The Welfare Effect of Product Incompatibility in Complementary Good Markets

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Abstract

This paper studies the welfare effect of product incompatibility in complementary goods markets. Complementary goods are often incompatible across brands. Incompatibility imposes a choice constraint and increases consumers' costs of switching or upgrading. Firms take advantage of incompatibility to lock in consumers. In this paper, I develop a dynamic consumer demand model and an oligopoly pricing game for complementary goods with incompatibility. The model is estimated by Simulated Maximum Likelihood and Expectation-Maximization (EM) algorithm using a large-scale individual-level consumer panel data in the U.S. men's shaving market. Estimates are used to quantify the impact of product incompatibility on price competition and consumer welfare. I solve for the counterfactual market equilibrium in which products are compatible across firms and/or technologies within firms. Results show that compatibility softens price competition in the men's shaving market. Two effects are presented: demand expansion effect and intensified competition effect. Razor prices go up since firms can't lock in consumers by razors. Blade prices go up since demand expansion effect dominates intensified competition effect. Consumer welfare is improved overall because the benefit consumers derive from expanded choices outweighs increased product costs. However, the welfare effect varies across consumers.

Keywords: Product Incompatibility, Price Competition, Consumer Welfare, Complementary Goods Markets

JEL Classification: L15, L42, L68, D23, C61

1   Introduction

Product incompatibility is commonplace in many complementary goods markets. In the market of smartphones and operating systems, IOS can only be installed in IPhones. Furthermore, in the market of game consoles and video games, exclusive contracts between game console manufacturers and game developers make some new video games available to be played only on certain game consoles[1]. Given the growing prevalence of complementary goods, addressing the effect of product incompatibility on market competition and consumer welfare is crucial for antitrust and public policies [2]. However, the welfare effect of product incompatibility in complementary goods markets is a source of active debate and an open empirical question (e.g., Matutes and Regibeau (1988), Economides (1989), Whinston (1990), Katz and Shapiro (1994), Choi (2010), and Zhou (2017)). [3]

This paper investigates the welfare effect of product incompatibility in complementary goods markets. The key questions to be answered are: How does product incompatibility affect price competition and consumer welfare in complementary goods markets? In particular, when products are incompatible across firms and also across successive technologies within firms, how does horizontal and vertical incompatibility, respectively, each affect price competition and consumer welfare? In addition, what are the implications on policies relevant to product (in)compatibility like standardization and patent protection? Answers to these questions will shed light on the regulation policy of antitrust and intellectual property, and firms' optimal pricing strategy and compatibility design in general complementary products markets.

Previous empirical studies on the welfare effect of incompatibility focus on consumer demand change ignoring the effect of incompatibility on price competition and therefore provide misleading welfare results. To the best of my knowledge, this paper is the first to empirically study the welfare effect of product incompatibility taking into account both consumer demand and price competition and incorporating the vertical incompatibility within firms. An oligopoly pricing game incorporating a dynamic demand model for complementary goods with incompatibility is developed and estimated using an individual-level consumer

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[1]Other examples of complementary goods with incompatiblity include tied goods (razors and blades, printers and ink cartridges, cameras and lenses), hardware and software (computers and software, IPhone and IOS), and platforms and applications (Windows and IE, Apple Store and Applications) and so on.

[2]It is at the heart of recent antitrust cases, e.g., U.S. v.s. Microsoft (2001), U.S. v.s. Visa/MasterCard (2003) and European Union v.s. Google (2016).

[3]For example, there is debate regarding the effect of product incompatibility on price competition. On the one hand, incompatibility may soften price competition since incompatibility allows firms to leverage market power from one market to the other and charge higher prices. On the other hand, incompatibility may intensify price competition when products are perfect complements (e.g., Matutes and Regibeau (1988), Zhou (2017)). Incompatibility reduces product variety and shrinks market demand, and it also allows firms to internalize the complementarity between two products and charge lower prices.

panel data in the U.S. men's shaving market. Counterfactual results show that, in contrast to traditional knowledge but consistent with the recent theoretical literature (e.g. Zhou (2017)), compatibility softens price competition in complementary goods markets since demand expansion effect dominates intensified competition effect. Accordingly, consumer welfare taking into account increased product costs is improved only marginally when products are compatible. My model and findings can be applied to general complementary goods markets with incompatibility like tied goods, hardware and software, platforms and applications. This suggests policies/practices in favor of compatibility like standardization may not improve consumer welfare since compatibility may soften price competition and hurt consumers by higher prices. On the other hand, policies in favor of incompatibility like intellectual property (patent protection) may intensify price competition and benefit consumers by lower prices.

The U.S. men's shaving market provides a suitable setting to study the welfare effect of product incompatibility in complementary good markets. Razors and blades are complementary goods. Moreover, razors and blades are incompatible across firms and across technologies within the same firm. The market is highly concentrated with two big brands. Gillette and Schick compete in both razors and blades. Each firm has three technologies and the disposable razor is the non-tied shaving option.

In this paper, I am using the Nielsen consumer panel data from the Kilts center of marketing at the University of Chicago, Booth School of Business. The data is at the household level and covers all retail channels and geographic markets. Household-level purchasing histories are observed from 2004 to 2014. Different from market-level data, individual-level consumer panel data provides much more information on consumers' preferences and heterogeneity which facilitates the identification of demand parameters.

Data patterns reveal how product incompatibility affects consumer demand for complementary goods and provide insights for empirical modeling. First, razor ownership affects blade choices since consumers need to own a compatible razor before purchasing a blade. However, about one-third consumers own multiple razors in hand after controlling the number of males in households. The choice constraint from incompatibility is mitigated by multi-homing in consumers. Second, consumers have stockpiling behaviors. The higher their blade inventory is, the less likely they are to engage in a new purchase. Third, consumers have heterogeneity in blade consumption rates, implying the effect of incompatibility on demand varies across consumers.

Motivated by data patterns, I develop a dynamic demand model for complementary goods incorporating product incompatibility, stockpiling behaviors, and unobserved consumers' heterogeneity. Product incompatibility affects consumer demand via different channels: imposing a choice constraint and increasing costs of switching or upgrading. On the one

hand, consumers' choices of blades are constrained by their razor ownerships. On the other hand, the costs of switching and upgrading are higher with incompatibility. Consumers need to buy a new razor before using a new blade. Moreover, unused and incompatible blades in the inventory also make consumers less likely to switch and/or upgrade.

To understand how incompatibility affects price competition, I develop an oligopoly pricing game in which firms compete in both primary goods (razors) and aftermarket goods (blades). Consumers are locked in by their razor ownerships and face high costs to switch and upgrade. Firms compete for a larger consumer base in razors by pricing razors at a lower markup and then gain a higher revenue stream by pricing blades at a higher markup. In contrast to single-product pricing, firms take into account the complementarity between razors and blades. Moreover, multiple-product firms also take into account the cannibalization effect between different technologies within the same firm.

I then estimate the structural model using the individual-level Nielsen consumer panel data in the U.S. men's shaving market. I first estimate the dynamic demand model by the simulated maximum likelihood method. To control for the unobserved consumers' types, I adopt Expectation-Maximization (EM) algorithm to estimate both the distribution of unobserved consumers' types and demand parameters. Given demand estimates, the oligopoly pricing game is estimated by the first order conditions of observed optimal prices. Individual-level consumer panel data facilitates the identification of unobserved type distribution, blade consumption rates and preference parameters.

Following the estimation strategy, I estimate the model and present estimation results. Estimates show that consumers have unobserved heterogeneity in blade consumption rates, price sensitivity, and other preference parameters. Firms price razors at a low markup and price blades at a higher markup. The model fit is checked by comparing the consumers' choice probabilities of each technology and package generated by the model with ones observed in the data. Long-run own- and cross-price elasticities are shown, implying the complementarity between razors and blades and the cannibalization effect between different technologies within the same firm.

To investigate the effect of product incompatibility on price competition and consumer welfare, I use the estimates to conduct counterfactual analysis in which razors and blades are compatible. Vertical and horizontal compatibility are explored separately[4]. Three counterfactual experiments are conducted: horizontal compatible across firms; vertical compatible across technologies within firms; all round compatible across firms and technologies.

In each counterfacutual experiment, product compatibility affects consumer demand

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[4]As defined in Katz and Shapiro (1994), "horizontal" compatibility is between two roughly comparable rival systems and "vertical" compatibility is between successive generations of similar technology.

in several ways. For example, there is less or no constraint on blade choices from razor ownership. The removal of choice constraint makes razors and blades more attractive and demand for razors and blades increases. Next, costs of switching or upgrading are lower since consumers now don't need to purchase a new razor and can use the old blades in the inventory with new purchases. Finally, because consumers only need one razor to use all blades, the demand for razors decreases and it's less likely for consumers to own multiple razors.

Product compatibility also changes firms' pricing strategies. Two effects determine the new prices of razors and blades: competition is intensified and demand for blades expands. On the one hand, razors can't lock in consumers and firms raise razor prices. Competition between firms is intensified in blades, blade prices may be lower. On the other hand, the demand for blades is higher. Increased product variety attracts some consumers from disposable razors and outside option. Moreover, the market size of blades expands since all consumers, no matter their razor ownership, have a demand for all blades. The demand expansion effect enables firms to charge higher prices of blades.

Counterfactual results suggest that product compatibility softens price competition and improves consumer welfare in the U.S. men's shaving market. Firms charge higher prices for both razors and blades since the demand expansion effect dominates the effect of intensified competition. Consumer welfare is improved since the benefit consumers derive from expanded choices outweighs increased product costs. However, the welfare effect varies across consumers since consumers have heterogeneity in preferences and blade consumption rates. Consumers with strong brand preferences and large blade consumption rates are more likely to be worse off.

This paper studies the welfare effect of product incompatibility which is a source of active debate and an open empirical question. My paper adds to the empirical literature by studying the complementary goods markets and addressing the effect of incompatibility on price competition. I find, in contrast to traditional knowledge but consistent with the recent theoretical literature (e.g. Zhou (2017)), compatibility softens price competition. Previous empirical studies have primarily studied markets of individual products with direct or indirect network effects where incompatibility is between networks [5]. There is little empirical literature in the complementary goods markets where incompatibility is between primary goods (e.g. razors, printers, game consoles) and aftermarket goods (e.g. blades, ink cartridges, video games). The majority of previous empirical studies evaluating consumer welfare effect have ignored the effect of incompatibility on price competition and provide misleading welfare

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[5]including Ohashi (2003) on VCRs; Rysman (2004) on yellow pages; Ho (2006) on insurer-hospital networks; and Knittel and Stango (2011) on ATM networks.

results. Lee (2013) studies the market of game consoles and video games, and simulates market outcomes of increasing compatibility between hardware (game consoles) and software (video games). He finds that consumer welfare is improved given prices are fixed. Differently, I also study the effect of incompatibility on price competition and find consumer welfare is improved overall taking into account price changes, but the improvement is marginal and the welfare effect varies across consumers.

This paper also contributes to the literature of dynamic demand by modeling the dynamic demand for complementary goods. Previous literature have primarily studied the dynamic demand for durable goods (e.g. Gowrisankaran and Rysman (2012), Goettler and Gordon (2011)) and storable goods (e.g. Hendel and Nevo (2006)). Complementary goods provide a new source of dynamics in consumer demand: consumers form expectations about the future availability and prices of aftermarket goods. Hartmann and Nair (2010) study the consumer purchasing patterns in the market of razors and blades in a short period, and assume consumers own only one type of blades and there is no replacement for razors. Differently, I investigate the effect of incompatibility on consumers' technology upgrading decisions over a long period. Consumers are also allowed to own blades of multiple brands in inventory. It provides insights into explaining vertical incompatibility decisions of multi-product firms like Gillette and Apple.

Few empirical literature studies the pricing of complementary goods. Previous papers have largely ignored or adopted a reduced form approach to one side of the market (e.g., Nair (2007), Dube et al. (2010)). Some recent papers study the pricing of complementary goods using a structural approach. Derdenger (2014) finds the technology tying intensifies price competition in hardware (game consoles), but assumes prices of software (video games) are fixed. My paper studies price competition of both primary goods and aftermarket goods. Li (2018) simulates the optimal prices of E-readers and E-books for a monopolist (Amazon). Differently, I model and estimate an oligopoly pricing game. Most relevant to this paper, Chintagunta et al. (2018) study the impact of licensing on price competition using an aggregate-level data. They adopt a static demand model for aftermarket goods (coffee pods) and assume a fixed durability of primary goods (coffee machines). My paper models the dynamic demand for both products and focuses on the effect of product incompatibility. Additionally, I look at firms with multiple technologies, and study consumers' technology upgrading patterns. Moreover, individual-level consumer panel data allows incorporating multi-homing (owning multiple products) and consumers' heterogeneity which are crucial to evaluate the consumer welfare effect.

The rest of this paper is structured as follows. Section 2 introduces the institutional background. Data patterns are summarized in section 3. A dynamic demand model of

complementary goods and an oligopoly pricing game are presented in section 4. The estimation and identification strategies are presented in section 5, with the estimation results shown in section 6. Section 7 conducts counterfactual experiments and discusses the policy implications. Section 8 concludes.

2   Institutional Background

The U.S. men's shaving market provides a suitable setting to study the welfare effect of product incompatibility in complementary good markets. Primary good (razor) and the aftermarket good (blade) are complementary goods and they are incompatible across brands and across technologies within the same brand. The incompatibility locks in consumers by the razor holdings to purchase compatible blades and contributes to the razor and blade pricing strategy: Razors are priced at a low markup and blades are priced at a higher markup.

There are three segments in the U.S. men's shaving market: non-disposable razors, refill blade cartridges, and disposable razors[6]. Razors and blades are complementary goods. Razor is the primary good and blade is the aftermarket good. Consumers have to use razor and blades together to shave, and only razor or only blade won't work. A tied shaving system is consisting of a razor and a blade, and the disposable razor is an non-tied alternative to the tied shaving system.

As shown in Table 1, razors and blades are incompatible across brands and also across different technologies within the same brand. For example, Gillette Sensor blade cartridge is incompatible with any Schick razor or any other Gillette razor like Mach. Therefore, the men's shaving market provides a suitable setting to study the impact of incompatibility in complementary good markets.

Table 1: Compatibility Matrix of Razors and Blades

			Non-disposable Razor					
	Brand			Gillette			Schick	
		Generation	Sensor	Mach	Fusion	Xtreme	Quattro	Hydro
Blade	Gillette	Sensor	✓					
		Mach		✓				
		Fusion			✓			
	Schick	Xtreme				✓		
		Quattro					✓	
		Hydro						✓

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[6]In the rest of paper, I use razor to denote the non-disposable razor and blade to denote the refill blade cartridge.

2.1 Market Structure

Among the three segments in the men's shaving market, blade cartridges have the largest share in sales, but disposable razors have the largest share in transactions. More consumers are switching from shaving system to disposable razors. Figure 1 shows the evolution of marker shares [7] in sales and transactions for each segment. The shaving system has a 72 % share of sales on average in the U.S. men's shaving market from 2004 to 2014, while blade cartridges have a 57% share of sales. However, consumers switch to disposable razors from the tied shaving system over time due to the rising prices of baldes and the incompatibility between razors and blades. In 2014, disposable razor has a about 60% share in transactions, while non-disposable razor and blade cartridges have a share of 27% and 13% respectively.
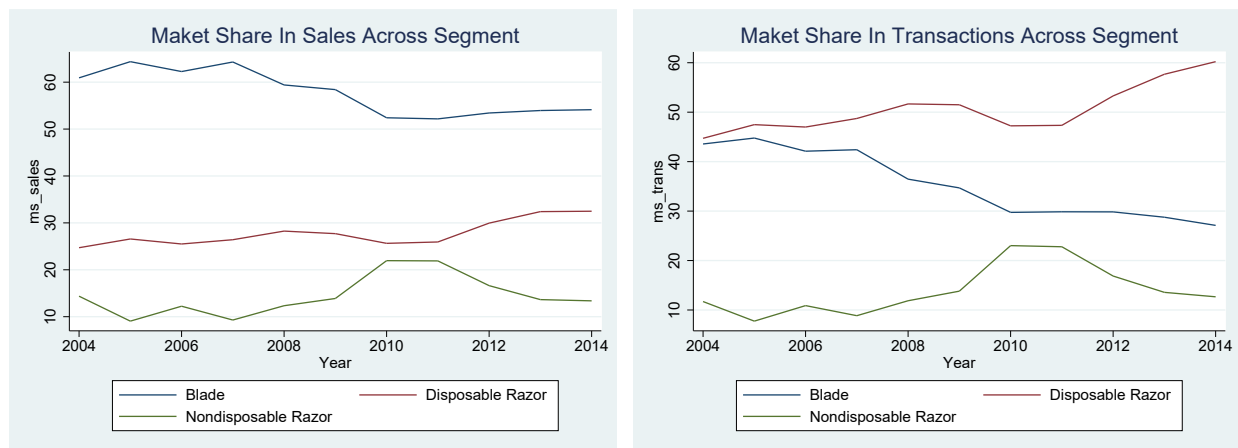
(a) Share in sales   (b) Share in transactions

Figure 1: Sales and Transaction Breakdown Across Segment

The U.S. men's shaving market is highly concentrated. Two big brands: Gillette and Schick dominate the market. Figure 2 presents the market structures of non-disposable razors and blades. It is a duopoly in the razor segment. The market share of Gillette is 60 % and Schick's share is 27% in 2014. Gillette dominates the market segment of blades with a share of about 80% in sales. As the second biggest brand in the blade market segment, Schick has a share of about 12% in sales in 2014.

