Job Market Paper

Optimal Financing Contracts in Venture Capital Partnerships*

Qing Liu
Boston University

January 12, 2020

Abstract

This paper analyzes a business venture's financing problem where an entrepreneur and an advisor exert complementary efforts to improve the productivity of a risky investment project under moral hazard. With verifiable contracts, it is optimal for the entrepreneur to sign contingent contracts directly with the advisor and the investors. With unverifiable contracts, the entrepreneur cannot write contingent contracts and she is constrained to employ either angel financing or venture capital financing. In the case of angel financing, the entrepreneur finds it expensive to raise money. In the case of venture capital financing, the entrepreneur contracts only with the advisor like a venture capitalist who has formed a limited partnership with the investors. In competitive capital and labor markets, the entrepreneur maximizes her expected payoff by choosing venture capital financing, which explains its prevalence in practice. The entrepreneur issues convertible securities to respond to different incentives from the venture capitalist and the investors.

Keywords: Angels, Convertible Securities, Double Moral Hazard, Venture Capital

JEL codes: G24, G32

*Contacts: `qingliu@bu.edu`. I am extremely grateful for my Ph.D. advisors Lucy White, Andrea Buffa, and Jerome Detemple for their invaluable guidance and support. I also thank Qianqian Yu and Chris Veld for many helpful comments. All errors are mine.

1 Introduction

"Time, Talent, and Treasure (the three Ts of venturing) are required by both the entrepreneur and the venture capitalist."

– Amit, Glosten, and Muller (1990)

Two major sources of financing for start-up companies in the United States are angel investors and venture capitalists. In an OECD study, Wilson (2011) estimates the size of the angel market globally and concludes that the amount of investments in the angel market is extremely similar to that in the venture capital market.[1] Moreover, Hellmann, Schure, and Vo (2019) show that angel financing and venture capital financing are dynamic substitutes, and it is the companies who self-select into the financing path that works the best for them. This paper studies the optimal source of financing for entrepreneurs who need to raise capital and seek expert advice to implement and run business ventures.

Venture capital financing is different from angel financing in at least two ways: (1) Angel investors are wealthy individuals who directly invest their own money into business ventures, whereas most of venture capitalists' funding comes from the limited partners (LPs), with the general partners (GPs) who make the investment decisions investing only limited amounts of their own money into the fund (Fenn, Liang, and Prowse (1997), Robinson and Sensoy (2013)). (2) The general partners of venture capital firms are more actively involved in the management of the start-up firms than angel investors (Ehrlich, De Noble, Moore, and Weaver (1994)).[2]

However, little is known about the financial intermediation role of venture capitalists aside from the services they provide to their portfolio companies (Gorman and Sahlman (1989)). This paper explores the role of venture capitalists as intermediaries between entrepreneurs and investors, shows how, by providing sufficient transparency of contracts, venture capitalists can add value. This helps us to understand the prevalence of venture capital financing, in particular compared to other possible structures such as the separation of investment and consulting services (Casamatta (2003), Chemmanur and Chen (2014)).

[1]For instance, Wilson (2011) estimates the size of the angel market in 2009 is $17,700 millions, compared to $18,275 millions of the total venture capital market, in the United States.

[2]The community of angel investors is quite diverse, including individual angels, angel groups, angel funds, etc. Although Kerr, Lerner, and Schoar (2011) find that angel groups have adopted a similar hands-on role like venture capitalists in the portfolio companies, we believe that the majority of angel investors, especially those work individually or work with only a couple of others, is less involved in the companies invested, compared to typical venture capitalists.

Suppose a financially constrained entrepreneur, endowed with an innovative project, is able to costlessly find a consultant who can provide the same *expert advice*[3] as a venture capitalist, will the entrepreneur fund her project through venture capital financing, or hire the consultant and fund the project through angel financing? Figure 1 describes these two financing options for the entrepreneur. Standard principal-agent theory implies that the entrepreneur should never choose venture capital financing. First, for whatever contract the venture capitalist offers to the investors, the entrepreneur can, if she wishes, always offer the same contract to the investors that the venture capitalist would have offered. Second, since the venture capitalist's interest is not perfectly aligned with the entrepreneur's, the entrepreneur can in general do strictly better by offering a different contract to investors from the one that the venture capitalist would have chosen. Thus, the prevalence of the venture capital structure, where the design of the financial contract offered to investors is delegated to the general partners represents something of a puzzle for corporate finance theory. In this paper, we show that the above reasoning holds true only when all contracts are *verifiable* by all parties. When instead verifiability is more difficult, the venture capital contracting structure dominates the angel financing and separate advising model.

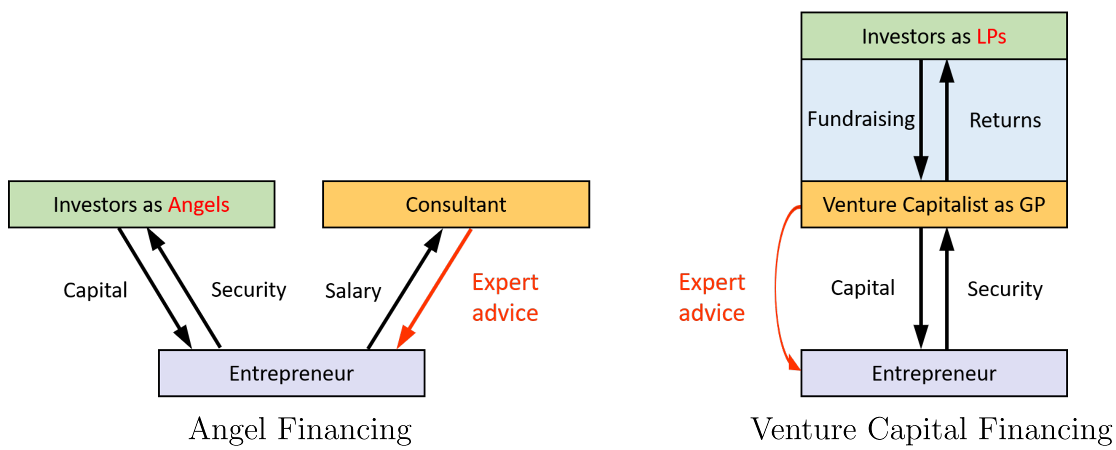

Figure 1: "Replicating" Venture Capital Financing using Angel Financing

When contracts between two contracting parties are always verifiable by third parties, the entrepreneur can essentially write a contingent contract directly with the consultant which specifies not only his compensation but also the investors' payment structure accordingly. Similarly, the investors' contract offered by the entrepreneur would also need to set out their pay package con-

[3]Expert advice is broadly defined here and can include access to platforms, infrastructures, connections to suppliers and users, and of course, human capital, including knowledge of the law, business, etc. This is in line with venture capitalists' different aspects of value-added role documented by Barry, Muscarella, Peavy III, and Vetsuypens (1990), Hellmann and Puri (2002), Hsu (2004), Gompers (1995), Kaplan and Strömberg (2004), Lerner (1994), Lerner (1995), etc. Moreover, Sørensen (2007) finds that the expert advice provided by venture capitalists is significant and brings companies public at a higher rate.

tingent on the consultant's. In this hypothetical world, both the consultant and the investors can verify how much the entrepreneur offers to the other, so payment to one can be made contingent on the contract offered to the other, and each can costlessly sue the entrepreneur for any contractual deviations.

However, the assumption of fully verifiable contingent contracts is too strong to implement in practice. We show that with *unverifiable* contracts, that is, when the entrepreneur cannot write contracts with one party contingent on contracts offered to the other, then venture capital financing can become optimal for the entrepreneur. With venture capital financing, the venture capitalist serves as the intermediary between the entrepreneur and the investors. This makes the venture capitalist the signatory to both contracts (one is with the entrepreneur, the other is with the investors), and so the venture capitalist can by default, verify both contracts. As long as the investors in the venture capital firm can also ultimately verify the payoffs from the securities that the venture capitalist holds in the fund in which they invest, then this is sufficient verifiability for the venture capitalist structure – in contrast to the angel financing structure – to obtain the second best optimum.

The model we develop has several features that are consistent with the private equity market. First, we assume that both the entrepreneur and the advisor (either the consultant or the venture capitalist) must engage in costly effort to improve the productivity of the project, and their efforts are *complementary*. This implies that both have to be given appropriate incentives to exert effort, and the advisor will choose to exert more effort if he believes the entrepreneur will also exert higher effort and vice versa. Second, we assume that when the investors form a limited partnership with the venture capitalist, they can verify how the entrepreneur implements the contract she signs with the venture capitalist. To be more precise, the LPs can eventually confirm the types and amounts of securities issued by the entrepreneur in exchange for their capital investment. Third, we assume competitive capital and labor markets, so that all the surplus is allocated to the entrepreneur.

In this setting, we derive several interesting results about the entrepreneur's optimal financing choice and the implementation of the optimal financing contract. First, with angel financing - where the entrepreneur contracts directly and separately with the consultant and investors - the entrepreneur finds it expensive to raise money compared to the second best with verifiable contracts. To understand the intuition for this result, suppose that the investors agreed to fund the project for the same return as with verifiable contracts. Then the entrepreneur would be tempted to reduce the rent that the consultant requires to exert higher effort. The entrepreneur, in this way, could be made better off without violating the consultant's participation constraint at the expense of sabotaging the investors' expected payoff. Anticipating the "opportunistic" behavior of the other pair, the investors require a higher return to provide the capital. If contracts were verifiable, the investors could confidently expect higher efforts from both the entrepreneur and the consultant. Knowing the project becomes more productive, the investors would be happy to receive lower compensation as a compromise to give the others higher incentives. However, once the verifiability of contracts by third parties assumption is dropped, raising capital becomes more expensive.

Next, we characterize the optimal contracts with venture capital financing. As the entrepreneur, in this case, cannot write a contract directly with the investors, she is constrained to offer a compensation budget with the venture capitalist which the latter decides how to distribute the budget between himself and the investors. Nonetheless, the entrepreneur knows that for whatever contract she writes, the venture capitalist will play a best response to that: he maximizes the residual surplus without violating the investors' participation constraint. On one hand, the entrepreneur loses control over the compensation structure; on the other hand, the investors, through verifying the compensation budget, can calculate (hence eventually verify) the venture capitalist's pay package by comparing the payoffs on the owned securities with the payoffs they themselves receive. Compared to angel financing, the entrepreneur trades off the potential rent extraction from the venture capitalist against the improved verifiability from the investors.

The result of the model is that the entrepreneur maximizes her expected payoff by choosing venture capital financing. We find that the investors get more return while the venture capitalist receives higher compensation with venture capital financing. The result of the analysis is that with both financing schemes, the investors have to obtain an expected payoff equal to their capital investment. They demand a larger share of the returns due to unverifiability in the case of angel financing, yet they are assured of increased productivity of the project from verifying the compensation budget in the case of venture capital financing. From the entrepreneur's perspective, an increase in the investors' compensation is a pure cost: it decreases her residual value with no effect on the project's productivity. Although an increase in the venture capitalist's compensation decreases the entrepreneur's residual value as well, there is an extra positive effect: the venture capitalist is incentivized to exert higher effort which increases the productivity of the project. This increased productivity can offset some cost from a lower residual which makes the entrepreneur in favor of venture capital financing. As for the advisor, a higher return to the investors decreases the entrepreneur's residual value which lowers her motivation to exert effort. This decreases the project's productivity which further decreases the advisor's expected payoff. On the other hand, a higher compensation to the advisor can completely offset the negative effect from lower productivity. The advisor, accordingly, also prefers to become a venture capitalist, instead of a consultant, so that he will receive a higher expected payoff. There is no conflict of interest between the entrepreneur and the advisor in terms of the financing choice.

A surprising result is that there is no rent extraction by the venture capitalist when the entrepreneur delegates contracting with the investors to the venture capitalist. In other words, venture capital financing recovers the second-best contracts (optimal verifiable contracts). The entrepreneur can essentially keep the optimal second-best amount of residual and distribute all the rest to the venture capitalist, and the venture capitalist will divide it between himself and the investors in a way that maximizes the entrepreneur's expected payoff in a second-best way. This result is again due to the investors' participation constraint and the fact that they can verify their investment in the business venture. The intuition for the result is that although both a higher return to the investors and a higher compensation to the venture capitalist can increase the investors' expected payoff, for the same amount of increase, the investors prefer a higher return. If the venture capitalist keeps more budget to himself, then the investors will decline this offer due to an insufficient return. If the venture capitalist distributes more to the investors, then this increases the investors' expected payoff at the price of distorting his own. As a result, the venture capitalist will stick to the entrepreneur's optimal distribution of the budget.

Last, we offer a rationale for the use of convertible securities (e.g. convertible preferred equity, convertible debt, etc.). With either financing choice, the investors' optimal contract follows a standard debt form, while the advisor's optimal contract follows a standard equity form. The intuition is that the investors, as the capital provider, maximize the entrepreneur's expected payoff in the most efficient way with debt compensation (Innes (1990)). The advisor, as an effort provider, aligns his personal interest to the entrepreneur's with equity compensation (Jensen and Meckling (1976)). Ideally, the entrepreneur would like to issue different securities. However, the entrepreneur's preference for venture capital financing prevents her from writing a separate contract with the investors. The use of convertible preferred equity (or convertible debt), with a carefully designed cumulative dividend (resp. deferred coupon) pledged and conversion ratio, can essentially assist the entrepreneur in compensating the investors and venture capitalist with different securities. When the performance of the common stocks is bad, the venture capitalist will not convert into common shares. The convertible securities act as debt and all the cumulative dividend or deferred coupon pledged will be distributed to the investors. When the common stocks' performance is good, the venture capitalist optimally chooses to convert his securities into common stocks and shares them with the investors. The venture capitalist gets paid only when the firm's equity performance is good enough which fundamentally depends on the productivity of the project. As a result, the venture capitalist would like to exert more effort to improve the value of the common stocks and this motivation aligns his interest with the entrepreneur. The investors, on the other hand, only provide capital. To incentivize both the entrepreneur and the venture capitalist to improve the expected value of the project, the investors demand all the *liquidity preference*. The contract between the venture capitalist and the investors describes the investors' liquidity preference and the venture capitalist's *carried interest*: the share of profits that will be distributed to the venture capitalist once the investors receive their entire capital investment. By issuing convertible securities, the venture capitalist gets equity-like securities while the investors get risky debt-like securities, and the entrepreneur maximizes her expected payoff accordingly.

Related Literature

This paper contributes to the existing literature in the following strands. First, although Sahlman (1990) and Gompers and Lerner (1999b) have pointed out a complete venture capital contractual relations includes (1) contracting between the investors and the venture capital firm, and (2) contracting between the venture capitalist and the entrepreneurial venture, existing literature tends to focus on analyzing only one of them, without considering both of them altogether. What the scholars doing are either making the venture capitalists solely responsible for the funds' performance (e.g., Gompers and Lerner (1999a); Axelson, Strömberg, and Weisbach (2009); Robinson and Sensoy (2013)), or approximating the venture capitalists as the investors of theory (e.g., Kaplan and Strömberg (2001); Kaplan and Strömberg (2003)), and investigating the contracts between the venture capitalists (GPs) and the LPs or the contracts between the venture capitalists and the entrepreneurs, respectively. To our knowledge, our paper is the first one considering a complete venture capital entities: the entrepreneur, the venture capitalist, and the LPs, and especially the delegated financing nature of the venture capital in the sense that the entrepreneur funds her project using the capital provided by the venture capitalist which is actually owned by the LPs. We explicitly take into account that the venture capitalist's incentive is not perfectly aligned with the entrepreneur's and he also needs to protect the LPs' interest.

Second, this paper is also related to the strand of literature on the positive roles of venture capitalists as intermediaries. Similar to the traditional financial intermediation theory (Diamond (1984) and Holmstrom and Tirole (1997)), one category of literature focuses on the venture capitalists' information-based role, such as the mitigation of moral hazard or adverse selection. Chan (1983) identifies the venture capitalists as informed agents who increase the investors' payoffs by inducing the entrepreneurs to offer high return projects. Another category of literature deals with the venture capitalist's active involvement in the portfolio companies in addition to providing capital. Gorman and Sahlman (1989) presents that the venture capitalist helps the entrepreneur raise additional funds and Hellmann and Puri (2002) shows the venture capitalist's assistance in the professionalization of the business venture. In contrast to the above literature, our paper rationalizes the role of a venture capitalist as endorsing the entrepreneur to offer the second-best contracts in a credible way. Making the venture capitalist as the intermediary between the entrepreneur and the LPs reduces the verifiability considerations of the contracts.

Third, motivated by the empirical evidence that both entrepreneurs and venture capitalists provide substantial efforts on implementing projects, our paper also contributes the theoretical literature on the double-sided moral hazard (Bhattacharyya and Lafontaine (1995)) in the sense that both have to be incentivized to exert costly effort. Casamatta (2003) and Casamatta (2010) consider an additive specification of simultaneous efforts on the project's productivity and conclude the joint advent of advising and financing by the venture capitalist. In a similar setting of Renucci (2006), he shows that the venture capitalist will be hired only if the moral hazard from the entrepreneur is limited enough and there is sufficient residual incentive to induce effort from the venture capitalist. Repullo and Suarez (2004) considers the joint realization of simultaneous efforts from the entrepreneur and the venture capitalist in a sequential investment setting and study the security design problem accordingly. Inderst and Müller (2004) considers the various degree of complementarity between the venture capitalist's and entrepreneur's efforts in an equilibrium framework and analyzes the effect of different capital market characteristics on the firm's value. Dessi (2005) studies the venture capitalist's monitoring role in reducing the entrepreneur's opportunistic behavior. Schmidt (2003) considers sequential actions setting where the entrepreneur exerts effort before the venture capitalist and how convertible securities induce both to invest efficiently into the project. Hori and Osano (2013) explore a continuous-time framework (based on DeMarzo and Sannikov (2006)) where the venture capitalist supplies effort and also chooses the optimal timing of IPO. Our paper also recognizes the indispensable but unverifiable actions chosen by the entrepreneur and the venture capitalist, and we consider a production technology which exhibits both complementarity of efforts and necessity of their joint realization, which is in line with Buffa, Liu, and White (2019). As the entrepreneur and the venture capitalist produce different tasks for the business venture, it is suitable to assume their efforts are complementary in the sense that one's effort is more productive if the other exerts higher effort. Since the main focus of our paper is to investigate the financing problem faced by the entrepreneur, we limit our attention to the necessity of their joint realizations of effort, emphasizing the team aspect of output. We examine the optimal financing scheme for the entrepreneur when she must (1) work on her project, (2) hire an advisor to work with, and (3) raise capital from investors to implement the project. We separate the role of advising and financing into two people which distinguishes us from the existing literature.

Fourth, our paper is also related to the literature offering various rationales for the pervasive use of convertible securities in venture capital financing, documented by Kaplan and Strömberg (2003). In general, there are three categories. The first category deals with moral hazard or adverse selection problems from entrepreneurs only. For example, Gompers (1993) and Gompers (1997) show that the equity feature of the convertible debt limits the entrepreneur's incentive to take risks, and the debt feature helps the investors to select those entrepreneurs with high ability. Cornelli and Yosha (2003) and Trester (1998) show that convertibles can be used as a way to prevent the entrepreneur from manipulating performance and shifting risk to the investors. The second category deals with moral hazard or adverse selection problems from venture capitalists only. For instance, Marx (1998) shows a mixture of debt and equity, where the equity is a convertible, enables an optimal level of intervention compared with pure debt or pure equity. The third category considers the double-sided moral hazard framework, such as Casamatta (2003), Casamatta (2010), Dessi (2005), Hellmann (2006), Inderst and Müller (2004), Repullo and Suarez (2004), and Schmidt (2003). All of these models recognize that both entrepreneurs and venture capitalists should be allocated some residual cash flows and offer different explanations regarding the liquidation cash flow rights that come with convertibles. This paper contributes to this category and provides another rationale for the use of convertible securities. Entrepreneurs and venture capitalists are both effort providers, while investors are capital providers. This implies that entrepreneurs when designing securities have to consider the different incentives from venture capitalists and investors. However, venture capital financing prevents entrepreneurs from contracting directly with investors. Our analysis shows that with a careful designed convertible preferred equity or convertible debt can essentially assist entrepreneurs in compensating venture capitalists with common equity-like securities and investors with risky debt-like securities.

Fifth, our paper contributes to the literature regarding optimal financing choice for new ventures, especially the comparison between angel financing and venture capital financing. Although venture capital and private equity have been extensively studied in the existing literature, angel financing, as one of the most common methods to fund new ventures, is least studied both empirically and theoretically (e.g. Wong, Bhatia, and Freeman (2009); Wilson (2011)). Literature differentiates angel financing from venture capital financing in the sense that venture capitalists provide not only capital but also effort while angels provide only capital. Renucci (2006) and Casamatta (2010) show that entrepreneurs choose angel financing for inefficient pure effort providing advisors and Casamatta (2003) finds venture capital financing is optimal when inefficient advisors also provide capital. Adding one assumption that venture capital financing is scarce relative to angel financing, Chemmanur and Chen (2014) characterize the optimal financing path: depending on firm characteristics, a firm may use angel financing in ite early stages and switch to venture capital financing in later stages, or vice versa. Hellmann and Thiele (2015), on the other hand, consider a special case where angel financing and venture capital financing are complementary: angels have limited funds which requires venture capital to provide follow-on funding for their companies. However, using a unique tax credit program data from British Columbia, Canada, Hellmann, Schure, and Vo (2019) actually show that angel financing and venture capital financing are dynamic substitutes and the substitutes relationship is indeed company-led. Van Osnabrugge (2000) empirically finds that when dealing with agency problems, angel investors and venture capitalists use different approaches. Angel investors recognize the incomplete nature of contracts and focus on *ex post* allocation of control (Hart and Moore (1990), Aghion and Bolton (1992), Hart (1995)). Venture capitalists, on the other hand, use the classical principal-agent approach and focus on ex ante screening and contract writing (Jensen and Meckling (1976), Holmstrom (1979), Innes (1990)). Our paper acknowledges his findings by recognizing that the verifiability of contracts is different between angel financing and venture capital financing. In the case of angel financing, entrepreneurs are unable to write angel investors' contracts contingent on advisors' contracts, and vice versa. Unveriable contracts are also one type of incomplete contracts. In the case of venture capital financing, venture capitalists form limited partnerships with investors and contracts become verifiable accordingly. This paper contributes to this strand of literature by expliciting solving and comparing the optimal contracts in both financing schemes.

Finally, our paper also connects to the literature on organizational structure where agents have to work in a team. Alchian and Demsetz (1972) argues that moral hazard from agents necessitates their supervision and moral hazard from the monitor can be eliminated by making him a residual claimant. Similarly, Holmstrom (1982) proposes an even simpler way, a "budget-breaking" scheme, that agents, all contracting with a principal, will not get paid if their output falls below the optimal level. Eswaran and Kotwal (1984) points out the problem of moral hazard in team that the monitor or principal can offer a secret contract to one of the agents which increases their joint payoffs at the expense of lowering others'. Hart and Tirole (1990) and McAfee and Schwartz (1994) formally introduce the notion of beliefs when contracts are unobservable (hence unverifiable) and Grossman and Hart (1986) recognize the incomplete nature of unverifiable contracts. They suggest a vertical integration between two parties as a solution to restore the higher profit. The most recent work by Buffa, Liu, and White (2019) formally solves the problem of moral hazard in teams and offers a trade-off between direct contracting and delegated contracting. In our paper, angel financing resembles direct contracting as entrepreneurs contract directly with advisors and investors, and venture capital financing resembles delegated contracting as entrepreneurs contract only with venture capitalists and venture capitalists contract with investors. Our paper contributes to this string of literature as an application to financial services.

2 The Model

We consider a one period economy. Ex-ante, an entrepreneur is endowed with a profitable but risky project which requires another two players, investors and an advisor, to implement. The investors are needed to fund the initial capital investment, denoted by K. The advisor exerts expert effort, denoted by a, which combined with the entrepreneur's innovative effort, denoted by e, affects the project's expected cash flow, which is realized ex-post.

Technology. We denote the cash flow of the risky project by X, and for simplicity, we assume the project either succeeds or fails. The realized cash flow is X_L in case of failure, and there is an additional amount of output x in case of success. If K is invested, the probability of high cash flow, denoted by π, is determined by the efforts exerted by the entrepreneur and the advisor:

$$X(e,a) = \begin{cases} X_L + x & \text{with prob. } \pi(e,a) \\ X_L & \text{with prob. } 1-\pi(e,a). \end{cases} \tag{1}$$

If K is not invested, π is equal to 0. The riskiness of the project implies that

$$X_L + x > K > X_L. \tag{2}$$

Effort. We assume efforts are *unobservable* (and hence unverifiable). For the sake of tractability, we model the probability of success π as the following:

$$\pi(e,a) = \min\{\varepsilon ea, 1\}, \tag{3}$$

where e and a are continuous in $[0,\infty)$ and $\varepsilon > 0$. This specification implies that efforts from the entrepreneur and the advisor are complementary, $\partial^2\pi/\partial e\partial a = \varepsilon > 0$: one's effort is more productive if the other exerts higher effort. ε measures the degree of complementarity. Moreover, $\pi(0,a) = \pi(e,0) = 0$ implies that both efforts are necessary for the project to succeed, emphasizing the team aspect of output.[4]

Finally, we assume that the entrepreneur's and the advisor's disutility cost of effort functions are given by

$$\beta\frac{e^m}{m}, \tag{4}$$

[4]Our production technology is in line with Buffa, Liu, and White (2019). In their setting, π follows a Cobb-Douglas functional form which also exhibits complementarity of efforts and necessity of their joint realization.

and

$$\gamma \frac{a^n}{n}, \tag{5}$$

where $\beta, \gamma > 0$ and $m, n > 2$, both of which are strictly increasing and convex in effort. Here, a larger m (resp. n) makes exerting innovative effort (resp. expert effort) more costly.[5]

The social value of investing in the project is

$$S(e, a) = \min\{\varepsilon ea, 1\}(X_L + x) + \max\{1 - \varepsilon ea, 0\}X_L - \beta \frac{e^m}{m} - \gamma \frac{a^n}{n} - K. \tag{6}$$

Note that if efforts are verifiable, then from the view of the social planner, the optimal levels of effort are given by the first-order condition of the maximization of S:

$$e^{FB} = \frac{\varepsilon^{\frac{n}{mn-m-n}}}{\beta^{\frac{n-1}{mn-m-n}} \gamma^{\frac{1}{mn-m-n}}} x^{\frac{n}{mn-m-n}}, \tag{7}$$

and

$$a^{FB} = \frac{\varepsilon^{\frac{m}{mn-m-n}}}{\beta^{\frac{1}{mn-m-n}} \gamma^{\frac{m-1}{mn-m-n}}} x^{\frac{m}{mn-m-n}}, \tag{8}$$

which are under the assumption that

$$\frac{\varepsilon^{\frac{mn}{mn-m-n}}}{\beta^{\frac{n}{mn-m-n}} \gamma^{\frac{m}{mn-m-n}}} x^{\frac{m+n}{mn-m-n}} < 1, \tag{9}$$

so the constraint $\min\{\varepsilon ea, 1\} \leq 1$ is not binding at the first-best. We implement assumption (9) throughout this paper. To avoid credit rationing, we require $S(e^{FB}, a^{FB}) \geq 0$, that is

$$x \geq x^{FB}. \tag{10}$$[6]

Financing. Although the entrepreneur needs another two players to implement the project, she can choose to finance the project from the investors directly and hire an advisor independently, or instead to work with a "team" of two members, one offering capital and one exerting effort. We

[5]Since we assume π to be affine in effort, the disutility of effort function has to be more convex than the commonly used quadratic form, and $m, n > 2$ is simply one sufficient condition.

