthermore $T > \frac{\sqrt{\delta}}{\sqrt{2}}$ (which again follows from (A34)), it is straightforward to show that

$$\left.\frac{\partial c_2^p}{\partial \theta_2}\right|_{\theta_2=\theta} > \left.\frac{\partial c_2^s}{\partial \theta_2}\right|_{\theta_2=\theta}. \tag{A48}$$

This proves that if ϵ is small enough, then good 2 (with the higher δ) is too complex in equilibrium than in the planner's choice. By the same logic this proves that good 1 (with the lower δ) is too simple.

The rest of the proofs follow a similar logic and we just report the main steps. We also assume in all the following that $T > \frac{\sqrt{\delta}}{\sqrt{2}}$.

Heterogeneity in θ: The derivatives of interest in the panner's optimum are

$$\left.\frac{\partial c_1^s}{\partial \theta_1}\right|_{\theta_1=\theta} = -\left.\frac{\partial c_2^s}{\partial \theta_2}\right|_{\theta_2=\theta} = 0, \tag{A49}$$

for the equilibrium

$$\left.\frac{\partial c_1^p}{\partial \theta_1}\right|_{\theta_1=\theta} = -\left.\frac{\partial c_2^p}{\partial \theta_2}\right|_{\theta_2=\theta} = \frac{\delta T}{2(1-\theta)\left(8T^2-3\delta\right)}. \tag{A50}$$

It follows that

$$\left.\frac{\partial c_2^p}{\partial \theta_2}\right|_{\theta_2=\theta} < \left.\frac{\partial c_1^p}{\partial \theta_1}\right|_{\theta_1=\theta} \tag{A51}$$

and

$$\left.\frac{\partial c_2^p}{\partial \theta_2}\right|_{\theta_2=\theta} < \left.\frac{\partial c_2^s}{\partial \theta_2}\right|_{\theta_2=\theta}. \tag{A52}$$

Heterogeneity in α: The derivatives of interest in the panner's optimum are

$$\left.\frac{\partial c_1^s}{\partial \alpha_1}\right|_{\alpha_1=\alpha} = -\left.\frac{\partial c_2^s}{\partial \alpha_2}\right|_{\alpha_2=\alpha} = -\frac{T^2}{2T^2-\delta}, \tag{A53}$$

for the equilibrium

$$\left.\frac{\partial c_1^p}{\partial \alpha_1}\right|_{\alpha_1=\alpha} = -\left.\frac{\partial c_2^p}{\partial \alpha_2}\right|_{\alpha_2=\alpha} = -\frac{4T^2}{8T^2-3\delta}. \tag{A54}$$

It follows that

$$\left.\frac{\partial c_2^p}{\partial \alpha_2}\right|_{\alpha_2=\alpha} > \left.\frac{\partial c_1^p}{\partial \alpha_1}\right|_{\alpha_1=\alpha} \tag{A55}$$

and

$$\left.\frac{\partial c_2^p}{\partial \alpha_2}\right|_{\alpha_2=\alpha} < \left.\frac{\partial c_2^s}{\partial \alpha_2}\right|_{\alpha_2=\alpha}. \tag{A56}$$

Heterogeneity in w: The derivatives of interest in the panner's optimum are

$$\left.\frac{\partial c_1^s}{\partial w_1}\right|_{w_1=w} = -\left.\frac{\partial c_2^s}{\partial w_2}\right|_{w_2=w} = \frac{T\left(\delta+2T^2-2\alpha T\right)}{2w\left(2T^2-\delta\right)}, \tag{A57}$$

for the equilibrium

$$\left.\frac{\partial c_1^p}{\partial w_1}\right|_{w_1=w} = -\left.\frac{\partial c_2^p}{\partial w_2}\right|_{w_2=w} = \frac{T\left(3\delta+8T^2-8\alpha T\right)}{2w\left(8T^2-3\delta\right)}. \tag{A58}$$

It follows that

$$\left.\frac{\partial c_2^p}{\partial w_2}\right|_{w_2=w} > \left.\frac{\partial c_1^p}{\partial w_1}\right|_{w_1=w} \tag{A59}$$

and

$$\left.\frac{\partial c_2^p}{\partial w_2}\right|_{w_2=w} > \left.\frac{\partial c_2^s}{\partial w_2}\right|_{w_2=w}. \tag{A60}$$

For the first we need that $T \in \left(\sqrt{\frac{3}{8}}\sqrt{\delta}, \frac{1}{2}\left(\sqrt{\alpha^2 - \frac{3\delta}{2}} + \alpha\right)\right)$, while for the second we need $T > \frac{\alpha}{2}$, both follow from (A34). $\square$