Figure 2 reveals Gillette's asymmetric market powers in the razor and blade segments. Due to product incompatibility, consumers who own one razor can only use compatible blades from the same brand. However, Gillette has a much larger market share in blade sales

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[7]Calculated (or Derived) based on the Nielsen Consumer Panel data from The Nielsen Company (US), LLC and marketing databases provided by the Kilts Center for Marketing Data Center at The University of Chicago Booth School of Business.

compared to the share of razor sales. There are three possible reasons beside of the brand preference behind the phenomenon.
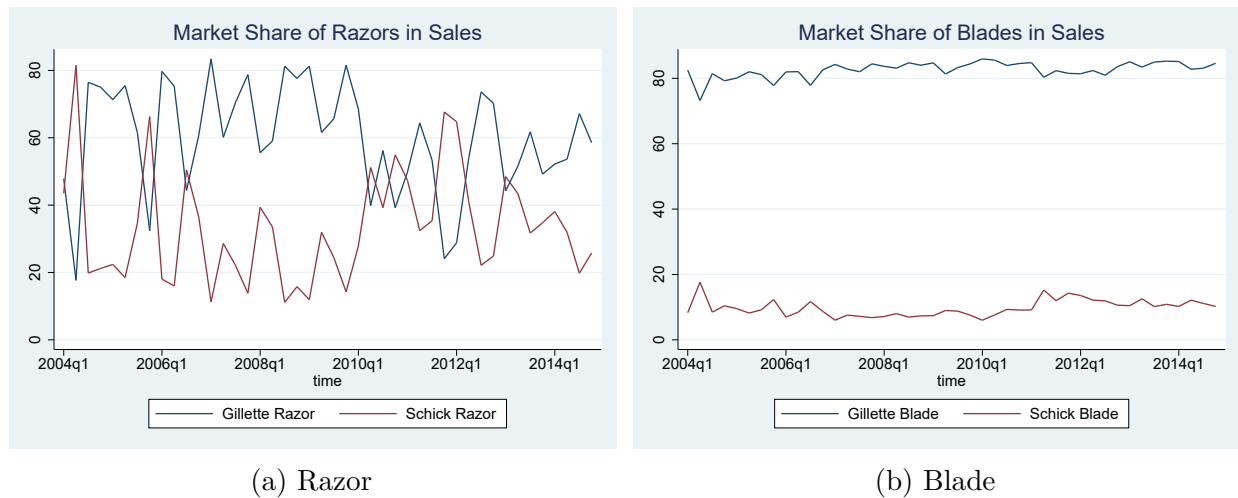

(a) Razor (b) Blade

Figure 2: Market Share in Sales of Razor and Blade

First, razors are more durable and blades are consumable. The dominance of Gillette in the market of refill blade cartridges reflects the success of Gillette razors in the history. Since the base of consumers owning Gillette razors is larger, the demand for compatible blades help Gillette dominate the market segment of blade cartridges. The pattern of asymmetric market shares indicates that consumers' razor ownership affects the demand for blade cartridges. Second, the average package size of Gillette blades is larger than that of Schick blades. Consumers buying the Gillette razors will buy a larger blade package, contributing to a larger sale of blades for Gillette. Last, the Gillette blades are more expensive than Schick blades which also leads to a larger share in sales of Gillette blades.

Figure 2 also shows that market shares in the market segment of blade cartridges have less variation than shares in non-disposable razor market segment. Market shares of non-disposable razors are affected by new product introductions. After Gillette introduced the Fusion razor in 2006, Gillette's share increased and Schick's share decreased. When Schick introduced the Hydro in 2010, Schick achieved a higher market share while Gillette experienced a large decline in the market share of non-disposable razors. However, the market shares of blade cartridges rely on the consumers' inventories of non-disposable razors. The short run variation of non-disposable razor shares didn't change the shares of blade cartridges equivalently.
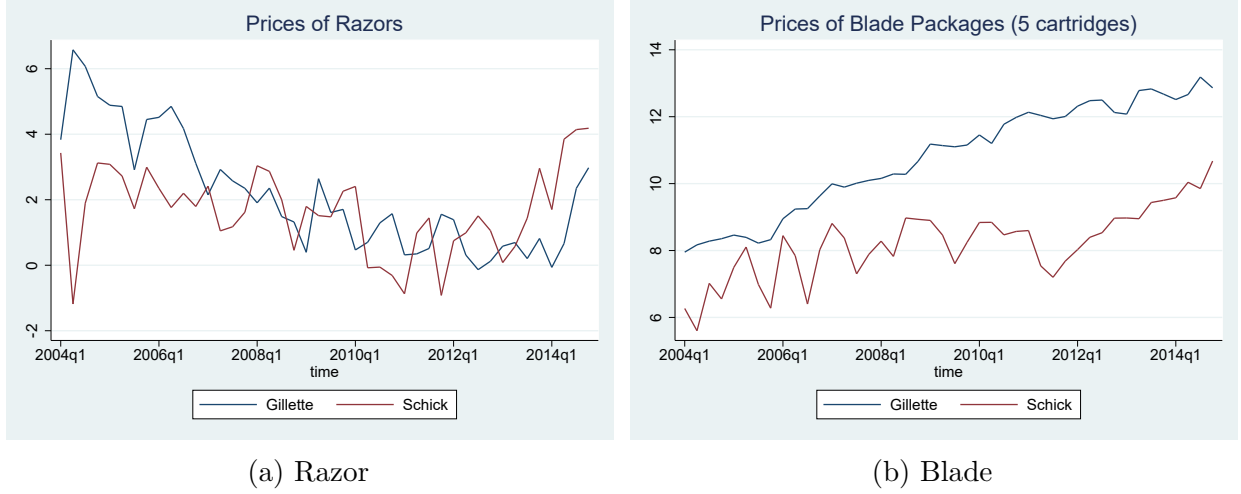
| (a) Razor   (b) Blade |

Figure 3: Prices of Razor and Blade

2.2 Pricing

The Figure 3 shows pricing trends of razors and blades over time. It shows the evolution of average razor prices (across technologies, exclude the price of blade cartridges in the razor package) after deals for each brand. Similar to what happens in other durable goods markets, razors experienced a gradual price decline over time. It also shows the evolution of average prices for a package of 5 blade cartridges (across technologies) after deals for each brand. Blades experienced a significant price increase over time.

Figure 4 presents pricing trends of razors and blades at the technology level. Razor prices experience a decline initially and then stayed stable. Razors of new technologies like Fusion are priced lower than razors of older technologies withing the same firm such as Mach and Sensor. Blade prices are quite stable and the new technology blades are more expensive than those of older technologies.

3 Data

In this section, I introduce the household-level Nielsen consumer panel data of shaving products including purchasing histories for a large panel of households over 11 years. Different from market-level data, household-level consumer panel data provides much more information on consumers' preferences and heterogeneity which facilitates the identification of demand parameters. Data patterns reveal how product incompatibility affects consumers' demand for razors and blades, and provide insights for the modeling of consumers' demand for complementary goods with incompatibility.
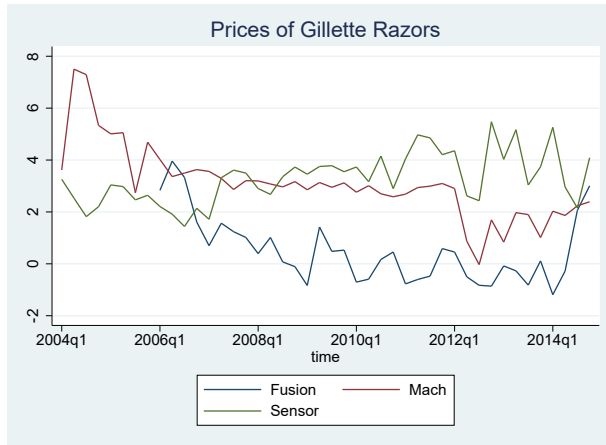
(a) Gillette Razor
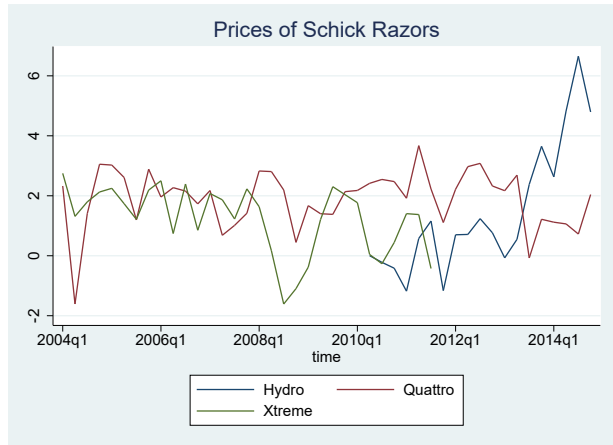
(b) Schick Razor
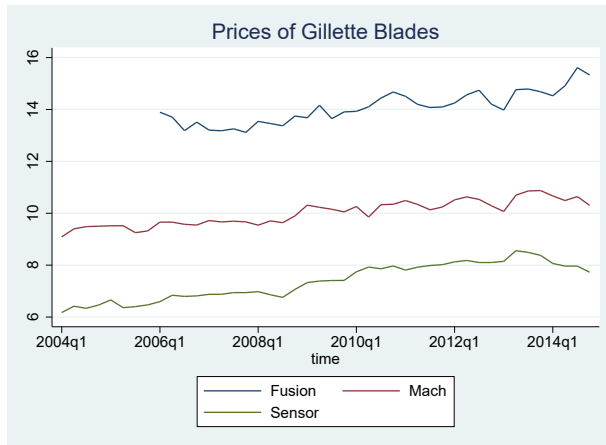
(c) Gillette Blade
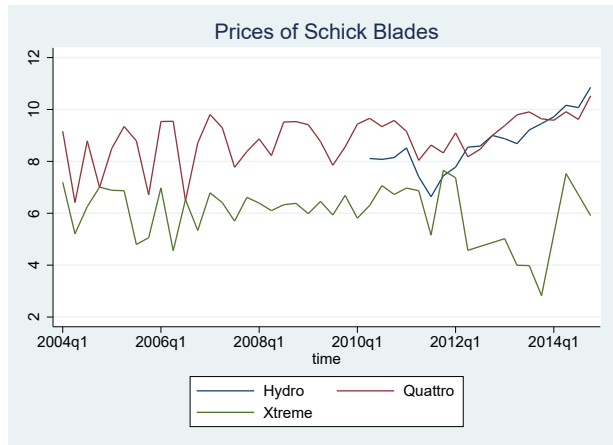
(d) Schick Blade

Figure 4: Prices of Razor and Blade across Technologies

3.1 Data Summary

The data I am using in this paper is the Nielsen consumer panel data from the Kilts center of marketing at the University of Chicago, Booth School of Business [8]. The Nielsen consumer panel data comprise a representative panel of households that continually provide information about their purchases in a longitudinal study in which panelists stay on as long as they continue to meet Nielsen's criteria. Nielsen Consumer panelists use in-home scanners to record all of their purchases (from any outlet) intended for personal, in-home use. Consumers provide information about their households and what products they buy, as well as when and where they make purchases.

The data starts from 2004 and includes annual updates. There are 40,000 - 60,000 active panelists (varies by year), projectable to the total United States using household projection factors. For each panelist, household demographic, geographic, and product ownership variables are included, as well as select demographics for the heads of household and other members. All products include UPC code and description, brand, multi-pack, and size, as well as Nielsen codes for department, product group, and product module. Each shopping trip contains the date, retail chain code, retail channel, first three digits of store zip code, and total amount spent. For each product purchased, the UPC code, quantity, price, and any deals/coupons are recorded.

The full coverage of retail channels and geographic markets is important to alleviate the sample selection problem. The Nielsen consumer panel data covers all retail channels in 52 major markets (covering entire U.S.)- grocery, drug, mass merchandise, superstores, club stores, convenience, health, and others.

The data of men's shaving products includes the purchasing histories of 74,710 households from 2004 to 2014. There are 517,315 observations of purchases including purchases of razors, blades and disposable razors. The median years households stay in the data is 5 years. On average, each household makes about 7 purchases.

The Table 2 provides summary statistics of the data. Average unit prices of razors and blades are $ 9.0348 and $ 2.2203 while 65% of razors are purchased in sales. The unit price of razors after sales is $ 6.0011. The blade purchases are less likely to have sales. Razor packages usually contain some amount of blades inside. The unit razor price controlling for the amount of blades in the razor package is low. The average number of blades in a razor package is 2. Taking account the unit price of blades, the unit razor price is only $ 1.7545, cheaper than the unit blade price.

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[8] The data is available for academic research through a partnership with the Kilts Center at the University of Chicago Booth School of Business. See http://research.chicagobooth.edu/nielsen for more details on the data.

Table 2: Summary Statistics

	Variable	Mean	Std. Dev.	Min	Max	# of obs
Razor	Package size	1.0865	0.4611	1	19	73,828
	Unit Price	9.0348	3.7755	0.01	49.99	73,828
	Sales	0.6454	0.4784	0	1	73,828
	Unit Price in Sales	6.0011	4.7155	0	49.99	73,828
Blade	Package Size	7.6690	5.7383	1	256	182,215
	Unit Price	2.2203	1.0597	0.01	9.245	182,215
	Sales	0.1690	0.3748	0	1	182,215
	Unit Price in Sales	2.1233	1.0493	0	9.245	182,215
Disposable razor	Package Size	9.9206	9.4817	1	416	261,092
	Unit Price	0.8137	0.9286	0.0008	15.98	261,092
	Sales	0.2614	0.4394	0	1	261,092
	Unit Price in Sales	0.6847	0.7769	0	14.98	261,092
All						517,135

Figure 3 shows the purchasing cycles (waiting time (days) between purchases) for both razors and blades. In the sample of households who participated the survey all years from 2004 to 2014, the average waiting time is 179 days and the median waiting time is 91 days. The average time waited to purchase another non-disposable razor is one year, and it's 9 months for new blade cartridge purchases. The median waiting time for razors and blades is 4 months and 4.5 months respectively.

Non-disposable razors are durable and consumers take a long time to purchase a new razor. The blades are less durable but they are sold in packages. Common blade packages contain about 4 or 8 blades, and consumers usually take some time to consume all blades before purchasing a new blade package.

Table 3: Purchasing Cylce (days)

	Mean	Standard Deviation	Median
Non-disposable Razor	376	587	119
Blade Cartridge	268	375	140
Disposable Razor	292	435	133
All	179	262	91

Consumers' switching and upgrading behaviors across brands and technologies are examined. In the blade segment, the majority of consumers stick with current technology instead of switching to a new technology. Over 85% of Gillette Fusion blade users keep using Fusion blade cartridges. On the other hand, switchings are more frequent in razors. Only 35% of Gillette Mach razor users bought the same product in the next purchase. I also find

more frequent upgradings in razors. 35% of Gillette Mach razor users and 25% of Gillette Sensor razor users upgrade to the newest Fusion razor.