[6]The formal derivation, including the value of x^{FB}, is in Appendix B.1

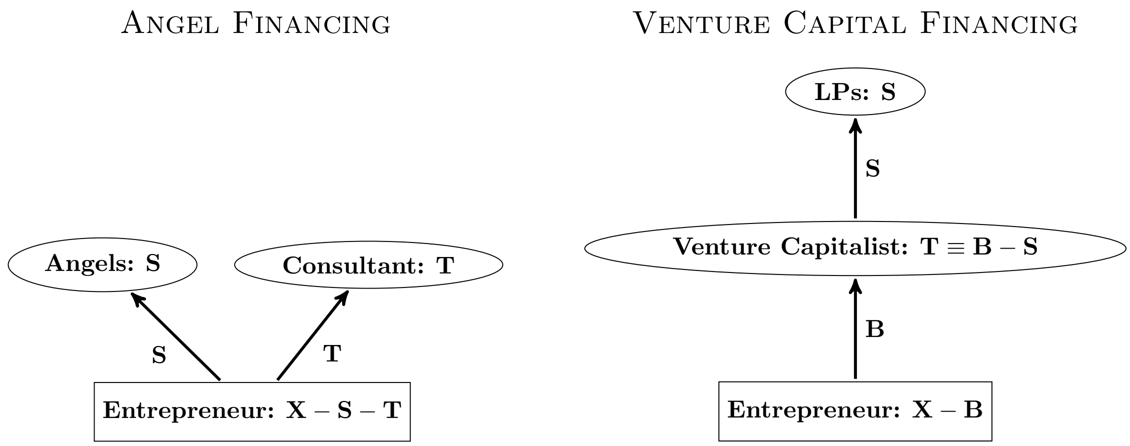

Figure 2: Financing Schemes and Compensation Structures

refer to the two financing schemes as *angel financing* and *venture capital financing*, respectively. In the angel financing scheme, the investors will be called angels and the advisor will be called a consultant. In the venture capital financing scheme, we call the investors who provide capital K LPs, which stands for limited partners, and the advisor who exerts effort a a venture capitalist. The investors form a limited partnership with the venture capitalist. Figure 2 illustrates the structure of the two financing schemes.

Contracts. Although both financing schemes consist of two bilateral contracts, the signatories of each are not the same. It is the entrepreneur who contracts with the other two through angel financing, while through venture capital financing, it is the venture capitalist. No contract is committed either between the angels and the consultant or between the entrepreneur and the LPs.

Moreover, the verifiability of the bilateral contract also differs in the two schemes. Regarding angel financing, we assume each contract is verifiable only by the two parties signing it. This means that neither the angels nor the consultant can verify the contract which the entrepreneur offers to the other. One can think of the consultant as an employee hired by the entrepreneur. A typical employment contract is not contingent on how the entrepreneur actually raises the capital. Similarly, the investors fund a project without specifying how much the entrepreneur should compensate her employee. In most cases, these contracts are private information to the signatories. Regarding venture capital financing, we assume that the entrepreneur cannot verify the contract between the venture capitalist and the LPs. It is very unlikely that the entrepreneur is completely informed of the true funding details of the specific fund which provides the initial capital investment. As for the LPs, according to the limited partnership agreement, they are not entitled to any firm-specific information. However, the venture capitalist, as a general partner, periodically reports some high-level descriptions regarding the portfolio companies, including but not limited to the shares owned. The entrepreneur normally receives capital from the venture capitalist by ceding a portion of ownership. Therefore, in our setting, the LPs can eventually verify the contract between the other two.

This implies that any enforceable contract has to be based on what is verifiable by all three players, which is the realization of the project's cash flow $X(e, a)$. We assume that all players are risk-neutral with limited liability. This means that the only cash flows that can be shared are no more than the outcome of the project. We denote by S (resp. T) as the amount of the cash flow accruing to the investors (resp. the advisor). $S = S_L$ (resp. $T = T_L$) in the case of project failure, and $S = S_L + s$ (resp. $T = T_L + t$) in the case of success. The entrepreneur collects the residuals in both states.

Payoffs. We assume competitive capital and labor markets: all capital (resp. labor) providers obtain expected payoffs equal to their opportunity cost of investment (resp. outside options), and the surplus is allocated to the entrepreneur. The intuition is that there are many wealthy investors and skillful laborers looking for good investment opportunities and only a few entrepreneurs with good projects. The entrepreneur has all the bargaining power and can make a take-it-or-leave-it offer to them. For the sake of simplicity, we normalize the risk-free interest rate and the advisor's reservation utility to zero. [7]

The expected payoff of the entrepreneur, denoted by E, is given by her expected residual claim minus her cost of effort,

$$E = \varepsilon ea(X_L - S_L - T_L + x - s - t) + (1 - \varepsilon ea)(X_L - S_L - T_L) - \beta\frac{e^m}{m}, \tag{11}$$

the expected payoff of the advisor, denoted by A, is given by her expected compensation minus his cost of effort,

$$A = \varepsilon ea(T_L + t) + (1 - \varepsilon ea)T_L - \gamma\frac{a^n}{n}, \tag{12}$$

[7] Ewens, Gorbenko, and Korteweg (2019) consider a bargaining game between heterogeneous qualities of entrepreneurs and VC investors, including certain contract terms and distribution of profit.

and the expected payoff of the investors, denoted by I, is given by their expected compensation

$$I = \varepsilon ea(S_L + s) + (1 - \varepsilon ea)S_L. \tag{13}$$

When both contracts are verifiable (and hence can be made contingent on each other) by all three players, the entrepreneur is indifferent between angel financing and venture capital financing. We will see later that this contrasts sharply with the conclusions once the verfiability of contract is an issue. Before studying the optimal choice that the entrepreneur makes at date 0 between the two financing schemes, we first characterize the optimal contracts in two schemes separately, and discuss the fundamental role played by the verifiability of these contracts.

3 Angel Financing

Optimal Contracting. In this section, we discuss the angel financing scheme in which the entrepreneur contracts with the angels and the consultant independently. We model the interactions among the three players as a noncooperative game. The timing of the game is as follows:

(i) The entrepreneur simultaneously offers two take-it-or-leave-it compensation contracts. Both the angels and the consultant can only verify their own offer.

(ii) Once the contracts are signed, the angels invest capital K, and both the entrepreneur and the consultant exert their levels of effort. Efforts are unobservable.

(iii) The outcome of the project is realized. Acccording to the realized state, the entrepeneur uses the project's cash flow to pay the angels and the consultant the compensation specified in the contracts accepted and collects the residual.

We solve for the optimal contracts by working backward. First, for any contract T committed to hire the consultant in advance, we derive the consultant's optimal effort choice a by maximizing his expected utility A given his *beliefs* about the contract S offered to the angels. Similarly, for any contract S, we derive the angels' expected payoff I given their *beliefs* about T. Second, we derive the entrepreneur's optimal effort level e which maximizes her expected payoff E by playing a best response to the contracts S and T agreed before as well as the consultant's optimal effort choice a determined in the first step. Third, given the effort levels e and a that determined, we derive the optimal contract S and T which maximizes the entrepreneur's expected payoff E

subject to delivering an expected payoff I, determined in the first step, to the angels no less than their capital investment K.

Notice that we use the word "beliefs" here to describe the unverifiable contract not signed by the player. This means that the consultant's optimal effort choice and the angels' decision to invest capital into the project cannot be made contingent on the unverifiable contract. This creates a role for beliefs about *unverifiable contracts* that has been missing from the literature so far but is central to our analysis.

When both contracts are verifiable by all three players, this noncooperative game has a proper subgame. As a best response to the two verifiable contracts, the optimal effort choice from the consultant will be a function of S and T, whereby he can derive his own effort and calculate the entrepreneur's equilibrium effort choice accordingly. As for the angels, they can also infer the entrepreneur's and the consultant's effort choices based on the verifiable contracts so that they can determine whether or not to invest into the project. Therefore, the entrepreneur's maximizing residual cash flow problem must be employed in a subgame perfect Nash equilibrium.[8]

Now suppose that the verifiability of the contracts is dropped, the entrepreneur tries to offer the optimal verifiable contracts and promises the angels and the consultant that she will also stick to offer the optimal verifiable contract to the other. Would the consultant exert the same level of effort as he would if the contracts are verifiable? Would the angels agree to provide capital into the project?

The answer is No to both questions. To see why, suppose that the consultant committed to exert the same level of effort as under the verifiable contracts, then it would be optimal for the entrepreneur to "collude" with the angels by deviating to a better contract which economizes on the incentive payments. The entrepreneur, in this way, could be made better off without violating the angels' participation constraint. The consultant is made worse off as his effort choice is not a best response to the deviating contract. Or, to put it in a different way, when the consultant puts in more effort, he creates a positive externality effect on both the entrepreneur and the angels, and the contract offered to the angels internalizes this externality on the entrepreneur but not on the consultant. The entrepreneur could conduct a similar "trick" to increase her expected payoff by deviating the consultant's contract through sabotaging the angels' expected payoff. Anticipating the "opportunistic" behavior of the other pair, the consultant demands more compensation to exert a certain effort than he would with verifiable contracts, and the angels also require a higher return to provide the capital. With verifiable contracts, the consultant could

[8]These are the second-best contracts in that the angels and the consultant can perfectly verify each others' contracts although efforts are still not verifiable. The formal derivation is in Appendix B.2.

confidently expect higher effort from the entrepreneur, which make the productivity of their own effort larger due to complementary effect, and so makes higher effort more worthwhile for the same level of compensation. Knowing that the project has a larger chance of reaching the higher cash flow, the angels are also happy to receive a lower compensation to compromise. However, with unverifiable contracts, incentive provision becomes more expensive.

As a result, this noncooperative game no longer has a proper subgame. Although the angels' (resp. the consultant's) expected payoff, in equilibrium, depends on the contract the entrepreneur offered to the other, whether to invest in the project (resp. how much effort to exert) cannot be conditioned on the unverifiable contract. Subgame perfection cannot be employed in this game. Of course, as there are many ways to form beliefs about the unverifiable contract, there must be many Nash equilibria. However, most of them are noncredible. For instance, the beliefs that the entrepreneur will stick to offering the verifiable contracts is not credible.

Therefore, as a refinement of Nash equilibrium, throughout, we consider only *perfect Bayesian Nash equilibrium* (PBE) in which consistency of beliefs with Bayes Rule is required both on and off the equilibrium path.[9] Notice, however, that Bayes Rule does not pin down beliefs after probability zero events, and so in principle different systems of out-of-equilibrium beliefs after such events exist and these could be used to support different PBEs, so the game could have multiple PBEs. In this game, a system of beliefs characterizes how the angels and the consultant revise their beliefs about the unverifiable contract *when receiving an out-of-equilibrium offer from the entrepreneur*. The willingness to accept an "unexpected" offer depends on what they think about how, if at all, the entrepreneur might change the contract she privately offered to the other. In this paper, we consider a PBE which is supported by so-called *passive beliefs*.[10]

When the consultant, with passive beliefs, receives an out-of-equilibrium offer from the entrepreneur, he does not revise his beliefs about the unverifiable offer made to the angels. In other words, the consultant believes that the entrepreneur sticks to offer the equilibrium contract to the angels, no matter what contract he receives from her. Therefore, his conjectures about the

[9] To be more precise, it should be weak perfect Bayesian Nash equilibrium (weak PBE). We stick to the perfect Bayesian Nash equilibrium (PBE), because first there is no subgame with private contracts, weak PBE is equivalent to PBE in this game. Second, PBE is what industrial organization literature uses, we use PBE here as well serving as a good reference to the existing literature.

[10] McAfee and Schwartz (1994) propose three possible beliefs to consider: symmetric beliefs, passive beliefs, and wary beliefs. Passive beliefs, originally introduced by Hart and Tirole (1990), is widely used in the industrial organization literature, including O'Brien and Shaffer (1992), McAfee and Schwartz (1994), Rey and Vergé (2004) etc. We only focus on passive beliefs because symmetric beliefs require homogeneous agents but we have heterogeneous players, the angels and the consultant, in our setting, and the equilibrium outcome with wary beliefs is the same as with passive beliefs. To be more precise, Rey and Vergé (2004) point out that wary beliefs are equivalent to passive beliefs only when there exists a perfect Bayesian equilibrium with passive beliefs. However, we do not discuss nonexistence problems in this paper.

angels' contract, denoted by $\hat{S}$, remains the equilibrium one which he expects the entrepreneur to offer. The consultant chooses his optimal effort level, denoted by $a(\hat{S}, T)$, and calculates the entrepreneur's effort level, denoted by $e(\hat{S}, T)$, using his contract T and the conjected contract $\hat{S}$. Similarly for the angels, they form passive beliefs about the unverfiable contract offered to the consultant, denoted by $\hat{T}$. Using S and $\hat{T}$, the angels can calculate the entrepreneur's effort level, denoted by $e(S, \hat{T})$, and the consultant's effort level, denoted by $a(S, \hat{T})$, and conclude the project's probability of success. Then, they determine their expected payoff and decide whether to invest K into the project.[11]

Having defined our notion of equilibrium and the system of off-equilibrium beliefs, we next discuss the optimization problems of the three players. We start with the consultant's problem in stage two. After receiving T from the entrepreneur, she forms beliefs about the contract offered to the angels, $\hat{S}$. Therefore, the optimal level of effort chosen by the consultant, $a(\hat{S}, T)$, is determined in the system of equations below:

$$\begin{cases} e(\hat{S}, T) = \arg\max\limits_{e} \ \varepsilon e a(\hat{S}, T)(X_L - \hat{S}_L - T_L + x - \hat{s} - t) + (1 - \varepsilon e a(\hat{S}, T))(X_L - \hat{S}_L - T_L) - \beta \frac{e^m}{m}, \\ a(\hat{S}, T) = \arg\max\limits_{a} \ \varepsilon e(\hat{S}, T) a(T_L + t) + (1 - \varepsilon e(\hat{S}, T) a) T_L - \gamma \frac{a^n}{n}, \end{cases} \tag{14}$$

and we get

$$a(\hat{S}, T) = \frac{\varepsilon^{\frac{m}{mn-m-n}} (x - \hat{s} - t)^{\frac{1}{mn-m-n}} t^{\frac{m-1}{mn-m-n}}}{\beta^{\frac{1}{mn-m-n}} \gamma^{\frac{m-1}{mn-m-n}}}. \tag{15}$$

The consultant's effort choice does not depend on T_L, which is how much he receives when the project fails. In order to induce effort from the consultant, the entrepreneur has to compensate him more when the project succeeds, which requires that

$$t > 0. \tag{16}$$

The intuition is the same as in the standard principal-agent problems under moral hazard: the agent would like to exert higher effort for a higher compensation. In our two-state setting, the higher the compensation in the high state of the world, the higher the consultant's desire to increase the likelihood of that state, and a lower state compensation, in fact, reduces effort incentive. This is the direct effect of t on incentive provision. However, due to our limited liability assumption, a higher t also reduces the entrepreneur's residual value, $x - s - t$, in the high state, which further decreases her desire to exert effort. The complementarity between efforts implies that

[11] Buffa, Liu, and White (2019) also discuss unverifiable contracts in their setting. Instead of forming beliefs on contracts, they form beliefs on the unverifiable effort directly.

the consultant's willingness to exert effort decreases with the entrepreneur's effort. This is the indrect effect of t. The entrepreneur has to tradeoff the increased direct effect against the decreased indirect effect for an increased value of t. Moreover, the consultant exerts lower effort if he believes that the entrepreneur compensates the angels more in the high state, $\hat{s}$. This is again due to the complementary effect of efforts as a higher beliefs of $\hat{s}$ lowers the entrepreneur's effort which further reduces the consultant's desire to exert effort.

Next, we consider the angels' problem in stage two after receiving S from the entrepreneur. The angels must form beliefs about the contract offered to the consultant, $\hat{T}$. Although the angels do not exert effort, they need to calculate their expected payoff so as to determine whether to invest K into the project. To be more precise, they need to derive the entrepreneur's and the consultant's effort levels, $e(S,\hat{T})$ and $a(S,\hat{T})$, in order to determine their expected payoff, denoted by $I(S,\hat{T})$. Therefore, the optimal effort levels are given by the system of equations below:

$$\begin{cases} e(S,\hat{T}) = \arg\max_{e} \ \varepsilon e a(S,\hat{T})(X_L - S_L - \hat{T}_L + x - s - \hat{t}) + (1 - \varepsilon e a(S,\hat{T}))(X_L - S_L - \hat{T}_L) - \beta \frac{e^m}{m}, \\ a(S,\hat{T}) = \arg\max_{a} \ \varepsilon e(S,\hat{T}) a(\hat{T}_L + \hat{t}) + (1 - \varepsilon e(S,\hat{T}) a)\hat{T}_L - \gamma \frac{a^n}{n}. \end{cases} \tag{17}$$

The associated value for the investors' expected payoff is

$$\begin{aligned} I(S,\hat{T}) &= \varepsilon e(S,\hat{T}) a(S,\hat{T})(S_L + s) + (1 - \varepsilon e(S,\hat{T}) a(S,\hat{T}))S_L, \\ &= \frac{\varepsilon^{\frac{mn}{mn-m-n}} (x - s - \hat{t})^{\frac{n}{mn-m-n}} \hat{t}^{\frac{m}{mn-m-n}} s}{\beta^{\frac{n}{mn-m-n}} \gamma^{\frac{m}{mn-m-n}}} + S_L. \end{aligned} \tag{18}$$

The angels' expected payoff increases with their lower state compensation, S_L. We know from our previous analysis that efforts only depend on how much *more* the entrepreneur and the consultant get in the higher state. This implies that how much the effort providers receive in the lower state has no effect on the project's probability of success. In other words, the entrepreneur can increase the angels' expected payoff without changing the incentive provision of efforts.

We also restrict our analysis to the case where the investors get a *monotonic contract*, that is the investors' compensation is constrained to be monotonically nondecreasing in the project's realized cash flow. Here, it simply means that

$$s \geq 0. \tag{19}$$

This assumption is used to prevent the entrepreneur from secretly diverting cash flows.[12] A nonnegative s also has two offsetting effects on the angels' expected payoff. A higher s increases what they actually get when the project is a success but also decreases the probability of reaching the higher state. The entrepreneur has to balance these two effects when offering s to the angels. The angels' beliefs $\hat{t}$ also have two opposite effects. On one hand, a higher beliefs $\hat{t}$ increases the direct incentive effect on the consultant's effort choice while it decreases the direct incentive effect on the entrepreneur's effort level. This is again due to the complementarity of efforts.

Then, we consider the entrepreneur's optimal choice in stage two after offering S and T. Being a signatory to both of the contracts, the entrepreneur does not need to form beliefs about the contracts. At the same time, she also knows that the consultant's optimal effort choice is derived based on his contract T and his beliefs about contract $\hat{S}$. As a result, the entrepreneur plays a best response to those beliefs by exerting an optimal effort level which is a function of S, T, and $\hat{S}$, denoted by $e(\hat{S}, S, T)$:

$$\begin{aligned} e(\hat{S}, S, T) &= \arg\max_{e} \ \varepsilon e a(\hat{S}, T)(X_L - S_L - T_L + x - s - t) + (1 - \varepsilon e a(\hat{S}, T))(X_L - S_L - T_L) - \beta \frac{e^m}{m}, \\ &= \frac{\varepsilon^{\frac{n}{mn-m-n}} (x - \hat{s} - t)^{\frac{1}{(m-1)(mn-m-n)}} (x - s - t)^{\frac{1}{m-1}} t^{\frac{1}{mn-m-n}}}{\beta^{\frac{n-1}{mn-m-n}} \gamma^{\frac{1}{mn-m-n}}}. \end{aligned} \tag{20}$$

Similar to the consultant's, the entrepreneur's optimal effort choice does not depend on S_L and T_L. Moreover, the entrepreneur has to save enough residual in the high state in order to exert effort, that is

$$x - s - t > 0. \tag{21}$$

She exerts higher effort if she compensates the angels less in the high state, s, or if the consultant believes that she compensates less, $\hat{s}$. How much the entrepreneur compensates the consultant, t, also has two offseting effects on her optimal effort choice, which is again due to the complementary effect between efforts.

Last, we move back to stage one. In this stage, the entrepreneur's optimization problem is to choose the angels' contract S and the consultant's contract T so as to maximize her expected residual cash flow from the project, denoted by $E(\hat{S}, \hat{T}, S, T)$, subject to the entrepreneur's and

[12] As argued by Innes (1990), even with verifiable output, the entrepreneur has the incentive to borrow risk freely and inject into the actual cash flow when the project turns out to be a failure. In this way, the entrepreneur's expected payoff increases as she compensates the investors less.

the consultant's incentive compatibility (IC), the three players' individual rationality (IR), the limited liability (LL), the angels' monotonic contract (MC), and positive effort (PE) constraints,

$$\begin{aligned}(S(\hat{S},\hat{T}),T(\hat{S},\hat{T})) &= \underset{S,\ T}{\arg\max}\ E(\hat{S},\hat{T},S,T),\\ &= \underset{S,\ T}{\arg\max}\ \Big(\varepsilon e(\hat{S},S,T)a(\hat{S},T)(X_L-S_L-T_L+x-s-t)\\ &\qquad + (1-\varepsilon e(\hat{S},S,T)a(\hat{S},T))(X_L-S_L-T_L)-\beta\frac{e(\hat{S},S,T)^m}{m}\Big). \end{aligned} \tag{22}$$

The IC constraint of the consultant is given by the optimal effort choice in (15) and that of the entrepreneur is in (20). The IR constraints of the entrepreneur and the consultant are always satisfied given their outside options of 0, and the IR constraint of the angels is that their expected payoff should be no smaller than the capital investment K,

$$I(S,\hat{T}) \geq K, \tag{23}$$

where their expected payoff $I(S,\hat{T})$ is defined in (18). Combining (LL), (MC), and (PE) constraints, we get

$$\begin{cases} s \geq 0, \\ t > 0, \\ x-s-t > 0, \\ S_L \geq 0, \\ T_L \geq 0, \\ X_L - S_L - T_L \geq 0. \end{cases} \tag{24}$$

The entrepreneur rationally takes into account, throught the IC and IR constraints, that the consultant and the angels each form passive beliefs about the unverifiable contract offered to the other. Therefore, the optimal level of contracts, S and T, are both a function of the beliefs, $\hat{S}$ and $\hat{T}$. In equilibrium, the consultant's (resp. the angels') conjecture about the contract offered to the angels $\hat{S}$ (resp. the consultant $\hat{T}$) must be correct and correspond to the equilibrium contract level, denoted by S^A (resp. T^A),

$$\hat{S} = S^A, \tag{25}$$

$$\hat{T} = T^A, \tag{26}$$

where the superscript A stands for angel financing. Imposing (25) and (26) after solving the optimization problem in (22) subject to (23) and (24), we obtain the optimal contracts in the angel financing scheme, which the following proposition characterizes.

Proposition 1. *With angel financing, there exists a threshold $x \geq x^A$ such that the optimal compensations for the angels and the consultant are respectively equal to*

$$S_L^A = X_L, \tag{27}$$

$$T_L^A = 0, \qquad t^A = \frac{x - s^A}{n}, \tag{28}$$

and s^A exists and is determined uniquely in

$$\frac{\varepsilon^{\frac{mn}{mn-m-n}} \left(x - s^A - \frac{x-s^A}{n}\right)^{\frac{n}{mn-m-n}} \left(\frac{x-s^A}{n}\right)^{\frac{m}{mn-m-n}} s^A}{\beta^{\frac{n}{mn-m-n}} \gamma^{\frac{m}{mn-m-n}}} + X_L - K = 0. \tag{29}$$

It follows that:

(i) The angels individual rationality constraint is binding, $I^A = K$;

(ii) when the project fails, the angels receive all the cash flow, $S_L^A = X_L$, while neither the consultant nor the entrepreneur receives anything, $T_L^A = X_L - S_L^A - T_L^A = 0$;

(iii) when the project succeeds, these three players share the extra cash flow x, where t^A decreases with n and s^A, and s^A decreases with t^A.

The optimal contracts obtained in Proposition 1 are based on the maintained assumptions of (9) and that contracts are not verifiable in the angel financing scheme. The IR constraint of the angels is obviously binding, because if it is not, the entrepreneur can always increase her expected payoff by decreasing the angels' compensation without affecting her and the consultant's incentives. Both the entrepreneur and the consultant receive nothing in the lower state, due to our monotonic contract assumption (19) and its "maximal high-profit-state payoff" property (Innes (1990)). As discussed in our analysis before, their incentives solely depend on how much more they would receive when the project succeeds, (21) and (16). The riskiness of the project, defined in (2), also limits the maximal amount of cash flow that can be distributed to the angels when the project fails. Combining these two factors, it is optimal to compensate the effort providers only in the higher cash flow state. In our setting, a larger n means that the consultant has a more convex cost function which implies that exerting effort becomes more costly for him. When

n increases, it means that the consultant is getting less productive, and the entrepreneur finds it better to decrease the project's probability of success with the benefit of an increased residual cash flow $x - s - t$ and an increase of compensation s. s and t being negatively correlated is due to the fact that the entrepreneur has to make sure the angels' expected payoff equal to K. As the angels collect $X_L < K$ in the both states, the expected value of how much extra they get in the higher state has to be strictly equal to $K - X_L$. A higher s increases their realized cash flow and has to be offset by a decreased probability of success which is a lower t, and vice versa.

In the next section, we first analyze the venture capital financing scheme, and then compare the optimal contracts between these two schemes.

4 Venture Capital Financing

Optimal Contracting. In this section, we discuss the venture capital financing scheme in which the entrepreneur contracts only with the venture capitalist, who has formed a limited partnership with the LPs to raise funds. As with angel financing, we model the interactions among the three players as a noncooperative game. The timing of the game is as follows:

(i) The venture capitalist forms a limited partnership with the LPs by offering a take-it-or-leave-it contract in exchange for a certain amount of committed capital.

(ii) The entrepreneur offers a take-it-or-leave-it contract to the venture capitalist, who has signed a limited partnership agreement with the LPs in stage one. The entrepreneur cannot verify the contract between the venture capitalist and the LPs.

(iii) Once the LPs and the venture capitalist have accepted contracts in stage one and stage two, respectively, capital K is invested, and both the entrepreneur and the venture capitalist exert their levels of effort. Efforts are unobservable.

(iv) The outcome of the project is realized. According to the realized state, the entrepreneur uses the project's cash flow and pay the venture capitalist the compensation specified in the contract accepted in stage two and collects the residual. After the LPs verify the contract between the entrepreneur and the venture capitalist, the venture capitalist uses what he receives from the entrepreneur to pay the LPs according to the contract signed in stage one and keeps the residual part.

A typical venture capital cycle begins with fund-raising, followed by investing and exiting stages. The venture capitalist raises funds from the LPs, by pooling their capital into a venture capital fund, and then invests the fund into a business venture, through contracting with the entrepreneur. Our timeline described above is also consistent with what happens in practice.

However, in our setting, since we assume competitive capital and labor markets, we can think of the problem as the entrepreneur offers a take-it-or-leave-it contract to the venture capitalist first and then the venture capitalist offers a take-it-or-leave-it contract to the LPs conditioning on his contract with the entrepreneur. The intuition is that the entrepreneur has all the bargaining power and she can always find a venture capitalist who would like to accept her offer. In practice, there are many venture capitalists who have already completed the fund-raising stage. Depending on time, the funding market, the relative bargaining power between the venture capitalists and LPs, and most importantly the venture capitalists' past performance, different venture capitalists can raise funds from different LPs under different contracts. The venture capitalist would like to accept the entrepreneur's offer if he knows that both his and the LPs' individual rationality constraints can be satisfied.[13] Therefore, we rewrite stage one and stage two of the game as the following:

(i)' The entrepreneur offers a take-it-or-leave-it contract to the venture capitalist, denoted by B, as an exchange of an investment capital K, in which the entrepreneur compensates the venture capitalist $B_L + b$ when the project succeeds and B_L when the project fails. The venture capitalist decides whether to accept the offer or not.

(ii)' After committing a contract in stage one, the venture capitalist makes a take-it-or-leave-it offer S to the LPs. $B - S$ is essentially his compensation T which is defined in Section 2. The LPs can verify the contract B and decide whether to accept the offer or not. The entrepreneur cannot verify S.

Similarly to the angel financing case, we solve the entrepreneur's venture capital financing problem by working backward. First, for any contracts B and S committed in advance, we derive the LPs' expected payoff I. Second, we derive the venture capitalist's optimal effort level a which maximizes his expected payoff A by playing a best response to the contracts B and S. Third, given the effort level a determined in the second step and B committed , we derive the optimal contract S which maximizes the venture capitalist's expected payoff I subject to delivering the LPs an

[13] Another way to rationalize our analysis is by assuming this is a simultaneous move game, as argued by Casamatta (2003), since efforts are unobservable (and unverifiable). For a simultaneous game, it is irrelevant to consider the ranking of the stages, such as which stage comes first.

expected payoff I determined in the first step no less than the capital investment K. Fourth, for any B, we derive the entrepreneur's optimal effort level e which maximizes her expected payoff E by playing a best response to the contract S determined in the third step. Last, given the effort levels e, a, and S determined, we derive the optimal contract B which maximizes the entrepreneur's expected payoff E subject to the venture capitalist's individual rationality constraint.

In the angel financing scheme that the entrepreneur contracts with the other two players, both of whom form beliefs about the contract offered to the other. By contrast, in the case of venture capital financing, it is the venture capitalist who contracts with the other two players. Obviously, the venture capitalist does not need to form beliefs since he is the signatory to both contracts. The LPs also do not need to form beliefs. Even though they do not contract directly with the entrepreneur, they can verify the contract signed between the entrepreneur and the venture capitalist. Then, the question is will the entrepreneur form beliefs about the contract between the venture capitalist and the LPs? Will the venture capitalist extract rent from the entrepreneur?[14]

The answer is No to both questions. Too see this, we begin with the LPs' problem in stage two. After receiving S from the venture capitalist, the LPs knows that the venture capitalist keeps $B-S$, since the contract B is verifiable to the LPs. Similar to the angels, although the LPs do not exert effort, they need to derive the entrepreneur's and the venture capitalist's effort levels, respectively denoted by $e(B,S)$ and $a(B,S)$, in order to determine their expected payoff, denoted by $I(B,S)$. Therefore, the optimal effort levels are determined in the system of equations below:

$$\begin{cases} e(B,S) = \arg\max\limits_{e} \ \ \varepsilon e a(B,S)(X_L - B_L + x - b) + (1-\varepsilon e a(B,S))(X_L - B_L) - \beta \frac{e^m}{m}, \\ a(B,S) = \arg\max\limits_{a} \ \ \varepsilon e(B,S) a(B_L - S_L + b - s) + (1-\varepsilon e(B,S) a)(B_L - S_L) - \gamma \frac{a^n}{n}. \end{cases} \tag{30}$$

The associated value for the LPs' expected payoff is

$$\begin{aligned} I(B,S) &= \varepsilon e(B,S) a(B,S)(S_L + s) + (1-\varepsilon e(B,S) a(B,S)) S_L, \\ &= \frac{\varepsilon^{\frac{mn}{mn-m-n}} (x-b)^{\frac{n}{mn-m-n}} b - s^{\frac{m}{mn-m-n}} s}{\beta^{\frac{n}{mn-m-n}} \gamma^{\frac{m}{mn-m-n}}} + S_L. \end{aligned} \tag{31}$$

We also restrict the LPs' contract to be monotonic, therefore,

$$s \geq 0. \tag{32}$$

[14]We define rent extraction here as the additional fraction of the compensation budget, $\frac{B^{VC}-S^{VC}}{B^{VC}}$, that the venture capitalist keeps for himself, compared to the second-best choice, $\frac{T^{SB}}{S^{SB}+T^{SB}}$, when the entrepreneur delegates contracting with the LPs to the venture capitalist. Please refer to Buffa, Liu, and White (2019) Section 4 for a further discussion.

Next, we consider the venture capitalist's problem in stage three. After receiving contract B from the entrepreneur in stage one and committing to pay the LPs S in stage two, the venture capitalist's optimal effort choice will be a best response to B and S, denoted by $a(B, S)$. Therefore, the optimal level of effort chosen by the venture capitalist is also determined in the system of equations (30), and we get

$$a(B, S) = \frac{\varepsilon^{\frac{m}{mn-m-n}}(x-b)^{\frac{1}{mn-m-n}}(b-s)^{\frac{m-1}{mn-m-n}}}{\beta^{\frac{1}{mn-m-n}}\gamma^{\frac{m-1}{mn-m-n}}}. \tag{33}$$

Similar to the angel financing, the venture capitalist's effort choice only depends on how much extra he will get when the project succeeds. In order to save enough residual cash flow to exert effort, he also needs

$$b - s > 0. \tag{34}$$

The venture capitalist exerts more effort if he distributes less to the LPs, a lower s, when the project is in higher state. b also has two effects on his effort choices: direct and indrect effect. A large b increases the venture capitalist compensation, $b - s$, which further increases his incentive to exert effort, while it decreases the entrepreneur's compensation, $s - b$, which lowers his effort incentive.

Next, we analyze the two optimal contracts. One is offered by the entrepreneur; the other is offered by the venture capitalist. Note that in the angel financing scheme, it is the entrepreneur who offers both two contracts S and T. Both contracts can be signed either simultaneously or sequentially; the timing of the two contracts does not matter. The entrepreneur can freely contract with either the angels or the consultant first or both at the same time. The reason for this is the contracts in the case of angel financing are unverifiable to them. In one words, the entrepreneur cannot offer one contract that is contingent on the other contract. As a result, the contract S and T must be determined jointly. However, in the venture capital financing scheme, a sequential commitment of contracts is necessary. In our setting, the entrepreneur must sign a contract B with the venture capitalist before him contracting with the LPs. We can think of B as the total compensation budget that the entrepreneur forfeit in order to implement the project, and the venture capitalist decides how to split the budget between him and the LPs. As a result, the LPs' contract S, offered by the venture capitalist, must be contingent on B. We donote the optimal contract which the venture capitalist offers to the LPs as a best response to B as $S(B)$.

We move back to stage two and analyze the venture capitalist's problem again. In this stage, his optimization problem is to choose the LPs' contract S so as to maximize his expected residual cash flow from B, denoted by $A(B, S)$, subject to his incentive compatibility (IC), the LPs' individual rationality (IR), the limited liability (LL), the LPs' monotonic contract (MC), and positive effort (PE) constraints,

$$
\begin{aligned}
S(B) &= \arg\max_{S} \; A(B,S), \\
&= \arg\max_{S} \; \varepsilon e(B,S)a(B,S)(B_L - S_L + b - s) + (1 - \varepsilon e(B,S)a(B,S))(B_L - S_L) - \gamma \frac{a(B,S)^n}{n}.
\end{aligned} \tag{35}
$$

The IC constraint of the venture capitalist is given by the optimal effort choice in (33) and $e(B,S)$ is determined in the system (30). The IR constraints of the LPs is that their expected payoff is no smaller than the capital investment K,

$$
I(B,S) \geq K, \tag{36}
$$

where their expected payoff $I(B,S)$ is defined in (31). Combining (LL), (MC), and (PE) constraints, we get

$$
\begin{cases}
s \geq 0, \\
b - s > 0, \\
S_L \geq 0, \\
B_L - S_L \geq 0.
\end{cases} \tag{37}
$$

After that, we consider the entrepreneur's optimal effort choice in stage three after committing to offering B to the venture capitalist. The entrepreneur knows that the venture capitalist will offer $S(B)$ to the LPs. At the same time, she also knows that the venture capitalist's optimal effort choice is derived based on B and $S(B)$, denoted by $a(B, S(B))$. As a result, the entrepreneur plays a best response to those by exerting an optimal effort level which is a function of B only, denoted by $e(B)$:

$$
\begin{aligned}
e(B) &= \arg\max_{e} \;\; \varepsilon e a(B,S(B))(X_L - B_L + x - b) + (1 - \varepsilon e a(B,S(B)))(X_L - B_L) - \beta \frac{e^m}{m}, \\
&= \frac{\varepsilon^{\frac{n}{mn-m-n}} (x-b)^{\frac{n-1}{mn-m-n}} (b - s(B))^{\frac{1}{mn-m-n}}}{\beta^{\frac{n-1}{mn-m-n}} \gamma^{\frac{1}{mn-m-n}}}.
\end{aligned} \tag{38}
$$

Similar to the angel financing scheme, the entrepreneur's optimal effort choice depends only on how much extra she has when the project succeeds. As a result, the entrepreneur has to save enough residual in the high state in order to exert effort, that is

$$x - b > 0. \tag{39}$$

Moreover, how much compensation budget she offers to the venture capitalist in the high state, b, has two offseting effects on her optimal effort choice, which is again due to the complementary effect between efforts.

Last, we move back to stage one. In this stage, the entrepreneur's optimization problem is to choose the compensation budget B so as to maximize her expected residual cash flow from the project, denoted by $E(B)$, subject to the entrepreneur's and the consultant's incentive compatibility (IC) and individual rationality (IR), the limited liability (LL), and positive effort (PE) constraints,

$$\begin{aligned} B &= \arg\max_{B} E(B), \\ &= \varepsilon e(B) a(B, S(B))(X_L - B_L + x - b) + (1 - \varepsilon e(B) a(B, S(B)))(X_L - B_L) - \beta \frac{e(B)^m}{m}. \end{aligned} \tag{40}$$

The IC constraint of the venture capitalist is given by the optimal effort choice in (33) and that of the entrepreneur is in (38). Their IR constraints of the them are always satisfied given their outside options of 0. Combining (LL), (MC), and (PE) constraints, we get

$$\begin{cases} b > 0, \\ x - b > 0, \\ B_L \geq 0, \\ X_L - B_L \geq 0. \end{cases} \tag{41}$$

Unlike in the case of angel financing, no player in the venture capital financing scheme forms beliefs about the contract committed between the other two players. We denote S^{VC} and T^{VC} as the equilibrium compensation levels for the LPs and the venture capitalist, respectively, where the superscript VC stands for venture capital financing. We solve the optimization problem in (40) subject to (41) and the following proposition summarizes the optimal contracts in the venture capital financing scheme.

Proposition 2. *With venture capital financing, there exists a threshold* $x \geq x^{VC}$ *such that the optimal compensations for the LPs and the venture capitalist are respectively equal to*

$$S_L^{VC} = X_L, \tag{42}$$

$$T_L^{VC} = 0, \qquad t^{VC} = \frac{x}{n}, \tag{43}$$

and s^{VC} *exists and is determined uniquely in*

$$\frac{\varepsilon^{\frac{mn}{mn-m-n}} \left(x - s^{VC} - \frac{x}{n}\right)^{\frac{n}{mn-m-n}} \left(\frac{x}{n}\right)^{\frac{m}{mn-m-n}} s^{VC}}{\beta^{\frac{n}{mn-m-n}} \gamma^{\frac{m}{mn-m-n}}} + X_L - K = 0. \tag{44}$$

It follows that:

(i) The LPs individual rationality constraint is binding, $I^{VC} = K$*;*

(ii) when the project fails, the LPs receive all the cash flow, $S_L^{VC} = X_L$*, while neither the venture capitalist nor the entrepreneur receives anything,* $T_L^{VC} = X_L - S_L^{VC} - T_L^{VC} = 0$*;*

(iii) when the project succeeds, these three players share the extra cash flow x*, where* t^{VC} *decreases with* n*, and* s^{VC} *decreases with* t^{VC}*.*

The optimal contracts derived in Proposition 2 are based on the maintained assumptions of (9) and the fact that the LPs can eventually verifiy the contract signed between the entrepreneur and the venture capitalist. Similar to the case of angel financing, the IR constraint of the LPs is binding. Both the effort providers, the entrepreneur and the venture capitalist, only receive compensation only in the higher state, and s^{VC} decreases with t^{VC}. The intuition remains the same as that given after Proposition 1.

However, an interesting result which differentiates venture capital financing from angel financing is that the venture capitalist's compensation, $t^{VC} = \frac{x}{n}$, depends only on how productive he is. The productivity, defined in our setting, corresponds to the convexity of his effort cost function n. In the case of angel financing, the consultant compensation, $t^A = \frac{x - s^A}{n}$, depends not only on his productivity level but also on the angels' compensation s^A.

Optimal Financing Scheme. Having analyzed the optimal contracts in the two financing schemes, a natural question arised is which scheme the entrepreneur should choose? What about the other equilibrium quantities between these two schemes? The following proposition and corollary answer these questions.

Proposition 3. *The entrepreneur maximizes her expected payoff by choosing the venture capital financing scheme,*

$$E^{VC} > E^{A}. \tag{45}$$

Moreover, the credit rationing threshold is smaller with venture capital financing scheme,

$$x^{VC} < x^{A}. \tag{46}$$

Corollary 1. *Compared to the angel financing scheme, in the venture capital financing scheme:*

(i) the entrepreneur gets lower residual value while exerting higher effort:

$$x - s^{VC} - t^{VC} < x - s^{A} - t^{A}, \tag{47}$$

$$e^{VC} > e^{A}; \tag{48}$$

(ii) the venture capitalist receivess higher compensation, exerts higher effort, and reaches a higher expected payoff:

$$t^{VC} > t^{A}, \tag{49}$$

$$a^{VC} > a^{A}, \tag{50}$$

$$A^{VC} > A^{A}; \tag{51}$$

(iii) the LPs receive lower compensation when the project succeeds while the probability of success is higher.

$$s^{VC} < s^{A}, \tag{52}$$

$$\pi^{VC} > \pi^{A}. \tag{53}$$

Proposition 3 states that the entrepreneur is willing to finance her project through a venture capitalist, instead of angel investors with whom she also needs to hire a consultant. To grasp the intuition, we first note that, in our setting, all three players' equilibrium expected payoffs - as well as the effect levels and the probability of success - admit the same functional form between the two financing schemes:

$$E(S,T) = \Big(1-\frac{1}{m}\Big)\frac{\varepsilon^{\frac{mn}{mn-m-n}}(x-s-t)^{\frac{m(n-1)}{mn-m-n}}t^{\frac{m}{mn-m-n}}}{\beta^{\frac{n}{mn-m-n}}\gamma^{\frac{m}{mn-m-n}}} + X_L - S_L - T_L, \tag{54}$$

$$I(S,T) = \frac{\varepsilon^{\frac{mn}{mn-m-n}}(x-s-t)^{\frac{n}{mn-m-n}}t^{\frac{m}{mn-m-n}}s}{\beta^{\frac{n}{mn-m-n}}\gamma^{\frac{m}{mn-m-n}}} + S_L, \tag{55}$$

$$A(S,T) = \Big(1-\frac{1}{n}\Big)\frac{\varepsilon^{\frac{mn}{mn-m-n}}(x-s-t)^{\frac{n}{mn-m-n}}t^{\frac{n(m-1)}{mn-m-n}}}{\beta^{\frac{n}{mn-m-n}}\gamma^{\frac{m}{mn-m-n}}} + T_L, \tag{56}$$

where $E^{VC} = E(S^{VC}, T^{VC})$, $E^A = E(S^A, T^A)$, $I^{VC} = I(S^{VC}, T^{VC})$, and $I^A = I(S^A, T^A)$. We also know from Proposition 1 and Proposition 2 that $S_L^{VC} = S_L^A = X_L$, $T_L^{VC} = T_L^A = 0$, and $I(S^{VC}, T^{VC}) = I(S^A, T^A) = K$. As a result, in the following analysis, we consider only the parts which differ between the two financing schemes and:

$$\bar{E}(s,t) = (x-s-t)^{\frac{m(n-1)}{mn-m-n}}t^{\frac{m}{mn-m-n}}, \tag{57}$$

$$\bar{I}(s,t) = (x-s-t)^{\frac{n}{mn-m-n}}t^{\frac{m}{mn-m-n}}s, \tag{58}$$

$$\bar{A}(s,t) = (x-s-t)^{\frac{n}{mn-m-n}}t^{\frac{n(m-1)}{mn-m-n}}, \tag{59}$$

where we call $\bar{E}(s,t)$, $\bar{I}(s,t)$, and $\bar{A}(s,t)$ the *effective* expected payoff of the entrepreneur, the investors and the advisor, respectively. Moreover, the investors' IR constraint being binding in both financing schemes implies that $\bar{I}(s^{VC}, t^{VC}) = \bar{I}(s^A, t^A) = \frac{\beta^{\frac{n}{mn-m-n}}\gamma^{\frac{m}{mn-m-n}}}{\varepsilon^{\frac{mn}{mn-m-n}}}(K - X_L) \equiv k$.

First, let's determine the levels of s and t which maximize the entrepreneur's effective expected payoff $\bar{E}(s,t)$. Figure 3 displays how $\bar{E}(s,t)$ responses to t for given levels of s. *Without* considering the investors' individual rationality condition, $\bar{I}(s,t) \geq k$, it is easy to see that the levels are $s^* = 0$ and $t^* = \frac{x}{n}$. The intuition is quite straight forward. The advisor's compensation in the higher state has two opposite effects on the entrepreneur's expected payoff. A higher t induces higher effort from the advisor which further increases the probability of success, while at the same time, a higher t decreases the entrepreneur's residual cash flow. $t^* = \frac{x}{n}$ is a result of balancing these two effects. However, compensating the investors is a pure cost for the entrepreneur. A higher s decreases the total amount of compensation that can be distributed to the entrepreneur and the advisor, $x - s - t$, while it has no direct impact on the probability of success since the investors are not effort providers. Hence, the entrepreneur will not leave anything for them $s^* = 0$.

However, it is obvious that the levels $s^* = 0$ and $t^* = \frac{x}{n}$ violate the investors' individual rationality constraint. Due to the riskiness of the project, defined in (2), the entrepreneur has to

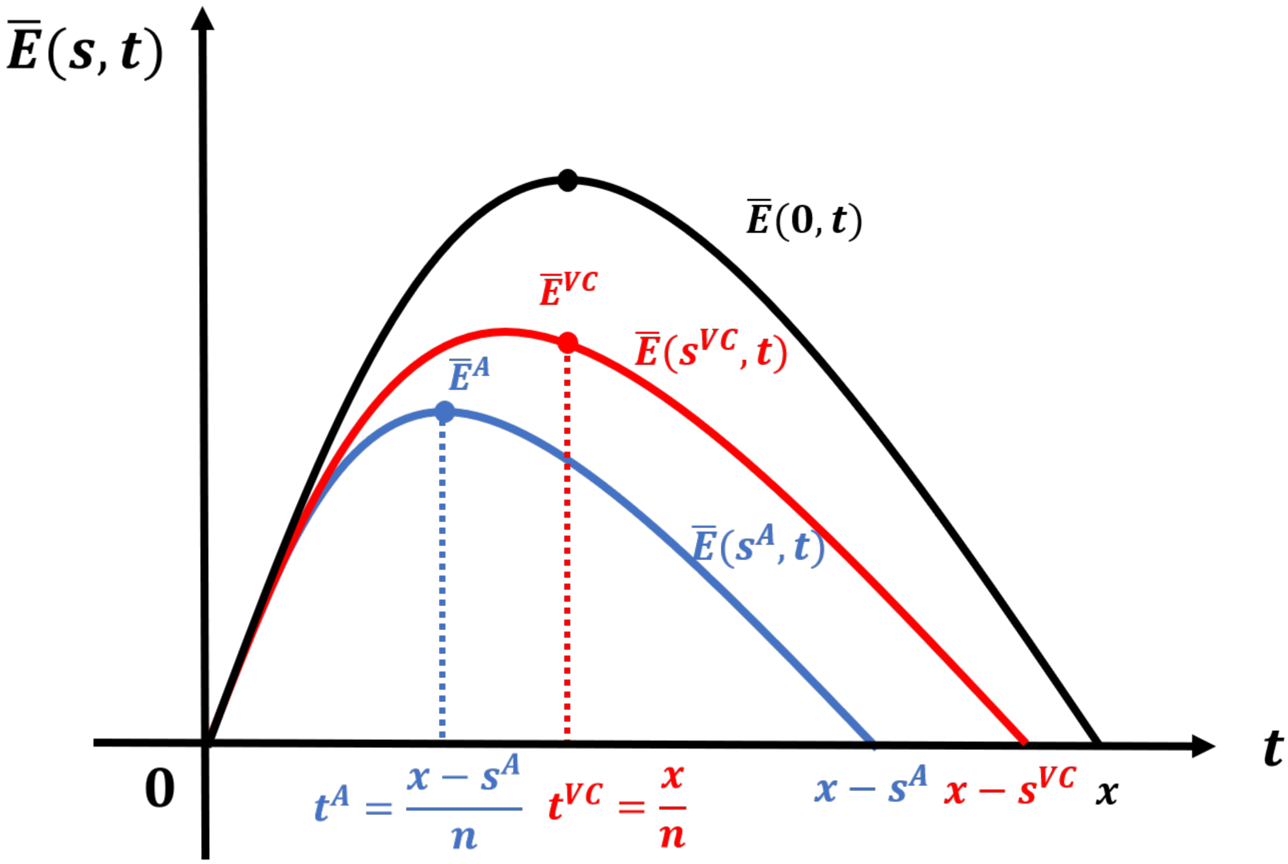

Figure 3: Entrepreneur's *Effective* Expected Payoff

In this figure, we plot the entrepreneur's effective expected payoff $\bar{E}(s,t)$ as a function of the advisor's compensation t, for a fixed level of the investors' compensation (1) $s = 0$ (black line), (2) $s = s^{VC}$ (red line), and (3) $s = s^A$ (blue line).

leave a positive amount for the investors in the higher state, $s > 0$. In the range $t \in [\frac{x-s}{n}, \frac{x}{n}]$, where $\frac{x-s}{n}$ corresponds to the optimal level in the angel financing and $\frac{x}{n}$ corresponds to that in the venture capital financing, the investors' effective expected payoff $\bar{I}(s,t)$ is increasing in both s and t. s has a direct effect while t has an indirect effect on their effective expected payoff: A higher s directly increases the investors' realized cash flow in the higher state; While, a higher t increases the advisor's compensation, inducing higher effort from him, which further increases the probability of success. As a result, the entrepreneur has to compensate either the investors or the advisor more in order to satisfy the investors' individual rationality constraint. Both the solutions (s^{VC}, t^{VC}) and (s^A, t^A) meet the investors' expectation, $\bar{I}(s^{VC}, t^{VC}) = \bar{I}(s^A, t^A) = k$.

Next, let's consider how s and t affect the entrepreneur's effective expected payoff $\bar{E}(s,t)$:

$$\frac{\partial \bar{E}(s,t)}{\partial s} = \underbrace{-\frac{m(n-1)}{mn-m-n}(x-s-t)^{\frac{n}{mn-m-n}} t^{\frac{m}{mn-m-n}}}_{\text{compensation effect}} < 0, \tag{60}$$

$$\frac{\partial \bar{E}(s,t)}{\partial t} = \underbrace{-\frac{m(n-1)}{mn-m-n}(x-s-t)^{\frac{n}{mn-m-n}} t^{\frac{m}{mn-m-n}}}_{\text{compensation effect}} + \underbrace{\frac{m}{mn-m-n}(x-s-t)^{\frac{m(n-1)}{mn-m-n}} t^{\frac{m}{mn-m-n}-1}}_{\text{complementary effect}} < 0. \tag{61}$$

We can see that a higher s decreases $\bar{E}(s,t)$ in the way that it lowers the entrepreneur's realized cash flow $x-s-t$, which we call a negative *compensation effect*. In the same way, a higher t also has a negative compensation effect. However, a higher t also increases the advisor's compensation which induces her to exert higher effort. Due to our complementary assumption of effort in our setting, a higher effort from the advisor increases the probability of the higher state which further increases $\bar{E}(s,t)$. We call this a *complementary effect*. As a result, although both s and t have a negative effect on entrepreneur's effective expected payoff, the effect of t is less severe, $\frac{\partial \bar{E}(s,t)}{\partial s} < \frac{\partial \bar{E}(s,t)}{\partial t} < 0 \; \forall s > 0$ and $t \in [\frac{x-s}{n}, \frac{x}{n}]$. A simple comparison of the optimal contracts between the two financing schemes, which are characterized in corollary 1, yields that

$$s^{VC} < s^{A}, \qquad t^{VC} > t^{A}. \tag{62}$$

The venture capitalist receives higher compensation while the angels receive more. Accordingly, the entrepreneur optimizes her expected payoff by choosing the venture capital financing scheme.

Then, a question raised is why the entrepreneur pays less to the consultant but more to the angels in the angel financing scheme? To put it in another way, can the entrepreneur replicate the optimal contracts in the venture capital financing scheme and compensate s^{VC} to the angels and t^{VC} to the consultant, respectively, to reach a higher expected payoff? The answer is NO due to the unverifiability of contracts. Figure 4 displays the entrepreneur's opportunistic behavior if angels agree to fund the project under compensation s^{VC}.