Moreover, consumers takes a long time to upgrade to a newer technology. On average, consumers take about 516 days to upgrade from a Gillette Mach razor to a Gillette Fusion razor, and 586 days to upgrade form a Schick Quatrro razor to a Hydro razor. And consumers wait about 1 year to upgrade their blade cartridges from Gillette Mach to Gillette Fusion, and 402 days to upgrade from Schick Quattro blades to Schick Hydro blades.

The incompatibility between razors and blades may deter consumers' switching and upgrading. There are two channels of the effect on switching and upgrading. First, consumers need to buy a new razor before using a new blade, which increases the switching costs. Furthermore, the unused old blades in the inventory also increase the costs to switch or upgrade. Consumers with a larger inventory of old and incompatible blades may postpone the new purchases (either switch or upgrade) before consuming up the old blades in the inventory.

3.2 Data Patterns

Data patterns are summarized in this section. Data reveals that razor ownership affects blade choice, consumers may own multiple razors, blade inventory affects the probabilities of making new purchases, and consumers have heterogeneity in blade consumption rates. These patterns provide insights for the empirical model and suggest a dynamic demand model incorporating incompatibility, stockpiling, and consumers' heterogeneity.

3.2.1   Razor Ownership Affects Blade Choice

The incompatibility between razors and blades implies that consumers have to purchase blades compatible with razors they own. Razors and blades are incompatible across brands and technologies. Gillette Fusion blades can only be used with Gillette Fusion razors. Consumers who don't own Gillette Fusion razors can't use the Gillette Fusion blades without purchasing a new Fusion razor. Consumers owning Fusion razors only can only purchase compatible Fusion blades instead of blades from other technologies or brands. Data shows that the probability of purchasing compatible blades is higher given the razor choice at the previous purchase.

Table 4: Probability of Blade Choice Given Last Razor Choice (%)

| | | Blade | | | |
		Fusion	Hydro	Mach	Quattro
Razor	Fusion	76.8	8.51	5.88	3.80
	Hydro	17.01	66.32	4.36	5.98
	Mach	24.78	4.85	53.83	7.31
	Quattro	17.46	11.88	10.77	49.89

Table 4 presents blade choice probabilities given the razor choice at the previous purchase. Diagonal numbers are the probabilities of purchasing compatible blades given the razor choice of last purchase. Large probabilities of purchasing compatible blades given the razor is owned (consumers own that razor after purchasing it) illustrates the effect of razor ownership on blade choices due to the product incompatibility.

Table 4 also implies consumers may own multiple razors at the same time. There are some probabilities that consumers bought one razor in the last purchase but purchased blades from other technologies. Non-diagonal numbers are the probability of purchasing blades from a different technology from the razor purchased in the last purchase. It implies consumers may own multiple razors in the inventory, and can purchase some blades without purchasing the compatible razors. For instance, consumers purchased a Schick Hydro razor in the last purchase, and purchased a package of Gillette Fusion blades, which implying the consumers hold Hydro and Fusion razors at the same period.

3.2.2 Consumers Own Multiple Razors

As implied by Table 4, households are observed in the data to hold multiple razors. However, households may have more than one adult males and hold multiple razors for multiple males. To separate from the effect of household size on razor holdings, I control for the number of males in households and calculate the fraction of consumers who own multiple razors. A subsample where households only have one adult male who is at least 18 years old is examined. From the new individual level data, I find that about 34% of consumers own both Gillette and Schick razors in 2014.

The fact that consumers may own multiple razors has an important implication on the effect of product incompatibility on consumer welfare and pricing strategies. If consumers own multiple razors in the holding, the choice constraint from the incompatibility on blades is limited since consumers who own multiple razors are free to choose blades which are compatible with any razor in the holding. Therefore, the incompatibility may have fewer harm on consumers and firms have fewer incentives to implement the razor and blade pricing

strategy.

3.2.3 Blade Inventory Affects Purchasing Probability

Table 5: Blade Inventory Affects Purchasing Probability

	(1) Purchase	(2) Purchase	(3) Purchase
Blade Inventory	-0.0304***	-0.0290***	
	(0.0021)	(0.0073)	
Squared Blade Inventory		-0.0001	
		(0.0007)	
No Purchase Last Period			0.1660***
			(0.0163)
Demographics	Yes	Yes	Yes
Observations	135,652	135,652	135,652

Conditional Logit Regression, dependent variable is the dummy variable of making purchases. Control for consumers' demographics. Standard errors in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 5 shows the blade inventory affects the probability of making new purchases. The results from Conditional Logit Regression find that there is a significant and negative effect of blade inventory on purchasing probability after controlling demographics variables like age, income, employment, marital status and so on. The third column shows the effect of blade inventory by estimating the effect of making no purchase in the last period on the probability of making new purchases today. A positive numbers implies if making no purchases in the last period and consuming blades in the inventory, blade inventory decreases and consumers are more likely to make new purchases today.

The fact that blade inventory affects purchasing probability inspires the demand model to incorporate the stockpiling behavior with the blade inventory. Consumers may consume blades in the inventory and make no purchases. It is also important to match the data that there is a larger portion of observations where consumers make no purchases.

3.2.4 Consumers Have Unobserved Heterogeneity

Consumers are found to have different blade consumption rates. Taking into account consumers' unobserved heterogeneity help get more accurate results of demand estimates and counterfactual analysis when products are compatible. As shown in Figure 5, there

exists a significant consumer heterogeneity in blade consumption rates: in a given year, some consumers purchased many more blades or disposable razors than others.
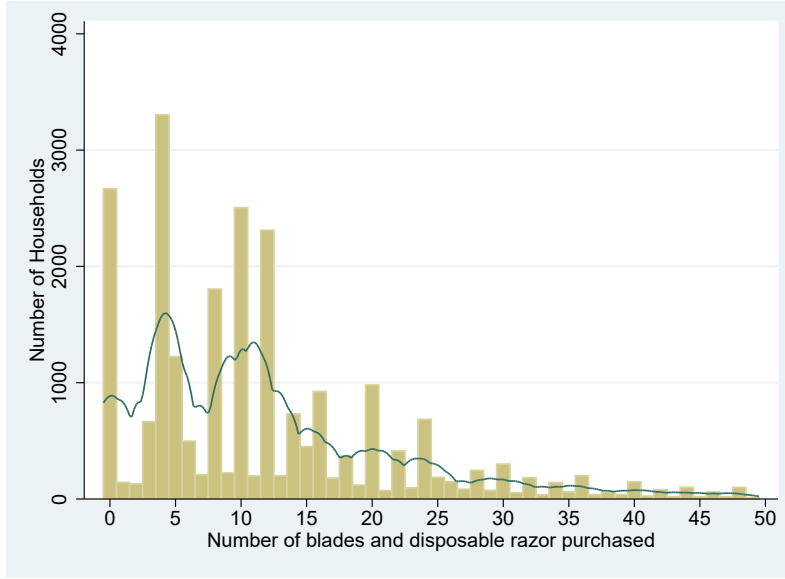
Figure 5: Consumer Heterogeneity in Number of Purchases in a Given Year (2011)

Consumers differ at the blade consumption rates and/or other preferences over products. High blade consumption rates imply more frequent purchases and larger number of blades and disposable razors purchased, and low blade consumption rates imply less frequent purchases and fewer blades and disposable razors purchased.

4 Model

Motivated by the data patterns found in the last section, I develop a dynamic demand model and an oligopoly pricing game for the complementary goods given incompatibility. Section 4.1 specifies a dynamic demand model for razors and blades incorporating the product incompatibility, stockpiling behavior, and unobserved consumers' heterogeneity. Section 4.2 presents an oligopoly pricing game where fully integrated firms simultaneously set prices for both razors and blades.

4.1 Demand Model

A dynamic demand model for razors and blades is presented here incorporating the product incompatibility, stockpiling behavior, and unobserved consumers' heterogeneity. Each quarter, consumers owning some razors and blades in the inventory make purchasing decisions of when to buy, what to buy, and how much to buy (choosing the package size).

There are $N_t + 1$ shaving options in the market: N_t tied razor technologies and one non-tied option: disposable razors. Index the technology by $r \in J_t \equiv \{1, .., N_t + 1\}$ where $r = N_t + 1$ denotes the disposable razor. Razors are always sold in packages containing blades. For each tied razor technology $r(r \neq 0, r \neq N_t + 1)$, index the package by $j \in J_r \equiv \{1, .., J_r\}$ where $j = 1$ denotes the razor package and $j > 1$ denotes the blade packages. Disposable razors are available in different packages where each pack $j \in J_{N_t+1} \equiv \{1, ..., J_{N_t+1}\}$ only contains disposable razors. Each package contains Q_{rj} blades. The tied nature of razors and blades requires the purchase of a razor before the compatible blades of the same technology can be used. Blades of a given technology are incompatible with razors of other technologies.

Let $r = 0$ denote the option of making no purchases. Consumers may shave with unused blades in the inventory or use other shaving products like electronic shavers or not shave. The outside option is defined to shave with other shaving products or not shave.

Demand for razors and blades are dynamic. Beside of razor durability and blades storability, two additional sources make the demand model dynamics: the complementarity of razors and blades, and the incompatibility between razors and blades. First, razors and blades are complementary. A set of blade cartridges are purchased after the adoption of a razor. This requires augmenting the demand model to accommodate the expectations of consumers about future availability and prices of blade cartridges. Moreover, razors and blades are incompatible. The purchase of razors changes the choice set of blades for the consumer tomorrow: by buying a Gillette razor, the set of blade cartridges compatible with the Gillette razor is added to the choice set. Thus individual current razor ownership is a state in the dynamic demand model. Furthermore, the inventory of blades affects consumers' future purchases: consumers with a high inventory of blades tend to buy a small blade package or make no new purchases.

A consumer's razor ownership at the beginning of each quarter is denoted by $\sigma \in \Sigma \equiv \{0, 1\}^{N_t}$, which is an N_t by 1 vector of indicators for whether or not each tied razor technology $(r \neq 0, r \neq N_t + 1)$ is owned. Razors are durable and consumers can own multiple razor technologies at the same time (i.e. multi-homing of razors). The razor ownership doesn't include the ownership status of disposable razors, since disposable razors don't have the problem of incompatibility and can be replaced as a whole.

A consumer's blade inventory at the beginning of each quarter is denoted by $B \in \Sigma^b \equiv R_+^{N_t+1}$, which is an $N_t + 1$ by 1 vector of quantities for each blade type including tied technologies and disposable razor. Consumers may own multiple types of blades at one period. Assume consumers use only one type of blade at one period.

4.1.1 Current Period Utility

The utility is defined from blade consumption. At the beginning of each quarter, consumer i owns some razors and some unused blades in the inventory. He may purchase one package of any tied technology, one disposable razor package, or make no purchases. After purchasing decisions, the consumer then consumes some amount of blades.

If the consumer purchases a new package j of technology $r(r \neq 0)$, he pays the price of the package and the new blades are added to the inventory. He will consume $C_{i,r}$ blades of type r if the new inventory after purchases $(B_{i,r,t} + Q_{r,j})$ is larger than the consumption $(C_{i,r})$, and consume all the blades of technology r in the inventory if otherwise. Assume $C_{i,r}$ is the average number of blades r consumed by consumer i in one quarter.

The flow utility of consumer i from purchasing a package j of technology $r(r \neq 0)$ and consuming some blades of technology r at time t is:

$$
\begin{aligned}
u_{i,r,j,t} = &\min\{C_{i,r}, B_{i,r,t} + Q_{r,j}\} + \gamma_{i,r} + X_{r,j,t}\lambda_i - \alpha_i P_{r,j,t} \\
&+ \eta_{1i}(B_{i,t} + Q_{r,j}) + \eta_{2i}(B_{i,t} + Q_{r,j})^2 + \epsilon_{i,r,t}
\end{aligned}
\tag{4.1}
$$

where $C_{i,r}$ is the average number of blades of technology r consumed by consumer i at one period, $B_{i,r,t}$ is the number of unused blade of type r owned by consumer i at the beginning of time t. $B_{i,t} \equiv \sum_r B_{i,r,t}$ is the total number of blades in the inventory of consumer i at time t. $Q_{r,j}$ is the size of package j of technology r. $\gamma_{i,r}$ is individual and technology specific fixed effect of technology r. $X_{r,j,t}$ are the product characteristics including the dummy variable of premium technology, and razor package dummy variable, $P_{r,j,t}$ is the price of package j of technology r at time t. α is the marginal utility of prices and λ are the tastes of product characteristics. $\epsilon_{i,r,t}$ is the individual, technology and time specific random utility shock.

$\min\{C_{i,r}, B_{i,r,t} + Q_{r,j}\}$ is the quantity of blades actually consumed by the consumer i at time t. The consumer is willing to consume $C_{i,r}$ blades, but is facing the constraint of blade inventory. If the quantity of desired blade consumption is larger than the blade inventory, the consumer will consume all blades in the inventory. Since razors and blades are incompatible, only compatible blades from the same technology in the inventory can be used with the razor of technology r. The technology specific fixed effects are assumed to enter the utility linearly to avoid the problem of separately identifying the blade consumption rates and the technology specific fixed effects [9]. $\eta_{1i}(B_{i,t} + Q_{r,j}) + \eta_{2i}(B_{i,t} + Q_{r,j})^2$ captures the dis-utility of keeping a high inventory due to storage cost. The quadratic function form reflects the fact

---

[9]I tried the alternative utility function form where the product of quantities of blade consumed and fixed effect enters the utility. Numerical simulation results suggest the blade consumption rates and the technology specific fixed effects can't be separately identified.

that the marginal storage cost is nonlinear with the inventory.

Consumers may make no purchases and consume unused blades in the inventory or choose not to shave or use other shaving products like electronic shavers. Not shaving or using other shaving products is defined as the outside option, and the utility from this outside option is normalized to 0.

If the consumer makes no new purchases and consumes unused blades in the inventory, he will choose the best blade technology which provides the highest utility if he owns blades of multiple technologies in that quarter. Let k denote the optimal choice of blade technology in the inventory which is defined as:

$$k = \arg\max_{l \in J_{i,t}} \min\{C_{i,l}, B_{i,l,t}\} + \gamma_{i,l} + X_{l,t}\lambda_i + \eta_{1i}B_{i,t} + \eta_{2i}B_{i,t}^2 \qquad (4.2)$$

where $J_{i,t} = \{r : B_{i,r,t} > 0\}$ is the set of available blade technologies in the inventory of consumer i at time t. If consumers only own blades of one technology, then he will use the blades of that particular technology.

When consumers make no purchases, the inventory remains the same. Consumer i will consume $C_{i,k}$ blades of type k if the inventory $(B_{i,k,t})$ is larger than the consumption $(C_{i,k})$, and consume all blades of technology k in the inventory otherwise.

The flow utility of consumer i from making no purchases at time t is

$$u_{i,0,t} = \max\{0, \min\{C_{i,k}, B_{i,k,t}\} + \gamma_{i,k} + X_{k,t}\lambda_i\} + \eta_{1i}B_{i,t} + \eta_{2i}B_{i,t}^2 + \epsilon_{i,0,t} \qquad (4.3)$$

where k is the blade technology in the inventory which provides the highest utility defined above, $B_{i,k,t}$ is the number of unused blade of type k owned by consumer i at time t. Consumers can choose not to shave or use other electric shavers as the outside option and get no utility or choose to consume some blades from the inventory and get some utility.

4.1.2 State Transition

Consumers' state vector is denoted by

$$S_{i,t} = \{\sigma_{i,t}, B_{i,t}, \Omega_t\}$$

where $\sigma_{i,t} \in \Sigma \equiv \{0,1\}^{N_t}$ is the individual razor ownership, $B_{i,t} \in \Sigma^b \equiv R_+^{N_t+1}$ is the individual inventory of blades, and $\Omega_t = \{P_t, X_t, J_t\}$ includes all the other information which is relevant to the future purchases including the prices, product characteristics, and the product set.