Suppose the entrepreneur offers s^{VC} to the angels and offers t^{VC} to the consultant in the angel financing scheme. How would the angels and the consultant repond to their out-of-the-equilibrium offers? For the angels, since they cannot verify the contract t, we think that they form passive beliefs about t. This means that when the angels receive an unexpected offer from the entrepreneur, they think that the entrepreneur has made an mistake but she will continue to offer the equilibrium contract to the consultant, which is t^{A}. Therefore, the angels calculate their effective expected

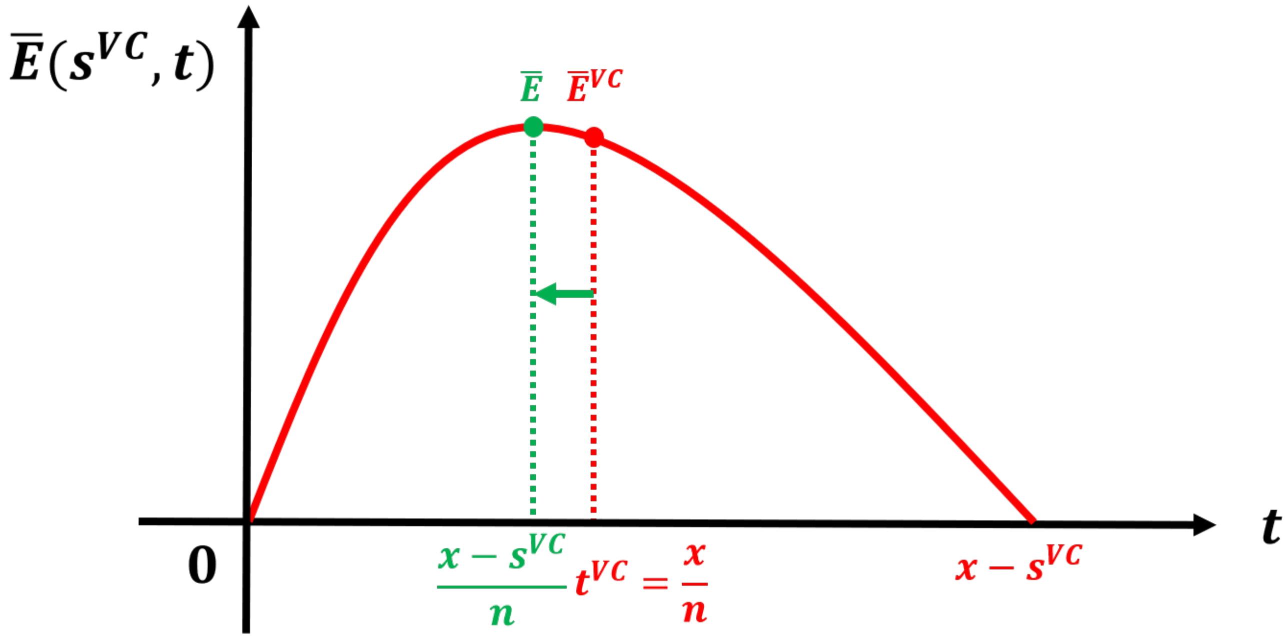

Figure 4: Entrepreneur's Opportunistic Behavior

In this figure, we plot the entrepreneur's effective expected payoff $\bar{E}(s^{VC}, t)$ as a function of the consultant's compensation t. If the angels agree to fund the project with s^{VC}, then the entrepreneur has the incentive to cut down on expert advice in order to save money which further increases her own expected payoff at the expense of making the angels worse off.

payoff as $\bar{I}(s^{VC}, t^A) < \bar{I}(s^A, t^A) = k$. As a result, they will decline the offer and not invest capital into the project. As for the consultant, he also thinks that the entrepreneur continue to offer s^A to the angels. From the consultant's prespective, he thinks that the entrepreneur keeps less compared to the equilibrium level, $x - s^A - t^{VC} < x - s^A - t^{VC}$, and the direct effect for her is to exert less effort accordingly. The complementary of effort implies that the consultant exerts less effort compared to what he will do with venture capital financing. In the end, the entrepreneur cannot commit s^{VC} and t^{VC} in the angel financing scheme as the angels would decline the offer and the consultant would not exert enough effort which further decreases the entrepreneur's expected payoff.

Last, we further examine the two financing schemes by comparing some other equilibrium quantities, given the results in Corollary 1. The venture capitalist receives a higher compensation compared with what the consultant gets, $t^{VC} > t^A$, which in turn makes him exert higher effort in equilibrium, $a^{VC} > a^A$. We also find out that the entrepreneur distributes a larger compensation budget, defined as $s + t$, through venture capital financing. This is consistent with Buffa, Liu, and White (2019). The intuition is again due to the verifiability of contracts. A large compensation budget has a direct income effect and indirect complementary effect on effort with verifiable contracts while it has only a direct income effect with unverifiable contracts. The venture capital as a signatory to both contracts would like to exert more effect under the same compensation as in the angel financing. The LPs are also not afraid of being expropriated since they can verify the contract t and are happy to receive a lower return in the case of success, $s^{VC} < s^{A}$. Regarding the compensation and effort level of the entrepreneur, we find that she keeps less residual, $x - s^{VC} - t^{VC} < x - s^{A} - t^{A}$, while exerts higher effort level, $e^{VC} > e^{A}$ in the case of venture capital financing. This is again a result of the additional complementary effect of t. Accordingly, the probability of success, $\pi^{VC} > \pi^{A}$, as well as the venture capitalist's expected payoff, $A^{VC} > A^{A}$, are larger in that scheme.

Delegated Contracting. So far, we have showed that the entrepreneur always chooses venture capital financing to fund her project and we have analyzed how the different compensation contracts affect her expected payoff. However, one key feature between the two financing schemes is that the entrepreneur contracts directly with the other two players in the angel financing scheme while she contracts only with the venture capitalist and delegates to him the contracting with the LPs in the case of venture capital financing. The conflict of interest between the entrepreneur and the venture capitalist leaves room for rent extraction by the ventue capitalist in his case. Similar to the definition introduced by Buffa, Liu, and White (2019), we define the rent extraction of the venture capitalist as the additional compensation that the venture capitalist keeps for himself, compared with his second-best compensation. In particular, we would like to know how the optimal contracts derived in the venture capital differs from the second-best contracts. The following proposition summarizes the finding.

Proposition 4. *The venture capital financing scheme recovers the second-best contracts.*

Proposition 4 states an interesting result: there is no rent extraction from the venture capitalist. The entrepreneur can essentially keep the optimal second-best amount of residual and distribute all of the rest to the venture capitalist, and the venture capitalist will divide it between himself and the LPs in a way that maximizes the entrepreneur's expected payoff in a second-best way. This outcome is due to two factors: competitive investors market and the verifiability of the contract. Since we assume a competitive capital market, neither the entrepreneur and the venture capitalist will compensate the LPs such that their expected payoff exceed their capital investment K. As the LPs can verify the amount the entrepreneur keeps, they can use their contract offered by the venture capitalist to determine the venture capitalist's compensation. Now, let's consider how s and t affect the venture capitalist's expected payoff:

$$\frac{\partial \bar{A}(s,t)}{\partial s} = \underbrace{-\frac{n}{mn-m-n}(x-s-t)^{\frac{n}{mn-m-n}-1}t^{\frac{n(m-1)}{mn-m-n}}}_{\text{compensation effect}} < 0, \tag{63}$$

$$\frac{\partial \bar{A}(s,t)}{\partial t} = \underbrace{-\frac{n}{mn-m-n}(x-s-t)^{\frac{n}{mn-m-n}-1}t^{\frac{n(m-1)}{mn-m-n}}}_{\text{compensation effect}} + \underbrace{\frac{n(m-1)}{mn-m-n}(x-s-t)^{\frac{n}{mn-m-n}}t^{\frac{m}{mn-m-n}}}_{\text{direct incentive effect}} > 0. \tag{64}$$

Similar to $\bar{E}(s,t)$, both s and t have a negative compensation effect on $\bar{A}$. However, t also has a positive direct incentive effect which can completely offset the negative compensation effect. As a result, the only way that the venture capitalist can generate a higher expected payoff is to increase the amount he keeps for himself out of the compensation budget B. Next, let's consider how s and t affect the venture capitalist's expected payoff:

$$\frac{\partial \bar{I}(s,t)}{\partial s} = \underbrace{-\frac{n}{mn-m-n}(x-s-t)^{\frac{n}{mn-m-n}-1}t^{\frac{m}{mn-m-n}}s}_{\text{compensation effect}} + \underbrace{(x-s-t)^{\frac{n}{mn-m-n}}t^{\frac{m}{mn-m-n}}}_{\text{direct incentive effect}} > 0, \tag{65}$$

$$\frac{\partial \bar{I}(s,t)}{\partial t} = \underbrace{-\frac{n}{mn-m-n}(x-s-t)^{\frac{n}{mn-m-n}-1}t^{\frac{m}{mn-m-n}}s}_{\text{compensation effect}} + \underbrace{\frac{m}{mn-m-n}(x-s-t)^{\frac{n}{mn-m-n}}t^{\frac{m}{mn-m-n}-1}s}_{\text{complementary effect}} > 0. \tag{66}$$

As before, both s and t have a negative compensation effect on $\bar{I}$. However, there is also a positive direct incentive effect from s and a positive complementary effect from t. Although either positive effect can offset the negative effect, the direct incentive effect is more prominent, $\frac{\partial \bar{E}(s,t)}{\partial s} > \frac{\partial \bar{E}(s,t)}{\partial t} > 0$. This means that, keeping the total value of $s+t$ as fixed, the entrepreneur would have a higher expected payoff if a part of t can be transferred to s, and she would have a lower expected payoff the other way around.

Suppose the venture capitalist keeps more budget for himself, denoted by $\delta > 0$, and offers a contract $s^{SB} - \delta$ which is strictly less than s^{SB}, then the LPs would decline that offer since $\bar{I}(s^{SB} - \delta, t^{SB} + \delta) < \bar{I}(s^{SB}, t^{SB}) = k$. However, if he distributes more to the LPs, this increases their expected payoff at the price of distorting his own. As a result, the venture capitalist will stick to offering the second-best contract to the LPs and the venture capital financing scheme coincides with the second-best contracts.

Throughout this paper, we have maintained the assumption that neither the angels nor the consultant can verify the other contract in the angel financing scheme, while both the LPs and the venture capitalist can verify both contracts in the venture capital financing scheme. This is a realistic assumption which describes exactly the financing schemes the entrepreneur of a startup can choose from in practice. One question is how do the other industries value these two financing schemes, in the sense that only the direct signatories can verify the bilateral contracts? The following corollary presents the result.

Corollary 2. *If the LPs cannot verify the contract signed between the entrepreneur and the venture capitalist, then the optimal contracts coincide with those in the angel financing scheme.*

The result is still a consequence of the competitive capital market assumption that the investors' expected payoff is strictly equal to K. Since the LPs cannot verify how much the entrepreneur distributes to the venture capitalist, they form passive beliefs about it and will not revise their beliefs even though they receive a different offer from the venture capitalist. $\frac{\partial \bar{I}(s,t)}{\partial s} > 0$ implies that the LPs agree to invest into the project only when they receive compensation no smaller than s^A, $\bar{I}(s, t^A) \leq \bar{I}(s^A, t^A) = k \ \forall s \leq s^A$. As analyzed before, the venture capital will not offer a higher compensation to the LPs unless he receives a larger budget from the entrepreneur, $\frac{\partial \bar{A}(s,t)}{\partial s} < 0 < \frac{\partial \bar{A}(s,t)}{\partial t}$. Now, let's suppose the entrepreneur indeed distributes a larger budget, $\frac{\partial \bar{E}(s,t)}{\partial s} < \frac{\partial \bar{E}(s,t)}{\partial t} < 0 \ \forall s > 0$ and $t \in [\frac{x-s}{n}, \frac{x}{n}]$ implies that she can reach a higher expected payoff with a larger budget if the venture capitalist keeps more to himself $t > t^A$, and compensates the LPs less, $s < s^A$. However, this is not feasible as the LPs only accept offers that are larger than or equal to s^A. As a result, the optimal contracts coincide with the optimal angel financing contracts.

Buffa, Liu, and White (2019) investigate a similar setting where two agents exert complementary efforts on a principal's project. The principal can choose to contract directly with both agents, or contract with only one agent and delegate this agent to contract with the other agent. They refer to them as "centralized contracting" and "delegated contracting" schemes, respectively. They have maintained the assumption that only the direct signatores to the contract observe (and hence can verify) the contract. Our setting in Corollary 2 is similar to their setting in the sense that the entrepreneur can be viewed as the principal, and the advisor and the investors can be treated as the two agents. The entrepreneur's financing scheme decision to fund the project: angel financing versus venture capital financing, is analogous to the principal's centralized contracting versus delegated contracting schemes, respectively.

However, as a comparison, in the setting of Buffa, Liu, and White (2019), the principal has to trade-off between the benefit of improved observability of contracts and cost of loss of control over the compensation budget when deciding between centralized contracting and delegated contracting. This is different from our setting in Corollary 2 as the entrepreneur is essentially indifferent between the two contracting schemes. There is no benefit or cost from delegating to the venture capitalist. The reason is that although the entrepreneur and the principal are both the residual claimants to their projects, the entrepreneur is also an effort provider while the principal is not. At the same time, the investors have to obtain an expected payoff at least equal to their investment K but the Subagent's reservation utility is zero.[15] This means that the Subagent will accept any offer from the Agent while the LPs won't. The Agent, as a result, can exploit the Subagent by keeping more budget to himself which decreases the principal's expected payoff. Meanwhile, the Agent observing both contracts gives the principal the ability to commit to distribute a larger budget which increases her expected payoff. However, in our setting of Corollary 2, the venture capitalist cannot exploit the LPs and the entrepreneur cannot increase her profit with a larger budget.

Up until now, our model provides a rationale for the existence of venture capital firms. If the venture capitalist provides the same quality of expert advice as an outside consultant and there is no differential cost of financing, the entreprenur should strictly prefer venture capital financing over angel financing in a competitive capital and labor markets. Note that the rationale provided here is different from the one introduced by Casamatta (2003). In her setting, the advisor provides less efficient effort compared to what the entrepreneur does and the existence of venture capital firms is the result of the joint provision of effort and wealth. She treats the venture capital firm as a whole who interacts with the entrepreneur altogether, while in our case, we work on a more realistic setting in which we consider the venture capital firm as a partnership: the venture capitalist (GP) forms a limited partnership with the investors (LPs), whereas the GP exerts managerial effort and the LPs provide capital.

The next section investigates the type of securities the entrepreneur would like to offer to the venture capitalist and how she markets those securities with the double-sided moral hazard financing problem studied here.

[15]Buffa, Liu, and White (2019) define Agent as the agent who contracts directly and Subagent as the agent who does not contract directly with the principal in the delegated contracting scheme.

5 Optimal Financial Contracts in Venture Capital Partnerships

So far, we have characterized the optimal contracts in two different financing schemes and demonstrated the optimality of the venture capital financing scheme in the entrepreneur's innovative project. In this section, we show how these contracts can be implemented using financial claims that are widely issued by venture capital backed business ventures.

For the sake of simplicity, we restrict ourselves to only one type of security. An illustration of this is that some securities can be replicated using other securities, for instance, the payoff of a convertible debt can be replicated using a combination of debt and equity plus a conversion option. Differences between these two types of claim usually the allocation of control rights in case of a liquidation event, which is irrelevant in our setting. The implementation we present in the following is just one but not the unique way for the entrepreneur to finance her project.

5.1 Optimal Security Design

The entrepreneur raises initial capital K by issuing a security ex-ante. Her objective is to determine which financial assets will not only replicate the exact payoff of the optimal contracts and thereby provide powerful incentives for the entrepreneur, the venture capitalist, and the LPs.[16]

The optimal contracts with venture capital financing derived in Proposition 2 imply that the LPs obtain X_L when the project fails, and the entrepreneur and the venture capitalist are compensated only when the project succeeds. All three players share the higher state outcome. These results are a direct consequence of the riskiness assumption, defined in (2), and the "maximal high-profit-state payoff" property, introduced by Innes (1990). The LPs' optimal contract is shown to take a standard debt form, while the entrepreneur's and the venture capitalist's contracts are shown to take a standard equity form. However, the entrepreneur contracts only with the venture capitalist and we restrict her to issue one type of security. As a result, the entrepreneur can implement the optimal contracts by *issuing convertible preferred stocks* to the venture capitalist.[17]

[16]Since the optimal contracts are written on the realized output, which is verifiable to all three players, the implementation described here holds regardless of who designs the security, as long as the entrepreneur has all the bargaining power.

[17]Note that convertible debt can also implement the optimal contracts in our setting as we focus on cash flow rights and optimal incentive provisions, instead of control rights and liquidation rights here. Although the payoffs and conversion to common equity triggers between convertible debt and convertible preferred equity are basically the same, the differences arise in the events of liquidation. First, convertible debt has priority over convertible preferred equity if both claims are issued. Second, if the convertible debt has a coupon which cannot be deferred,

Define D as the *cumulative* dividend pledged on each convertible preferred stock multiplied by the total number of convertible preferred stocks issued.[18] Denote α as the fraction of convertible preferred stock in the firm's equity. Therefore, the fraction of common stock is $1-\alpha$. For the sake of simplicity, we assume the conversion ratio is one: one share of convertible preferred stock can be exchanged into one share of common stock. In order to distinguish between the convertible preferred stock and common stock, we assume

$$\alpha X_L < D < \alpha(X_L + x), \tag{67}$$

and

$$D \leq X_L. \tag{68}$$

Here, αX_L is the convertible preferred shareholders' payoff if they convert their shares in the lower state of the world and $\alpha(X_L + x)$ is their payoff if converting in the higher state. (67) implies that the venture capitalist will not convert his shares only if the project succeeds. (68) implies that the outcome in the lower state is sufficient enough to cover the cumulative dividend required for the convertible preferred stock. The contracts must match the payoffs of the convertible preferred stocks, therefore

$$\begin{cases} S_L^{VC} = D, \\ S_L^{VC} + s^{VC} + T_L^{VC} + t^{VC} = \alpha(X_L + x). \end{cases} \tag{69}$$

When the project fails, the entrepreneur uses the cumulative dividend as the LPs' compensation; when the project succeeds, the entrepreneur shares the higher output with the LPs and the venture capitalist as their compensation. The riskiness of the project, defined in (2), also implies that the LPs would also like to convert the preferred shares into common shares. There is no conflict of interest beween the LPs and venture capitalist regarding the conversion time. Denote by β the

fail to honor the coupon payment would put the firm into a default stage, while fail to distribute the dividend pledged is a less severe problem. Third, when issuing convertible debt, collateral is required. Collateral is not required when issuing convertible preferred equity.

[18] Kaplan and Strömberg (2003) find cumulative preferred dividends are present in 43.8 % of the venture capital financings. In general, start-ups rarely pay out dividends or coupons as they are cash poor. Paying dividends or coupons might cause them to raise another round of financing. However, sometimes venture capitalists can ask entrepreneurs to make their preferred dividends cumulative (rather than non-cumulative) as a way of making the liquidation rights stronger. Even though these are dividends that do not have to be paid out periodically, they accumulate and are added to the liquidation claims. If we implement the optimal contracts using convertible debt, D will be defined as deferred coupon or deferred interest. Our implementation is also in line of Casamatta (2003) who defines D as *minimum* dividend pledged.

fraction of those converted shares that will be distributed to the venture capitalist. In practice, β is frequently referred to as *carried interest*. To match the payoff, we also need

$$T_L^{VC} + t^{VC} = \beta(\alpha(X_L + x) - K). \tag{70}$$

Replacing S^{VC} and T^{VC} by their values derived in Proposition 2, the following proposition describes the financial claim that is optimally issued by the entrepreneur.

Proposition 5. *The entrepreneur can raise initial capital K by issuing convertible preferred stocks to the venture capitalist, which represents*

$$\alpha = \frac{X_L + s^{VC} + \frac{x}{n}}{X_L + x} \tag{71}$$

of the firm's equity. The cumulative dividend pledged on these convertible preferred stocks is

$$D = X_L, \tag{72}$$

and the conversion ratio is 1*. The convertible preferred stocks will be optimally converted into common stocks when the project is a success. The venture capitalist charges a carried interest of*

$$\beta = \frac{\frac{x}{n}}{X_L + s^{VC} + \frac{x}{n} - K}, \tag{73}$$

and the left portion will be distributed to the LPs,

$$1 - \beta = \frac{X_L + s^{VC}}{X_L + s^{VC} + \frac{x}{n} - K}. \tag{74}$$

Our implementation is consistent with the empirical observation that convertible preferred stocks are extensively used in the venture capital industry, as evidenced by Kaplan and Strömberg (2003) and Sahlman (1990). The use of carried interest is also widely documented in the alternative management literature, such as venture capital, private equity, mutual funds, and hedge funds.

The intuition for the convertible preferred stocks implementation is the following. If the venture capitalist and the LPs inside the venture capital firm contract individually with the entrepreneur, like the angel financing scheme we described in Section 3, then based on the optimal contracts, the entrepreneur would like to issue common stocks to the venture capitalist and standard debt to the LPs. The venture capitalist, as an effort provider, aligns his personal interest to the entrepreneur's with equity compensation, according to Jensen and Meckling (1976). The LPs, as the capital provider, maximize the entrepreneur's expected payoff in the most efficient way with debt compensation, according to Innes (1990). However, due to the lack of verifiability of the contracts, the entrepreneur is unable to reach for highest profit using angel financing scheme.

The use of venture capital financing scheme, although it helps the entrepreneur recover the second-best contracts (Proposition 4), prevents the entrepreneur from contracting directly with the LPs. A direct consequence is that the entrepreneur cannot issue common stocks to the venture capitalist. The optimal contracts derived in Proposition 2 imply that the entrepreneur does not receive any cash flow if the project fails. As a result, the entrepreneur must issue some securities that are senior to common stocks, such as preferred stocks or debt. But straight debt-like or debt securities do not align the venture capitalist's interest to the entrepreneur's.

The convertible preferred stocks essentially assist the entrepreneur in compensating the venture capitalist and the LPs with different securities. Figure 5 shows the return pattern for a typical convertible security as a function of the realized output X. When the project fails, it means that the performance of the common stocks is bad, the venture capitalist will not convert his shares. The convertible preferred stocks act as debt and the LPs receive all the cumulative pledged dividend. When the project succeeds, the venture capitalist optimally chooses to convert shares into common stocks and shares the outcome with the LPs. The venture capital gets paid only on the success of the project. In other words, the venture capitalist's compensation depends on the firm's equity performance and he would like to exert effort to improve the value of the common stocks. This motivation aligns his interest with the entrepreneur. By issuing convertible preferred stocks, the venture capitalist gets equity-like securities while the LPs get risky debt-like securities, and the entrepreneur maximizes her expected payoff accordingly. We also respectively depict the payoffs for the entrepreneur (Figure 6), the LPs (Figure 7), and venture capitalist (Figure 8) below.

Several theoretical papers on venture capital contracts also offer different explanations for the use of convertible preferred stocks in the venture capital financing. Gompers (1993) and Gompers (1997) show that convertible debt dominates common equity and standard debt in the sense that convertible debt combines the advantages of the other two. Using a model of venture capital financing combining both moral hazard and adverse selection, he shows that the equity feature of the convertible debt limits the entrepreneur's incentive to take risks, and the debt feature helps the investors to select those entrepreneurs with high ability. Similarly, Marx (1998) shows that pure debt gives the venture capitalist too much incentive to intervene, and pure equity too little. A mixture of debt and equity, where the equity is a convertible, enables an optimal level of intervention.