Individual razor ownership changes if a consumer purchases a new razor package of the tied technology he is not owning at time t. The new razor ownership evolves to $\sigma_{i,t+1}$ where the rth element denoting the ownership status of razor $r(r \neq 0, r \neq N_t + 1)$ is given by

$$\sigma_{i,r,t+1} = \begin{cases} 1, & \text{if} \quad d_{i,r,1,t} = 1 \\ \sigma_{i,r,t}, & \text{otherwise} \end{cases}$$

where $d_{i,r,1,t} \in \{0, 1\}$ is the decision of consumer i to purchase a new razor package of the tied technology r at time t.

Individual blade inventory at the next time period is affected by the product choice and blade consumption at current time period. The individual blade inventory of consumer i at time $t + 1$ is $B_{i,t+1}$, where the rth element denoting the number of blades of technology $r \in J_t (r \neq 0)$ in the inventory is given by

$$B_{i,r,t+1} = \begin{cases} \max\{B_{i,r,t} + Q_{r,j} - C_{i,r}, 0\}, & \text{if} \quad d_{i,r,j,t} = 1, \quad \forall j \in J_r \\ \max\{B_{i,r,t} - C_{i,r}, 0\}, & \text{if} \quad d_{i,0,t} = 1, \quad r = k \\ B_{i,r,t}, & \text{otherwise} \end{cases}$$

where $d_{i,r,j,t}$ is the decision of consumer i to purchase the package j of technology r at time t, $Q_{r,j}$ is the size of the package purchased, and $C_{i,r}$ is the desired number of blades of technology r consumed at time t by consumer i. Defined in equation (2), k is the optimal blade technology consumers will choose to consume in the inventory if making no new purchases.

Prices also evolve over time. Razor prices decrease over time as razors are durable, and blade prices increase over time as consumers are locked in by the incompatibility of razors and blades. Based on the pricing trends of different technologies, I assume prices evolve in AR(1) process.

$$P_{r,j,t} = \kappa_0 + \kappa_1 P_{r,j,t-1} + \nu_{r,j,t}$$

where $\nu_{r,j,t} \sim \mathcal{N}(0, 1)$ follows the standard normal distribution.

The product set J_t evolves exogenous according to the new product introductions. When a new technology was introduced, the new technology was added to the product set, and the product set remained the same otherwise. Consumers form the expectation of new product introductions which change the product set. There is an exogenous probability that a new technology was introduced at each period. The probability can be estimated using the ratio of the number of periods when a new technologies was introduced to the number of total

periods.

$$\Pr(J_{t+1} = J_t + 1) = \frac{\text{number of periods with a new technology introduced}}{\text{number of time periods}}$$

Consumers have rational expectations about how prices and product set will evolve in the future. The simplifying assumptions above allow me to treat consumer expectations in a reduced-form way, and estimate demand separately from the supply side.

The incompatibility of razors and blades across technologies kicks in the demand model by two ways. First, individual razor ownership determines the choice set of available blades individuals can buy. Second, consumers can only use the blades in the inventory compatible with the razor they are using. Blades of other technologies in the inventory can't be used with the specific razor. The utility of consume old blades of technology r in the inventory depends on the number of compatible blades of technology r, instead of all the blades in the inventory.

If the razor technology r is owned by the consumer i, he faces no constraint of razor ownership and can purchase any package. If the razor technology r is not owned by the consumer i, he has to purchase a razor package ($j = 1$) before purchasing any blade package due to the tied nature of razor and blade.

The individual choice set of available packages $J_{i,r,t}$ given any tied technology choice r $(r \neq 0, r \neq N_t + 1)$ for consumer i depends on his razor ownership at time t.

$$J_{i,r,t} = \begin{cases} \{1\}, & \text{if} \quad \sigma_{i,r,t} = 0 \\ \{1, 2, ..., J_r\}, & \text{if} \quad \sigma_{i,r,t} = 1 \end{cases}$$

where $\sigma_{i,r,t}$ is the indicator of ownership status of razor technology r. Consumer i owns razor technology r at time t if $\sigma_{i,r,t} = 1$, and vice versa.

4.1.3 Value Function

A consumer's value function of being able to choose packages of different technologies is given by

$$\tilde{V}(S_{i,t}, \epsilon_{i,t}) = \max_{r \in J_t, j \in J_{i,r,t}} u_{i,r,j,i} + \beta E[V(S_{i,t+1}, \epsilon_{i,t+1} | S_{i,t}, \epsilon_{i,t}, d_{i,r,j,t} = 1]$$

where β is the discounting rate of future payoffs, $S_{i,t}$ is the vector of state variables for consumer i at time t, and $\epsilon_{i,t} \equiv \{\epsilon_{i,r,j,t}\}_{r \in J_t, j \in J_r}$ is the vector of random utility shocks for consumer i at time t.

Assume ϵ are independently and identically distributed according to the Type I Extreme

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Value distribution, demeaned by Euler's constant. Following Rust (1987), the value function above can be integrated over ϵ to provide an expected value function for consumer i.

$$
\begin{aligned}
V(S_{i,t}) &\equiv \int_{\epsilon_{i,t}} \tilde{V}(S_{i,t}, \epsilon_{i,t}) dF(\epsilon_{i,t}) \\
&= \ln(\sum_{r \in J_t} \sum_{j \in J_{i,r,t}} \exp(\bar{u}_{i,r,j,t} + \beta E[V(S_{i,t+1}|S_{i,t}, d_{i,r,j,t} = 1)]))
\end{aligned}
\tag{4.4}
$$

where $\bar{u}_{i,r,j,t} = u_{i,r,j,t} - \epsilon_{i,r,j,t}$ is the mean utility from purchasing the package j of technology r.

The conditional choice probability of purchasing the package j of razor technology r for a consumer i at time t is

$$
\begin{aligned}
\mathrm{CCP}_{i,r,j,t} &= \Pr(d_{i,r,j,t} = 1|S_{i,t}) \\
&= \frac{\exp(\bar{u}_{i,r,j,t} + \beta E[V(S_{i,t+1}|S_{i,t}, d_{i,r,j,t} = 1)])}{\sum_{r' \in J_t} \sum_{j' \in J_{i,r',t}} \exp(\bar{u}_{i,r',j',t} + \beta E[V(S_{i,t+1}|S_{i,t}, d_{i,r',j',t} = 1)])}.
\end{aligned}
\tag{4.5}
$$

Aggregating the individual choice probability, the market share of product (r, j) is derived as

$$
s_{r,j,t} = \sum_i \mathrm{CCP}_{i,r,j,t}/N.
\tag{4.6}
$$

4.2 Supply Model

In the supply side, firms compete in both primary goods (razors) and aftermarket goods (blades). Each period, firms play a Bertrand pricing game and they simultaneously chooses the the unit prices of razors and blade for different technologies and packages. I focus on the firm's optimal pricing decisions. Thus, other firm's strategic considerations are abstracted away, such as entry or exit, innovation, and new products introductions. Assume the product set determined by firms' product introduction decisions evolves exogenously.

The period profit function for firm $l \in \{1, 2\}$ at time t beside of the fixed costs is

$$
\Pi_{l,t} = \sum_{r \in N_{l,t}} \sum_{j \in J_r} M_t s_{r,j,t} (P_{r,j,t} - mc_{r,j,t}),
\tag{4.7}
$$

where $N_{l,t}$ is the product set of firm l, M_t is the market size, and $s_{r,j,t}$ is the market share of technology r and package j at time t from equation (4.6).

Marginal costs are assumed to have the following forms.

$$
mc_{r,j,t} = mc_{r,j} + \omega_{r,j,t}
$$

where $mc_{r,j}$ are the average marginal cost of technology r and package j, and ω are the marginal cost shocks.

Given the states at each period, each firm maximizes its period profit by optimally choosing prices of razors and blades for all technologies. Assume marginal costs shocks are public information that both firms can observe the marginal costs shocks of the other firm at each period.

Firms simultaneously choose razor and blade prices of all technologies to satisfy the first order conditions. For each firm l, the optimal condition for the price of any product (r, j) where $r \in N_{l,t}, j \in J_{l,t}^r$ is

$$
\begin{aligned}
M_t s_{r,j,t} + M_t (P_{r,j,t} - mc_{r,j,t}) \frac{\partial s_{r,j,t}}{\partial P_{r,j,t}} \\
+ \sum_{(r',j') \neq (r,j)} M_t (P_{r',j',t} - mc_{r',j',t}) \frac{\partial s_{r',j',t}}{\partial P_{r,j,t}} = 0
\end{aligned}
\tag{4.8}
$$

where the first two terms capture the direct effect of price change on the profit of that product, and the third term sums up the indirect effects of product's (r, j) price on all other products in the same firm l at time t. Firms have multiple products, and aim to maximize the total profits by choosing the optimal prices for all products.

Different from the single product pricing, multi-product firms need to take into account the cannibalization effects between different products since the price change of one product could also affect market shares of other products in the same firm. For instance, when the price of fusion blades go down, the market share of Fusion blades will increase. However, at the same time the market shares of Gillette Sensor and Mach blades are decreasing since they are substitutes for consumers.

5   Estimation

I estimate the model of consumers' demand and firms' pricing using a two-step sequential approach (as in Chintagunta et al. (2018), Derdenger (2014), among others). More specifically, I estimate the dynamic demand model first, and given the demand estimates, I recover marginal costs by assuming that the observed prices represent equilibrium outcomes of a Bertrand pricing game. This two-step sequential approach ensures that the possible misspecification of the pricing model will not contaminate the demand parameter estimates.

Section 5.1 summarizes the vector of parameters to be estimated in the demand model and describes the estimation procedure, and Section 5.2 discusses the identification of the demand parameters from data variation, and Section 5.3 presents how to deal with the

problem of unobserved state variables and a large dimension of state space.

5.1 Estimate Procedure

To recover the dynamic demand model structural parameters, I follow Hartman and Nair (2010) to estimate the demand of complementary goods using simulated maximum likelihood method with individual level consumer data. Moreover, I extend Hartman and Nair (2010) by allowing the technology specific blade inventory and accounting for consumers' expectation of new technology introductions in this market. Following Arcidiacono and Johns (2003), Expectation and Maximization (EM) algorithm is applied to account for the unobserved consumers' types.

5.1.1 Demand Parameters

The parameters in the state transitions including prices and product set $\{\kappa_0, \kappa_1, \Pr(J_{t+1} = J_t + 1)\}$ are estimated from the data in the first stage.

The vector of demand parameters in the utility function to be estimated for consumer i is $\theta_i = \{\{\gamma_{i,r}, C_{i,r}\}_{r=1}^J, \alpha_i, \lambda_i, \eta_{1,i}, \eta_{2,i}\}$, where $C_{i,r}$ is the desired average number of blades of technology r consumed in one time period by consumer i.

The parameters of period blade consumption rates $\{C_{i,r}\}_{r=1}^J$ also enter the state transition process of blade inventory, but they will be estimated with other preference parameters in the second stage. The main reason is that the individual blade inventories are unobserved to econometricians and can only be estimated by the observed purchasing choices and waiting times between purchases.

Based on the data pattern, consumers' unobserved heterogeneity is accounted for by adopting Latent Class method. Each consumer belongs to one of M classes (type). θ_i are class specific and can take M possible sets of values. Assume there are two types of consumers who have different blade consumption rates and other preferences parameters [10]. From the data, the consumers may differ at the blade consumption rates, one type of consumers are frequent shavers and the other consumers are normal users. The probability of being type I is denoted by ρ.

Consumers' types are unobserved. The parameter vector to be estimated is $\{\rho, \theta\}$ where $\theta = \{\theta^1, \theta^2\}$ and θ^m is the preference parameter vector for consumers of type $m (m \in M \equiv \{1, 2\})$.

---

[10]I tried allowing the consumers to have three types and compare the estimation results with those when consumers have two types. The non-nested hypothesis test rejects the three types.

5.1.2 Estimation Strategy

Following Arcidiacono and Johns (2003), I use the Expectation and Maximization (EM) algorithm to estimate the dynamic demand model with unobserved types.

From the demand model, the conditional choice probability that product (r, j) is chosen by consumer i at time t is

$$\Pr(d_{i,r,j,t} = 1 | S_{i,t}, \theta, \rho) = \rho(\Pr(d_{i,r,j,t} = 1 | S_{i,t}, \theta^1)) + (1 - \rho)(\Pr(d_{i,r,j,t} = 1 | S_{i,t}, \theta^2)) \quad (5.1)$$

where ρ is the population probability of being type I with the utility parameters θ^1, and $1 - \rho$ is the probability of being type II with the utility parameters θ^2. The individual choice probability depends on the consumers' type, different types of consumers have different choice probability.

Individual likelihood function for consumer i at time t is then derived as

$$\mathcal{L}_t(d_{i,t} | S_{i,t}, \theta, \rho) = \prod_r \prod_j \Pr(d_{i,r,j,t} = 1 | S_{i,t}, \theta, \rho)^{d_{i,r,j,t}=1}$$

Summing up all the consumers and all the time periods in the data, the log likelihood function is derived as

$$\mathcal{LL}(\theta, \rho) = \ln \left[\prod_{i=1}^N \prod_{t=1}^T \mathcal{L}_t(d_{i,t} | S_{i,t}, \theta, \rho) \Pr(S_{i,t} | S_{i,t-1}, d_{i,t-1}, \theta) \Pr(S_{i,1}) \right] \quad (5.2)$$

where $S_{i,t} = \{\sigma_{i,t}, B_{i,t}, \Omega_t\}$, and $S_{i,1}$ are the initial razor ownership, blade inventory of consumer i and the initial prices, product characteristics and product set.

Because consumers' types are unobserved to researchers, the traditional maximum likelihood method doesn't work here. The equation 5.2 can't be directly maximized to find the optimal parameters ρ and θ. To account for the unobserved types, I use Expectation-Maximization (EM) algorithm to estimate the dynamic demand model.

At the Expectation step, the individual conditional probability that the consumer is being of type I is updated by Bayes' rule:

$$h(m_i = 1 | d_i, \theta, \rho) = \frac{\rho(\prod_t \mathcal{L}_t(d_{i,t} | S_{i,t}, \theta^1))}{\rho(\prod_t \mathcal{L}_t(d_{i,t} | S_{i,t}, \theta^1)) + (1 - \rho)(\prod_t \mathcal{L}_t(d_{i,t} | S_{i,t}, \theta^2))} \quad (5.3)$$

where $d_i = \{d_{i,t}\}_{t=1}^T$ are the observations of consumer i over time in the data, and $\mathcal{L}_t(d_{i,t} | S_{i,t}, \theta^m)$

is the individual likelihood of consumer i at time t given the consumer is being type m.

$$\mathcal{L}_t(d_{i,t}|S_{i,t},\theta^m) = \prod_r \prod_j \Pr(d_{i,r,j,t}=1|S_{i,t},\theta^m)^{d_{i,r,j,t}=1}$$

Given the individual probability of being type I, the population probability of being type I is updated.

$$\rho' = \sum_i h(m_i = 1|d_i,\theta,\rho)/N \tag{5.4}$$

At the Maximization step, given the updated probability of being type I, the estimates θ' maximizes the log likelihood function weighted by the type probabilities.

$$\theta' = \arg\max_{\theta\in\Theta} \sum_i \sum_t \sum_{m\in M} h(m_i = m|d_i,\theta,\rho')\ln(\mathcal{L}_t(d_{i,t}|S_{i,t},\theta^m))) \tag{5.5}$$

The maximization step of the EM algorithm can be interpreted as consumers' types are observed and the $h(m_i = m|d_i,\theta,\rho)$ is used as population weights.

Given the new parameters θ', I update the probability of being type I again. Repeat the expectation and maximization steps until the type probability ρ and the demand parameters $\theta = \{\theta^1,\theta^2\}$ converge.

5.2 Identification

The discount factor β is assumed to be a fixed value as in the literature. The discount factor is nonparametrically unidentified from a dynamic discrete choice problem (Magnac and Thesmar, 2002). I also impose an assumption on expectation that consumers form rational expectations following literature convention. Several different values of discount factor are tried to check whether the main counterfactual results change within a reasonable range. I find the results are invariant in a reasonable range and stick with the discount factor of 0.95.

A brief informal discussion of identification of consumption rates and preferences parameters is presented as below. The availability of consumer panel data over a long time period, including both the product choice as well as the quantity choices (i.e., pack sizes), facilitates the identification of blade consumption rates, product preferences and heterogeneity. This part of the identification argument is similar to Hartmann and Nair (2010).

Period blade consumption rates are unobserved in the data and can be identified by the joint distribution of the waiting times between purchases and the package size choices. Large pack size choices with shorter waiting times between purchases imply high blade consumption rates. Small pack size choices with longer waiting times between purchases imply lower blade consumption rates.

The blade consumption rates vary across types: non-disposable blades and disposable razors. The heterogeneity of blade consumption rates is identified by the difference of waiting times to purchase a new blade or disposable package after previous purchases. If consumers wait a longer time to purchase a new package after using a blade package than a disposable razor package of the same size, then it's inferred that the consumption rate of disposable razor is higher than that of non-disposable blades.

Variations of prices, characteristics, blade inventories and market shares of each product choice help identify the other preference parameters $\{\alpha_i, \gamma_{i,r}, \lambda_i, \eta_{1,i}, \eta_{2,i}\}$. The repeated observations of the same consumer over time help identify the type probability ρ. If consumers are observed to make fewer purchases and have longer waiting time between purchases, they are implied to have a lower blade consumption rate and a higher price sensitivity. The observed differences in the choices of when to purchase, what to purchase, and how much to purchase of the same consumer over a long time help identify the consumer's type.

A numerical simulation exercise is also conducted to provide the numerical argument for the identification of the demand parameters. Simulation results show the demand parameters including the type probability, and the blade consumption rates can be numerically identified.

5.3   Unobserved State Variables and the Dimension of State Space

The vector of state variables for consumer i at time t is denoted by

$$S_{i,t} = \{\sigma_{i,t}, B_{i,t}, \Omega_t\}$$

where $\sigma_{i,t} \in \Sigma \equiv \{0,1\}^{N_t}$ is the individual razor ownership, $B_{i,t} \in \Sigma^b \equiv R_+^{N_t+1}$ is the individual blade inventory, and $\Omega_t = \{P_t, X_t, J_t\}$ is all the other information which is relevant to the future purchases including the prices, product characteristics, and the product set.

The blade inventory is unobserved by researchers. Given the product choices, blade consumption rates and the initial inventory, I can infer the blade inventory at each period according to the state transition process of blade inventory defined above. Consumer may own some blades in the inventory, and may purchase some blades or use old blades in the inventory, the new inventory will equal to the sum of old inventory and new blades purchased minus the blades consumed.

The initial state is also unobserved to researchers which generates the standard problem of initial condition. I infer the initial razor ownership from consumers' purchasing history. If a consumer purchases a blade without purchasing a razor of the same technology, then he owns that razor at the beginning of period. The initial blade inventory is inferred from the number of periods until the first purchase. Assume consumers have no blade inventory at the

period when they made the first purchase in the data. The product of number of periods before the first purchase and the average number of blades consumed in a period provides the initial blade inventory.

Since the state space includes a number of state variables like the razor ownership, blade inventory, prices for each technology, the dimension of state space is large and increases the computation burden significantly. To keep the problem tractable, I make some assumptions to reduce the dimension of state space.

Since consumers form expectations on future prices and there are 7 technologies and 3 packages for each technologies. The dimension of price vectors is too large. As shown in Figure 4, different technologies in the same firm share the similar pricing trend of razors and blades. I assume only prices of razor and blade of the premium technology for each firm and the price of disposable razors enter the state space. Consumers form expectation of these five prices in the future and the expected future prices of other technology and packages can be backed out using empirical price ratios of other technologies to the premium technology in each firm.

Suppose consumers expect the future razor price of Gillette Fusion to be $P_{\text{fusion},t+1}^{\text{razor}}$ and empirical price ratio of Mach and Fusion razors over the data period $R_{\text{fusion}}^{\text{mach}}$, then the expected future razor price of Gillette Mach is backed out by the empirical price ratio.

$$P_{\text{mach},t+1}^{\text{razor}} = P_{\text{fusion},t+1}^{\text{razor}} * R_{\text{fusion}}^{\text{mach}}$$

Suppose consumers expect the future price of Gillette Fusion small blade package to be $P_{\text{fusion},t+1}^{\text{blade,small}}$ and empirical price ratio of Fusion small and large blade packages over the data period $R_{\text{fusion,small}}^{\text{large}}$, then the expected future razor price of Gillette Mach is backed out by the empirical price ratio.

$$P_{\text{fusion},t+1}^{\text{blade,large}} = P_{\text{fusion},t+1}^{\text{blade,small}} * R_{\text{fusioin,small}}^{\text{large}}$$

Because there are 7 technologies available in the market, allowing consumers to hold blades of all 7 technologies at one period is not realistic and also increases the computation burden significantly. Instead, I assume besides of disposable razors, consumers hold at most two different non-disposable technologies of razors and blades at one time period. If consumers hold more than two tied technologies in the inventory, they would prefer to keep the newer technology of each firm, and discard the older technology in the same firm. For example, if one consumers hold Gillette Sensor and Mach, and also Schick Hydro, he would drop the Gillette Sensor razors and blades, and keep Gillette Mach and Schick Hydro in the inventory.

Last, since Gillette dominates the blade market with a share of about 80%, and Schick including three technologies has a share of about 15%, I group three Schick technologies

(Xtreme, Quattro and Hydro) into one technology: Schick. The grouping doesn't change the market position of Schick as the second largest firm in the market. However, since three technologies in Schik are grouped into one, then Schick razors and blades are now compatible with each other and the vertical incompatibility between different technologies within Schick is abstracted awsy under this assumption. The effect of grouping is expected to be small since Schick has a small market share relatively to Gillette.

5.4 Sample Data For Estimation

The Nielsen Consumer panel data is unbalanced and at the household level. To control for the time periods when households were in the data, I focus on a subsample of individual consumers who were in the data from 2004 to 2014. To control for the family size, I restrict the number of adult males in the household to be one. By focusing on the individual-level data, the possible effects of multiple males in a household on purchases choices are controlled. In the sample data for estimation, the total number of consumers is 3086 and the time period is 11 years (44 quarters).

6 Estimation Results

This section presents the estimate results of a dynamic demand model where consumers have unobserved heterogeneity and a static oligopoly pricing game where fully integrated firms set prices of razors and blades simultaneously. Several robustness checks including the homogeneous consumers, myopic consumers and single homing are conducted.

6.1 Demand Estimation Results

The table 6 presents the estimate results of the dynamic demand model of razors and blades where consumers have unobserved heterogeneity. The probability of being type I is estimated to be 0.4649 and the probability of being Type II is then 0.5351. Two types of consumers differ at all the parameters including blade consumption rates and the other preference parameters.

Consumers of Type I consume fewer blades and disposable razors in a quarter and are more price sensitive to prices relative to consumers of Type II. The technology specific fixed effects are higher for Type I consumers since they value each blade more and consume fewer blades in a quarter. The parameters representing the durability of razors $\lambda_{Gillette}$ and λ_{Schick} [11] are also higher for consumers of Type I since they consume fewer blades and replace the

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[11]Razor durability variables enter the product characteristics and the parameters λ are type specific: non-disposable blades and disposable razors.

Table 6: Demand Estimates

Variable	Parameter	Type I		Type II	
		Estimate	Std. Err.	Estimate	Std. Err.
Type Probability	ρ	0.4649***	0.0613		
Price	α	-0.2737***	0.0009	-0.0507***	0.0018
Blade consumption rate					
Non-disposable	C_{Blade}	3.3476***	0.0141	8.8238***	0.0082
Disposable razor	C_{Disp}	3.9718***	0.0184	13.5662***	0.0220
Technology Fixed Effect					
Sensor	γ_{Sensor}	-3.3377***	0.0196	-2.8539***	0.0290
Mach	γ_{Mach}	-2.6096***	0.0004	-2.4063***	0.0282
Fusion	γ_{Fusion}	-1.4265***	0.0205	-1.7921***	0.0329
Schick	γ_{Schick}	-4.8937***	0.0316	-3.4663***	0.0297
Disposable Razor	γ_{Disp}	-6.0362***	0.0187	-3.2802***	0.0275
New Razor Fixed Effect					
Gillette	$\lambda_{Gillette}$	-1.0458***	0.0210	-2.3766***	0.0270
Schick	λ_{Schick}	1.3693***	0.0400	-1.0746***	0.0412
Inventory Effect					
Linear	η_1	-0.0094***	0.0007	-0.7353***	0.0086
Quadratic	η_2	0.0002***	0.0000	-0.0107***	0.0005

Note: Significance level *** (1%) ** (5%) * (10%)

razors less frequently than Type II consumers. Consumers of Type I also have lower inventory costs than Type II consumers since they usually hold more inventories when their blade consumption rates are lower.

The desired period blade consumption rates C are type specific: non-disposable blades and disposable razors. The consumption rates of disposable razors are higher than that of non-disposable blades, which is also consistent with the data where the mean frequency of purchasing a new disposable package is higher than that of razor or blade packages and the package size of disposable razors is also bigger.

6.2 Model Fit

I check the model fit by comparing the consumers' choice probabilities of each technology and package generated by the model with the ones observed in the data. The results are shown in Table 12.

Table 7: Estimates of Marginal Costs

Firm	Technology	Package	Average Prices	Marginal Costs	Std. Err.
Gillette	Sensor	Razor	5.3435	2.6962	0.4199
		Blade Small Package	1.7997	0.4371	0.2924
		Blade Large Package	1.6504	0.4728	0.2100
	Mach	Razor	8.1008	5.1132	0.8823
		Blade Small Package	2.4797	0.9806	0.2891
		Blade Large Package	2.1058	0.9315	0.1242
	Fusion	Razor	7.2243	4.1670	1.0549
		Blade Small Package	3.4874	1.8836	0.3579
		Blade Large Package	3.0071	1.6850	0.1853
Schick	Schick	Razor	5.9070	2.0086	0.8538
		Blade Small Package	2.0153	0.5734	0.2308
		Blade Large Package	1.9450	0.9315	0.2436

6.3 Supply Estimate

Given the demand estimates, a static oligopoly pricing game is estimated to match the observed razors and blades prices. Consistent with the data and the well known razor and blade pricing strategy in the men's shaving market, razor prices are relatively cheaper, and prices of blades are more expensive.

Table 7 presents the estimates of marginal costs for each technology and each package in the two firms [12]. Razor packages have larger marginal costs since razor packages usually contain one blade inside. Different blade packages contain different number of blades. Small blade packages contain 4 blades and large blade packages contain 8 blades. The estimated marginal costs for blade packages are unit marginal costs, and the marginal costs of blade packages equals to the product of unit marginal costs and the package size.

Table 7 indicates razor markup (razor handle) is lower than blade markup. The marginal cost of a razor handle is calculated using the marginal cost of a razor package which contains two blades minus the total marginal costs of these two blades. For example, the marginal cost of Sensor handle is about $ 1.81 and the price of Senor razor handle is about $ 1.74. The price of Sensor razor handle is lower than the marginal costs. Firms are losing money in razor sales. The average price of one Sensor blade is about $ 1.8 and the marginal cost of Sensor blade is only $ 0.44. Firms are making high profits from blades sales and losing money in razor sales. This is consistent with the razor and blade pricing strategy: sell razors

---

[12]Marginal cost of blades is the unit marginal cost for one blade, and marginal cost of disposable razors is the unit marginal cost for one disposable razor. Marginal cost of razors is the marginal cost for the razor package

in a lower markup and sell blades in a higher markup.

6.4 Robustness Check

Three robustness checks are conducted to assess the impact of the assumptions on demand estimates. First, consumers are myopic. Second, consumers are homogeneous. Third, consumers are single homing in razors and blades. The non-nested hypothesis tests reject all the three specifications and favor the assumptions made in the model.

7 Counterfactual Analysis

To investigate the effect of product incompatibility on price competition and consumers' welfare in the men's shaving market, counterfactual analysis are conducted when razors and blades are compatible across firms and/or technologies within firms. In the counterfactual analysis, I assume there is no patent protection since product incompatibility comes from patent protection in the men's shaving market. In particular, I distinguish two dimensions of compatibility. Product are "horizontal" compatible if they are compatible across firms. Products are "vertical" compatible if they are compatible across successive technologies within the same firm. Without the protection of patents, each firm can choose horizontal and/or vertical compatibility. Different from pricing decisions, firms cannot decide compatibility by their own. If one firm chooses incompatibility while the other firm chooses compatibility, products may still be compatible across firms.

Three counterfactual experiments are conducted to investigate the welfare effect of product incomparability and firms' optimal compatibility design. First, razors and blades are incompatible across firms but compatible across technologies within the firm. Second, razors and blades are compatible across firms but incompatible across technologies within the firm. Third, razors and blades are compatible across firms and technologies. New market equilibrium prices, profits and consumers' welfare are calculated given the compatibility decisions in each experiment.

When razors and blades are compatible, consumers demand changes in three ways. First, there is no or less constraint on blade choices from razor ownership. Consumers can buy any blade no matter of their razor ownership since now blades can be used with any razor. The removal of choice constraint will make the razors and blades more attractive and the demand for razors and blades may increase. Second, the costs of switching or upgrading to a new technology are lower. Consumers don't need to buy a razor before purchasing a new blade. Moreover, the old blades in the inventory are now compatible with any razor. Thus, there is

no cost of switching or upgrading from old blades in the inventory. Third, because consumers only need one razor to use all blades, the demand for razors will decrease and it's less likely for consumers to own multiple razors.

Firms will re-optimize pricing strategies given the effect of compatibility on consumers demand. Two channels determines the new prices of razors and blades: intensified competition and demand expansion. On one hand, since razors can't lock in consumers to purchase compatible blades, firms lose the market power in the blade market. Thus, firms have fewer incentives to lower the razor prices in order to gain more profits in the blade market. Accordingly, razor prices may be higher than before. Since there is more competition in the blade market, blade prices may be lower. On the other hand, since razors and blades now are compatible, they are more attractive for consumers compared to the disposable razors or outside option. Demand for blades expands because some consumers will switch from disposable razors or outside option to blades. Furthermore, market size of blades increases for each firm since firms now can attract consumers owing razors from the competitor in the blade market. All consumers no matter of the razor ownership have potential demand for both blades. The demand expansion effect of product compatibility helps firms set higher prices of blades.

Comparing the prices and consumers welfare in each experiment and the real world, I find that product compatibility softens price competition and improves the consumers' welfare in the men's shaving market. Razor prices are higher since firms can't lock in consumers. Blade prices are higher since demand expansion effect dominates intensified competition effect. In addition, firms may prefer the compatibility both horizontally and vertically in the men's shaving market abstracting away the cost of compatibility and the potential effect on entry/exit.

7.1   Horizontal Compatibility: Compatible across Firms

This first counterfactual experiment abstracts away product incompatibility across firms but keeps the incompatibility across technologies within the same firm. Razors and blades now are compatible across firms. The constraint of razor ownership on blade choice is mitigated since consumers can use razors of one firm and blades of another firm.

Gillette has three technologies: Sensor, Mach and Fusion, and these three technologies are incompatible vertically. The horizontal compatibility changes the demand model in a similar way to all round compatibility except the vertical incompatibility. First, the individual choice set of blades are constrained by the individual technology specific razor ownership. For instance, if consumers own a Gillette Sensor razor, they can match the razor with the blades

of Schick but not with the blades of other technologies in Gillette like Mach and Fusion. Second, the inventory is also technology specific within the same firm. For example, Sensor blades in the inventory can be used with a new Schick razor but not with a new razor from other technologies of Gillette.

The choice set of consumers changes when razor and blade are compatible across firms but incompatible across technologies within the same firm. Consumers can choose any blade compatible with the razors they own and the choice set depends on individual technology specific razor ownership.

In the notation, I assume Gillette technologies are denoted by $r = \{1, 2, 3\}$ where $r = 1$ denotes the Sensor technology, $r = 2$ denotes the Mach technology, $r = 3$ denotes the Fusion technology. Schick technology is denoted by $r = 4$, and the disposable razor is denoted by $r = 5$.