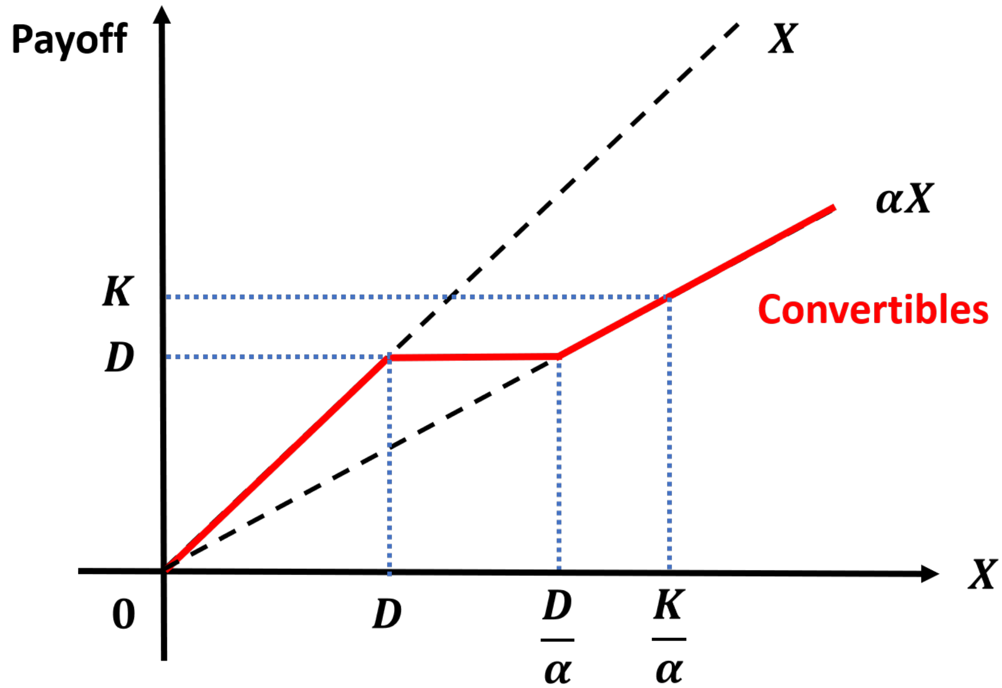

Figure 5: Convertible security's payoff (red line) responses to realized output

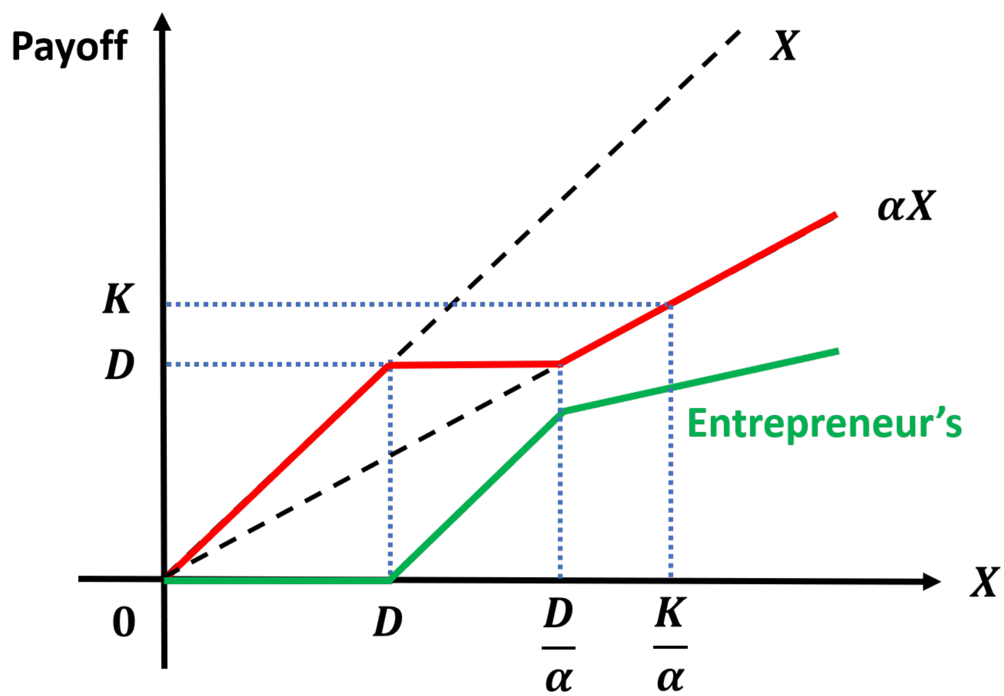

Figure 6: Entrepreneur's payoff (green line) responses to realized output

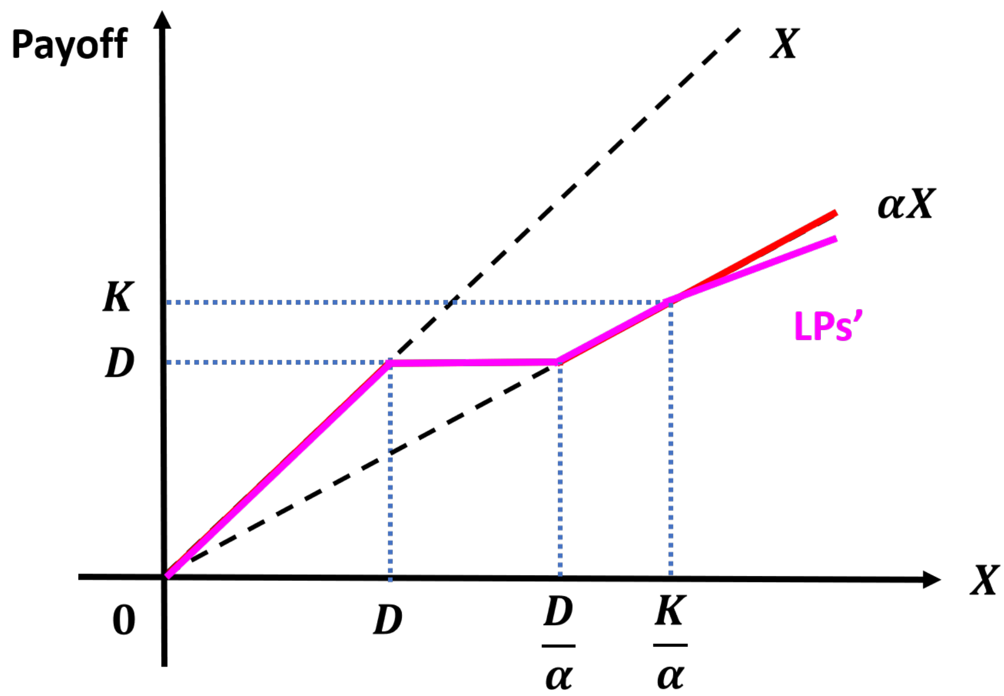

Figure 7: LPs' payoff (pink line) responses to realized output

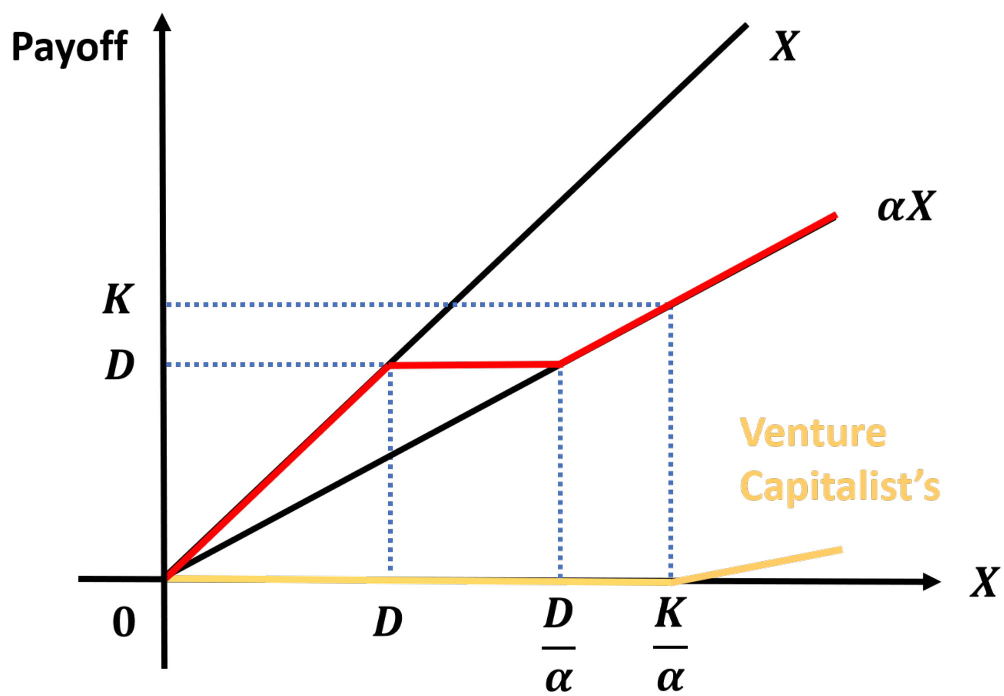

Figure 8: Venture capitalist's payoff (yellow line) responses to realized output

Recognizing the effort provision from both the entrepreneur and the venture capitalist, some other work, such as Casamatta (2003), Dessi (2005), Inderst and Müller (2004), Renucci (2006), Repullo and Suarez (2004), and Schmidt (2003), also consider a framework of double-sided moral hazard. All of these models emphasize the importance of having both the effort providers be residual claimants when the project succeeds, and they offer different explanations of the use of convertible preferred stocks accordingly. In Casamatta (2003), Dessi (2005), Inderst and Müller (2004), and Renucci (2006), they find that in parts of the parameter space can the convertible preferred stocks be issued optimally. In the setting of Casamatta (2003), when the amount of venture capital financing is large enough do the optimal contracts resemble convertible bonds or preferred stocks and the entrepreneur's resemble common shares. In a similar setting, Renucci (2006) shows that a mix of debt and equity financing is optimal only when the entrepreneur's private benefit from shirking is within a certain range. In the setting of Dessi (2005), the entrepreneur uses a convertible or a combination of debt and equity when the entrepreneur's endowed capital bounded between a certain range. In the setting of Inderst and Müller (2004), the venture capitalist holds debt when his equilibrium equity share is large enough. Repullo and Suarez (2004) consider the a sequential investment model. They find that when the venture capital investing happens at a later stage, it is optimal to compensate the initial stage investors with convertible claims while preserving the incentives for the venture capitalist later. The analysis of Schmidt (2003) also explains that a convertible induce the venture capitalist to intervene precisely in those states where his action is desirable.

Similar to the convertible securities' mitigating risk-shifting motivation, introduced by Brennan and Schwartz (1988) and Green (1984), another strand of theoretical work, such as Cornelli and Yosha (2003) and Trester (1998), shows that convertibles can be used as a way to prevent the entrepreneur from manipulating performance and shifting risk to the investors. In the setting of Cornelli and Yosha (2003), the entrepreneur has the incentive to boost interim performance in order to attract additional rounds of financing. As a high performance makes the venture capitalist convert his convertible debt into equity, this dilutes the entrepreneur's equity stake and prevents her from wanting to manipulate earnings. Trester (1998) also recognizes the higher probability of asymmetric information between the entrepreneur and the venture capitalist. The foreclosure feature of debt will push the entrepreneur into opportunistic behavior by taking all assets available which decreases the venture capitalist's expected return. Preferred equity, acting like a debt but without the foreclosure right, allows the venture capitalist to receive some positive return rather than a big loss.

One more theoretical approach, represented by Berglof (1994) and Hellmann (2006), considers convertible securities as a way to avoid inefficient exit from ventures. In the model of Berglof

(1994), convertible debt enables the entrepreneur and the venture capital to capture the maximum amount of rents in the presence of a sale. Built on Aghion and Bolton (1992), Hellmann (2006) mentions the conflicts between the entrepreneur and the venture capitalist about cash flow rights, depending on whether exit occurs by an IPO or acquisition. The venture capitalist has to give up a cumulative amount of equity in the case of an IPO, which makes him in favor of an acquisition. The use of (participating) convertible preferred equity, including automatic conversion at IPO, aligns his interest to the entrepreneur's.

5.2 Empirical Predictions

The analysis of our model presented here implies the following testable predictions:

- *Entrepreneur's financing choice between the angel and venture capital financing:* First, there should be a relationship between the performance of the business venture and choice of financing scheme, conditioning on the involvement of the entrepreneur as well as an advisor are needed. Our model predicts that with competitive capital and labor markets, the entrepreneur should always prefer venture capital financing rather than angel financing. An innovative venture which has shown a sequence of outstanding past performance but is still showing great potential (e.g. not close to an IPO) indicates that most of the bargaining power should be allocated to the entrepreneur. If the project still has a large room for improvement and the firm's management team is still incomplete, the entrepreneur should accept venture capital financing, instead of funding from a pure financier (e.g. an angel) which the entrepreneur has to hire one additional consultant.
- *Securities used between the angel and venture capital financing:* Second, convertible preferred stocks (or any securities that display the same incentives and payoffs as convertible preferred stocks) should be more frequently issued with venture capital financing when the entrepreneur still needs managerial advice from the venture capitalist. In this setting, the entrepreneur cannot contract directly with the LPs, and the venture capitalist exerts managerial effort which combines with the entrepreneur's innovative effort. Convertible preferred stocks essentially allow the entrepreneur to compensate the venture capitalist with common stocks and the LPs with straight debt. Thus, our model predicts that if an advisor's intervention is unnecessary, the entrepreneur would like to either go directly with angel financing or lower the venture capitalist's incentive to exert effort. In eithe case, convertible preferred equities are less likely to be issued. A debt-like security is more likely to be employed.

- *Term sheet specification in venture capital financing contracts:* Third, since contracts are written contingent on the performance of the company, when the business venture becomes riskier, controlling for the expected profitability, we should expect a lower cumulative pledged dividend as part of the convertible preferred equity issued by the entrepreneur, and higher fraction of the equity (of the business venture) upon conversion. There are a couple of ways to measure risk, such as the number of funding rounds, capital raised in each round, time of the most recent funding, the management team of the company, equity structure, etc. For example, fewer stages of financing or smaller capital raised previously could indicate a higher risk level of the business venture.

- *Efficiency and compensation of managerial effort:* Fourth, the advisor's compensation should be positively correlated with his effort efficiency in both the angel and venture capital financing schemes. Our model predicts that as an advisor gets more productive, the entrepreneur finds it profitable to increase the project's expected payoff by compensating the advisor more to induce higher effort. One way to measure the advisor's ability is his past performance. An advisor who has demonstrated a successful track record with business ventures indicates that he is more skillful. In other words, in a competitive labor market, a less experienced financial advisor or a newly established venture capital firm should demand less compensation or a smaller carried interest in order to attract investment opportunities.

- *Advising service provided between consultant and venture capitalist:* Fifth, our model concludes that the advisor receives higher expected payoff as a venture capitalist compared to that as a consultant if he provides the same kind of managerial service in a competitive labor market. As a result, we predict that, when the supply of venture capital increases, more advisors are willing to form a limited partership with investors, instead of being an independent consultant. We should witness more venture capital funds being established in boom fundraising periords.

- *Verifiability of contract and compensation of the LPs:* Sixth, our model predicts that if the LPs are unable to verify their specific investment in the venture capital fund, such as how much of the money has been invested and for the amount invested what are the associated securities the venture capitalist are holding, the LPs will ask for a larger return at the expense of decreasing the venture capitalist's compensation and expected payoff. As a result, we should see that for the LPs who have been in the venture capital industry before or a country who has a well defined legal system regarding venture capital partnerships, the LPs will demand a lower return for the same amount of capital invested, compared to those LPs who have a hard time verify their investment.

6 Conclusions

In this paper, we have analyzed a double-sided moral hazard problem in which the entrepreneur must (1) work on her project, (2) hire an expert advisor to work with her, and (3) raise capital from investors to implement the project. Both the entrepreneur and the advisor must individually exert unverifiable effort to increase the project's expected cash flow and their efforts exhibit complementarity. In the angel financing scheme, the entrepreneur offers contracts individually to the consultant and the angels where neither agent can verify the contract offere to the other agent. In the setting of venture capital financing, the entrepreneur offers a verifiable contract to a venture capital firm. This venture capital firm is structured as a partnership in which the venture capitalist is the general partner, while the investors are the limited partners (LPs).

Because angels cannot verify the contract offered to the consultant, and vice versa, both financing and incentive provisions become more expensive, and it is difficult for the entrepreneur to offer the optimal verifiable contracts. In the case of angel financing, the angels fear that the entrepreneur has the tendency to economize her incentive payment to the consultant and hence would only agree to fund the project with a strictly higher return; the consultant is afraid that the entrepreneur and the investors would deviate to another contract which internalizes his effort and hence demands more compensation to exert the same level of effort. As a result, the entrepreneur offers suboptimal contracts. In the venture capital setting, both the venture capitalist's and the LPs' concern for verifiability disappears. The venture capitalist, being the signatory to both contracts, can verify the contracts by default, and the LPs can eventually verify the specific ownership of their investment in the venture capital fund according to limited partnership agreement through periodic reports or maturity of the fund. This implies that the entrepreneur always prefers to fund her project through venture capital financing rather than angel financing.

Our result challenges the standard principal-agent theory that the entrepreneur should always do at least weekly better by contracting directly with the advisor and investors. This result holds only when the entrepreneur can commit to verifiable contracts with different contractors, which is rarely the case in practice. A contract is usually private information, verifiable only by its direct signatories. This implies that each agent's compensation cannot be made contingent on the structure of the other agents' contracts. We offers an explanation to the prevalence of venture capital industry in the sense that the limited partnership agreement, although forbids the LPs from participating in daily operations (not exerting effort), mandates the venture capitalist (the general partner) to disclose some high-level details about the portfolio companies, including the specific ownership of their investment. This implies that the institutional structure of venture capital financing assists the entrepreneur in restoring the verifiability of contracts, which is very hard to achieve in the contracting structure of angel financing.

This paper also offers an explanation of the pervasiveness of convertible securities in venture capital financing. The payoff structure of the optimal contracts implies that the entrepreneur would like to issue an equity-like security to the venture capitalist while providing debt-like securities to the LPs to maximize the higher state payoffs for herself and the venture capitalist. This is hard to implement as the entrepreneur neither contracts directly with the LPs nor can verify the contract signed between the venture capitalist and the LPs. Properly designed convertible securities can help the entrepreneur essentially issue two different securities even she only signs one contract with the venture capitalist. When the project has a lower output, it means that the performance of the common stocks is bad. The venture capitalist will not convert his shares. The convertibles act as a straight debt, and the LPs collect all the cumulative dividend pledged. When the project generates a higher output, it means that the common stocks are worth more. The venture capitalist optimally converts his shares into common stocks and shares the cash flows with the entrepreneur and the LPs. The venture capitalist gets paid only when the firm's equity performance is good enough and he would like to exert effort to improve the value of the common stocks. This motivation aligns his interest with the entrepreneur. The LPs, on the other hand, only provide capital. To incentivize both the entrepreneur and the venture capitalist to improve the productivity of the project, it is optimal to distribute all liquidation cash flows to the LPs.

It is natural to extend our framework by introducing an intermediate period between the project implementation date and the project harvest date. Stage financing is a widely used financing technique by the venture capitalist in which the venture capitalist is given an option to liquidate an unpromising project earlier, something the entrepreneur will never do. Similarly, for the LPs, they typically only commit to providing a certain amount of capital, and they always have the option to decline a capital call from the venture capitalist for poor portfolio performance. It is interesting to explore how the abandonment threat affects the entrepreneur's and the venture capitalist's incentive provision, and how the early and delayed compensation aligns their interest with the LPs. Another interesting dimension is to consider how the relative efficiency of incentive provision varies with the characteristics of projects as well as the economic environment for start-up ventures. We could obtain comparative static analyses regarding the frequency of venture capital financing and the type of securities issued by the entrepreneur.

Appendix A: Proofs

Proof of Proposition 1. In the angel financing scheme, the consultant's maximization problem is

$$\begin{cases} e(\hat{S},T) = \arg\max_{e} \ \ \varepsilon e a(\hat{S},T)(X_L - \hat{S}_L - T_L + x - \hat{s} - t) + (1 - \varepsilon e a(\hat{S},T))(X_L - \hat{S}_L - T_L) - \beta \frac{e^m}{m}, \\ a(\hat{S},T) = \arg\max_{a} \ \ \varepsilon e(\hat{S},T) a (T_L + t) + (1 - \varepsilon e(\hat{S},T) a) T_L - \gamma \frac{a^n}{n}. \end{cases} \tag{A.1}$$

The first order conditions are

$$\begin{cases} \varepsilon a(\hat{S},T)(x - \hat{s} - t) - \beta e^{m-1} = 0, \\ \varepsilon e(\hat{S},T) t - \gamma a^{n-1} = 0, \end{cases}$$

and solving the system of first order conditions in $(e(\hat{S},T), a(\hat{S},T))$, we obtain

$$a(\hat{S},T)) = \frac{\varepsilon^{\frac{m}{mn-m-n}} (x - \hat{s} - t)^{\frac{1}{mn-m-n}} t^{\frac{m-1}{mn-m-n}}}{\beta^{\frac{1}{mn-m-n}} \gamma^{\frac{m-1}{mn-m-n}}}. \tag{A.2}$$

The angels' maximization problem in stage two is

$$\begin{cases} e(S,\hat{T}) = \arg\max_{e} \ \ \varepsilon e a(S,\hat{T})(X_L - S_L - \hat{T}_L + x - s - \hat{t}) + (1 - \varepsilon e a(S,\hat{T}))(X_L - S_L - \hat{T}_L) - \beta \frac{e^m}{m}, \\ a(S,\hat{T}) = \arg\max_{a} \ \ \varepsilon e(S,\hat{T}) a (\hat{T}_L + \hat{t}) + (1 - \varepsilon e(S,\hat{T}) a) \hat{T}_L - \gamma \frac{a^n}{n}. \end{cases} \tag{A.3}$$

The first order conditions are

$$\begin{cases} \varepsilon a(S,\hat{T})(x - s - \hat{t}) - \beta e^{m-1} = 0, \\ \varepsilon e(S,\hat{T}) \hat{t} - \gamma a^{n-1} = 0, \end{cases}$$

and solving the system of first order conditions in $(e(S,\hat{T}), a(S,\hat{T}))$, we obtain

$$\begin{cases} e(S,\hat{T}) = \dfrac{\varepsilon^{\frac{n}{mn-m-n}} (x-s-\hat{t})^{\frac{n-1}{mn-m-n}} \hat{t}^{\frac{1}{mn-m-n}}}{\beta^{\frac{n-1}{mn-m-n}} \gamma^{\frac{1}{mn-m-n}}}, \\ a(S,\hat{T}) = \dfrac{\varepsilon^{\frac{m}{mn-m-n}} (x-s-\hat{t})^{\frac{1}{mn-m-n}} \hat{t}^{\frac{m-1}{mn-m-n}}}{\beta^{\frac{1}{mn-m-n}} \gamma^{\frac{m-1}{mn-m-n}}}, \end{cases} \tag{A.4}$$

and the probability of success, $P(S,\hat{T})$, is

$$P(S,\hat{T}) = \varepsilon e(S,\hat{T}) a(S,\hat{T}) = \frac{\varepsilon^{\frac{mn}{mn-m-n}} (x - s - \hat{t})^{\frac{n}{mn-m-n}} \hat{t}^{\frac{m}{mn-m-n}}}{\beta^{\frac{n}{mn-m-n}} \gamma^{\frac{m}{mn-m-n}}}. \tag{A.5}$$

The angels' expected payoff is

$$\begin{aligned} I(S,\hat{T}) &= P(S,\hat{T})(S_L + s) + (1 - P(S,\hat{T})) S_L, \\ &= \frac{\varepsilon^{\frac{mn}{mn-m-n}} (x - s - \hat{t})^{\frac{n}{mn-m-n}} \hat{t}^{\frac{m}{mn-m-n}} s}{\beta^{\frac{n}{mn-m-n}} \gamma^{\frac{m}{mn-m-n}}} + S_L. \end{aligned} \tag{A.6}$$

The entrepreneur's optimal effort choice in stage two is

$$e(\hat{S}, S, T) = \arg\max_{e} \ \varepsilon e a(\hat{S}, T)(X_L - S_L - T_L + x - s - t) + (1 - \varepsilon e a(\hat{S}, T))(X_L - S_L - T_L) - \beta \frac{e^m}{m},$$
$$= \frac{\varepsilon^{\frac{n}{mn-m-n}} (x - \hat{s} - t)^{\frac{1}{(m-1)(mn-m-n)}} (x - s - t)^{\frac{1}{m-1}} t^{\frac{1}{mn-m-n}}}{\beta^{\frac{n-1}{mn-m-n}} \gamma^{\frac{1}{mn-m-n}}}, \tag{A.7}$$

and therefore her expected payoff is

$$E(\hat{S}, S, T) = \varepsilon e(\hat{S}, S, T) a(\hat{S}, T)(X_L - S_L - T_L + x - s - t) + (1 - \varepsilon e(\hat{S}, S, T) a(\hat{S}, T))(X_L - S_L - T_L)$$
$$- \beta \frac{e(\hat{S}, S, T)^m}{m},$$
$$= \left(1 - \frac{1}{m}\right) \frac{\varepsilon^{\frac{mn}{mn-m-n}} (x - \hat{s} - t)^{\frac{m}{(m-1)(mn-m-n)}} (x - s - t)^{\frac{m}{m-1}} t^{\frac{m}{mn-m-n}}}{\beta^{\frac{n}{mn-m-n}} \gamma^{\frac{m}{mn-m-n}}} + X_L - S_L - T_L. \tag{A.8}$$

The entrepreneur's maximization problem in stage one becomes

$$\max_{S, T} \ \left(1 - \frac{1}{m}\right) \frac{\varepsilon^{\frac{mn}{mn-m-n}} (x - \hat{s} - t)^{\frac{m}{(m-1)(mn-m-n)}} (x - s - t)^{\frac{m}{m-1}} t^{\frac{m}{mn-m-n}}}{\beta^{\frac{n}{mn-m-n}} \gamma^{\frac{m}{mn-m-n}}} + X_L - S_L - T_L,$$

subject to

$$\begin{cases} \frac{\varepsilon^{\frac{mn}{mn-m-n}} (x - s - \hat{t})^{\frac{n}{mn-m-n}} \hat{t}^{\frac{m}{mn-m-n}} s}{\beta^{\frac{n}{mn-m-n}} \gamma^{\frac{m}{mn-m-n}}} + S_L \geq K, \\ s \geq 0, \\ t > 0, \\ x - s - t > 0, \\ S_L \geq 0, \\ T_L \geq 0, \\ X_L - S_L - T_L \geq 0, \end{cases}$$

where, in equilibrium,

$$\begin{cases} \hat{s} = s^A, \\ \hat{t} = t^A. \end{cases}$$

Suppose

$$\mathcal{L} = \left(1 - \frac{1}{m}\right) \frac{\varepsilon^{\frac{mn}{mn-m-n}} (x - \hat{s} - t)^{\frac{m}{(m-1)(mn-m-n)}} (x - s - t)^{\frac{m}{m-1}} t^{\frac{m}{mn-m-n}}}{\beta^{\frac{n}{mn-m-n}} \gamma^{\frac{m}{mn-m-n}}} + X_L - S_L - T_L,$$

then $\frac{\partial \mathcal{L}}{\partial S_L} < 0$ and $\frac{\partial \mathcal{L}}{\partial s} < 0$ imply that $S_L = s = 0$ which violates the investors' individual rationality condition. Therefore, the investors' participation constraint must be binding. Now, suppose

$$\mathcal{L} = \Big(1 - \frac{1}{m}\Big) \frac{\varepsilon^{\frac{mn}{mn-m-n}} (x - \hat{s} - t)^{\frac{m}{(m-1)(mn-m-n)}} (x - s - t)^{\frac{m}{m-1}} t^{\frac{m}{mn-m-n}}}{\beta^{\frac{n}{mn-m-n}} \gamma^{\frac{m}{mn-m-n}}} + X_L - S_L - T_L + \lambda\Big(\frac{\varepsilon^{\frac{mn}{mn-m-n}} (x - s - \hat{t})^{\frac{n}{mn-m-n}} \hat{t}^{\frac{m}{mn-m-n}} s}{\beta^{\frac{n}{mn-m-n}} \gamma^{\frac{m}{mn-m-n}}} + S_L - K\Big),$$

then

$$\frac{\partial \mathcal{L}}{\partial S_L} = -1 + \lambda,$$

$$\frac{\partial \mathcal{L}}{\partial s} = - \frac{\varepsilon^{\frac{mn}{mn-m-n}} (x - \hat{s} - t)^{\frac{m}{(m-1)(mn-m-n)}} (x - s - t)^{\frac{1}{m-1}} t^{\frac{m}{mn-m-n}}}{\beta^{\frac{n}{mn-m-n}} \gamma^{\frac{m}{mn-m-n}}} + \lambda\Big(\frac{\varepsilon^{\frac{mn}{mn-m-n}} (x - s - \hat{t})^{\frac{n}{mn-m-n}} \hat{t}^{\frac{m}{mn-m-n}}}{\beta^{\frac{n}{mn-m-n}} \gamma^{\frac{m}{mn-m-n}}} - \frac{n}{mn-m-n} \frac{\varepsilon^{\frac{mn}{mn-m-n}} (x - s - \hat{t})^{\frac{n}{mn-m-n}-1} \hat{t}^{\frac{m}{mn-m-n}} s}{\beta^{\frac{n}{mn-m-n}} \gamma^{\frac{m}{mn-m-n}}}\Big).$$

If $\lambda = 1$, then

$$\frac{\partial \mathcal{L}}{\partial S_L} = 0,$$

$$\frac{\partial \mathcal{L}}{\partial s} = - \frac{n}{mn-m-n} \frac{\varepsilon^{\frac{mn}{mn-m-n}} (x - s - t)^{\frac{n}{mn-m-n}-1} t^{\frac{m}{mn-m-n}} s}{\beta^{\frac{n}{mn-m-n}} \gamma^{\frac{m}{mn-m-n}}} < 0.$$

Hence, $s = 0$ and no matter what value of $S_L \in [0, X_L]$ be, still violates investors' individual rationality condition due to (2), the riskiness of the project.
If $\lambda < 1$, then

$$\frac{\partial \mathcal{L}}{\partial S_L} < 0,$$

$$\frac{\partial \mathcal{L}}{\partial s} = -Y + \lambda Z,$$

where

$$Y = \frac{\varepsilon^{\frac{mn}{mn-m-n}} (x - \hat{s} - t)^{\frac{m}{(m-1)(mn-m-n)}} (x - s - t)^{\frac{1}{m-1}} t^{\frac{m}{mn-m-n}}}{\beta^{\frac{n}{mn-m-n}} \gamma^{\frac{m}{mn-m-n}}},$$

$$Z = \frac{\varepsilon^{\frac{mn}{mn-m-n}} (x - s - \hat{t})^{\frac{n}{mn-m-n}} \hat{t}^{\frac{m}{mn-m-n}}}{\beta^{\frac{n}{mn-m-n}} \gamma^{\frac{m}{mn-m-n}}} - \frac{n}{mn-m-n} \frac{\varepsilon^{\frac{mn}{mn-m-n}} (x - s - \hat{t})^{\frac{n}{mn-m-n}-1} \hat{t}^{\frac{m}{mn-m-n}} s}{\beta^{\frac{n}{mn-m-n}} \gamma^{\frac{m}{mn-m-n}}}.$$

If $Z \leq 0$, then $\frac{\partial \mathcal{L}}{\partial s} < 0$; if $Z > 0$, then $\frac{\partial \mathcal{L}}{\partial s} < \frac{\partial \mathcal{L}}{\partial s}|_{\lambda=1} < 1$. Hence, $S_L = s = 0$ still violates the investors' individual rationality condition.
Therefore, we must have $\lambda > 1$. Then, since

$$\frac{\partial \mathcal{L}}{\partial S_L} = -1 + \lambda > 0,$$

we get $S_L = X_L - T_L$, and

$$\frac{\partial \mathcal{L}}{\partial T_L} = -1 < 0,$$

we get $T_L = 0$. As a result, we have $S_L^A = X_L$, and $T_L^A = 0$. To simplify the expressions, we define

$$K - X_L = k \frac{\varepsilon^{\frac{mn}{mn-m-n}}}{\beta^{\frac{n}{mn-m-n}} \gamma^{\frac{m}{mn-m-n}}}. \tag{A.9}$$

Let's rewrite the optimization problem as

$$\mathcal{L} = \Big(1 - \frac{1}{m}\Big)(x - \hat{s} - t)^{\frac{m}{(m-1)(mn-m-n)}} (x - s - t)^{\frac{m}{m-1}} t^{\frac{m}{mn-m-n}} + \lambda\Big((x - s - \hat{t})^{\frac{n}{mn-m-n}} \hat{t}^{\frac{m}{mn-m-n}} s - k\Big), \tag{A.10}$$

where

$$\begin{cases} \frac{\partial \mathcal{L}}{\partial s} = 0, \\ \frac{\partial \mathcal{L}}{\partial t} = 0, \\ \frac{\partial \mathcal{L}}{\partial \lambda} = 0, \\ \lambda > 1, \\ \hat{s} = s^A, \\ \hat{t} = t^A. \end{cases} \tag{A.11}$$

$\frac{\partial \mathcal{L}}{\partial t}|_{\hat{s}=s^A, \hat{t}=t^A} = 0$ implies that $t^A = \frac{x-s^A}{n}$.