Now, the choice set of packages given the Gillette technology choice ($r \in \{1, 2, 3\}$) of consumer i at time t is

$$
J'_{i,r,t} = \begin{cases} \{1\}, & \text{if} \quad \sigma_{i,r,t} = 0, \quad \sigma_{i,4,t} = 0, \quad \forall r \in \{1, 2, 3\} \\ \{1, 2, ..., J_r\}, & \text{otherwise} \end{cases} \tag{7.1}
$$

Consumers who want to purchase blades from Gillette technology have to own a compatible razor or a Schick razor, because Schick razors can be used with any Gillette blades.

If the consumer chooses the Schick technology ($r = 4$), his new choice set of packages at time t is

$$
J'_{i,r,t} = \begin{cases} \{1\}, & \text{if} \quad \sigma_{i,1,t} = 0, \quad \sigma_{i,2,t} = 0, \quad \sigma_{i,3,t} = 0, \quad \sigma_{i,4,t} = 0 \\ \{1, 2, ..., J_r\}, & \text{otherwise} \end{cases} \tag{7.2}
$$

As long as the consumers own a razor from any technology including three Gillette technologies and Schick, he has no choice constraint. Because any razor can be used with Schick blades.

Moreover, when razors and blades are compatible across firms, old blades in the inventory can be used with the razor of the other firm. For instance, the consumer owning Sensor blades in the inventory can use the Sensor blades with a Schick razor. The consumers may be more likely to switch to Schick since the old blades of Gillette in the inventory are not costs of switching to Schick anymore.

The new flow utility of consumer i from purchasing a package j of any Gillette technology

$r(r \in \{1, 2, 3\})$ at time t when razors and blades are compatible is:

$$u_{i,r,j,t} = \min\{C_{i,r}, B_{i,r,t} + B_{i,4,t} + Q_{r,j}\} + \gamma_{i,r} + X_{r,j,t}\lambda_i - \alpha_i P_{r,j,t}$$
$$+ \eta_{1i}(B_{i,t} + Q_{r,j}) + \eta_{2i}(B_{i,t} + Q_{r,j})^2 + \epsilon_{i,r,t} \tag{7.3}$$

Consumers can use the Schick blades in the inventory if they purchases a Gillette razor or blade, since razors and blades are compatible across firms.

The new flow utility of consumer i from purchasing a package j of Schick technology $r(r = 4)$ at time t is:

$$u_{i,r,j,t} = \min\{C_{i,r}, B_{i,t} + Q_{r,j}\} + \gamma_{i,r} + X_{r,j,t}\lambda_i - \alpha_i P_{r,j,t}$$
$$+ \eta_{1i}(B_{i,t} + Q_{r,j}) + \eta_{2i}(B_{i,t} + Q_{r,j})^2 + \epsilon_{i,r,t} \tag{7.4}$$

Different from the flow utility function of Gillette technology and package choices, consumers purchasing Schick technology can use all the blades in the inventory no matter of what technology the blades are. Because razors and blades are compatible across firms.

The flow utility of purchasing disposable razors doesn't change since the horizontal incompatibility affects only non-disposable razors and blades.

The new flow utility of consumer i from making no purchases at time t is

$$u_{i,0,t} = \max\{0, \max_{k \in \{1,2,3\}} \min\{C_{i,k}, B_{i,k,t} + B_{i,4,t}\} + (\gamma_{i,k} + X_{k,t}\lambda_i) * 1\!\!1(B_{i,k,t} + B_{i,4,t} > 0),$$
$$\min\{C_{i,4}, B_{i,t}\} + (\gamma_{i,4} + X_{4,t}\lambda_i) * 1\!\!1(B_{i,t} > 0),$$
$$\min\{C_{i,N_t+1}, B_{i,N_t+1,t}\} + (\gamma_{i,N_t+1} + X_{N_t+1,t}\lambda_i) * 1\!\!1(B_{i,N_t+1,t} > 0)\}$$
$$+ \eta_{1i}B_{i,t} + \eta_{2i}B_{i,t}^2 + \epsilon_{i,0,t}$$
$$\tag{7.5}$$

When making no new purchases, consumers choose to consume the blades of tied technologies, the disposable razors, or choose the outside option which is not shaving or using other shaving products like electronic shavers.

Then a consumer's value function of being able to choose packages of different technologies is given by

$$\tilde{V}(S_{i,t}, \epsilon_{i,t}) = \max_{r \in J_t, j \in J'_{i,r,t}} u_{i,r,j,i} + \beta E[\tilde{V}(S_{i,t+1}, \epsilon_{i,t+1}|S_{i,t}, \epsilon_{i,t}, d_{i,r,j,t} = 1)] \tag{7.6}$$

where the choice set of package choice $J'_{i,r,t}$ depends on individual technology specific razor ownership due to the vertical incompatibility, and is different from the old choice set $J_{i,r,t}$. Consumers owning any Gillette razor can buy the Schick blades and vice versa.

Given the distribution assumption of ϵ, the expected value function of consumer i is

$$V(S_{i,t}) = \ln(\sum_{r \in J_t} \sum_{j \in J'_{i,r,t}} \exp(\bar{u}_{i,r,j,t} + \beta E[V(S_{i,t+1}|S_{i,t}, d_{i,r,j,t} = 1)])). \tag{7.7}$$

The new conditional choice probability of purchasing the package j of razor technology r for a consumer i at time t is

$$\begin{aligned} CCP_{i,r,j,t} &= \Pr(d_{i,r,j,t} = 1|S_{i,t}) \\ &= \frac{\exp(\bar{u}_{i,r,j,t} + \beta E[V(S_{i,t+1}|S_{i,t}, d_{i,r,j,t} = 1)])}{\sum_{r' \in J_t} \sum_{j' \in J_{r'}} \exp(\bar{u}_{i,r',j',t} + \beta E[V(S_{i,t+1}|S_{i,t}, d_{i,r',j',t} = 1)])}. \end{aligned} \tag{7.8}$$

where $\bar{u}_{i,r,j,t} = u_{i,r,j,t} - \epsilon_{i,r,j,t}$ is the mean utility.

Firms will re-optimize pricing strategies of razors and blades given the effect of horizontal compatibility on consumers demand. Two channels determines the new prices of razors and blades: increasing competition and demand expansion. On one hand, since razors are less effective in locking in consumers to purchase compatible blades, firms have fewer incentives to lower down the razor prices in order to gain more profits in the blade market. Accordingly, razor prices may be higher than before. Moreover, there is more competition between firms in the blade market, blade prices may be lower. On the other hand, since razors and blades now are horizontal compatible, they are more valuable for consumers compared to the disposable razors or outside option. Demand for blades expands because some consumers will switch from disposable razors or outside option to blades. Furthermore, market size of blades for each firm expands since firms now can attract consumers owning razors of the competitor in the blade market. All consumers no matter of the razor ownership have potential demand for blades. The demand expansion effect of product compatibility may help firms set higher prices of blades.

The two panels in Table 13 presents the new equilibrium prices and the choice probabilities when razors and blades are compatible across firms but incompatible with technologies within the same firm. Both razors and blades prices are higher than those where razors and blades are incompatible across firms and technologies. Under the new equilibrium prices, choice probabilities of razors and blades for technologies within the brand of Gillette are lower, blades of Schick have higher choice probabilities than before since they are compatible with all razors of Gillette while razors and blades within the Gillette brand are still incompatible across technologies. More consumers switch from Gillette blades to Schick blades due to the horizontal compatibility. Schick is better off with a higher market share when razors and blades are compatible across brands but incompatible across technologies within the brand.

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7.2 Vertical Compatibility: Compatible across Technologies within Firms

The vertical incompatibility is common in many complementary good markets. For instance, Apple develops new operating systems (IOS) and make the new system incompatible with some older versions of Iphones. It's important to understand why firms choose to make technologies vertically incompatible. A comparison of two cases where products are vertical compatible and incompatible helps explore the effect of vertical incompatibility on firms' prices and profits.

In the men's shaving market, razors and blades are also incompatible across technologies within the same firm. Gillette has three technologies: Sensor, Mach and Fusion. The new technology razor can be used only with the new technology blades instead of blades of older technologies. The vertical compatibility is a decision for firms. When a new technology is introduced, the firm decides whether to make the new technology compatible with older technologies.

In this section, it is assumed that razors and blades are compatible across technologies within the same brand but still incompatible across firms. Thus, any razor of a Gillette technology can be used with blades of any other Gillette technologies, but can't be used with Schick blades.

Similar to the previous section where razors and blades are compatible across brands but not technologies within the brand, consumers' blade choices are partially constrained by the firm specific razor ownership. And the firm specific blade inventory matters when consumers make the purchasing decisions.

The vertical compatibility changes the demand model in a similar way to horizontal compatibility. First, the individual choice set of blades are constrained by the individual firm specific razor ownership. For instance, if consumers own a Gillette Sensor razor, they can match the razor with blades of any Gillette technology but not with the blades of Schick. Second, the inventory is also firm specific. For example, all Gillette blades in the inventory can be used with razors of any Gillette technology but not with a razor from Schick.

Using the same notation as in the last section, the new choice set of packages given the Gillette technology choice $r(r \in \{1, 2, 3\})$ of consumer i at time t is

$$
J'_{i,r,t} = \begin{cases} \{1\}, & \text{if} \quad \sigma_{i,1,t} = 0, \quad \sigma_{i,2,t} = 0, \quad \sigma_{i,3,t} = 0 \\ \{1, 2, ..., J_r\}, & \text{otherwise} \end{cases} \tag{7.9}
$$

Consumers who don't own Gillette razors can't choose blades of any Gillette technology, but can always choose the razor package $(j = 1)$.

The choice set of packages given the Schick technology doesn't change since the vertical compatibility within Gillette doesn't affect Schick.

Moreover, when razors and blades are compatible across technologies within the firm, old blades in the inventory can be used with the razor belonging to the same firm. For instance, the consumer owning Sensor blades in the inventory can use the Sensor blades with razors from other technologies of Gillette. The consumers may be more likely to upgrade to a new technology since the blades of older Gillette technologies in the inventory are not costs of upgrading to a new Gillette technology anymore.

The new flow utility of consumer i from purchasing a package j of any Gillette technology $r(r \in \{1,2,3\})$ at time t when razors and blades are compatible is:

$$
\begin{aligned}
u_{i,r,j,t} = {}& \min\{C_{i,r}, \sum_{r\in\{1,2,3\}} B_{i,r,t} + Q_{r,j}\} + \gamma_{i,r} + X_{r,j,t}\lambda_i - \alpha_i P_{r,j,t} \\
& + \eta_{1i}(B_{i,t} + Q_{r,j}) + \eta_{2i}(B_{i,t} + Q_{r,j})^2 + \epsilon_{i,r,t}
\end{aligned}
\tag{7.10}
$$

Consumers can use all the Gillette blades in the inventory with any razor of Gillette since razors and blades are compatible across technologies within Gillette.

The flow utilities of purchasing Schick and disposable razors don't change since the vertical incompatibility affects only Gillette razors and blades.

The new flow utility of consumer i from making no purchases at time t is

$$
\begin{aligned}
u_{i,0,t} = {}& \max\{0, \max_{k\in\{1,2,3\}} \min\{C_{i,k}, \sum_{k\in\{1,2,3\}} B_{i,k,t}\} + (\gamma_{i,k} + X_{k,t}\lambda_i) * 1\!\!\!/\!K(\sum_{k\in\{1,2,3\}} B_{i,k,t} > 0), \\
& \min\{C_{i,4}, B_{i,4,t}\} + (\gamma_{i,4} + X_{4,t}\lambda_i) * 1\!\!\!/\!K(B_{i,4,t} > 0), \\
& \min\{C_{i,N_t+1}, B_{i,N_t+1,t}\} + (\gamma_{i,N_t+1} + X_{N_t+1,t}\lambda_i) * 1\!\!\!/\!K(B_{i,N_t+1,t} > 0)\} \\
& + \eta_{1i}B_{i,t} + \eta_{2i}B_{i,t}^2 + \epsilon_{i,0,t}
\end{aligned}
\tag{7.11}
$$

When making no new purchases, consumers choose to consume the blades of tied technologies, the disposable razors, or choose the outside option which is not shaving or using other shaving products like electronic shavers.

Then a consumer's value function of being able to choose packages of different technologies is given by

$$
\tilde{V}(S_{i,t}, \epsilon_{i,t}) = \max_{r\in J_t, j\in J'_{i,r,t}} u_{i,r,j,i} + \beta E[\tilde{V}(S_{i,t+1}, \epsilon_{i,t+1}|S_{i,t}, \epsilon_{i,t}, d_{i,r,j,t} = 1)]
\tag{7.12}
$$

where the choice set of package choice $J'_{i,r,t}$ depends on individual technology specific razor ownership due to the vertical incompatibility, and is different from the old choice set $J_{i,r,t}$.

Consumers owning any Gillette razor can buy the Schick blades and vice versa.

Given the distribution assumption of ϵ, the expected value function of consumer i is

$$V(S_{i,t}) = \ln(\sum_{r \in J_t} \sum_{j \in J'_{i,r,t}} \exp(\bar{u}_{i,r,j,t} + \beta E[V(S_{i,t+1}|S_{i,t}, d_{i,r,j,t} = 1)])). \tag{7.13}$$

The new conditional choice probability of purchasing the package j of razor technology r for a consumer i at time t is

$$
\begin{aligned}
CCP_{i,r,j,t} &= \Pr(d_{i,r,j,t} = 1|S_{i,t}) \\
&= \frac{\exp(\bar{u}_{i,r,j,t} + \beta E[V(S_{i,t+1}|S_{i,t}, d_{i,r,j,t} = 1)])}{\sum_{r' \in J_t} \sum_{j' \in J_{r'}} \exp(\bar{u}_{i,r',j',t} + \beta E[V(S_{i,t+1}|S_{i,t}, d_{i,r',j',t} = 1)])}.
\end{aligned}
\tag{7.14}
$$

where $\bar{u}_{i,r,j,t} = u_{i,r,j,t} - \epsilon_{i,r,j,t}$ is the mean utility.

Table 14 presents the new equilibrium prices of razors and blades and the new equilibrium choice probabilities when razors and blades are vertical compatible and horizontal incompatible. Both razors and blades have higher prices under the vertical compatibility. Under the new equilibrium prices, all razor and blade packages have lower choice probabilities and the disposable razors and the no purchases have higher choice probabilities. Consumers switch from using non-disposable razors and blades to using disposable razors or making no purchases due to the higher prices.

7.3 All Round Compatibility

The last counterfactual experiment abstracts away both the horizontal and vertical incompatibility. Razors and blades now are compatible across firms and technologies. Consumers can mix and match razors and blades since the constraint of razor ownership on blade choices is fully removed. All round compatibility affects consumer demand and firms' pricing strategy in a similar way which is shown in the appendix.

Panel A in Table 15 represents the firms' new optimal prices at the first period when razors and blades are compatible across firms and technologies. The first column provides the real prices in the data when razors and blades are incompatible across firms and technologies. And the second column show the firms' new optimal prices when razors and blades are compatible. I find prices are higher for both razors and blades when razors and blades are compatible. Firms rise up razors prices because they can't lock in consumers to buy the compatible blades and razors are more valuable. Higher blade prices means the market expansion effect of compatible design dominates the effect of increasing competition. Firms face higher demand of blades and rise up blade prices even though the higher prices will

lower some demand due to increasing competition.

The Panel B shows the choice probabilities in three scenarios. Baseline denotes the scenario where razors and blades are incompatible. The second column represents the scenario where razors and blades are compatible and prices are fixed. The last column represents the scenario where razors and blades are compatible and prices are the new equilibrium prices. The higher prices of razors and blades decrease the choice probabilities of razors and blades, and increase the choice probabilities of disposable razors and no purchases.

7.4 Summary of Counterfactual Results

A summary of the three counterfactual results is presented in Table 8. The new equilibrium prices of razors and blades are higher than before if razors and blades are horizontal, or vertical, or all round compatible.

All round compatibility gives highest prices for razor packages and blade packages for Schick. Horizontal compatibility gives the lowest price for razor and blade packages for Gillette as there is more competition within the brand of Gillette. Vertical compatibility gives the highest prices for blade packages of Gillette, since Gillette blades are more valuable as they can be used with any Gillette razor.

Panel B in Table 8 calculates the profit of each product given the new equilibrium prices under each scenario. All three counterfactuals give firms higher profits compared to those in the baseline case. Since Gillette is the market leader and Schick is the follower, they have different market positions and may prefer different compatibility strategies. Gillette prefer the all round compatibility the most, and the horizontal compatibility the least. Schick prefers the horizontal compatibility the most as it can attract more consumers switching from Gillette and prefers the vertical compatibility the least, as it will make Schick products less competitive than Gillette products.

Panel C in Table 8 calculates the consumer welfare change in each connterfactual experiment. Consumer welfare is improved overall when razors and blades are compatible. However, the welfare effect varies with consumers. Some consumers are worse off because they may have strong preferences over firms and/or technologies and would not change their choices even facing a higher price when razors and blades are compatible. Other consumers with weak preferences will benefit more from compatibility.

Given the results of three counterfactual experiments, I find that all round compatibility gives firms the highest profit. Assume there is no additional cost of making products compatible with other products, firms will prefer the compatibility both horizontally and vertically, because the demand expansion effect dominates the effect of intensified competition

when razors and blades are compatible.

We should be careful when interpreting the result that firms will also prefer the vertical compatibility. However, firms are observed to have horizontal and vertical incompatibility in the real world. There are several potential reasons. First, firms are assumed to be myopic and maximize period profits which doesn't capture the forward looking behavior of firms (e.g. entry/exit, new product introductions). Second, the results here depend on the costs of compatibility design. If the costs of compatibility design are high enough, it may dominate the benefit from compatibility. Firms then choose incompatibility.

Table 8: Counterfactual Results: Equilibrium Prices and Profits

Panel A: Equilibrium Prices

Firm	Technology	Package	Baseline	Compatible All Round	Compatible Horizontal	Compatible Vertical
Gillette	Sensor	Razor	5.7300	10.2364	9.7989	10.1241
		Blade Small Package	1.3164	3.1275	2.7058	3.1683
		Blade Large Package	1.1495	2.2082	1.9623	2.2272
	Mach	Razor	7.2537	11.4694	11.0593	11.3539
		Blade Small Package	1.9006	3.4522	3.1501	3.4861
		Blade Large Package	1.6898	2.6877	2.4711	2.7063
Schick	Schick	Razor	5.9365	9.7006	9.5891	9.5453
		Blade Small Package	1.2536	2.6193	2.4864	2.3949
		Blade Large Package	1.2536	2.0609	1.9707	1.9224

Panel B: Equilibrium Profits

Firm	Technology	Package	Baseline	Compatible All Round	Compatible Horizontal	Compatible Vertical
Gillette	Sensor	Razor	0.0123	0.0131	0.0134	0.0134
		Blade Small Package	0.0294	0.1016	0.0576	0.0916
		Blade Large Package	0.0185	0.0786	0.0428	0.0722
	Mach	Razor	0.0205	0.0226	0.0230	0.0230
		Blade Small Package	0.0821	0.1640	0.1317	0.1471
		Blade Large Package	0.0568	0.1029	0.0864	0.0943
	Total		0.2197	0.4829	0.3549	0.4415
Schick	Schick	Razor	0.0374	0.0393	0.0397	0.0400
		Blade Small Package	0.0079	0.0512	0.0528	0.0124
		Blade Large Package	0.0036	0.0342	0.0352	0.0080
	Total		0.0488	0.1247	0.1276	0.0604

Panel C: Consumers Welfare Change (%)

Compatible Horizontal	Compatible Vertical	Compatible All Round
0.57	0.13	0.44

8 Conclusion

This paper studies the welfare effect of product incompatibility in complementary good markets. I develop and estimate a structural model of dynamic consumer demand and oligopoly pricing game for complementary goods using an individual-level consumer panel data from the U.S. men's shaving industry.

The dynamic demand model for complementary goods incorporates complementarity, incompatibility, stockpiling behaviors and consumers' unobserved heterogeneity. i estimate the dynamic demand model by maximum likelihood method and EM algorithm. Given demand estimates, I estimate an oligopoly pricing game where fully integrated firms set prices of razors and blades and recover the marginal costs.

I then use estimates to quantify the effect of product incompatibility on price competition, and consumer welfare in the industry. In the counterfactual experiments, razors and blades are compatible across firms and/or technologies. Vertical and horizontal compatibility are explored separately. In each counterfactual experiment, consumers have a higher demand for blades since the constraint of razor ownership on choices is partially or fully removed. Consumers also switch and upgrade more frequently since the costs of switching or upgrading are lower when razors and blades are compatible.

Firms also adjust pricing strategies of razors and blades in the counterfactual analyses. There are two effects presented: competition is intensified and demand is expanded. On the one hand, competition in blades is intensified between firms since firms can't lock in consumers by razors. On the other hand, demand of blades increases and market size of blades expands. Firms can attract all consumers no matter their razor ownerships. Results show firms charge higher prices of razors and blades since the demand expansion effect dominates the intensified competition effect.

Combining the changes in consumer demand and firms' prices, the effect of product incompatibility on consumer welfare is evaluated. Overall, consumer welfare is improved since the benefit consumers derive from expanded choices outweighs the increased product costs. However, the welfare effect varies from consumers. Consumers with strong preferences bear the increased product costs without changing the choice and are worse off.

References

[1] ARCIDIACONO, P., AND JONES, J. B. Finite mixture distributions, sequential likelihood and the em algorithm. *Econometrica 71*, 3 (2003), 933–946.

[2] CHINTAGUNTA, P. K., QIN, M., AND VITORINO, M. A. Licensing and price competition in tied-goods markets: An application to the single-serve coffee system industry. *Marketing Science, Forthcoming* (2018). Available at SSRN: https://ssrn.com/abstract=2823221 or http://dx.doi.org/10.2139/ssrn.2823221.

[3] CHOI, J. P. Tying in two-sided markets with multi-homing. *The Journal of Industrial Economics 58*, 3 (2010), 607–626.

[4] DERDENGER, T. Technological tying and the intensity of price competition: An empirical analysis of the video game industry. *Quantitative Marketing and Economics 12*, 2 (2014), 127–165.

[5] DUBÉ, J.-P. H., HITSCH, G. J., AND CHINTAGUNTA, P. K. Tipping and concentration in markets with indirect network effects. *Marketing Science 29*, 2 (2010), 216–249.

[6] ECONOMIDES, N. Desirability of compatibility in the absence of network externalities. *The American Economic Review* (1989), 1165–1181.

[7] GOETTLER, R. L., AND GORDON, B. R. Does amd spur intel to innovate more? *Journal of Political Economy 119*, 6 (2011), 1141–1200.

[8] GOWRISANKARAN, G., AND RYSMAN, M. Dynamics of consumer demand for new durable goods. *Journal of political Economy 120*, 6 (2012), 1173–1219.

[9] HARTMANN, W. R., AND NAIR, H. S. Retail competition and the dynamics of demand for tied goods. *Marketing Science 29*, 2 (2010), 366–386.

[10] HENDEL, I., AND NEVO, A. Measuring the implications of sales and consumer inventory behavior. *Econometrica 74*, 6 (2006), 1637–1673.

[11] HENDEL, I., AND NEVO, A. Intertemporal price discrimination in storable goods markets. *American Economic Review 103*, 7 (2013), 2722–51.

[12] HO, K. The welfare effects of restricted hospital choice in the us medical care market. *Journal of Applied Econometrics 21*, 7 (2006), 1039–1079.

[13] HUANG, Y. The value of compatibility to a tied-good market. Working Paper, 2018.

[14] IGAMI, M. Estimating the innovator's dilemma: Structural analysis of creative destruction in the hard disk drive industry, 1981–1998. *Journal of Political Economy 125*, 3 (2017), 798–847.

[15] KATZ, M. L., AND SHAPIRO, C. Systems competition and network effects. *Journal of economic perspectives 8*, 2 (1994), 93–115.

[16] KNITTEL, C. R., AND STANGO, V. Strategic incompatibility in atm markets. *Journal of Banking & Finance 35*, 10 (2011), 2627–2636.

[17] LEE, R. S. Vertical integration and exclusivity in platform and two-sided markets. *The American Economic Review 103*, 7 (2013), 2960–3000.

[18] LI, H. Intertemporal price discrimination with complementary products: E-books and e-readers. *Management Science, Forthcoming* (2018).

[19] LIU, H. Dynamics of pricing in the video game console market: skimming or penetration? *Journal of Marketing Research 47*, 3 (2010), 428–443.

[20] MATUTES, C., AND REGIBEAU, P. " mix and match": product compatibility without network externalities. *The RAND Journal of Economics* (1988), 221–234.

[21] NAIR, H. Intertemporal price discrimination with forward-looking consumers: Application to the us market for console video-games. *Quantitative Marketing and Economics 5*, 3 (2007), 239–292.

[22] OHASHI, H. The role of network effects in the us vcr market, 1978–1986. *Journal of Economics & Management Strategy 12*, 4 (2003), 447–494.

[23] RYSMAN, M. Competition between networks: A study of the market for yellow pages. *The Review of Economic Studies 71*, 2 (2004), 483–512.

[24] TRAIN, K. E. *Discrete choice methods with simulation.* Cambridge university press, 2009.

[25] WHINSTON, M. D. Tying, foreclosure, and exclusion. *American Economic Review 80*, 4 (1990), 837–859.

[26] ZHOU, J. Competitive bundling. *Econometrica 85*, 1 (2017), 145–172.

A   Product Portfolio and Life Cycle

Figure 6 and 7 show the product portfolio for each firm, and also depict the product life cycle, in particular, the impact of new product introductions on competitors' products and also the old products within the same firm. Gillette introduced Fusion shaving system (both razor and blades) in 2006 and Schick introduced Hydro shaving system in 2010.

Sales of new razors will increase first and then decline gradually, while sales of new blade cartridges keep increasing over time. Sales of older razors and blades decline gradually over time, in particular after a new technology was introduced.

The cannibalization effects of new product introductions on old products within the same brand are depicted. For instance, the sales of Gillette Mach non-disposable razors and blade cartridges declined greatly after Gillette Fusion was introduced in 2006. Similar story applies to Schick Hydro and Schick Quattro. The competition effect of new non-disposable razors on those of the other firm are also significant. For instance, the sales of Schick Quattro non-disposable razors declined heavily after Gillette introduced Fusion non-disposable razors in 2006. However, the competition effect of new blade cartridges is not as significant as that of non-disposable razors.

Furthermore, the oldest non-disposable razor would be discontinued after the newest razor was introduced. As shown in Figure 6 and 7, Gillette Sensor and Schick Xtreme razors gradually exited the market of non-disposable razor after 2006. However, when the non-disposable razors were discontinued, there still exists the demand for compatible blade cartridges since some consumers will keeping using their old razors by purchasing compatible blade cartridges. Gillette Sensor is a good example. After Gillette introduced Fusion in 2006, the share of Sensor non-disposable razors declined from 6% in 2005 to 2.6% in 2006, and was less than 1% after 2008. However, Sensor blade cartridges still held a large market share.

B   Robustness Checks

2.1   Myopic Consumers

Since razors and blades are incompatible across technologies, consumers owning some razors face the constraint in blade choices: they can only choose the blades compatible with razors they own, otherwise they need to buy a new razor before consuming the new blades. Furthermore, consumers owning some unused blades in the inventory may deter the purchases of new blades since they can just consume the blades in the inventory. As shown in the section 2, prices of razors decline over time and prices of blades increase over time. And as

new products are introduced occasionally, the product set changes over time.

The incompatibility of razors and blades, the blade inventory, and the movement of prices and product set make consumers forward looking. Consumers learn their razor ownership and blade inventory at the beginning of the period, and form expectations of future razor ownership and blade inventory after current choices, beside of the future prices, product characteristics and product set. For example, consumers purchase a new razor not because the razor provides the highest utility at the current period, but it guarantees they could choose the compatible blades in the future. The value of razors in the future matters.

If consumers are myopic, they have no expectations of future states (razor ownership and blade inventory) beside of future prices, product characteristics, and product set. Consumers compare the current utility of each choice and makes the decision that maximize the current period utility, ignoring the possible effects in the future. Table 9 presents the estimate results of a static demand model with myopic consumers where consumers have unobserved types.

Consistent with the previous literature which compares the demand estimates in a static model and a dynamic model, the marginal utility of prices in static setting where consumers are myopic are bigger than those in dynamic demand model. The reason is when current prices go up, by the state transition process, consumers form the expectation that future prices will also go up and may make fewer purchases. However, the effect of future price increase is ignored in static setting and it is added to the effect of current price increase, then the estimated α will be larger in the static demand model where consumers are myopic and have no future expectations.

2.2 Homogeneous Consumers

From the data pattern, consumers have different blade consumption rates and may have different preferences. Ignoring the unobserved consumers' heterogeneity may lead to inaccurate demand estimates including the blade consumption rates. Table 10 presents the demand estimate results if consumers are assumed to be homogeneous.

The estimates of homogeneous consumers mix up the estimates for two types of consumers. For instance, the estimated marginal utility of prices is larger then that of Type II consumers but smaller than that of Type I consumers. The estimated blade consumption rates in a quarter for homogeneous consumers are also located in the middle of the blade consumption rates of two types of consumers.

Comparing the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC), I find that the model assuming consumers are homogeneous has a lower goodness of fit than the baseline model which allows unobserved heterogeneity in consumers' preferences

and blade consumption rates.

2.3 Single-homing in Razors and Blades

According to the analysis of the data, about 1/3 consumers own both Gillette and Schick razors in 2014, implying the multi-homing in razors is common in this market. Incorporating the multi-homing in razors in the demand model is important and has crucial implications on consumers' choices and firms' pricing strategy. Assuming away the multi-homing in razors implies consumers can only own one technology of razors and blades at each period. When consumers purchase razors or blades from a new technology, they discard the old razors and blades in the inventory and only keep the new technology in the inventory.

This restriction in razor holdings overestimate the effect of product incompatibility on consumers' choices. If consumers own multiple razors in the inventory, they can choose any blade compatible with razors they own. If consumers are single homing in razors, they can only choose blades from the technology of which they own razors. Table 11 presents the demand estimates if consumers are assumed to own only one technology razors and blades.

Comparing the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC), I find that the model with single-homing in razors and blades has a lower goodness of fit than the baseline model which allows the multi-homing in razors and blades. In particular, the fraction of consumers of Type I is much higher than the estimated number in the baseline model and the marginal utility of prices for consumers of Type II is even positive when assuming single-homing.

C   All Round Compatibility

When razors and blades are all round compatible, consumers demand changes in the following ways. First, there is no more constraint on blade choices from razor ownership. Consumers can buy any blade no matter of their razor ownership since now blades can be used with any razor. The removal of choice constraint will make the razors and blades more valuable and the demand for razors and blades may increase. Second, the switching costs are lower. Consumers don't need to buy a razor before purchasing a blade. Moreover, the old blades in the inventory are now compatible with any razor. Thus, there is no additional cost from old blades in the inventory if switching or upgrading to a new technology. Third, because consumers only need one razor to use all blades, the demand for razors will decrease and it's less likely for consumers to own multiple razors.

The choice set of consumers changes when razor and blade are compatible across firms

and technologies. Consumers can choose any blade they prefer and the choice set doesn't depend on individual razor ownership anymore. Now, the choice set of consumer i at time t is

$$J_{i,r,t} = J_r = \{1, 2, ..., J_r\}, \quad \forall \sigma_{i,t} \in \Sigma_t.$$

The individual choice set doesn't have the subscripts of i and t since all products are available to consumers no matter what razors they own.

Moreover, when razors and blades are compatible all round, old blades in the inventory can be used with any razor, not restricted by technology anymore. When razors and blades are incompatible, consumers can only use the blades in the inventory compatible with razor they are currently using. If compatible, consumers can consume all the blades in the inventory with any razor.