Define

$$f(s,t) \equiv (x - s - t)^{\frac{n}{mn-m-n}} t^{\frac{m}{mn-m-n}} s \tag{A.12}$$

where $t \in [\frac{x-s}{n}, \frac{x}{n}]$. Take the first order partial derivative with respect to s, we get

$$\frac{\partial f(s,t)}{\partial s} = \frac{1}{mn-m-n}(x-s-t)^{\frac{n}{mn-m-n}-1} t^{\frac{m}{mn-m-n}} \Big(-ns + (mn-m-n)(x-s-t)\Big),$$

and from the previous steps, we know that $\frac{\partial f(s,t)}{\partial s} > 0$ from $\lambda > 1$. Therefore, we get

$$(mn-m-n)(x-s-t) > ns, \qquad \forall \quad t \in [\frac{x-s}{n}, \frac{x}{n}]. \tag{A.13}$$

Hence, we must have $(mn-m-n)(x-s-\frac{x}{n}) > ns$, which implies $s < \frac{mn-m-n}{mn}x$. Take the first order partial derivative with respect to t, we get

$$\begin{aligned} \frac{\partial f(s,t)}{\partial t} &= \frac{1}{mn-m-n}(x-s-t)^{\frac{n}{mn-m-n}-1} t^{\frac{m}{mn-m-n}-1} s\Big(-nt + m(x-s-t)\Big), \\ &> \frac{1}{mn-m-n}(x-s-t)^{\frac{n}{mn-m-n}-1} t^{\frac{m}{mn-m-n}-1} s\Big(m(x-s) - \frac{m+n}{n}x\Big), \\ &> \frac{1}{mn-m-n}(x-s-t)^{\frac{n}{mn-m-n}-1} t^{\frac{m}{mn-m-n}-1} s\Big(mx - \frac{mn-m-n}{n}x - \frac{m+n}{n}x\Big) = 0, \end{aligned}$$

where the first inequality is due to $t \in [\frac{x-s}{n}, \frac{x}{n}]$ and the second inequality is due to $s \in (0, \frac{mn-m-n}{mn}x)$. As a result, we get

$$\begin{cases} \frac{\partial f(s,t)}{\partial s} & > 0, \\ \frac{\partial f(s,t)}{\partial t} & > 0, \qquad \forall \quad s \in [0, \frac{mn-m-n}{mn}x] \quad \text{and} \quad t \in [\frac{x-s}{n}, \frac{x}{n}]. \end{cases} \tag{A.14}$$

This implies that the investos' expected payoff increases with s and t and for a fixed level of k, a lower t requires a higher s, and vice versa. Since $f(0,t) = 0 < k$ and in order to have $f(s,t) = k$, using the Mean Value Theorem, we must have

$$\begin{aligned} &f(\frac{mn-m-n}{mn}x, \frac{x - \frac{mn-m-n}{mn}x}{n}) \\ =&\Big(\frac{mn+n^2-m-n}{n^2}\Big)^{\frac{n}{mn-m-n}}\Big(\frac{m+n}{mn}\Big)^{\frac{m}{mn-m-n}}\Big(\frac{mn-m-n}{mn}\Big)\Big(\frac{x}{m}\Big)^{\frac{n}{mn-m-n}}\Big(\frac{x}{n}\Big)^{\frac{m}{mn-m-n}}x \geq k. \end{aligned} \tag{A.15}$$

Therefore, in order to avoid credit rationing, we require that

$$x \geq x^A \equiv \Big(\frac{n^2}{mn+n^2-m-n}\Big)^{\frac{1}{m}}\Big(\frac{mn}{m+n}\Big)^{\frac{1}{n}} m^{\frac{1}{m}} n^{\frac{1}{n}}\Big(\frac{mn}{mn-m-n}\Big)^{\frac{mn-m-n}{mn}}\frac{\beta^{\frac{1}{m}}\gamma^{\frac{1}{n}}}{\varepsilon}(K-X_L)^{\frac{mn-m-n}{mn}}. \tag{A.16}$$

Under conditions $x \geq x^A$ and (9), s^A exists and uniquely determined in $f(s^A, \frac{x-s^A}{n}) = k$.

□

Proof of Proposition 2. In the venture capital financing scheme, the LPs' maximization problem in stage two and the venture capitalist's maximization problem in stage three is

$$\begin{cases} e(B,S) = \arg\max\limits_{e} \;\; \varepsilon e a(B,S)(X_L - B_L + x - b) + (1-\varepsilon e a(B,S))(X_L - B_L) - \beta\frac{e^m}{m}, \\ a(B,S) = \arg\max\limits_{a} \;\; \varepsilon e(B,S)a(B_L - S_L + b - s) + (1-\varepsilon e(B,S)a)(B_L - S_L) - \gamma\frac{a^n}{n}. \end{cases} \tag{A.17}$$

The first order conditions are

$$\begin{cases} \varepsilon a(B,S)(x-b) - \beta e^{m-1} = 0, \\ \varepsilon e(B,S)(b-s) - \gamma a^{n-1} = 0, \end{cases}$$

and solving the system of first order conditions in $(e(B,S), a(B,S))$, we obtain

$$e(B,S) = \frac{\varepsilon^{\frac{n}{mn-m-n}}(x-b)^{\frac{n-1}{mn-m-n}}(b-s)^{\frac{1}{mn-m-n}}}{\beta^{\frac{n-1}{mn-m-n}}\gamma^{\frac{1}{mn-m-n}}}, \tag{A.18}$$

$$a(B,S) = \frac{\varepsilon^{\frac{m}{mn-m-n}}(x-b)^{\frac{1}{mn-m-n}}(b-s)^{\frac{m-1}{mn-m-n}}}{\beta^{\frac{1}{mn-m-n}}\gamma^{\frac{m-1}{mn-m-n}}}. \tag{A.19}$$

The LPs' expected payoff, therefore, is

$$\begin{aligned} I(B,S) &= \varepsilon e(B,S)a(B,S)(S_L + s) + (1 - \varepsilon e(B,S)a(B,S))S_L, \\ &= \frac{\varepsilon^{\frac{mn}{mn-m-n}}(x-b)^{\frac{n}{mn-m-n}}(b-s)^{\frac{m}{mn-m-n}}s}{\beta^{\frac{n}{mn-m-n}}\gamma^{\frac{m}{mn-m-n}}} + S_L, \end{aligned} \tag{A.20}$$

and the venture capitalist's expected payoff is

$$A(B,S) = \varepsilon e(B,S)a(B,S)(B_L - S_L + b - s) + (1 - \varepsilon e(B,S)a(B,S))(B_L - S_L) - \gamma \frac{a(B,S)^n}{n},$$
$$= \left(1 - \frac{1}{n}\right) \frac{\varepsilon^{\frac{mn}{mn-m-n}} (x-b)^{\frac{n}{mn-m-n}} (b-s)^{\frac{n(m-1)}{mn-m-n}}}{\beta^{\frac{n}{mn-m-n}} \gamma^{\frac{m}{mn-m-n}}} + B_L - S_L. \tag{A.21}$$

The venture capitalist's maximization problem in stage two becomes

$$\max_S \ \left(1 - \frac{1}{n}\right) \frac{\varepsilon^{\frac{mn}{mn-m-n}} (x-b)^{\frac{n}{mn-m-n}} (b-s)^{\frac{n(m-1)}{mn-m-n}}}{\beta^{\frac{n}{mn-m-n}} \gamma^{\frac{m}{mn-m-n}}} + B_L - S_L,$$

subject to

$$\begin{cases} \frac{\varepsilon^{\frac{mn}{mn-m-n}} (x-b)^{\frac{n}{mn-m-n}} (b-s)^{\frac{m}{mn-m-n}} s}{\beta^{\frac{n}{mn-m-n}} \gamma^{\frac{m}{mn-m-n}}} + S_L \geq K, \\ s \geq 0, \\ b - s > 0, \\ S_L \geq 0, \\ B_L - S_L \geq 0. \end{cases}$$

Suppose

$$\mathcal{L} = \left(1 - \frac{1}{n}\right) \frac{\varepsilon^{\frac{mn}{mn-m-n}} (x-b)^{\frac{n}{mn-m-n}} (b-s)^{\frac{n(m-1)}{mn-m-n}}}{\beta^{\frac{n}{mn-m-n}} \gamma^{\frac{m}{mn-m-n}}} + B_L - S_L,$$

then $\frac{\partial \mathcal{L}}{\partial S_L} < 0$ and $\frac{\partial \mathcal{L}}{\partial s} < 0$ imply that $S_L = s = 0$ which violates the LPs' individual rationality condition. Therefore, the LPs' participation constraint must be binding. Now, suppose

$$\mathcal{L} = \left(1 - \frac{1}{n}\right) \frac{\varepsilon^{\frac{mn}{mn-m-n}} (x-b)^{\frac{n}{mn-m-n}} (b-s)^{\frac{n(m-1)}{mn-m-n}}}{\beta^{\frac{n}{mn-m-n}} \gamma^{\frac{m}{mn-m-n}}} + B_L - S_L$$
$$+ \lambda \left(\frac{\varepsilon^{\frac{mn}{mn-m-n}} (x-b)^{\frac{n}{mn-m-n}} (b-s)^{\frac{m}{mn-m-n}} s}{\beta^{\frac{n}{mn-m-n}} \gamma^{\frac{m}{mn-m-n}}} + S_L - K \right),$$

then

$$\frac{\partial \mathcal{L}}{\partial S_L} = -1 + \lambda,$$
$$\frac{\partial \mathcal{L}}{\partial s} = -\frac{(m-1)(n-1)}{mn-m-n} \frac{\varepsilon^{\frac{mn}{mn-m-n}} (x-b)^{\frac{n}{mn-m-n}} (b-s)^{\frac{m}{mn-m-n}}}{\beta^{\frac{n}{mn-m-n}} \gamma^{\frac{m}{mn-m-n}}}$$
$$+ \lambda \left(\frac{\varepsilon^{\frac{mn}{mn-m-n}} (x-b)^{\frac{n}{mn-m-n}} (b-s)^{\frac{m}{mn-m-n}}}{\beta^{\frac{n}{mn-m-n}} \gamma^{\frac{m}{mn-m-n}}} - \frac{m}{mn-m-n} \frac{\varepsilon^{\frac{mn}{mn-m-n}} (x-b)^{\frac{n}{mn-m-n}} (b-s)^{\frac{m}{mn-m-n}-1} s}{\beta^{\frac{n}{mn-m-n}} \gamma^{\frac{m}{mn-m-n}}} \right).$$

If $\lambda = 1$, then

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial S_L} &= 0,\\ \frac{\partial \mathcal{L}}{\partial s} &= -\frac{1}{mn-m-n}\frac{\varepsilon^{\frac{mn}{mn-m-n}}(x-b)^{\frac{n}{mn-m-n}}(b-s)^{\frac{m}{mn-m-n}}}{\beta^{\frac{n}{mn-m-n}}\gamma^{\frac{m}{mn-m-n}}}\\ &\quad -\frac{m}{mn-m-n}\frac{\varepsilon^{\frac{mn}{mn-m-n}}(x-b)^{\frac{n}{mn-m-n}}(b-s)^{\frac{m}{mn-m-n}-1}s}{\beta^{\frac{n}{mn-m-n}}\gamma^{\frac{m}{mn-m-n}}} < 0.\end{aligned}$$

Hence, $s = 0$ and no matter what value of $S_L \in [0, B_L]$ be, still violates LPs' individual rationality condition due to (2), the riskiness of the project.
If $\lambda < 1$, then

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial S_L} &< 0,\\ \frac{\partial \mathcal{L}}{\partial s} &= -Y + \lambda Z,\end{aligned}$$

where

$$\begin{aligned}Y &= \frac{(m-1)(n-1)}{mn-m-n}\frac{\varepsilon^{\frac{mn}{mn-m-n}}(x-b)^{\frac{n}{mn-m-n}}(b-s)^{\frac{m}{mn-m-n}}}{\beta^{\frac{n}{mn-m-n}}\gamma^{\frac{m}{mn-m-n}}},\\ Z &= \frac{\varepsilon^{\frac{mn}{mn-m-n}}(x-b)^{\frac{n}{mn-m-n}}(b-s)^{\frac{m}{mn-m-n}}}{\beta^{\frac{n}{mn-m-n}}\gamma^{\frac{m}{mn-m-n}}} - \frac{m}{mn-m-n}\frac{\varepsilon^{\frac{mn}{mn-m-n}}(x-b)^{\frac{n}{mn-m-n}}(b-s)^{\frac{m}{mn-m-n}-1}s}{\beta^{\frac{n}{mn-m-n}}\gamma^{\frac{m}{mn-m-n}}}.\end{aligned}$$

If $Z \leq 0$, then $\frac{\partial \mathcal{L}}{\partial s} < 0$; if $Z > 0$, then $\frac{\partial \mathcal{L}}{\partial s} < \frac{\partial \mathcal{L}}{\partial s}|_{\lambda=1} < 1$. Hence, $S_L = s = 0$ still violates the LPs' individual rationality condition.
Therefore, we must have $\lambda > 1$. Then, since

$$\frac{\partial \mathcal{L}}{\partial S_L} = -1 + \lambda > 0,$$

we get $S_L(B) = B_L$ and $s(B)$ is determined in

$$\frac{\varepsilon^{\frac{mn}{mn-m-n}}(x-b)^{\frac{n}{mn-m-n}}(b-s(B))^{\frac{m}{mn-m-n}}s(B)}{\beta^{\frac{n}{mn-m-n}}\gamma^{\frac{m}{mn-m-n}}} + B_L - K = 0. \tag{A.22}$$

Therefore, S_L only depends on B_L while s depends on both B_L and b.

Define $g(B_L, b, s) = \frac{\varepsilon^{\frac{mn}{mn-m-n}}(x-b)^{\frac{n}{mn-m-n}}(b-s)^{\frac{m}{mn-m-n}}s}{\beta^{\frac{n}{mn-m-n}}\gamma^{\frac{m}{mn-m-n}}} + B_L - K$. According to the Implicit Function Theorem, we know

$$\frac{\partial s(B)}{\partial B_L} = -\frac{\frac{\partial g(B_L,b,s)}{\partial B_L}}{\frac{\partial g(B_L,b,s)}{\partial s}} = -\frac{1}{\frac{\varepsilon^{\frac{mn}{mn-m-n}}(x-b)^{\frac{n}{mn-m-n}}(b-s(B))^{\frac{m}{mn-m-n}-1}}{\beta^{\frac{n}{mn-m-n}}\gamma^{\frac{m}{mn-m-n}}}\left((b-s(B)) - \frac{m}{mn-m-n}s(B)\right)}, \tag{A.23}$$

$$\frac{\partial s(B)}{\partial b} = -\frac{\frac{\partial g(B_L,b,s)}{\partial b}}{\frac{\partial g(B_L,b,s)}{\partial s}} = -\frac{-\frac{n}{mn-m-n}(b-s(B))s(B) + \frac{m}{mn-m-n}(x-b)s(B)}{(x-b)(b-s(B)) - \frac{m}{mn-m-n}(x-b)s(B)}. \tag{A.24}$$

The entrepreneur's optimal effort choice in stage one is

$$\begin{cases} e(B) = \arg\max_{e} \ \varepsilon e a(B)(X_L - B_L + x - b) + (1 - \varepsilon e a(B))(X_L - B_L) - \beta \frac{e^m}{m}, \\ a(B) = \arg\max_{a} \ \varepsilon e(B) a(b - s(B)) - \gamma \frac{a^n}{n}. \end{cases} \tag{A.25}$$

The first order conditions are

$$\begin{cases} \varepsilon a(B)(x-b) - \beta e^{m-1} = 0, \\ \varepsilon e(B)(b - s(B)) - \gamma a^{n-1} = 0, \end{cases}$$

and solving the system of first order conditions in $(e(B), a(B))$, we obtain

$$e(B) = \frac{\varepsilon^{\frac{n}{mn-m-n}} (x-b)^{\frac{n-1}{mn-m-n}} (b - s(B))^{\frac{1}{mn-m-n}}}{\beta^{\frac{n-1}{mn-m-n}} \gamma^{\frac{1}{mn-m-n}}}, \tag{A.26}$$

$$a(B) = \frac{\varepsilon^{\frac{m}{mn-m-n}} (x-b)^{\frac{1}{mn-m-n}} (b - s(B))^{\frac{m-1}{mn-m-n}}}{\beta^{\frac{1}{mn-m-n}} \gamma^{\frac{m-1}{mn-m-n}}}. \tag{A.27}$$

Therefore her expected payoff is

$$\begin{aligned} E(B) =& \varepsilon e(B) a(B)(X_L - B_L + x - b) + (1 - \varepsilon e(B) a(B))(X_L - B_L) - \beta \frac{e(B)^m}{m} \\ =& \Big(1 - \frac{1}{m}\Big) \frac{\varepsilon^{\frac{mn}{mn-m-n}} (x-b)^{\frac{m(n-1)}{mn-m-n}} (b - s(B))^{\frac{m}{mn-m-n}}}{\beta^{\frac{n}{mn-m-n}} \gamma^{\frac{m}{mn-m-n}}} + X_L - B_L. \end{aligned} \tag{A.28}$$

The entrepreneur's maximization problem in stage one becomes

$$\max_{B} \ \Big(1 - \frac{1}{m}\Big) \frac{\varepsilon^{\frac{mn}{mn-m-n}} (x-b)^{\frac{m(n-1)}{mn-m-n}} (b - s(B))^{\frac{m}{mn-m-n}}}{\beta^{\frac{n}{mn-m-n}} \gamma^{\frac{m}{mn-m-n}}} + X_L - B_L,$$

subject to

$$\begin{cases} b > 0, \\ x - b > 0, \\ B_L \geq 0, \\ X_L - B_L \geq 0. \end{cases}$$

Consider

$$\mathcal{L} = \Big(1 - \frac{1}{m}\Big) \frac{\varepsilon^{\frac{mn}{mn-m-n}} (x-b)^{\frac{m(n-1)}{mn-m-n}} (b - s(B))^{\frac{m}{mn-m-n}}}{\beta^{\frac{n}{mn-m-n}} \gamma^{\frac{m}{mn-m-n}}} + X_L - B_L,$$

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial B_L} &= -\frac{m-1}{m} \frac{m}{mn - m - n} \frac{\varepsilon^{\frac{mn}{mn-m-n}} (x-b)^{\frac{m(n-1)}{mn-m-n}} (b - s(B))^{\frac{m}{mn-m-n} - 1}}{\beta^{\frac{n}{mn-m-n}} \gamma^{\frac{m}{mn-m-n}}} \frac{\partial s(B)}{\partial B_L} - 1, \\ &= \frac{(m-1)(x-b)}{(mn - m - n)b - (mn - n)s(B)} - 1, \end{aligned}$$

and

$$\frac{\partial \mathcal{L}}{\partial b} = \frac{m-1}{m}\frac{m}{mn-m-n}\frac{\varepsilon^{\frac{mn}{mn-m-n}}(x-b)^{\frac{n}{mn-m-n}}(b-s(B))^{\frac{m}{mn-m-n}-1}}{\beta^{\frac{n}{mn-m-n}}\gamma^{\frac{m}{mn-m-n}}}\times$$
$$\Big(-(n-1)(b-s(B))+(x-b)\big(1-\frac{\partial s(B)}{\partial b}\big)\Big),$$
$$= \frac{m-1}{m}\frac{m}{mn-m-n}\frac{\varepsilon^{\frac{mn}{mn-m-n}}(x-b)^{\frac{n}{mn-m-n}}(b-s(B))^{\frac{m}{mn-m-n}-1}}{\beta^{\frac{n}{mn-m-n}}\gamma^{\frac{m}{mn-m-n}}}\times$$
$$\Big(-(n-1)(b-s(B))+x-b+\frac{-n(b-s(B))s(B)+m(x-b)s(B)}{(mn-m-n)b-(mn-n)s(B)}\Big).$$

We can check that when $s(B)=b-\frac{x}{n}$, $\frac{\partial \mathcal{L}}{\partial B_L}>1$ and $\frac{\partial \mathcal{L}}{\partial b}=0$.

As a result, we have $B_L^{VC}=X_L$, and $b^{VC}-s^{VC}=\frac{x}{n}$. This implies that $t^{VC}=\frac{x}{n}$.

Define

$$f(s,t)\equiv (x-s-t)^{\frac{n}{mn-m-n}}t^{\frac{m}{mn-m-n}}s \tag{A.29}$$

where $t\in[\frac{x-s}{n},\frac{x}{n}]$. Take the first order partial derivative with respect to s, we get

$$\frac{\partial f(s,t)}{\partial s}=\frac{1}{mn-m-n}(x-s-t)^{\frac{n}{mn-m-n}-1}t^{\frac{m}{mn-m-n}}\Big(-ns+(mn-m-n)(x-s-t)\Big),$$

and from the previous steps, we know that $\frac{\partial f(s,t)}{\partial s}>0$ from $\lambda>1$. Therefore, we get

$$(mn-m-n)(x-s-t)>ns,\qquad \forall \quad t\in[\frac{x-s}{n},\frac{x}{n}]. \tag{A.30}$$

Hence, we must have $(mn-m-n)(x-s-\frac{x}{n})>ns$, which implies $s<\frac{mn-m-n}{mn}x$. Take the first order partial derivative with respect to t, we get

$$\frac{\partial f(s,t)}{\partial t}=\frac{1}{mn-m-n}(x-s-t)^{\frac{n}{mn-m-n}-1}t^{\frac{m}{mn-m-n}-1}s\Big(-nt+m(x-s-t)\Big),$$
$$>\frac{1}{mn-m-n}(x-s-t)^{\frac{n}{mn-m-n}-1}t^{\frac{m}{mn-m-n}-1}s\Big(m(x-s)-\frac{m+n}{n}x\Big),$$
$$>\frac{1}{mn-m-n}(x-s-t)^{\frac{n}{mn-m-n}-1}t^{\frac{m}{mn-m-n}-1}s\Big(mx-\frac{mn-m-n}{n}x-\frac{m+n}{n}x\Big)=0,$$

where the first inequality is due to $t\in[\frac{x-s}{n},\frac{x}{n}]$ and the second inequality is due to $s\in(0,\frac{mn-m-n}{mn}x)$. As a result, we get

$$\begin{cases}\frac{\partial f(s,t)}{\partial s} & >0,\\ \frac{\partial f(s,t)}{\partial t} & >0, \qquad \forall \quad s\in[0,\frac{mn-m-n}{mn}x] \quad \text{and} \quad t\in[\frac{x-s}{n},\frac{x}{n}].\end{cases} \tag{A.31}$$

This implies that the LPs' expected payoff increases with s and t and for a fixed level of k, a lower t requires a higher s, and vice versa. Since $f(0,t)=0<k$ and in order to have $f(s,t)=k$, using the Mean Value Theorem, we must have

$$f(\frac{mn-m-n}{mn}x,\frac{x}{n})=\frac{mn-m-n}{mn}\Big(\frac{x}{m}\Big)^{\frac{n}{mn-m-n}}\Big(\frac{x}{n}\Big)^{\frac{m}{mn-m-n}}x\geq k. \tag{A.32}$$

Therefore, in order to avoid credit rationing, we require that

$$x \geq x^{VC} \equiv m^{\frac{1}{m}} n^{\frac{1}{n}} \Big(\frac{mn}{mn-m-n}\Big)^{\frac{mn-m-n}{mn}} \frac{\beta^{\frac{1}{m}}\gamma^{\frac{1}{n}}}{\varepsilon}(K-X_L)^{\frac{mn-m-n}{mn}}. \tag{A.33}$$

Under conditions $x \geq x^{VC}$ and (9), s^{VC} exists and uniquely determined in $f(s^{VC}, \frac{x}{n}) = k$. □

Proof of Proposition 3. Following the derivations before, we know that in both financing scheme

$$E = \Big(1-\frac{1}{m}\Big)\frac{\varepsilon^{\frac{mn}{mn-m-n}}(x-s-t)^{\frac{m(n-1)}{mn-m-n}}t^{\frac{m}{mn-m-n}}}{\beta^{\frac{n}{mn-m-n}}\gamma^{\frac{m}{mn-m-n}}}, \tag{A.34}$$

and therefore, we get

$$\frac{E^{VC}}{E^A} = \frac{(x-s^{VC}-t^{VC})^{\frac{m(n-1)}{mn-m-n}}(t^{VC})^{\frac{m}{mn-m-n}}}{(x-s^A-t^A)^{\frac{m(n-1)}{mn-m-n}}(t^A)^{\frac{m}{mn-m-n}}}, \tag{A.35}$$

$$\Big(\frac{E^{VC}}{E^A}\Big)^{\frac{mn-m-n}{m}} = \frac{(x-s^{VC}-t^{VC})^{n-1}t^{VC}}{(x-s^A-t^A)^{n-1}t^A}. \tag{A.36}$$

Since the right hand side of the last equation increases in s^A, using the fact that $s^{VC} < s^A$ (we prove this in the Corollary 1), we get

$$\Big(\frac{E^{VC}}{E^A}\Big)^{\frac{mn-m-n}{m}} > \frac{(x-s^{VC}-t^{VC})^{n-1}t^{VC}}{(x-s^{VC}-t^A)^{n-1}t^A}. \tag{A.37}$$

Since the right hand side of the above equation decreases in t^A,

$$\frac{\partial \frac{(x-s^{VC}-t^{VC})^{n-1}t^{VC}}{(x-s^{VC}-t^A)^{n-1}t^A}}{\partial t^A} = -\frac{(x-s^{VC}-t^{VC})^{n-1}t^{VC}}{(x-s^{VC}-t^A)^n(t^A)^2}(-nt^A+x-s^{VC}) < 0, \tag{A.38}$$

as $-nt^A + x - s^{VC} = -(x-s^A) + x - s^{VC} = s^A - s^{VC} > 0$, combining $t^A < t^{VC}$ (we prove this in the Corollary 1), we get

$$\Big(\frac{E^{VC}}{E^A}\Big)^{\frac{mn-m-n}{m}} > \frac{(x-s^{VC}-t^{VC})^{n-1}t^{VC}}{(x-s^{VC}-t^{VC})^{n-1}t^{VC}} = 1. \tag{A.39}$$

Hence,

$$E^{VC} > E^A.$$

We know from Proposition 1 and Proposition 2 that

$$x^A = \Big(\frac{n^2}{mn+n^2-m-n}\Big)^{\frac{1}{m}}\Big(\frac{mn}{m+n}\Big)^{\frac{1}{n}} m^{\frac{1}{m}} n^{\frac{1}{n}}\Big(\frac{mn}{mn-m-n}\Big)^{\frac{mn-m-n}{mn}}\frac{\beta^{\frac{1}{m}}\gamma^{\frac{1}{n}}}{\varepsilon}(K-X_L)^{\frac{mn-m-n}{mn}},$$
$$x^{VC} = m^{\frac{1}{m}} n^{\frac{1}{n}}\Big(\frac{mn}{mn-m-n}\Big)^{\frac{mn-m-n}{mn}}\frac{\beta^{\frac{1}{m}}\gamma^{\frac{1}{n}}}{\varepsilon}(K-X_L)^{\frac{mn-m-n}{mn}},$$

and we get

$$\frac{x^A}{x^{VC}} = \Big(\frac{n^2}{mn + n^2 - m - n}\Big)^{\frac{1}{m}} \Big(\frac{mn}{m+n}\Big)^{\frac{1}{n}}. \tag{A.40}$$

Define

$$g(m,n) \equiv \Big(\frac{n^2}{mn + n^2 - m - n}\Big)^{n} \Big(\frac{mn}{m+n}\Big)^{m}, \qquad \text{where} \quad m \geq 2, n \geq 2, \tag{A.41}$$

and define

$$h(m,n) \equiv \log g(m,n) = n(2\log n - \log(m+n) - \log(n-1)) + m(\log m + \log n - \log(m+n)). \tag{A.42}$$

Since

$$\frac{\partial h(m,n)}{\partial m} = \log \frac{mn}{m+n}, \tag{A.43}$$

$$\frac{\partial^2 h(m,n)}{\partial m^2} = \frac{1}{m} - \frac{1}{m+n} > 0, \tag{A.44}$$

we get that $\frac{\partial h(m,n)}{\partial m}$ increases in $m \geq 2$,

$$\frac{\partial h(m,n)}{\partial m} \geq \log \frac{2n}{2+n} \geq \log 1 = 0, \tag{A.45}$$

therefore, $h(m,n)$ also increases in $m \geq 2$. Define

$$f(n) \equiv h(2,n) = n(2\log n - \log(2+n) - \log(n-1)) + 2(\log 2 + \log n - \log(2+n)), \qquad \text{where} \quad n \geq 2. \tag{A.46}$$

Since

$$f'(n) = \frac{n-2}{n(n-1)} - \log \frac{(n-1)(n+2)}{n^2}, \tag{A.47}$$

$$f''(n) = \frac{-3(n-\frac{5}{3})^2 + \frac{13}{3}}{n^2(n-1)^2(2+n)}, \tag{A.48}$$

we know that $f''(n) \geq 0 \ \forall n \in [2, 2.8685]$ and $f''(n) < 0 \ \forall n \in (2.8685, \infty)$. Therefore, $f'(n)$ increases in $n \in [2, 2.8685]$ and decreases in $n \in (2.8685, \infty)$. As $f'(2) = 0$ and $\lim_{n\to\infty} f'(n) = 0$, we know that $f'(n) \geq 0$ $\forall n \in [2, \infty)$. Hence, $f(n)$ increases in $n \in [2, \infty)$. As a result, $h(m,n) > h(2,2) = 0 \ \forall m, n \in (2, \infty)$. This implies that $g(m,n) > 1$. Hence,

$$x^{VC} < x^A.$$

□

Proof of Corollary 1. We know from Proposition 1 and Proposition 2 that

$$t^A = \frac{x - s^A}{n},$$
$$t^{VC} = \frac{x}{n}.$$

- The venture capitalist's compensation t,

$$t^{VC} > t^A,$$

since $s^A > 0$.

- We know that

$$I(S,T) = \frac{\varepsilon^{\frac{mn}{mn-m-n}} (x - s - t)^{\frac{n}{mn-m-n}} t^{\frac{m}{mn-m-n}} s}{\beta^{\frac{n}{mn-m-n}} \gamma^{\frac{m}{mn-m-n}}} + S_L. \tag{A.49}$$

Since $I(S^{VC}, T^{VC}) = I(S^A, T^A) = K$, we know that it must be

$$(x - s^{VC} - t^{VC})^{\frac{n}{mn-m-n}} (t^{VC})^{\frac{m}{mn-m-n}} s^{VC} = (x - s^A - t^A)^{\frac{n}{mn-m-n}} (t^A)^{\frac{m}{mn-m-n}} s^A. \tag{A.50}$$

From the previous steps, we also know that

$$f(s,t) \equiv (x - s - t)^{\frac{n}{mn-m-n}} t^{\frac{m}{mn-m-n}} s \tag{A.51}$$

increases in s and t. As $t^{VC} > t^A$, it must be that

$$s^{VC} < s^A. \tag{A.52}$$

- We know that

$$I(S,T) = \pi s + S_L, \tag{A.53}$$

and since

$$s^{VC} < s^A, \tag{A.54}$$
$$S_L^{VC} = S_L^A = X_L, \tag{A.55}$$

it must be that

$$\pi^{VC} > \pi^A. \tag{A.56}$$

- Following from the previous step, using the Mean Value Theorem, we know that

$$\frac{ds}{dt} = -\frac{\frac{\partial f(s,t)}{\partial t}}{\frac{\partial f(s,t)}{\partial s}} = -\frac{s[m(x - s - t) - nt]}{t[(mn - m - n)(x - s - t) - ns]}. \tag{A.57}$$

Define

$$g(s,t) \equiv s[m(x-s-t)-nt] - t[(mn-m-n)(x-s-t)-ns], \quad = [ms-(mn-m-n)t](x-s-t). \tag{A.58}$$

Suppose $ms-(mn-m-n)t \geq 0$, then since $t \in [\frac{x-s}{n}, \frac{x}{n}]$, we must have $ms-(mn-m-n)\frac{x}{n} > 0$, so $s > \frac{mn-m-n}{mn}x$ which contradicts to (A.31). Therefore $g(s,t) < 0$. This implies that $-1 < \frac{ds}{dt} < 0$. This means that when t increases by Δ, s decreases less than Δ. As a result, $s+t$ increases. Hence, $t^{VC} > t^A$ implies that

$$s^{VC} + t^{VC} > s^A + t^A. \tag{A.59}$$

- (A.59) implies that

$$x - s^{VC} - t^{VC} < x - s^A - t^A. \tag{A.60}$$

- We know that

$$A(S,T) = \left(1-\frac{1}{n}\right)\frac{\varepsilon^{\frac{mn}{mn-m-n}}(x-s-t)^{\frac{n}{mn-m-n}}t^{\frac{n(m-1)}{mn-m-n}}}{\beta^{\frac{n}{mn-m-n}}\gamma^{\frac{m}{mn-m-n}}} + T_L, \tag{A.61}$$

$$I(S,T) = \frac{\varepsilon^{\frac{mn}{mn-m-n}}(x-s-t)^{\frac{n}{mn-m-n}}t^{\frac{m}{mn-m-n}}s}{\beta^{\frac{n}{mn-m-n}}\gamma^{\frac{m}{mn-m-n}}} + S_L. \tag{A.62}$$

Since $T_L^{VC} = T_L^A = 0$, $S_L^{VC} = S_L^A = X_L$, and $I(S^{VC}, T^{VC}) = I(S^A, T^A) = K$, we know it must be that

$$A = \left(1-\frac{1}{n}\right)\frac{t}{s}(I - X_L), \tag{A.63}$$

therefore

$$\frac{A^{VC}}{A^A} = \frac{t^{VC}}{t^A}\frac{s^A}{s^{VC}}. \tag{A.64}$$

As $t^{VC} > t^A$ and $s^{VC} < s^A$, it must be that

$$A^{VC} > A^A. \tag{A.65}$$

- Since

$$e = \frac{\varepsilon^{\frac{n}{mn-m-n}}(x-s-t)^{\frac{n-1}{mn-m-n}}t^{\frac{1}{mn-m-n}}}{\beta^{\frac{n-1}{mn-m-n}}\gamma^{\frac{1}{mn-m-n}}}, \tag{A.66}$$

therefore from (A.39), we know that

$$\left(\frac{e^{VC}}{e^A}\right)^{mn-m-n} = \frac{(x-s^{VC}-t^{VC})^{n-1}t^{VC}}{(x-s^A-t^A)^{n-1}t^A} = \left(\frac{E^{VC}}{E^A}\right)^{\frac{mn-m-n}{m}} > 1. \tag{A.67}$$

Hence,

$$e^{VC} > e^{A}. \tag{A.68}$$

- We know that

$$a = \frac{\varepsilon^{\frac{m}{mn-m-n}}(x-s-t)^{\frac{1}{mn-m-n}} t^{\frac{m-1}{mn-m-n}}}{\beta^{\frac{1}{mn-m-n}}\gamma^{\frac{m-1}{mn-m-n}}}, \tag{A.69}$$

then

$$\frac{a^{VC}}{a^{A}} = \left(\frac{x-s^{VC}-t^{VC}}{x-s^{A}-t^{A}}\right)^{\frac{1}{mn-m-n}} \left(\frac{t^{VC}}{t^{A}}\right)^{\frac{1}{mn-m-n}}, \tag{A.70}$$

$$> \left(\frac{x-s^{VC}-t^{VC}}{x-s^{A}-t^{A}}\right)^{\frac{m-1}{mn-m-n}} \left(\frac{t^{VC}}{t^{A}}\right)^{\frac{1}{mn-m-n}}, \tag{A.71}$$

since $x -^{VC} -t^{VC} < x - s^{A} - t^{A}$. This implies that

$$\left(\frac{a^{VC}}{a^{A}}\right)^{\frac{mn-m-n}{m-1}} > \frac{x-s^{VC}-t^{VC}}{x-s^{A}-t^{A}}\frac{t^{VC}}{t^{A}} > \frac{(x-s^{VC}-t^{VC})^{n-1}t^{VC}}{(x-s^{A}-t^{A})^{n-1}t^{A}} > 1, \tag{A.72}$$

where the second to the last inequality is derived using $x -^{VC} -t^{VC} < x - s^{A} - t^{A}$ again and the last inequality is derived due to (A.39). Hence,

$$a^{VC} > a^{A}. \tag{A.73}$$

□

Proof of Proposition 4. From Proposition 2 and Proposition B.2 in Section B.2, we know that

$$t^{VC} = t^{SB} = \frac{x}{n}.$$

Therefore $E^{VC} = E^{SB}$. □

Proof of Corollary 2. When the LPs cannot verify B, the LPs' maximization problem in stage two becomes

$$\begin{cases} e(\hat{B},S) = \arg\max\limits_{e} \ \ \varepsilon e a(\hat{B},S)(X_L - \hat{B}_L + x - \hat{b}) + (1-\varepsilon e a(\hat{B},S))(X_L - \hat{B}_L) - \beta\frac{e^m}{m}, \\ a(\hat{B},S) = \arg\max\limits_{a} \ \ \varepsilon e(\hat{B},S)a(\hat{B}_L - S_L + \hat{b} - s) + (1-\varepsilon e(\hat{B},S)a)(\hat{B}_L - S_L) - \gamma\frac{a^n}{n}. \end{cases} \tag{A.74}$$

The first order conditions are

$$\begin{cases} \varepsilon a(\hat{B},S)(x-\hat{b}) - \beta e^{m-1} = 0, \\ \varepsilon e(\hat{B},S)(\hat{b}-s) - \gamma a^{n-1} = 0, \end{cases}$$

and solving the system of first order conditions in $(e(\hat{B},S), a(\hat{B},S))$, we obtain

$$e(\hat{B},S) = \frac{\varepsilon^{\frac{n}{mn-m-n}} (x-\hat{b})^{\frac{n-1}{mn-m-n}} (\hat{b}-s)^{\frac{1}{mn-m-n}}}{\beta^{\frac{n-1}{mn-m-n}} \gamma^{\frac{1}{mn-m-n}}}, \tag{A.75}$$

$$a(\hat{B},S) = \frac{\varepsilon^{\frac{m}{mn-m-n}} (x-\hat{b})^{\frac{1}{mn-m-n}} (\hat{b}-s)^{\frac{m-1}{mn-m-n}}}{\beta^{\frac{1}{mn-m-n}} \gamma^{\frac{m-1}{mn-m-n}}}. \tag{A.76}$$

The LPs' expected payoff, therefore, is

$$\begin{aligned} I(\hat{B},S) &= \varepsilon e(\hat{B},S)a(\hat{B},S)(S_L+s) + (1-\varepsilon e(\hat{B},S)a(\hat{B},S))S_L, \\ &= \frac{\varepsilon^{\frac{mn}{mn-m-n}} (x-\hat{b})^{\frac{n}{mn-m-n}} (\hat{b}-s)^{\frac{m}{mn-m-n}} s}{\beta^{\frac{n}{mn-m-n}} \gamma^{\frac{m}{mn-m-n}}} + S_L. \end{aligned} \tag{A.77}$$

The venture capitalist's expected payoff stays the same as that in the venture capital financing scheme,

$$\begin{aligned} A(B,S) &= \varepsilon e(B,S)a(B,S)(B_L-S_L+b-s) + (1-\varepsilon e(B,S)a(B,S))(B_L-S_L) - \gamma\frac{a(B,S)^n}{n}, \\ &= \left(1-\frac{1}{n}\right)\frac{\varepsilon^{\frac{mn}{mn-m-n}} (x-b)^{\frac{n}{mn-m-n}} (b-s)^{\frac{n(m-1)}{mn-m-n}}}{\beta^{\frac{n}{mn-m-n}} \gamma^{\frac{m}{mn-m-n}}} + B_L - S_L \end{aligned} \tag{A.78}$$

therefore, his maximization problem in stage two becomes

$$\max_{S} \ \left(1-\frac{1}{n}\right)\frac{\varepsilon^{\frac{mn}{mn-m-n}} (x-b)^{\frac{n}{mn-m-n}} (b-s)^{\frac{n(m-1)}{mn-m-n}}}{\beta^{\frac{n}{mn-m-n}} \gamma^{\frac{m}{mn-m-n}}} + B_L - S_L,$$

subject to

$$\begin{cases} \frac{\varepsilon^{\frac{mn}{mn-m-n}} (x-\hat{b})^{\frac{n}{mn-m-n}} (\hat{b}-s)^{\frac{m}{mn-m-n}} s}{\beta^{\frac{n}{mn-m-n}} \gamma^{\frac{m}{mn-m-n}}} + S_L \geq K, \\ s \geq 0, \\ b-s > 0, \\ S_L \geq 0, \\ B_L - S_L \geq 0. \end{cases}$$

Suppose

$$\mathcal{L} = \left(1-\frac{1}{n}\right)\frac{\varepsilon^{\frac{mn}{mn-m-n}} (x-b)^{\frac{n}{mn-m-n}} (b-s)^{\frac{n(m-1)}{mn-m-n}}}{\beta^{\frac{n}{mn-m-n}} \gamma^{\frac{m}{mn-m-n}}} + B_L - S_L,$$

then $\frac{\partial \mathcal{L}}{\partial S_L} < 0$ and $\frac{\partial \mathcal{L}}{\partial s} < 0$ imply that $S_L = s = 0$ which violates the LPs' individual rationality condition. Therefore, the LPs' participation constraint must be binding. Now, suppose

$$\mathcal{L} = \Big(1 - \frac{1}{n}\Big) \frac{\varepsilon^{\frac{mn}{mn-m-n}} (x-b)^{\frac{n}{mn-m-n}} (b-s)^{\frac{n(m-1)}{mn-m-n}}}{\beta^{\frac{n}{mn-m-n}} \gamma^{\frac{m}{mn-m-n}}} + B_L - S_L + \lambda\Big(\frac{\varepsilon^{\frac{mn}{mn-m-n}} (x-\hat{b})^{\frac{n}{mn-m-n}} (\hat{b}-s)^{\frac{m}{mn-m-n}} s}{\beta^{\frac{n}{mn-m-n}} \gamma^{\frac{m}{mn-m-n}}} + S_L - K \Big),$$

then

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial S_L} &= -1 + \lambda, \\
\frac{\partial \mathcal{L}}{\partial s} &= - \frac{(m-1)(n-1)}{mn-m-n} \frac{\varepsilon^{\frac{mn}{mn-m-n}} (x-b)^{\frac{n}{mn-m-n}} (b-s)^{\frac{m}{mn-m-n}}}{\beta^{\frac{n}{mn-m-n}} \gamma^{\frac{m}{mn-m-n}}} \\
&\quad + \lambda\Big(\frac{\varepsilon^{\frac{mn}{mn-m-n}} (x-\hat{b})^{\frac{n}{mn-m-n}} (\hat{b}-s)^{\frac{m}{mn-m-n}}}{\beta^{\frac{n}{mn-m-n}} \gamma^{\frac{m}{mn-m-n}}} - \frac{m}{mn-m-n} \frac{\varepsilon^{\frac{mn}{mn-m-n}} (x-\hat{b})^{\frac{n}{mn-m-n}} (\hat{b}-s)^{\frac{m}{mn-m-n}-1} s}{\beta^{\frac{n}{mn-m-n}} \gamma^{\frac{m}{mn-m-n}}} \Big).
\end{aligned}$$

If $\lambda = 1$, then

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial S_L} &= 0, \\
\frac{\partial \mathcal{L}}{\partial s}\Big|_{\hat{b}=b=b^U} &= - \frac{1}{mn-m-n} \frac{\varepsilon^{\frac{mn}{mn-m-n}} (x-b^U)^{\frac{n}{mn-m-n}} (b^U-s)^{\frac{m}{mn-m-n}}}{\beta^{\frac{n}{mn-m-n}} \gamma^{\frac{m}{mn-m-n}}} \\
&\quad - \frac{m}{mn-m-n} \frac{\varepsilon^{\frac{mn}{mn-m-n}} (x-b^U)^{\frac{n}{mn-m-n}} (b^U-s)^{\frac{m}{mn-m-n}-1} s}{\beta^{\frac{n}{mn-m-n}} \gamma^{\frac{m}{mn-m-n}}} < 0.
\end{aligned}$$

Hence, $s = 0$ and no matter what value of $S_L \in [0, B_L]$ be, still violates LPs' individual rationality condition due to (2), the riskiness of the project.
If $\lambda < 1$, then

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial S_L} &< 0, \\
\frac{\partial \mathcal{L}}{\partial s}\Big|_{\hat{b}=b=b^U} &= -Y + \lambda Z,
\end{aligned}$$

where

$$\begin{aligned}
Y &= \frac{(m-1)(n-1)}{mn-m-n} \frac{\varepsilon^{\frac{mn}{mn-m-n}} (x-b^U)^{\frac{n}{mn-m-n}} (b^U-s)^{\frac{m}{mn-m-n}}}{\beta^{\frac{n}{mn-m-n}} \gamma^{\frac{m}{mn-m-n}}}, \\
Z &= \frac{\varepsilon^{\frac{mn}{mn-m-n}} (x-b^U)^{\frac{n}{mn-m-n}} (b^U-s)^{\frac{m}{mn-m-n}}}{\beta^{\frac{n}{mn-m-n}} \gamma^{\frac{m}{mn-m-n}}} - \frac{m}{mn-m-n} \frac{\varepsilon^{\frac{mn}{mn-m-n}} (x-b^U)^{\frac{n}{mn-m-n}} (b^U-s)^{\frac{m}{mn-m-n}-1} s}{\beta^{\frac{n}{mn-m-n}} \gamma^{\frac{m}{mn-m-n}}}.
\end{aligned}$$

If $Z \leq 0$, then $\frac{\partial \mathcal{L}}{\partial s} < 0$; if $Z > 0$, then $\frac{\partial \mathcal{L}}{\partial s} < \frac{\partial \mathcal{L}}{\partial s}|_{\lambda=1} < 1$. Hence, $S_L = s = 0$ still violates the LPs' individual rationality condition.
Therefore, we must have $\lambda > 1$. Then, since

$$\frac{\partial \mathcal{L}}{\partial S_L} = -1 + \lambda > 0,$$

we get $S_L(\hat{B}, B) = B_L$ and $s(\hat{B}, B)$ is determined in

$$\frac{\varepsilon^{\frac{mn}{mn-m-n}} (x - \hat{b})^{\frac{n}{mn-m-n}} (\hat{b} - s(B))^{\frac{m}{mn-m-n}} s(B)}{\beta^{\frac{n}{mn-m-n}} \gamma^{\frac{m}{mn-m-n}}} + B_L - K = 0. \tag{A.79}$$

Therefore, S_L only depends on B_L while s depends on both B_L and $\hat{b}$. From now on, we suppress the dependent variables, and denote $S_L(B_L)$ and $s(\hat{b}, B_L)$.

The entrepreneur's optimal effort choice in stage one is

$$\begin{cases} e(\hat{b}, B_L, b) = \arg\max\limits_e \ \ \varepsilon e a(\hat{b}, B_L, b)(X_L - B_L + x - b) + (1 - \varepsilon e a(\hat{b}, B_L, b))(X_L - B_L) - \beta \frac{e^m}{m}, \\ a(\hat{b}, B_L, b) = \arg\max\limits_a \ \ \varepsilon e(\hat{b}, B_L, b) a (b - s(\hat{b}, B)) - \gamma \frac{a^n}{n}. \end{cases} \tag{A.80}$$

The first order conditions are

$$\begin{cases} \varepsilon a(\hat{b}, B_L, b)(x - b) - \beta e^{m-1} = 0, \\ \varepsilon e(\hat{b}, B_L, b)(b - s(\hat{b}, B)) - \gamma a^{n-1} = 0, \end{cases}$$

and solving the system of first order conditions in $(e(\hat{b}, B_L, b), a(\hat{b}, B_L, b))$, we obtain

$$e(\hat{b}, B_L, b) = \frac{\varepsilon^{\frac{n}{mn-m-n}} (x - b)^{\frac{n-1}{mn-m-n}} (b - s(\hat{b}, B_L))^{\frac{1}{mn-m-n}}}{\beta^{\frac{n-1}{mn-m-n}} \gamma^{\frac{1}{mn-m-n}}}, \tag{A.81}$$

$$a(\hat{b}, B_L, b) = \frac{\varepsilon^{\frac{m}{mn-m-n}} (x - b)^{\frac{1}{mn-m-n}} (b - s(\hat{b}, B_L))^{\frac{m-1}{mn-m-n}}}{\beta^{\frac{1}{mn-m-n}} \gamma^{\frac{m-1}{mn-m-n}}}. \tag{A.82}$$

Therefore her expected payoff is

$$\begin{aligned} E(\hat{b}, B_L, b) =& \varepsilon e(\hat{b}, B_L, b) a(\hat{b}, B_L, b)(X_L - B_L + x - b) + (1 - \varepsilon e(\hat{b}, B_L, b) a(\hat{b}, B_L, b))(X_L - B_L) - \beta \frac{e(\hat{b}, B_L, b)^m}{m} \\ =& \Big(1 - \frac{1}{m}\Big) \frac{\varepsilon^{\frac{mn}{mn-m-n}} (x - b)^{\frac{m(n-1)}{mn-m-n}} (b - s(\hat{b}, B_L))^{\frac{m}{mn-m-n}}}{\beta^{\frac{n}{mn-m-n}} \gamma^{\frac{m}{mn-m-n}}} + X_L - B_L. \end{aligned} \tag{A.83}$$

The entrepreneur's maximization problem in stage one becomes

$$\max_B \ \Big(1 - \frac{1}{m}\Big) \frac{\varepsilon^{\frac{mn}{mn-m-n}} (x - b)^{\frac{m(n-1)}{(mn-m-n}} (b - s(\hat{b}, B_L))^{\frac{m}{mn-m-n}}}{\beta^{\frac{n}{mn-m-n}} \gamma^{\frac{m}{mn-m-n}}} + X_L - B_L,$$

subject to

$$\begin{cases} b > 0, \\ x - b > 0, \\ B_L \geq 0, \\ X_L - B_L \geq 0. \end{cases}$$

where, in equilibrium,

$$\hat{b} = b^U.$$

Suppose

$$\mathcal{L} = \left(1 - \frac{1}{m}\right) \frac{\varepsilon^{\frac{mn}{mn-m-n}} (x-b)^{\frac{m(n-1)}{(mn-m-n}} (b - s(\hat{b}, B_L))^{\frac{m}{mn-m-n}}}{\beta^{\frac{n}{mn-m-n}} \gamma^{\frac{m}{mn-m-n}}} + X_L - B_L,$$

then, $\frac{\partial \mathcal{L}}{\partial b}|_{\hat{b}=b^U, s(\hat{b}, B_L^U)=s^U} = o$ implies that $b^U = \frac{x-s^U}{n} + s^U$. Therefore,

$$t^U = B^U - s^U = \frac{x - s^U}{n} = t^A, \tag{A.84}$$

and $E^U = E^A$.

□

Proof of Proposition 5. Replacing S^{VC} and T^{VC} by their values derived in Proposition 2, (69) can be rewritten as

$$\begin{cases} X_L = D, \\ X_L + s^{VC} + 0 + \frac{x}{n} = \alpha(X_L + x), \end{cases} \tag{A.85}$$

and we get $D = X_L$ and $\alpha = \frac{X_L + s^{VC} + \frac{x}{n}}{X_L + x}$. Similarly, (70) can be rewritten as

$$\frac{x}{n} = \beta(X_L + s^{VC} + 0 + \frac{x}{n} - K), \tag{A.86}$$

and we get $\beta = \frac{\frac{x}{n}}{X_L + s^{VC} + \frac{x}{n} - K}$.

□

Appendix B: Optimal Contracts with Verifiable Efforts and/or Contracts

In this appendix, we derive the optimal contracts when both efforts and contracts are verifiable (Section B.1) or only contracts are verifiable (Section B.2).

B.1 Optimal Contracts with Verifiable Efforts and Contracts (First-Best)

We consider a financing scheme where both efforts and contracts are verifiable. We refer to the optimal contracts in this scheme as *first-best*, and we denote the corresponding compensation by (S^{FB}, T^{FB}).