The new flow utility of consumer i from purchasing a package j of tied technology $r(r \neq 0, r \neq N_t + 1)$ at time t when razors and blades are compatible is:

$$
\begin{aligned}
u_{i,r,j,t} = &\min\{C_{i,r}, \tilde{B}_{i,t} + Q_{r,j}\} + \gamma_{i,r} + X_{r,j,t}\lambda_i - \alpha_i P_{r,j,t} \\
&+ \eta_{1i}(B_{i,t} + Q_{r,j}) + \eta_{2i}(B_{i,t} + Q_{r,j})^2 + \epsilon_{i,r,t}
\end{aligned}
\tag{C.1}
$$

where $\tilde{B}_{i,t} = \sum_{r \neq N_t+1} B_{i,r,t}$ is the total number of unused blades for consumer i at time t. Different from the original utility function, consumers can consume all blades with any razor and face no constraint from the incompatibility.

The utility of purchasing disposable razors doesn't change since the incompatibility affects only non-disposable razors and blades.

The new flow utility of consumer i from making no purchases at time t is

$$
\begin{aligned}
u_{i,0,t} = &\max\{0, \max_{k \neq N_t+1} \min\{C_{i,k}, \tilde{B}_{i,t}\} + (\gamma_{i,k} + X_{k,t}\lambda_i) * \mathbb{K}(\tilde{B}_{i,t} > 0), \\
&\min\{C_{i,N_t+1}, B_{i,N_t+1,t}\} + (\gamma_{i,N_t+1} + X_{N_t+1,t}\lambda_i) * \mathbb{K}(B_{i,N_t+1,t} > 0)\} \\
&+ \eta_{1i}B_{i,t} + \eta_{2i}B_{i,t}^2 + \epsilon_{i,0,t}
\end{aligned}
\tag{C.2}
$$

When making no new purchases, consumers choose to consume the blades of tied technologies, the disposable razors, or choose the outside option which is not shaving or using other shaving products like electronic shavers.

Different from previous demand model, consumers can consume all the blades in the inventory instead of the compatible blades with razor they are using. There are two folds of effects on consumers' demand. First, the utility of making no purchases increases, and consumers may deter purchases. Second, consumers are more willing to switch to a new razor

which he is not owning, because all the blades can be used with the new razor.

Then a consumer's value function of being able to choose packages of different technologies is given by

$$\tilde{V}(S_{i,t}, \epsilon_{i,t}) = \max_{r \in J_t, j \in J_r} u_{i,r,j,i} + \beta E[\tilde{V}(S_{i,t+1}, \epsilon_{i,t+1}|S_{i,t}, \epsilon_{i,t}, d_{i,r,j,t} = 1)] \qquad \text{(C.3)}$$

where β is the discounting rate of future payoffs, and package choice j doesn't depend on individual razor ownership due to the compatibility, implying there is no consumer subscript in the choice set of packages.

Given the distribution assumption of ϵ, the expected value function of consumer i is

$$V(S_{i,t}) = \ln(\sum_{r \in J_t} \sum_{j \in J_r} \exp(\bar{u}_{i,r,j,t} + \beta E[V(S_{i,t+1}|S_{i,t}, d_{i,r,j,t} = 1)])). \qquad \text{(C.4)}$$

The new expected values of choosing non-disposable razors and blades are higher than before, because consumers now can choose any product in the future without the constraint on choice set anymore. If razors and blades are incompatible, consumers can only choose the compatible blades in the future. However, if compatible, consumers can buy any product and don't need to worry about the compatibility. The valuation of razors and blades increases. However, the compatibility doesn't change the valuation of disposable razors and making no purchases since consumers face no constraint to buy a disposable razor or making no purchases.

The new conditional choice probability of purchasing the package j of razor technology r for a consumer i at time t is

$$\begin{aligned}
CCP_{i,r,j,t} &= \Pr(d_{i,r,j,t} = 1|S_{i,t}) \\
&= \frac{\exp(\bar{u}_{i,r,j,t} + \beta E[V(S_{i,t+1}|S_{i,t}, d_{i,r,j,t} = 1)])}{\sum_{r' \in J_t} \sum_{j' \in J_{r'}} \exp(\bar{u}_{i,r',j',t} + \beta E[V(S_{i,t+1}|S_{i,t}, d_{i,r',j',t} = 1)])}.
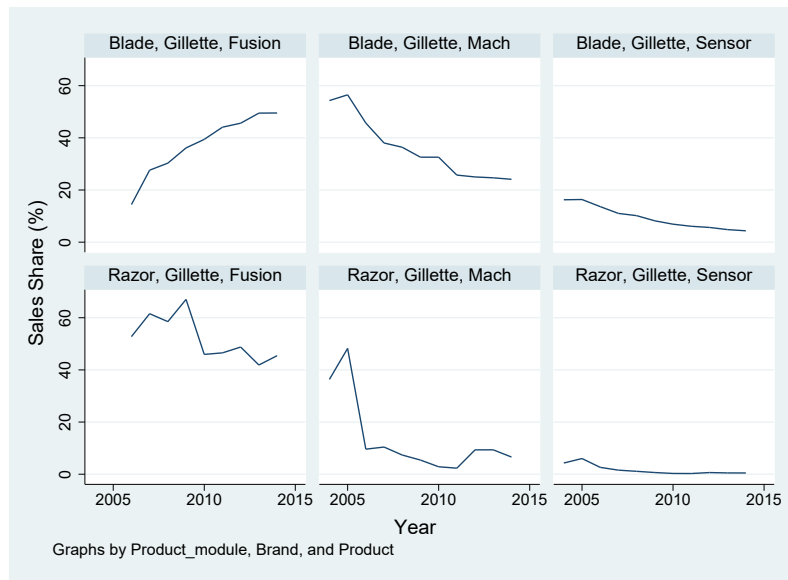\end{aligned} \qquad \text{(C.5)}$$

D Graphs

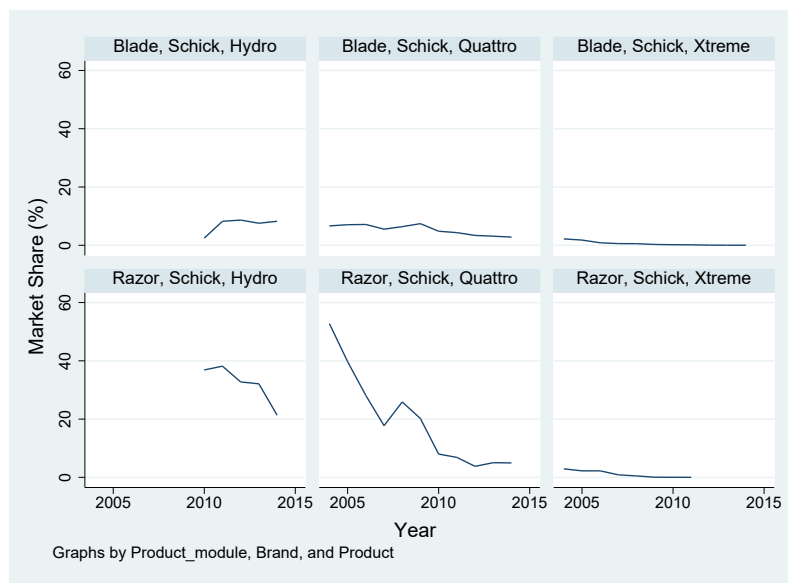Figure 6: Market Share (%) of Gillette in Sales of Razors and Blades

Figure 7: Market Share (%) of Schick in Sales of Razors and Blades

E Tables

Table 9: Demand Estimates of Myopic Consumers

		Type I		Type II	
Variable	Parameter	Estimate	Std. Err.	Estimate	Std. Err.
Type Probability	ρ	0.4127***	0.0548		
Price	α	-0.6599***	0.0025	-0.0818***	0.0024
Blade consumption	C_{Blade}	2.5376***	0.0168	8.0500***	0.0113
Disposable razor consumption	C_{Disp}	1.9065***	0.1539	8.9494***	0.0001
Sensor FE	γ_{Sensor}	-1.4496***	0.0373	-1.8809***	0.0214
Mach FE	γ_{Mach}	-0.2073***	0.0436	-1.5004***	0.0218
Fusion FE	γ_{Fusion}	0.3326***	0.0227	-0.8636***	0.0289
Schick FE	γ_{Schick}	-1.2388***	0.0377	-2.3534***	0.0230
Disposable Razor FE	γ_{Disp}	-1.9065***	0.1541	-3.0475***	0.0224
Gillette Razor	$\lambda_{Gillette}$	-2.4950***	0.0387	-2.3988***	0.0216
Schick Razor	λ_{Schick}	-1.9752***	0.0481	-1.3784***	0.0278
Inventory cost Linear	η_1	-0.0184***	0.0014	-1.2274***	0.0055
Inventory cost quadratic	η_2	0.0008***	0.0000	0.0335***	0.0003

Note: Significance level *** (1%) ** (5%) * (10%)

Table 10: Demand Estimates of Homogeneous Consumers

Variable	Parameter	Estimate	Standard Error
Price	α	-0.2884***	0.0016
Blade consumption	C_{Blade}	8.3196***	0.1977
Disposable razor consumption	C_{Disp}	7.6596***	0.0020
Sensor FE	γ_{Sensor}	-2.9922***	0.0158
Mach FE	γ_{Mach}	-1.7238***	0.0171
Fusion FE	γ_{Fusion}	-0.9162***	0.0243
Schick FE	γ_{Schick}	-2.6437***	0.0177
Disposable Razor FE	γ_{Disp}	-4.2757***	0.0082
Gillette Razor	$\lambda_{Gillette}$	-2.0324***	0.0198
Schick Razor	λ_{Schick}	-1.5394***	0.0231
Inventory cost Linear	η_1	-0.6729***	0.0013
Inventory cost quadratic	η_2	0.0147***	0.0000

Note: Significance level *** (1%) ** (5%) * (10%)

Table 11: Demand Estimates if Single-homing

Variable	Parameter	Type I Estimate	Type I Std. Err.	Type II Estimate	Type II Std. Err.
Type Probability	ρ	0.8094***	0.0324		
Price	α	−0.5442***	0.0017	0.0194***	0.0025
Blade consumption	C_{Blade}	4.3556***	0.0131	9.1077***	0.2895
Disposable razor consumption	C_{Disp}	3.8370***	0.0019	13.7904***	0.4583
Sensor FE	γ_{Sensor}	−2.6711***	0.0224	−2.1555***	0.0432
Mach FE	γ_{Mach}	−1.7718***	0.0246	−2.0967***	0.0353
Fusion FE	γ_{Fusion}	−0.6049***	0.0304	−2.2012***	0.0433
Schick FE	γ_{Schick}	−4.3556***	0.0390	−2.9596***	0.0399
Disposable Razor FE	γ_{Disp}	−5.6606***	0.0241	−3.1820***	0.0322
Gillette Razor	$\lambda_{Gillette}$	−0.6000***	0.0175	−3.0087***	0.0499
Schick Razor	λ_{Schick}	1.4836***	0.0222	−1.4819***	0.0529
Inventory cost Linear	η_1	−0.0050***	0.0002	−0.6053***	0.0104
Inventory cost quadratic	η_2	0.0012***	0.0000	−0.0242***	0.0007

Note: Significance level *** (1%) ** (5%) * (10%)

Table 12: Model Fit: Choice Probabilities

Firm	Technology	Package	Data	Estimated
		Razor	0.0021	0.0027
	Sensor	Blade Small Package	0.0050	0.0096
		Blade Large Package	0.0096	0.0078
		Razor	0.0033	0.0033
Gillette	Mach	Blade Small Package	0.0177	0.0193
		Blade Large Package	0.0218	0.0180
		Razor	0.0130	0.0117
	Fusion	Blade Small Package	0.0129	0.0130
		Blade Large Package	0.0118	0.122
		Razor	0.0094	0.0116
Schick	Schick	Blade Small Package	0.0062	0.0046
		Blade Large Package	0.0036	0.0044
		Small Package	0.0407	0.0367
	Disposable Razor	Median Package	0.0477	0.0532
		Large Package	0.0612	0.0565
No Purchase			0.7426	0.7461

Table 13: Counterfactual Results: Horizontal Compatibility

Panel A: Equilibrium Prices

Firm	Technology	Package	Baseline	Equilibrium Prices
Gillette	Sensor	Razor	5.7300	9.7989
		Blade Small Package	1.3164	2.7058
		Blade Large Package	1.1495	1.9623
	Mach	Razor	7.2537	11.0593
		Blade Small Package	1.9006	3.1501
		Blade Large Package	1.6898	2.4711
Schick	Schick	Razor	5.9365	9.5891
		Blade Small Package	1.7017	2.4864
		Blade Large Package	1.5386	1.9707

Panel B: Equilibrium Choice Probabilities

Firm	Technology	Package	Baseline	Compatible Fixed Prices	Compatible New Prices
Gillette	Sensor	Razor	0.0030	0.0029	0.0016
		Blade Small Package	0.0092	0.0127	0.0066
		Blade Large Package	0.0064	0.0088	0.0046
	Mach	Razor	0.0055	0.0052	0.0030
		Blade Small Package	0.0247	0.0286	0.0158
		Blade Large Package	0.0138	0.0153	0.0083
Schick	Schick	Razor	0.0086	0.0083	0.0050
		Blade Small Package	0.0031	0.0123	0.0071
		Blade Large Package	0.0020	0.0080	0.0047
	Disposable Razor	Small Package	0.0486	0.0467	0.0489
		Median Package	0.0513	0.0492	0.0516
		Large Package	0.0744	0.0822	0.0867
No Purchase			0.7494	0.7200	0.7561

Table 14: Counterfactual Results: Vertical Compatibility

Panel A: Equilibrium Prices

Firm	Technology	Package	Baseline	Equilibrium Prices
Gillette	Sensor	Razor	5.7300	10.1241
		Blade Small Package	1.3164	3.1683
		Blade Large Package	1.1495	2.2272
	Mach	Razor	7.2537	11.3539
		Blade Small Package	1.9006	3.4861
		Blade Large Package	1.6898	2.7063
Schick	Schick	Razor	5.9365	9.5453
		Blade Small Package	1.2536	2.3949
		Blade Large Package	1.2536	1.9224

Panel B: Equilibrium Choice Probabilities

Firm	Technology	Package	Baseline	Compatible Fixed Prices	Compatible New Prices
Gillette	Sensor	Razor	0.0030	0.0029	0.0016
		Blade Small Package	0.0092	0.0202	0.0086
		Blade Large Package	0.0064	0.0145	0.0063
	Mach	Razor	0.0055	0.0052	0.0029
		Blade Small Package	0.0247	0.0319	0.0152
		Blade Large Package	0.0138	0.0169	0.0077
Schick	Schick	Razor	0.0086	0.0083	0.0050
		Blade Small Package	0.0031	0.0030	0.0017
		Blade Large Package	0.0020	0.0019	0.0011
	Disposable Razor	Small Package	0.0486	0.0466	0.0492
		Median Package	0.0513	0.0491	0.0520
		Large Package	0.0744	0.0820	0.0874
No Purchase			0.7494	0.7176	0.7611

Table 15: Counterfactual Results: All Round Compatibility

Panel A: Equilibrium Prices

Firm	Technology	Package	Baseline	Equilibrium Prices
Gillette	Sensor	Razor	5.7300	10.2364
		Blade Small Package	1.3164	3.1275
		Blade Large Package	1.1495	2.2082
	Mach	Razor	7.2537	11.4694
		Blade Small Package	1.9006	3.4522
		Blade Large Package	1.6898	2.6877
Schick	Schick	Razor	5.9365	9.7006
		Blade Small Package	1.2536	2.6193
		Blade Large Package	1.2536	2.0609

Panel B: Equilibrium Choice Probabilities

Firm	Technology	Package	Baseline	Compatible Fixed Prices	Compatible New Prices
Gillette	Sensor	Razor	0.0030	0.0028	0.0015
		Blade Small Package	0.0092	0.0222	0.0097
		Blade Large Package	0.0064	0.0156	0.0069
	Mach	Razor	0.0055	0.0051	0.0028
		Blade Small Package	0.0247	0.0352	0.0172
		Blade Large Package	0.0138	0.0182	0.0085
Schick	Schick	Razor	0.0086	0.0081	0.0048
		Blade Small Package	0.0031	0.0112	0.0064
		Blade Large Package	0.0020	0.0072	0.0042
	Disposable Razor	Small Package	0.0486	0.0456	0.0486
		Median Package	0.0513	0.0478	0.0512
		Large Package	0.0744	0.0791	0.0853
No Purchase			0.7494	0.7020	0.7527