Proposition B.1. *When both efforts and contracts are verifiable, under the assumption of (9) and the condition $x \geq x^{FB}$, the optimal compensation for the investors and the advisor are respectively equal to*

$$S_L^{FB} = X_L, \qquad s^{FB} = \frac{\beta^{\frac{n}{mn-m-n}} \gamma^{\frac{m}{mn-m-n}} (K - X_L)}{\varepsilon^{\frac{mn}{mn-m-n}} x^{\frac{m+n}{mn-m-n}}}, \tag{B.2}$$

$$T_L^{FB} = 0, \qquad t^{FB} = \frac{x}{n}. \tag{B.3}$$

Proof. According to the social value of investing the project (6) and under assumption (9),

$$\begin{aligned} S(e,a) &= \varepsilon ea(X_L + x) + (1 - \varepsilon ea)X_L - \beta \frac{e^m}{m} - \gamma \frac{a^n}{n} - K, \\ &= \varepsilon eax - \beta \frac{e^m}{m} - \gamma \frac{a^n}{n} + X_L - K, \end{aligned} \tag{B.4}$$

efforts from the entrepreneur and the advisor are determined as

$$\begin{cases} e^{FB} = \arg\max\limits_e \ \ \varepsilon e a^{FB} x - \beta \frac{e^m}{m}, \\ a^{FB} = \arg\max\limits_a \ \ \varepsilon e^{FB} a x - \gamma \frac{a^n}{n}. \end{cases} \tag{B.5}$$

The first order conditions are

$$\begin{cases} \varepsilon a^{FB} x - \beta e^{m-1} = 0, \\ \varepsilon e^{FB} x - \gamma a^{n-1} = 0, \end{cases}$$

and solving the system of first order conditions in (e^{FB}, a^{FB}), we obtain

$$\begin{cases} e^{FB} = \dfrac{\varepsilon^{\frac{n}{mn-m-n}}}{\beta^{\frac{n-1}{mn-m-n}} \gamma^{\frac{1}{mn-m-n}}} x^{\frac{n}{mn-m-n}}, \\ a^{FB} = \dfrac{\varepsilon^{\frac{m}{mn-m-n}}}{\beta^{\frac{1}{mn-m-n}} \gamma^{\frac{m-1}{mn-m-n}}} x^{\frac{m}{mn-m-n}}. \end{cases} \tag{B.6}$$

Clearly, both the entrepreneur and the consultant exert effort in equilibrium, since $x > 0$, and the probability of high cash flow is

$$\varepsilon e^{FB} a^{FB} = \frac{\varepsilon^{\frac{mn}{mn-m-n}}}{\beta^{\frac{n}{mn-m-n}} \gamma^{\frac{m}{mn-m-n}}} x^{\frac{m+n}{mn-m-n}} < 1,$$

by (9). Since we assume competitive capital and labor markets, the entrepreneur will offer S^{FB} and T^{SB} to the investors and the advisor such that their expected payoffs equal to their outside options, which are K and 0, respectively:

$$\begin{cases} I^{FB} = \varepsilon e^{FB} a^{FB}(S_L + s) + (1 - \varepsilon e^{FB} a^{FB}) S_L = K, \\ A^{FB} = \varepsilon e^{FB} a^{FB}(T_L + t) + (1 - \varepsilon e^{FB} a^{FB}) T_L - \gamma \frac{(a^{FB})^n}{n} = 0. \end{cases} \tag{B.7}$$

There are multiple solutions to the above equations, and one particular solution is

$$S_L^{FB} = X_L, \qquad s^{FB} = \frac{\beta^{\frac{n}{mn-m-n}} \gamma^{\frac{m}{mn-m-n}} (K - X_L)}{\varepsilon^{\frac{mn}{mn-m-n}} x^{\frac{m+n}{mn-m-n}}}, \qquad T_L^{FB} = 0, \qquad t^{FB} = \frac{x}{n}.$$

Therefore, the entrepreneur's expected payoff is

$$\begin{aligned} E^{FB} &= \varepsilon e^{FB} a^{FB}(X_L - S_L^{FB} - T_L^{FB} + x - s^{FB} - t^{FB}) + (1 - \varepsilon e^{FB} a^{FB})(X_L - S_L^{FB} - T_L^{FB}) - \beta \frac{(e^{FB})^m}{m}, \\ &= \frac{mn - m - n}{mn} \frac{\varepsilon^{\frac{mn}{mn-m-n}}}{\beta^{\frac{n}{mn-m-n}} \gamma^{\frac{m}{mn-m-n}}} x^{\frac{mn}{mn-m-n}} + X_L - K, \\ &= S(e^{FB}, a^{FB}), \end{aligned} \tag{B.8}$$

which coincides with the optimal social value of investing the project. To avoid credit rationing, we require that

$$E^{FB} \geq 0, \tag{B.9}$$

that is

$$x \geq x^{FB} \equiv \left(\frac{mn}{mn - m - n} \right)^{\frac{mn-m-n}{mn}} \frac{\beta^{\frac{1}{m}} \gamma^{\frac{1}{n}}}{\varepsilon} (K - X_L)^{\frac{mn-m-n}{mn}}.$$

□

B.2 Optimal Contracts with Verifiable Contracts

Next, we consider a financing scheme where only contracts are verifiable. We refer to the optimal contracts in this scheme as *second-best*, and we denote the corresponding compensation by (S^{SB}, T^{SB}).

Proposition B.2. *When the contracts are verifiable, under the assumption of (9) and the condition $x \geq x^{VC}$, the optimal compensation for the investors and the advisor are respectively equal to*

$$S_L^{SB} = X_L, \tag{B.10}$$

$$T_L^{SB} = 0, \qquad t^{SB} = \frac{x}{n}, \tag{B.11}$$

and s^{SB} exists and is determined uniquely in

$$\frac{\varepsilon^{\frac{mn}{mn-m-n}}\left(x - s^{SB} - \frac{x}{n}\right)^{\frac{n}{mn-m-n}}\left(\frac{x}{n}\right)^{\frac{m}{mn-m-n}} s^{SB}}{\beta^{\frac{n}{mn-m-n}}\gamma^{\frac{m}{mn-m-n}}} + X_L - K = 0. \tag{B.12}$$

Proof. The entrepreneur's and the advisor's maximization problems in stage two are

$$\begin{cases} e(S,T) = \arg\max\limits_{e} \ \ \varepsilon e a(S,T)(X_L - S_L - T_L + x - s - t) + (1 - \varepsilon e a(S,T))(X_L - S_L - T_L) - \beta \frac{e^m}{m}, \\ a(S,T) = \arg\max\limits_{a} \ \ \varepsilon e(S,T) a(T_L + t) + (1 - \varepsilon e(S,T) a) T_L - \gamma \frac{a^n}{n}. \end{cases} \tag{B.13}$$

The first order conditions are

$$\begin{cases} \varepsilon a(S,T)(x - s - t) - \beta e^{m-1} = 0, \\ \varepsilon e(S,T) t - \gamma a^{n-1} = 0, \end{cases}$$

and solving the system of first order conditions in $(e(S,T), a(S,T))$, we obtain

$$\begin{cases} e(S,T) = \dfrac{\varepsilon^{\frac{n}{mn-m-n}}(x-s-t)^{\frac{n-1}{mn-m-n}} t^{\frac{1}{mn-m-n}}}{\beta^{\frac{n-1}{mn-m-n}}\gamma^{\frac{1}{mn-m-n}}}, \\ a(S,T) = \dfrac{\varepsilon^{\frac{m}{mn-m-n}}(x-s-t)^{\frac{1}{mn-m-n}} t^{\frac{m-1}{mn-m-n}}}{\beta^{\frac{1}{mn-m-n}}\gamma^{\frac{m-1}{mn-m-n}}}. \end{cases} \tag{B.14}$$

In order to induce a strictly positive probability of success of the risky project, both the entrepreneur and the advisor need to exert effort in equilibrium, which requires

$$x - s - t > 0, \tag{B.15}$$

$$t > 0, \tag{B.16}$$

and the probability of success is

$$\varepsilon e(S,T) a(S,T) = \frac{\varepsilon^{\frac{mn}{mn-m-n}}(x - s - t)^{\frac{n}{mn-m-n}} t^{\frac{m}{mn-m-n}}}{\beta^{\frac{n}{mn-m-n}}\gamma^{\frac{m}{mn-m-n}}} < \frac{\varepsilon^{\frac{mn}{mn-m-n}}}{\beta^{\frac{n}{mn-m-n}}\gamma^{\frac{m}{mn-m-n}}} x^{\frac{m+n}{mn-m-n}} < 1,$$

where the first inequality is due to limited liability, $S \in [0, X]$ and $T \in [0, X - S]$, and the second inequality is due to (9).

The expected payoffs of the three players are

$$\begin{aligned}
E(S,T) &= \varepsilon e(S,T)a(S,T)(X_L - S_L - T_L + x - s - t) + (1 - \varepsilon e(S,T)a(S,T))(X_L - S_L - T_L) - \beta\frac{e(S,T)^m}{m}, \\
&= \left(1 - \frac{1}{m}\right)\frac{\varepsilon^{\frac{mn}{mn-m-n}}(x-s-t)^{\frac{m(n-1)}{mn-m-n}}t^{\frac{m}{mn-m-n}}}{\beta^{\frac{n}{mn-m-n}}\gamma^{\frac{m}{mn-m-n}}} + X_L - S_L - T_L > 0, \qquad \text{(B.17)}\\
A(S,T) &= \varepsilon e(S,T)a(S,T)(T_L + t) + (1 - \varepsilon e(S,T)a(S,T))T_L - \gamma\frac{a(S,T)^n}{n}, \\
&= \left(1 - \frac{1}{n}\right)\frac{\varepsilon^{\frac{mn}{mn-m-n}}(x-s-t)^{\frac{n}{mn-m-n}}t^{\frac{n(m-1)}{mn-m-n}}}{\beta^{\frac{n}{mn-m-n}}\gamma^{\frac{m}{mn-m-n}}} + T_L > 0, \qquad \text{(B.18)}\\
I(S,T) &= \varepsilon e(S,T)a(S,T)(S_L + s) + (1 - \varepsilon e(S,T)a(S,T))S_L, \\
&= \frac{\varepsilon^{\frac{mn}{mn-m-n}}(x-s-t)^{\frac{n}{mn-m-n}}t^{\frac{m}{mn-m-n}}s}{\beta^{\frac{n}{mn-m-n}}\gamma^{\frac{m}{mn-m-n}}} + S_L, \qquad \text{(B.19)}
\end{aligned}$$

since $m > 2$ and $n > 2$, and combining limited liability, monotonic contract, and positive efforts, we get

$$\begin{cases} s \geq 0, \\ t > 0, \\ x - s - t > 0, \\ S_L \geq 0, \\ T_L \geq 0, \\ X_L - S_L - T_L \geq 0. \end{cases} \qquad \text{(B.20)}$$

The entrepreneur's maximization problem in stage one becomes

$$\max_{S,T} \left(1 - \frac{1}{m}\right)\frac{\varepsilon^{\frac{mn}{mn-m-n}}(x-s-t)^{\frac{m(n-1)}{mn-m-n}}t^{\frac{m}{mn-m-n}}}{\beta^{\frac{n}{mn-m-n}}\gamma^{\frac{m}{mn-m-n}}} + X_L - S_L - T_L,$$

subject to

$$\frac{\varepsilon^{\frac{mn}{mn-m-n}}(x-s-t)^{\frac{n}{mn-m-n}}t^{\frac{m}{mn-m-n}}s}{\beta^{\frac{n}{mn-m-n}}\gamma^{\frac{m}{mn-m-n}}} + S_L \geq K \quad \text{and} \quad \text{(B.20)}.$$

Suppose

$$\mathcal{L} = \left(1 - \frac{1}{m}\right)\frac{\varepsilon^{\frac{mn}{mn-m-n}}(x-s-t)^{\frac{m(n-1)}{mn-m-n}}t^{\frac{m}{mn-m-n}}}{\beta^{\frac{n}{mn-m-n}}\gamma^{\frac{m}{mn-m-n}}} + X_L - S_L - T_L,$$

then $\frac{\partial \mathcal{L}}{\partial S_L} < 0$ and $\frac{\partial \mathcal{L}}{\partial s} < 0$ imply that $S_L = s = 0$ which violates the investors' individual rationality condition. Therefore, the investors' participation constraint must be binding. Now, suppose

$$\mathcal{L} = \Big(1 - \frac{1}{m}\Big) \frac{\varepsilon^{\frac{mn}{mn-m-n}} (x-s-t)^{\frac{m(n-1)}{mn-m-n}} t^{\frac{m}{mn-m-n}}}{\beta^{\frac{n}{mn-m-n}} \gamma^{\frac{m}{mn-m-n}}} + X_L - S_L - T_L + \lambda\Big(\frac{\varepsilon^{\frac{mn}{mn-m-n}} (x-s-t)^{\frac{n}{mn-m-n}} t^{\frac{m}{mn-m-n}} s}{\beta^{\frac{n}{mn-m-n}} \gamma^{\frac{m}{mn-m-n}}} + S_L - K\Big),$$

then

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial S_L} &= -1 + \lambda, \\
\frac{\partial \mathcal{L}}{\partial s} &= -\frac{(m-1)(n-1)}{mn-m-n} \frac{\varepsilon^{\frac{mn}{mn-m-n}} (x-s-t)^{\frac{n}{mn-m-n}} t^{\frac{m}{mn-m-n}}}{\beta^{\frac{n}{mn-m-n}} \gamma^{\frac{m}{mn-m-n}}} \\
&\quad + \lambda\Big(\frac{\varepsilon^{\frac{mn}{mn-m-n}} (x-s-t)^{\frac{n}{mn-m-n}} t^{\frac{m}{mn-m-n}}}{\beta^{\frac{n}{mn-m-n}} \gamma^{\frac{m}{mn-m-n}}} - \frac{n}{mn-m-n} \frac{\varepsilon^{\frac{mn}{mn-m-n}} (x-s-t)^{\frac{n}{mn-m-n}-1} t^{\frac{m}{mn-m-n}} s}{\beta^{\frac{n}{mn-m-n}} \gamma^{\frac{m}{mn-m-n}}}\Big).
\end{aligned}$$

If $\lambda = 1$, then

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial S_L} &= 0, \\
\frac{\partial \mathcal{L}}{\partial s} &= -\frac{1}{mn-m-n} \frac{\varepsilon^{\frac{mn}{mn-m-n}} (x-s-t)^{\frac{n}{mn-m-n}} t^{\frac{m}{mn-m-n}}}{\beta^{\frac{n}{mn-m-n}} \gamma^{\frac{m}{mn-m-n}}} \\
&\quad - \frac{n}{mn-m-n} \frac{\varepsilon^{\frac{mn}{mn-m-n}} (x-s-t)^{\frac{n}{mn-m-n}-1} t^{\frac{m}{mn-m-n}} s}{\beta^{\frac{n}{mn-m-n}} \gamma^{\frac{m}{mn-m-n}}} < 0.
\end{aligned}$$

Hence, $s = 0$ and no matter what value of $S_L \in [0, X_L]$ be, still violates nvestors' individual rationality condition due to (2), the riskiness of the project.
If $\lambda < 1$, then

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial S_L} &< 0, \\
\frac{\partial \mathcal{L}}{\partial s} &= -Y + \lambda Z,
\end{aligned}$$

where

$$\begin{aligned}
Y &= \frac{(m-1)(n-1)}{mn-m-n} \frac{\varepsilon^{\frac{mn}{mn-m-n}} (x-s-t)^{\frac{n}{mn-m-n}} t^{\frac{m}{mn-m-n}}}{\beta^{\frac{n}{mn-m-n}} \gamma^{\frac{m}{mn-m-n}}}, \\
Z &= \frac{\varepsilon^{\frac{mn}{mn-m-n}} (x-s-t)^{\frac{n}{mn-m-n}} t^{\frac{m}{mn-m-n}}}{\beta^{\frac{n}{mn-m-n}} \gamma^{\frac{m}{mn-m-n}}} - \frac{n}{mn-m-n} \frac{\varepsilon^{\frac{mn}{mn-m-n}} (x-s-t)^{\frac{n}{mn-m-n}-1} t^{\frac{m}{mn-m-n}} s}{\beta^{\frac{n}{mn-m-n}} \gamma^{\frac{m}{mn-m-n}}}.
\end{aligned}$$

If $Z \leq 0$, then $\frac{\partial \mathcal{L}}{\partial s} < 0$; if $Z > 0$, then $\frac{\partial \mathcal{L}}{\partial s} < \frac{\partial \mathcal{L}}{\partial s}|_{\lambda=1} < 1$. Hence, $S_L = s = 0$ still violates the investors' individual rationality condition.
Therefore, we must have $\lambda > 1$. Then, since

$$\frac{\partial \mathcal{L}}{\partial S_L} = -1 + \lambda > 0,$$

we get $S_L = X_L - T_L$, and

$$\frac{\partial \mathcal{L}}{\partial T_L} = -1 < 0,$$

we get $T_L = 0$. As a result, we have $S_L^{SB} = X_L$, and $T_L^{SB} = 0$. To simplify the expressions, we define

$$K - X_L = k \frac{\varepsilon^{\frac{mn}{mn-m-n}}}{\beta^{\frac{n}{mn-m-n}} \gamma^{\frac{m}{mn-m-n}}}. \tag{B.21}$$

Let's rewrite the optimization problem as

$$\mathcal{L} = \left(1 - \frac{1}{m}\right)(x - s - t)^{\frac{m(n-1)}{mn-m-n}} t^{\frac{m}{mn-m-n}} + \lambda\left((x - s - t)^{\frac{n}{mn-m-n}} t^{\frac{m}{mn-m-n}} s - k\right), \tag{B.22}$$

where

$$\begin{cases} \frac{\partial \mathcal{L}}{\partial s} = 0, \\ \frac{\partial \mathcal{L}}{\partial t} = 0, \\ \frac{\partial \mathcal{L}}{\partial \lambda} = 0, \\ \lambda > 1. \end{cases} \tag{B.23}$$

$\frac{\partial \mathcal{L}}{\partial s} = 0$ and $\frac{\partial \mathcal{L}}{\partial t} = 0$ imply that $t^{SB} = \frac{x}{n}$. □

References

Aghion, Philippe, and Patrick Bolton, 1992, An incomplete contracts approach to financial contracting, *The review of economic Studies* 59, 473–494.

Alchian, Armen A, and Harold Demsetz, 1972, Production, information costs, and economic organization, *American Economic Review* 62, 777–795.

Amit, Raphael, Lawrence Glosten, and Eitan Muller, 1990, Does venture capital foster the most promising entrepreneurial firms?, *California Management Review* 32, 102–111.

Axelson, Ulf, Per Strömberg, and Michael S Weisbach, 2009, Why are buyouts levered? the financial structure of private equity funds, *The Journal of Finance* 64, 1549–1582.

Barry, Christopher B, Chris J Muscarella, John W Peavy III, and Michael R Vetsuypens, 1990, The role of venture capital in the creation of public companies: Evidence from the going-public process, *Journal of Financial economics* 27, 447–471.

Berglof, Erik, 1994, A control theory of venture capital finance, *JL Econ. & Org.* 10, 247.

Bhattacharyya, Sugato, and Francine Lafontaine, 1995, Double-sided moral hazard and the nature of share contracts, *RAND Journal of Economics* pp. 761–781.

Brennan, Michael J, and Eduardo S Schwartz, 1988, The case for convertibles, *Journal of Applied Corporate Finance* 1, 55–64.

Buffa, Andrea M., Qing Liu, and Lucy White, 2019, Optimal delegated contracting, Working paper Boston University.

Casamatta, Catherine, 2003, Financing and advising: optimal financial contracts with venture capitalists, *Journal of Finance* 58, 2059–2085.

——— , 2010, Financial contracts and venture capitalists' value-added, in Douglas J. Cumming, ed.: *Venture Capital Investment Strategies, Structures, and Policies* . pp. 153–167 (John Wiley & Sons).

Chan, Yuk-Shee, 1983, On the positive role of financial intermediation in allocation of venture capital in a market with imperfect information, *The Journal of Finance* 38, 1543–1568.

Chemmanur, Thomas J, and Zhaohui Chen, 2014, Venture capitalists versus angels: The dynamics of private firm financing contracts, *The Review of Corporate Finance Studies* 3, 39–86.

Cornelli, Francesca, and Oved Yosha, 2003, Stage financing and the role of convertible securities, *The Review of Economic Studies* 70, 1–32.

DeMarzo, Peter M, and Yuliy Sannikov, 2006, Optimal security design and dynamic capital structure in a continuous-time agency model, *The Journal of Finance* 61, 2681–2724.

Dessi, Roberta, 2005, Start-up finance, monitoring, and collusion, *RAND Journal of Economics* pp. 255–274.

Diamond, Douglas W, 1984, Financial intermediation and delegated monitoring, *The review of economic studies* 51, 393–414.

Ehrlich, Sanford B, Alex F De Noble, Tracy Moore, and Richard R Weaver, 1994, After the cash arrives: A comparative study of venture capital and private investor involvement in entrepreneurial firms, *Journal of Business Venturing* 9, 67–82.

Eswaran, Mukesh, and Ashok Kotwal, 1984, The moral hazard of budget-breaking, *RAND Journal of Economics* pp. 578–581.

Ewens, Michael, Alexander S. Gorbenko, and Arthur Korteweg, 2019, Venture capital contracts, Working paper Caltech and USC Marshall School of Business.

Fenn, George W, Nellie Liang, and Stephen Prowse, 1997, The private equity market: An overveiw, *Financial Markets, Institutions & Instruments* 6, 1–106.

Gompers, Paul, and Josh Lerner, 1999a, An analysis of compensation in the us venture capital partnership, *Journal of Financial Economics* 51, 3–44.

——— , 1999b, *The Venture Capital Cycle* (Cambridge, MA: MIT Press).

Gompers, Paul Alan, 1993, The theory, structure, and performance of venture capital, Ph.D. thesis Harvard University.

Gompers, Paul A, 1995, Optimal investment, monitoring, and the staging of venture capital, *The journal of finance* 50, 1461–1489.

——— , 1997, Ownership and control in entrepreneurial firms: an examination of convertible securities in venture capital investments, Unpublished working paper Harvard Business School.

Gorman, Michael, and William A Sahlman, 1989, What do venture capitalists do?, *Journal of business venturing* 4, 231–248.

Green, Richard C, 1984, Investment incentives, debt, and warrants, *Journal of financial Economics* 13, 115–136.

Grossman, Sanford J, and Oliver D Hart, 1986, The costs and benefits of ownership: A theory of vertical and lateral integration, *Journal of political economy* 94, 691–719.

Hart, Oliver, 1995, *Firms, contracts, and financial structure* (Clarendon press).

——— , and John Moore, 1990, Property rights and the nature of the firm, *Journal of political economy* 98, 1119–1158.

Hart, Oliver, and Jean Tirole, 1990, Vertical integration and market foreclosure, *Brookings papers on economic activity. Microeconomics* pp. 205–286.

Hellmann, Thomas, 2006, Ipos, acquisitions, and the use of convertible securities in venture capital, *Journal of Financial Economics* 81, 649–679.

——— , and Manju Puri, 2002, Venture capital and the professionalization of start-up firms: Empirical evidence, *The journal of finance* 57, 169–197.

Hellmann, Thomas, Paul Schure, and Dan Vo, 2019, Angels and venture capitalists: Substitutes or complements?, Working paper University of Oxford Saïd Business School.

Hellmann, Thomas, and Veikko Thiele, 2015, Friends or foes? the interrelationship between angel and venture capital markets, *Journal of Financial Economics* 115, 639–653.

Holmstrom, Bengt, 1979, Moral hazard and observability, *Bell journal of Economics* 10, 74–91.

——— , 1982, Moral hazard in teams, *Bell Journal of Economics* pp. 324–340.

——— , and Jean Tirole, 1997, Financial intermediation, loanable funds, and the real sector, *Quarterly Journal of Economics* 112, 663–691.

Hori, Keiichi, and Hiroshi Osano, 2013, Managerial incentives and the role of advisors in the continuous-time agency model, *Review of Financial Studies* 26, 2620–2647.

Hsu, David H, 2004, What do entrepreneurs pay for venture capital affiliation?, *The Journal of Finance* 59, 1805–1844.

Inderst, Roman, and Holger M Müller, 2004, The effect of capital market characteristics on the value of start-up firms, *Journal of Financial Economics* 72, 319–356.

Innes, Robert D, 1990, Limited liability and incentive contracting with ex-ante action choices, *Journal of economic theory* 52, 45–67.

Jensen, Michael C, and William H Meckling, 1976, Theory of the firm: Managerial behavior, agency costs and ownership structure, *Journal of financial economics* 3, 305–360.

Kaplan, Steven N, and Per Strömberg, 2001, Venture capitalists as principals: Contracting, screening, and monitoring, *The American Economic Review Papers and Proceedings* 91, 426–430.

——— , 2003, Financial contracting theory meets the real world: An empirical analysis of venture capital contracts, *The review of economic studies* 70, 281–315.

Kaplan, Steven N, and Per ER Strömberg, 2004, Characteristics, contracts, and actions: Evidence from venture capitalist analyses, *The Journal of Finance* 59, 2177–2210.

Kerr, William R, Josh Lerner, and Antoinette Schoar, 2011, The consequences of entrepreneurial finance: Evidence from angel financings, *The Review of Financial Studies* 27, 20–55.

Lerner, Joshua, 1994, Venture capitalists and the decision to go public, *Journal of financial Economics* 35, 293–316.

Lerner, Josh, 1995, Venture capitalists and the oversight of private firms, *the Journal of Finance* 50, 301–318.

Marx, Leslie M, 1998, Efficient venture capital financing combining debt and equity, *Review of Economic Design* 3, 371–387.

McAfee, R Preston, and Marius Schwartz, 1994, Opportunism in multilateral vertical contracting: Nondiscrimination, exclusivity, and uniformity, *American Economic Review* pp. 210–230.

O'Brien, Daniel P., and Greg Shaffer, 1992, Vertical control with bilateral contracts, *RAND Journal of Economics* pp. 299–308.

Renucci, Antoine, 2006, Optimal relationships with value-enhancing investors, Working paper UPPA.

Repullo, Rafael, and Javier Suarez, 2004, Venture capital finance: A security design approach, *Review of Finance* 8, 75–108.

Rey, Patrick, and Thibaud Vergé, 2004, Bilateral control with vertical contracts, *RAND Journal of Economics* pp. 728–746.

Robinson, David T, and Berk A Sensoy, 2013, Do private equity fund managers earn their fees? compensation, ownership, and cash flow performance, *The Review of Financial Studies* 26, 2760–2797.

Sahlman, William A, 1990, The structure and governance of venture-capital organizations, *Journal of financial economics* 27, 473–521.

Schmidt, Klaus M, 2003, Convertible securities and venture capital finance, *The Journal of Finance* 58, 1139–1166.

Sørensen, Morten, 2007, How smart is smart money? a two-sided matching model of venture capital, *The Journal of Finance* 62, 2725–2762.

Trester, Jeffrey J, 1998, Venture capital contracting under asymmetric information, *Journal of Banking & Finance* 22, 675–699.

Van Osnabrugge, Mark, 2000, A comparison of business angel and venture capitalist investment procedures: an agency theory-based analysis, *Venture Capital: An international journal of entrepreneurial finance* 2, 91–109.

Wilson, Karen, 2011, Financing high growth firms: the role of angel investors, Oecd publishing http://dx.doi.org/10.1787/9789264118782-en.

Wong, Andrew, Mihir Bhatia, and Zachary Freeman, 2009, Angel finance: the other venture capital, *Strategic Change: Briefings in Entrepreneurial Finance* 18, 221–230.